Contents

	Intro	page 1	
1	Preli	3	
	1.1	Notation	3
	1.2	Hausdorff Measures	3
	1.3	Lipschitz Maps	5
2	Rect	ifiable Curves	6
3	One-	9	
	3.1	Definitions and Tangents	9
	3.2	Densities	11
	3.3	Projections	13
	3.4	Analyst's Travelling Salesman Problem	14
4	Higher-Dimensional Rectifiable Sets		18
	4.1	Definitions and Area and Coarea Formulas	18
	4.2	Tangent Planes	20
	4.3	Tangent Measures	21
	4.4	Densities	23
	4.5	Projections	27
	4.6	Multiscale Approximations	29
	4.7	Reifenberg-Type Results	30
	4.8	Lebesgue Null-Sets and Singular Measures	32
	4.9	Minkowski Content and Discrete Energies	34
5	Uniform Rectifiability		37
	5.1	One-Dimensional Sets	37
	5.2	Lipschitz Maps and Approximation by Planes	38
	5.3	Density Ratios	41
	5.4	Projections	42

vi Contents

	5.5	Basic Tools	43		
	5.6	Parabolic Rectifiability	44		
6	Recti	fiability of Measures	47		
	6.1	Some Basic Facts and Examples	47		
	6.2	Square Functions in General Dimensions	48		
	6.3	Square Functions and One-Dimensional Measures	49		
	6.4	Square Functions and Distance of Measures	51		
7	Rectifiable Sets in Metric Spaces				
	7.1	Definition and Norm	52		
	7.2	Densities when $m = 1$	52		
	7.3	Densities and Area Formula for General m	54		
	7.4	Tangent Planes	56		
	7.5	Cheeger's Differentiability Spaces and Alberti			
		Representations	56		
	7.6	Projections as Lipschitz Images	58		
	7.7	Metric Tangents	60		
	7.8	Menger Curvature	62		
8	Heisenberg and Carnot Groups				
	8.1	The Heisenberg Group \mathbb{H}^n	63		
	8.2	Some Analytic Tools in Heisenberg and Carnot Groups	65		
	8.3	Definitions of Rectifiability	65		
	8.4	Rectifiable Sets and Tangent Subgroups	70		
	8.5	Densities and Tangent Measures	71		
	8.6	Projections	75		
	8.7	Uniform Rectifiability	75		
9	Bounded Analytic Functions and the Cauchy Transform				
	9.1	Removable Sets and Menger Curvature	78		
	9.2	Projections	83		
	9.3	Principal Values	84		
	9.4	Square Functions	85		
	9.5	Other Related Kernels	86		
10	Singular Integrals				
	10.1	A Few Words in General	89		
	10.2	L^2 -Boundedness and Uniform Rectifiability	90		
	10.3	Principal Values	94		
	10.4	F The state of the	95		
	10.5	Parabolic Singular Integrals	97		
	10.6	Heisenberg Groups	97		

		Contents	vii
11	Harn	nonic Measure and Elliptic Measures	99
	11.1	Harmonic Measure	99
	11.2	Elliptic Measures in Codimension 1	103
	11.3	Elliptic Measures in Codimension Bigger Than One	103
12	Sets of Finite Perimeter and Functions of Bounded Variation		
	12.1	Sets of Finite Perimeter	107
	12.2	Plateau-Type Problems	110
	12.3	Functions of Bounded Variation	111
	12.4	Perimeter in Heisenberg and Carnot Groups	114
13	Currents and Varifolds		
	13.1	Currents in Euclidean Spaces	115
	13.2	Currents in Metric Spaces	120
	13.3	Varifolds	121
14	Mini	mizers and Quasiminimizers	125
	14.1	Quasiminimizers	125
	14.2	Mumford-Shah Functional	126
	14.3	Some Free Boundary Problems	127
15	Recti	fiability of Singularities	129
	15.1	Mass Minimizing Currents and Stationary Varifolds	129
	15.2	Energy Minimizing Maps	132
	15.3	Mean Curvature Flow	135
	15.4	Gromov-Hausdorff Limits and Related Matters	137
	15.5	Measure Solutions of PDEs	139
	15.6	A Free Boundary Problem	142
16	Miscellaneous Topics Related to Rectifiability		
	16.1	Curvature Measures	143
	16.2	Dynamical Systems	144
	16.3	Higher-Order Rectifiability	146
	16.4	Fractal Rectifiability	148
Dafa	rancas		140

171

Index