


# 3D PIC Simulations for Relativistic Jets with a Toroidal Magnetic Field

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**Abstract.** Particle-in-Cell simulations can provide a possible answer to an important key issue for astrophysical plasma jets; namely on how a toroidal magnetic field affects the evolution of pair and electron-ion jets. We show that Weibel, mushroom, and kinetic Kelvin-Helmholtz instabilities excited at the linear stage, generate a quasi-steady  $x$ -component of the electric field which accelerates and decelerates electrons. We observe significant differences in the structure of the strong electromagnetic fields that are driven by the kinetic instabilities with the pair jet. We find that the two different jet compositions ( $e^\pm$  and  $e^- - i^+$ ) generate different instability modes respectively. Moreover, the magnetic field in the non-linear stage generated by different instabilities is dissipated and reorganized into new topologies. A 3D magnetic field topology depiction indicates possible reconnection sites in the non-linear stage where the particles are significantly accelerated by the dissipation of the magnetic field associated to a possible reconnection manifestation.

**Keywords.** relativistic jets, kinetic instabilities, particle acceleration, magnetic reconnection

## 1. Introduction

Relativistic astrophysical jets are ubiquitous in astrophysical systems, and have been observationally associated with the activity of central black holes in Active Galactic Nuclei (AGN, e.g., [EHT Collaboration 2019](#)). It is also theorized that relativistic jets may occur in Gamma-ray Bursts (GRBs, e.g., [Ruiz et al. 2018](#)). The formation and powering of these astrophysical jets are highly complex phenomena involving relativistic plasmas and twisted magnetic fields which are organized in such a manner as to ultimately launch an outflow from a central compact source (e.g., [McKinney & Uzdensky 2012](#)).

Relativistic jets interact with the plasma environment of an astrophysical source and subsequently instabilities occur which are responsible for the acceleration of particles (e.g., [Nishikawa et al. 2021](#)). In some cases, e.g. for unmagnetized jets, previous computer simulations have shown that the Weibel instability (WI) excites relativistic shocks, which results in particle acceleration (e.g., [Nishikawa et al. 2009](#), and references therein). Other instabilities such as the kinetic Kelvin-Helmholtz (kKHI) and the mushroom instability (MI), are driven by the velocity-shear at the boundary between the jet and the ambient medium, which were first observed in using a slab model for plasma sheets (e.g., [Alves et al. 2012](#); [Nishikawa et al. 2014](#)). PIC simulation studies have been performed for the evolution of cylindrical jets with a helical magnetic-field topology (e.g., [Alves et al. 2015](#); [Nishikawa et al. 2021](#)).

Moreover, using the so-called Harris model in a slab geometry, magnetic reconnection has commonly been studied with PIC simulations. It was observed to produce a significant particle acceleration (e.g., [Sironi & Spitkovsky 2014](#); [Guo et al. 2016](#)).

Global 3D PIC modeling of relativistic jets allows for a self-consistent investigation of the complex kinetic processes occurring in the jet and the surrounding medium. These processes can reveal electron-scale short-wavelength instabilities, their saturation and associated phenomena. Such studies have been first performed for unmagnetized jets ([Nishikawa et al. 2016a](#)). PIC simulations of relativistic jets containing helical magnetic fields were, for the first time, presented by [Nishikawa et al. \(2016b\)](#). These initial studies addressed the early, linear growth of kinetic instabilities in the electron-ion and electron-positron jets. However, such simulations were limited by the size of the computational box.

### 1.1. Importance of toroidal magnetic field

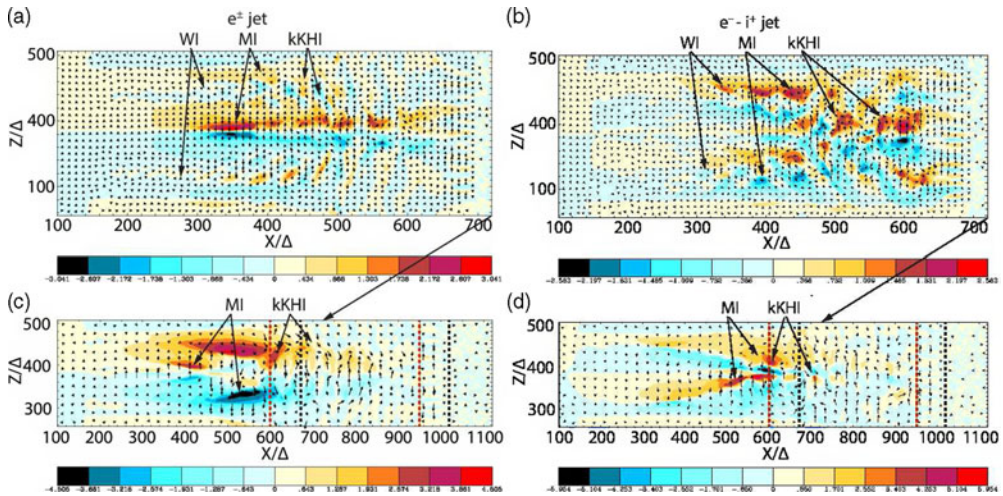
To address the importance of toroidal magnetic fields for growing instabilities, the present study involves a much larger jet radius and longer simulation times than previous works (e.g., [Nishikawa et al. 2016a](#); [Nishikawa et al. 2016b](#)), allowing for a non-linear evolution of the jets with a toroidal magnetic field aiming the following key questions: (i) How does a toroidal magnetic field affect the growth of kKHI, MI, and WI within the jet and in the jet-ambient plasma boundary? (ii) How do jets composed of electrons and positrons and jets composed of electrons and ions evolve in the presence of a large-scale toroidal magnetic field? (iii) How and where are particles accelerated in jets with different plasma compositions?

## 2. Simulation set-up

Our 3D PIC code is a modified version of the relativistic electromagnetic PIC code TRISTAN ([Buneman 1993](#)) with MPI-based parallelization ([Niemić et al. 2008](#); [Nishikawa et al. 2009](#)). The numerical grid is set to  $(L_x, L_y, L_z) = (1285\Delta, 789\Delta, 789\Delta)$  and is twice as long as that in our previous simulation studies ([Nishikawa et al. 2016b](#)). Here  $\Delta = 1$  is the size of an individual grid cell. Open boundaries are used on the surfaces at  $x/\Delta = 0$  and 1285, whilst periodic boundary conditions are implemented along the transverse directions  $y$  and  $z$ . Since the jet is located in the center of the simulation box far from the boundaries, the effect of periodic boundaries is negligible.

## 3. Simulation results

We present simulation results for  $e^\pm$  and  $e^- - i^+$  jets with a toroidal magnetic field, applying a new and improved jet injection scheme (for details, see [Meli et al. 2023](#)). We are in particular interested in the differences in the dynamical behavior of the jets of different plasma compositions and in the way these jets interact with the surrounding



**Figure 1.** Color maps of the  $B_y$  magnetic field with arrows depicting the magnetic field components in the  $x - z$  plane, at  $t = 900 \omega_{pe}^{-1}$ . Upper panels (a) and (b) show unmagnetized jets and lower panels (c) and (d) jets with a toroidal magnetic field. The maximum and minimum are (a):  $\pm 1.525$ , (b):  $\pm 4.073$ , (c):  $\pm 4.505$  and (d):  $\pm 5.954$ . Adapted from Fig. 4 in Meli *et al.* (2023).

environment. In order to give an overview of how the toroidal magnetic field affects the evolution of the jet, in Fig. 1 we present simulation results for jets with a toroidal field (shown in the lower panels of each figure) and compare them to the results obtained for an unmagnetized jet.

Figure 1 shows the amplitude of  $B_y$  component with the arrows indicating the magnetic field components in the  $x - z$  plane. For the magnetized jets, we apply  $B_0 = 0.5$  initially at the jet orifice while  $B_y$  is measured in the same units. In the presence of the toroidal magnetic field for the  $e^\pm$  jet, we find a maximum value of  $B_y = 4.505$ , which means that  $B_0$  is amplified by a factor of 9.01 over the initial value. It is a factor of 2.95 stronger than magnetic fields in the unmagnetized  $e^\pm$  jet. The evident differences in the magnetic field structure in cases with and without the initial field, indicate a significant impact of the toroidal field on the development of the kinetic instabilities. The same is true for the  $e^- - i^+$  jet, in which the magnetic field amplification is comparably stronger and the field amplitudes reach  $B/B_0 \approx 10.6$ , a factor of 1.3 stronger compared to the unmagnetized case. It should be noted that for unmagnetized  $e^- - p^+$  jet the large mass ratio contributes to a stronger growth of MI, this is the reason why we needed to reduce the mass ratio to control the growth of MI. We note that for the  $e^\pm$  jet the magnetic field dissipates, i.e., becomes considerably weakened, at the jet region  $x/\Delta \gtrsim 680$  and similarly for the  $e^- - i^+$  jet the weakening occurs around  $x/\Delta \gtrsim 700$ . By comparing both magnetized jet species, at the non-linear stage close to  $x/\Delta \gtrsim 950$ , one discerns that for the electron-ion jet the  $B_y$  field almost dissipates and becomes disorganized (turbulent).

#### 4. Summary

We have conducted extensive 3D PIC simulations to study the spatiotemporal evolution of magnetized relativistic electron-positron and electron-ion jets, examining their kinetic instabilities and the associated particle acceleration. We investigated the excited kinetic instabilities and the associated magnetic fields at the linear and non-linear stage, as they may occur in astrophysical relativistic jets. The dissipation of the magnetic fields

was observed to generate electric fields that are sufficiently strong to further accelerate particles to Lorentz factors of up to around 35.

In this work we used a new jet injection scheme. We injected both  $e^\pm$  and  $e^- - i^+$  jets, with a co-moving toroidal magnetic field while we used a top-hat jet density profile. The current was self-consistently carried by the jet particles,  $\mathbf{J} = \nabla \times \mathbf{B}$ . In order to sustain the toroidal magnetic field carried by the jet, the current was applied at the jet orifice and a motional electric field was applied in order to compensate the bending by the applied toroidal magnetic field. Both jets were initially moderately magnetized while the ambient medium remained unmagnetized. We have run the simulations sufficiently long in order to examine the non-linear effects of the jet evolution.

We found that the dominant excited modes of instabilities depend on the different jet compositions. Three different instabilities (WI, MI, kKHI) grow in similar timescales, however, depending on the plasma conditions (unmagnetized, different species) some instabilities grow faster and stronger. Particularly the MI grows faster and stronger with larger mass ratio (4 and 1836) and an excitation of MI or kKHI is weaker for the unmagnetized case. In general, stronger MI and kKHI grow in the jets with toroidal magnetic fields. The MI and kKHI are associated with a quasi-steady electric field ( $E_x$ ) for both jet species. These accompanied electric fields accelerate and decelerate electrons and positrons. Additionally we found that the electrons can be further accelerated by the development of twisted and turbulent magnetic fields which are generated by the dissipation of the toroidal magnetic fields possibly accompanying reconnection.

We have identified potential sites of magnetic reconnection in our simulations; however, an unambiguous determination in 3D is not trivial (see, [Meli et al. 2023](#)). The magnetic field structure of a reconnection site in 2D simulations consists of  $X$  and  $O$  shapes which can be recognized rather easily by the changes of the magnetic field direction and the position of null (very weak) magnetic fields in the 2D projections. The complex structures of 3D reconnections have been investigated in (e.g., [Lazarian et al. 2020](#)). In order to determine the reconnection locations analytically, we would need to investigate the eigenvalues of Jacobian matrix, which is beyond the scope of this work, for more details, see [Cai et al. \(2007\)](#).

For an  $e^\pm$  jet, the MI is excited combined with a kKHI, while the produced quasi steady  $E_x$  modulates the jet particles. For the electron-ion jet, the jet electrons are pinched dominantly grown MI at the later linear stage. Further simulations will be important to decisively confirm possible supplemental acceleration mechanisms with varying simulation parameters such as, jet radius, magnetization factor, jet density profile, etc.

From the present extensive simulation studies we conclude, that a moderate initial magnetic field can change the development of the kinetic plasma instabilities, for different jet species, even if the instabilities significantly amplify the magnetic field, here by Lorentz factors reaching 50.

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