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A CROSS-SECTIONAL METHOD FOR RIGHT-TAILED PANIC TESTS UNDER A MODERATELY LOCAL TO UNITY FRAMEWORK

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The left-tailed unit-root tests of the panel analysis of nonstationarity in idiosyncratic and common components (PANIC) proposed by Bai and Ng (2004, *Econometrica* 72, 1127–1177) have standard local asymptotic power. We assess the size and power properties of the right-tailed version of the PANIC tests when the common and/or the idiosyncratic components are moderately explosive. We find that, when an idiosyncratic component is moderately explosive, the tests for the common components may have considerable size distortions, and those for an idiosyncratic component may suffer from the nonmonotonic power problem. We provide an analytic explanation under the moderately local to unity framework developed by Phillips and Magdalinos (2007, *Journal of Econometrics* 136, 115–130). We then propose a new cross-sectional (CS) approach to disentangle the common and idiosyncratic components in a relatively short explosive window. Our Monte Carlo simulations show that the CS approach is robust to the nonmonotonic power problem.

1. INTRODUCTION

Large dimensional common factor models are a driving force in recent empirical analysis in various fields of economics. Bai (2003) and Bai and Ng (2006) provide sufficient conditions under which the principal component estimator is consistent for the common and idiosyncratic components when the series have no time trends. When the series have stochastic trends of integration of the order one, the standard practice is to induce stationarity by transforming the original data by

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first differencing before identifying and estimating the common and idiosyncratic components.[1] If one is interested in identifying whether the stochastic trends lie in the common or idiosyncratic components, Bai and Ng (2004) suggest applying augmented Dickey–Fuller (ADF) tests (Dickey and Fuller, 1979) for these components estimated by first-differenced data. This method is called the panel analysis of nonstationarity in idiosyncratic and common components (PANIC). One main advantage of this method is that the common and idiosyncratic components are separately identified under the null hypothesis of a random walk. In addition, the ADF test has nontrivial power when testing against the alternative hypothesis of stationarity (hereafter, the left-tailed test) because, under such a hypothesis, the first-differenced series may be over-differenced, but has no time trends; hence, the common and idiosyncratic components are correctly identified. Bai and Ng (2004) show that the test for common components has good size and power despite stationary or random walk idiosyncratic components. The same can be said of the test for idiosyncratic components. Therefore, the PANIC approach successfully disentangles the common and idiosyncratic components.

In this study, we investigate whether this convenient property of the PANIC approach is available even when the right-tailed version of the ADF test (hereafter, the right-tailed test) is used. The right-tailed unit-root tests are used in various applications. For example, testing for speculative bubbles in asset prices is a long-standing problem for which numerous econometric techniques have been developed. The most recent studies include the seminal work of Phillips, Wu, and Yu (2011), in which they pay attention to the link between speculative bubbles and the explosive behaviors of asset price data. Their strategy is to fit a univariate autoregressive (AR) model and test whether the root is greater than unity. The present study considers situations where speculative bubbles may be present in large dimensional panel data of financial assets. It is important to investigate whether these bubbles are an economy-wide phenomenon or market-specific events. This study takes a step toward answering such a question.

Consistent with Becheri and van den Akker (2015) and Westerlund (2015), we first show that both left-tailed and right-tailed PANIC tests for common and idiosyncratic explosive behaviors exhibit the standard local asymptotic power when the AR coefficient shrinks to one at a fast rate of T^{-1}, where T is the time dimension of the panel dataset (see Appendix I of the Supplementary Material). A potential problem of such a local to unity (LTU) asymptotic framework is that it only considers small deviations from the unit root. The recent literature establishes that the asymptotic results under such local asymptotic frameworks may not adequately approximate the finite sample behaviors of the test statistics (see, e.g., Deng and Perron, 2008). With this caveat in mind, we take an approach that considers the AR root that shrinks to one at a slower rate than T^{-1}. In particular,

[1] Bai (2004) proposes estimating common stochastic trends by using the principal components of level data when none of the idiosyncratic errors have stochastic trends, but the common factors do. See also the seminal work of Stock and Watson (2002, 2005) for empirical examples.

we use the moderately local to unity (MLTU) framework developed by Phillips and Magdalinos (2007). Under this framework, we find that the explosive idiosyncratic components may be identified as the common component. As a result, the tests for the common and idiosyncratic components have size distortions and power loss.

Our Monte Carlo simulations illustrate the analytic findings. We first confirm Bai and Ng's (2004) results—that is, as far as the left-tailed tests are concerned, the PANIC approach provides good size and power. However, the right-tailed tests behave very differently from the left-tailed tests. First, the test for common components shows significant size distortions when some idiosyncratic components are explosive because the explosive idiosyncratic components are misidentified as the common factor. Second, the test for idiosyncratic components suffers from size distortions when the common components are explosive for the same reason. Finally, and most importantly, the test for the idiosyncratic components shows an upward power function when the AR coefficient is slightly larger than one. However, the power function starts to decline toward zero as the AR coefficient further increases. This phenomenon is the well-known nonmonotonic power problem widely documented in the context of structural change tests (Perron, 1991; Vogelsang, 1999). What is new in this study is that the source of nonmonotonic power is the identification failure between the common factors and explosive idiosyncratic components under the alternative hypothesis.

This study provides a new method of testing for explosive behavior in the common and idiosyncratic components. In many empirical situations, explosive behaviors appear only in a certain subperiod, and the series are not explosive in the remaining sample period—we take advantage of this fact. Therefore, we can set a training sample during which no, or only weak, explosive behavior exists. We then use cross-sectional (CS) regressions to estimate the common components in the explosive window as the coefficients attached to the factor loadings, whereas the factor loadings are estimated in the training sample. We call this the CS method. It is shown that the tests for the common components and the tests for the idiosyncratic components achieve the correct asymptotic size and are consistent under the MLTU framework. A Monte Carlo simulation shows that the CS test for common components considerably reduces size distortions. More importantly, the CS test for idiosyncratic components is robust to the nonmonotonic power problem.

The structure of the remaining paper is as follows. Section 2 introduces the model, assumptions, and existing PANIC tests. Section 3 presents the finite sample size and power of the right-tailed PANIC tests and investigates their theoretical properties under the MLTU framework. Section 4 proposes a new CS method and investigates its theoretical and finite sample properties. Section 5 concludes the paper. The details of technical derivations and additional results are provided in the Supplementary Material, including further details on the Results under the LTU Framework (Appendix I), Proof of Theorem SA-1 and Theorem 1 (Appendix II), Proof of Factor Estimation Errors in Theorem 1(i) (Appendix III), Finite Sample Properties of the CS Tests when the Training Sample is Selected by a Statistical

Method (Appendix IV), and Proof of Theorem SA-2 and Theorem 2 (Appendix V). Throughout the paper, the following notations are used. The euclidean norm of vector x is denoted by $\|x\|$. For the matrices, the vector-induced norm is used. The symbols $O(\cdot)$ and $o(\cdot)$ denote the standard asymptotic orders of sequences. The symbol $\xrightarrow{p}$ represents convergence in probability under the probability measure P and the symbol $\Rightarrow$ denotes convergence in distribution. $O_p(\cdot)$ and $o_p(\cdot)$ are the orders of convergence in probability under P as $N,T\to\infty$ (or $N,T,h\to\infty$). We use the symbol "$x\approx y$" when $\|x-y\| = o_p(1)$, for two vectors of random variables x and y.

2. MODEL AND TEST STATISTICS

We consider the common factor model:

$$X_{i,t} = \mu_i + \lambda_i' F_t + U_{i,t}, \quad \text{for } i = 1,\ldots,N \text{ and } t = 1,\ldots,T, \tag{1}$$

where $X_{i,t}$ is a scalar of the observed random variable, μ_i is an intercept, F_t and λ_i are the $r\times 1$ vectors of the common factors and factor loadings, respectively, and $U_{i,t}$ is a scalar idiosyncratic component. We focus on the essence of the problem by assuming the number of factors is one with no loss of substance and is known by the econometrician, so the estimation of r is not needed.[2,3] The common factor follows $(1-\alpha L)F_t = C(L)e_t$, where $C(L) = \sum_{j=0}^{\infty} C_j L^j$ with $C_0 = 1$ and e_t is a white noise disturbance. The idiosyncratic components follow $(1-\rho_i L)U_{i,t} = D_i(L)z_{i,t}$, where ρ_i is the AR coefficient of the ith cross section, $D_i(L) = \sum_{j=0}^{\infty} D_{ij}L^j$, $D_{i0} = 1$, and $z_{i,t}$ is a white noise disturbance.

We consider the following assumptions in this model. Let $M<\infty$ be a generic constant.

Assumption 1. For every $t = 0,1,\ldots,T$, $e_t \sim i.i.d.(0,\sigma^2)$, $\mathbb{E}|e_t|^4 \le M$, and $\sum_{j=0}^{\infty} j\left|C_j\right| < M$. Furthermore, $\mathbb{E}|F_0| \le M$.

Assumption 2.

(a) λ_i is a nonrandom quantity satisfying $|\lambda_i| \le M$ or a random quantity satisfying $\mathbb{E}|\lambda_i|^2 \le M$.
(b) $N^{-1}\sum_{i=1}^{N}\lambda_i^2 \xrightarrow{p} \sigma_\lambda^2$, where σ_λ is a positive constant.

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[2]When $r>1$, one can implement the right-tailed test series by series with the individual factors to investigate whether the common factor space is explosive. This is adequate because at least one rejection implies that the whole space is explosive. This is in contrast to the left-tailed tests. As Bai and Ng (2004) contemplate, rejections of individual factors do not necessarily imply a rejection for the common factor space if they have a cointegration relationship. Note that since the estimated factors are uncorrelated with each other, the size of the testing for series by series is controlled.

[3]The cross-section-specific intercepts, μ_i, are eliminated in the first-differenced data such that they do not affect inference on α and ρ_i. When (1) includes linear time trends, one can work with the demeaned first-differenced data and the equivalent principal components are obtained.

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Assumption 3. For every $t, s = 0, 1, \ldots, T$ and $i = 1, \ldots, N$, the following hold.

(a) $z_{i,t} \sim i.i.d.(0, \sigma_i^2)$, $\mathbb{E}\left|z_{i,t}\right|^8 \leq M$, and $\sum_{j=0}^{\infty} j\left|D_{ij}\right| < M$.
(b) Let $\phi_{i,j} = \mathbb{E}(z_{i,t}z_{j,t})$. Then, $\sum_{i=1}^{N}\left|\phi_{i,j}\right| \leq M$ for all j and $N^{-1}\sum_{i=1}^{N}\sum_{j=1}^{N}\left|\phi_{i,j}\right| \leq M$.
(c) Let $\zeta_{s,t} = \mathbb{E}\left|N^{-1/2}\sum_{i=1}^{N}[z_{i,s}z_{i,t} - \mathbb{E}(z_{i,s}z_{i,t})]\right|^4$. Then, $\zeta_{s,t} \leq M$.
(d) $\mathbb{E}\left|U_{i,0}\right| \leq M$ for every $i = 1, \ldots, N$.

Assumption 4. $z_{i,s}$, e_t, and λ_j are mutually independent for every (i, j, s, t).

The model and assumptions follow those of Bai and Ng (2004). In particular, Assumption 3(a) permits weak serial correlations in the idiosyncratic errors $(1 - \rho_i L)U_{i,t}$, whereas Assumption 3(b,c) allows weak cross-sectional correlations. Bai and Ng (2004) consider the unit-root test against the alternative hypothesis of stationarity for the common and idiosyncratic components. In this study, we are interested in the test against the alternative hypothesis of an explosive process. For the common component, $H_0 : \alpha = 1$ versus $H_1 : \alpha > 1$, and for the ith idiosyncratic component, $H_0 : \rho_i = 1$ versus $H_1 : \rho_i > 1$. Under the restriction of $\alpha = 1$, the model is the same as Bai and Ng's (2004) PANIC. They propose a method of separately identifying the common factors and idiosyncratic errors under the null hypothesis that the common factors follow random walks. This is based on first-differenced data; therefore, $x_{i,t} = \lambda_i f_t + u_{i,t}$, where $x_{i,t} = X_{i,t} - X_{i,t-1}$, $f_t = F_t - F_{t-1}$, and $u_{i,t} = U_{i,t} - U_{i,t-1}$. In the following, we assume that there are $T + 1$ observations $t = 0, 1, \ldots, T$ for $X_{i,t}$ (so that F_t and $U_{i,t}$) for notational simplicity. The common factors and factor loadings can be estimated by using $x_{i,t}$ following the principal component method such that

$$(\hat{f}_t, \hat{\lambda}_i) = \arg \min_{\{\lambda_i\}_{i=1}^{N}, \{f_t\}_{t=1}^{T}} \sum_{i=1}^{N}\sum_{t=1}^{T}(x_{i,t} - \lambda_i f_t)^2, \tag{2}$$

with normalization $T^{-1}\sum_{t=1}^{T}\hat{f}_t^2 = 1$. This minimization problem provides a common factor estimate $\hat{f} = [\hat{f}_1, \ldots, \hat{f}_T]'$ as the $\sqrt{T}$-times eigenvectors of xx' corresponding to the largest eigenvalue, where x is a $T \times N$ matrix with the (t, i)th element being $x_{i,t}$. The factor loadings are estimated by $\hat{\lambda}_i = \frac{1}{T}\sum_{t=1}^{T}\hat{f}_t x_{i,t}$, the level common factor is estimated by $\hat{F}_t = \sum_{s=1}^{t}\hat{f}_s$, and the level idiosyncratic errors are estimated by $\hat{U}_{i,t} = \sum_{s=1}^{t}\hat{u}_{i,s}$, where $\hat{u}_{i,s} = x_{i,s} - \hat{\lambda}_i\hat{f}_s$.

The unit-root test for the common component (hereafter, the common test) can be implemented by using a t-test for $H_0 : \delta = 0$ in the regression $\hat{f}_t = \delta\hat{F}_{t-1} + error$ such that $t_{\hat{F}} = \hat{\delta}/se(\hat{\delta})$, where $\hat{\delta}$ is an ordinary least-squares estimator for δ and $se(\hat{\delta})$ represents its standard errors. The regression may include an intercept and a time trend with appropriate adjustment to the critical values to take account of the time trend. When an intercept is included, we denote the t-test by $\bar{t}_{\hat{F}}$. When the errors are suspected of being serially correlated, we can include the lags of $\hat{f}_t$ in the regression. However, a model with no lags is relevant for asset price data in which

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no serial correlations are present in their first differences.[4] If necessary, we can extend the framework to the model with p lags. The lag order selection follows the conventional method, such as the information criteria based on estimated components. As shown in Said and Dickey (1984), the asymptotic distributions of the t-tests are not affected by including p lags if $p^3/T \to \infty$. Here, this condition must consider that factor estimation errors vanish if $N,T \to \infty$; hence, we require $p^3/\min\{N,T\} \to 0$. When $r > 1$, we propose testing the estimated common factors series by series to determine whether any of the common factors are explosive. This is a sufficient treatment for the present study because we are only interested in the space spanned by the common factors.[5] The unit-root test for the ith idiosyncratic component (hereafter, the idiosyncratic test) is implemented by using a t-test for $H_0 : \delta_i = 0$ in the regression $\hat{u}_{i,t} = \delta_i \hat{U}_{i,t-1} + error$ so that $t_{\hat{U}}(i) = \hat{\delta}_i / se(\hat{\delta}_i)$ where the same note as $t_{\hat{F}}$ applies. When an intercept is included, we denote the t-test by $\bar{t}_{\hat{U}}(i)$.

As Bai and Ng (2004) note, this approach is convenient because the common and idiosyncratic components are separately identified by using the first-differenced data. This way, both common and idiosyncratic tests have the standard Dickey and Fuller (1979) distribution under the null hypothesis. If the alternative hypothesis of stationarity is true, the series become over-differenced, but they remain stationary; hence, the tests have nontrivial power. Furthermore, their simulation study shows that the common test demonstrates good size and power despite stationary or random walk idiosyncratic components. The same can be said of the idiosyncratic test. Therefore, the PANIC approach successfully disentangles the common and idiosyncratic components.

Remark 1 (Bai and Ng, 2004)**.** Let Assumptions 1–4 hold. (i) Under $\alpha = 1$ and $|\rho_i| \le 1$, for all i, $t_{\hat{F}} \Rightarrow [\int_0^1 W(r)dW(r)]/[\int_0^1 W(r)^2 dr]^{1/2}$ and $\bar{t}_{\hat{F}} \Rightarrow [\int_0^1 \bar{W}(r)dW(r)]/[\int_0^1 \bar{W}(r)^2 dr]^{1/2}$ as $N,T \to \infty$, where $W(r)$ is the standard Wiener process defined on $r \in [0,1]$ and $\bar{W}(r) = W(r) - \int_0^1 W(r)dr$. (ii) Under $\rho_i = 1$, $\alpha = 1$, and $|\rho_j| \le 1$, for all $j \ne i$, $t_{\hat{U}}(i) \Rightarrow (\int_0^1 W_i(r)dW_i(r))/[\int_0^1 W_i(r)^2 dr]^{1/2}$ and $\bar{t}_{\hat{U}}(i) \Rightarrow [\int_0^1 \bar{W}_i(r)dW_i(r)]/[\int_0^1 \bar{W}_i(r)^2 dr]^{1/2}$ as $N,T \to \infty$, where $W_i(r)$ are standard Wiener processes defined on $r \in [0,1]$ and $\bar{W}_i(r) = W_i(r) - \int_0^1 W_i(r)dr$.

These null distributions are applicable to both left-tailed and right-tailed tests, as long as the common and idiosyncratic components are consistently estimated. This is warranted in Bai and Ng's (2004) framework, where all components are $I(1)$ or $I(0)$ such that their first differences are stationary. However, this is not necessarily the case if explosive processes are present. When some idiosyncratic components are moderately explosive, the common components may not be consistently

[4] Phillips and Yu (2011) also consider only the model with $p = 0$.

[5] Bai and Ng (2004) consider the method proposed by Stock and Watson (1988) to determine the number of common trends in the factor space in the setting of $I(0)$ and $I(1)$. However, the number of explosive common trends is not our direct interest. Hence, their method is not used in this study.

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estimated, and a consistent estimate for the idiosyncratic components is not warranted either. Hence, size distortions in the common and idiosyncratic tests are concerned. We discuss the properties of PANIC tests in explosive environments in the next section.

3. PROPERTIES OF THE PANIC TESTS

3.1. Finite Sample Properties

We begin our analysis by investigating the finite sample properties of the PANIC tests via Monte Carlo simulations. Although we focus on the empirical size and power of the right-tailed tests, those of the left-tailed tests are also presented for reference. Although the latter experiment overlaps with Bai and Ng's (2004) results, it is instructive to illustrate how differently the left-tailed and right-tailed tests behave. The data are generated from (1) with $F_t = \alpha F_{t-1} + e_t$ and $U_{i,t} = \rho_i U_{i,t-1} + z_{i,t}$ with $r = 1$, where λ_i, e_t, z_{it}, F_0, and $U_{0,i}$ are independently drawn from the standard normal quasi-random variables in each replication.[6] To evaluate size and power, we vary the values of α and ρ_i from 1.0 to 1.1 for the right-tailed test and from 1.0 to 0.0 for the left-tailed test. Results using the regression models that include (A) no deterministic components, (B) an intercept but no time trend, and (C) an intercept and a linear time trend are produced. Since they are almost identical, we only report case (B). We use the first idiosyncratic component to evaluate the idiosyncratic tests; however, this choice is trivial because the Monte Carlo design is symmetric for any i. We use $(N,T) = (100,100), (100,150), (150,100), (150,150)$ to investigate size and $(N,T) = (100,100)$ to investigate power. The number of replications is 5,000, and the nominal level 5% is used.

We first consider size. We set $\alpha = 1.0$ to evaluate the size of the common test and $\rho_i = 1.0$ to evaluate that of the idiosyncratic test. The upper panel of Table 1 reports the size of the common test as a function of ρ_i, and the lower panel shows the size of the idiosyncratic test as a function of α. The left-tailed tests exhibit good size properties—along the same lines as in Bai and Ng (2004)—confirming that the PANIC approach successfully disentangles the common and idiosyncratic components. However, the results of the right-tailed tests are markedly different. The size of the common test is close to the nominal level when ρ_i is approximately smaller than 1.02; however, it quickly reaches one as ρ_i increases. Furthermore, the size of the idiosyncratic test is also distorted toward zero as α increases. These size distortions suggest that the convenient property of Bai and Ng (2004), that is, the common and idiosyncratic components are separately identified, no longer applies to the right-tailed tests. Regarding the effect of sample size, the size of the left-tailed test is good regardless of N and T, whereas the size of the right-tailed test deteriorates as T increases.

[6] We also computed the size and power of the right-tailed PANIC test using models with $p = 4[\frac{\min\{N,T\}}{100}]^{1/4}$. The results are qualitatively the same and are, thus, not reported.

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TABLE 1. Size of the PANIC tests.

Common tests

	Left-tailed tests					Right-tailed tests			
N	100	150	100	150	N	100	150	100	150
T	100	100	150	150	T	100	100	150	150
ρ_i=1.0	0.049	0.050	0.048	0.054	ρ_i =1.00	0.049	0.050	0.048	0.045
0.8	0.043	0.046	0.049	0.046	1.02	0.069	0.063	0.243	0.206
0.6	0.051	0.046	0.043	0.051	1.04	1.000	1.000	1.000	1.000
0.4	0.047	0.050	0.050	0.049	1.06	1.000	1.000	1.000	1.000
0.2	0.053	0.048	0.049	0.052	1.08	1.000	1.000	1.000	1.000
0.0	0.051	0.044	0.049	0.045	1.10	1.000	1.000	1.000	1.000

Idiosyncratic tests

	Left-tailed tests					Right-tailed tests			
N	100	150	100	150	N	100	150	100	150
T	100	100	150	150	T	100	100	150	150
α=1.0	0.047	0.049	0.052	0.050	α=1.00	0.048	0.049	0.045	0.051
0.8	0.050	0.054	0.052	0.051	1.02	0.035	0.039	0.025	0.025
0.6	0.050	0.050	0.052	0.050	1.04	0.011	0.010	0.002	0.002
0.4	0.050	0.045	0.053	0.052	1.06	0.001	0.001	0.002	0.001
0.2	0.049	0.049	0.053	0.044	1.08	0.001	0.001	0.005	0.006
0.0	0.051	0.056	0.051	0.050	1.10	0.002	0.002	0.010	0.008

Next, we consider power. The upper panels of Figure 1 report the power functions of the common test under $\rho_i = 1$ for all i and show that the common test has a standard power function.[7] Our interest is the power functions of the idiosyncratic test presented in the lower panels. The left-tailed test again has the standard power function; however, the right-tailed test shows a clear nonmonotonic pattern. When the explosive coefficient ρ_i is slightly larger than one, the power increases as ρ_i increases; however, the power function starts to decline toward zero as ρ_i further increases. This means the PANIC approach fails to detect explosive behaviors in an individual idiosyncratic component unless they are quite small.[8] Regarding the effect of sample size, the power increases as T increases

[7]Setting at $\rho_i > 1$ does not show any unique power features of the common tests, except for the size distortions already reported in Table 1. That is, the power functions of the right-tailed test in the case of $\rho_i > 1$ start at a point above the nominal level, but draw an upward curve.

[8]Bai and Ng (2004) also propose a pooled test for the idiosyncratic components and investigate the properties of the left-tailed tests under the assumption that idiosyncratic components are cross-sectionally independent. This is not our direct interest. However, our unreported Monte Carlo results show that the pooled version of the right-tailed PANIC tests have qualitatively similar finite sample properties to those of the individual idiosyncratic tests reported in Figures 1 and 4.

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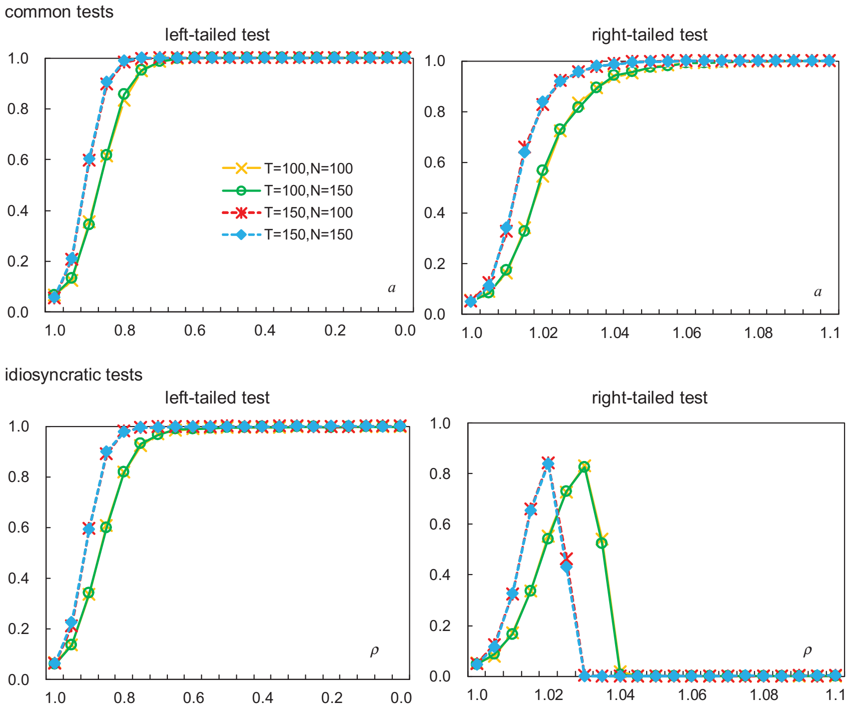

FIGURE 1. Power of the PANIC tests.

when the function is monotonic, but remains the same as N increases. When it is nonmonotonic, the peak of the power shifts leftward as T increases, but again, remains the same as N increases. We also present simulation results in which only one of the idiosyncratic components is explosive, that is, $\rho_N \geq 1.0$ and $\rho_i = 1.0$, for all $i \neq N$. Table 2 shows the size of the common and the idiosyncratic right-tailed tests, whereas Figure 2 presents the power of the idiosyncratic test for $i = N$. The results are consistent with the previous case in which all the idiosyncratic components are explosive, i.e., the common test has considerable size distortions and the idiosyncratic test shows nonmonotonic power. These findings motivate us to theoretically investigate the PANIC methods under explosive environments in the following subsections.

3.2. Analytic Investigation

Becheri and van den Akker (2015) and Westerlund (2015) derive the standard local asymptotic power of the pooled panel unit-root tests in which common factors are extracted by the PANIC method. In doing so, the first-order AR coefficients are

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TABLE 2. Size of the PANIC right-tailed tests when one idiosyncratic component is explosive.

	Common tests					Idyosyncratic tests			
N	100	150	100	150	N	100	150	100	150
T	100	100	150	150	T	100	100	150	150
ρ_N=1.00	0.047	0.050	0.048	0.048	α =1.00	0.050	0.047	0.047	0.046
1.02	0.050	0.054	0.045	0.047	1.02	0.043	0.039	0.025	0.026
1.04	0.059	0.056	0.471	0.389	1.04	0.011	0.012	0.003	0.003
1.06	0.529	0.445	0.966	0.962	1.06	0.002	0.002	0.001	0.003
1.08	0.920	0.900	0.997	0.998	1.08	0.001	0.001	0.006	0.004
1.10	0.986	0.983	1.000	1.000	1.10	0.004	0.003	0.010	0.011

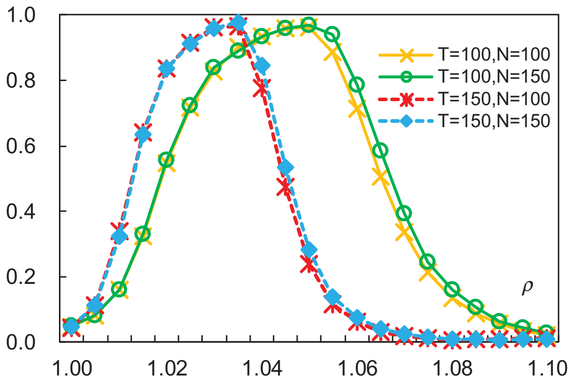

FIGURE 2. Power of the PANIC idiosyncratic test when one idiosyncratic component is explosive.

assumed to shrink to one at a fast rate of T^{-1}.[9] We also investigate the power properties of the common and individual idiosyncratic right-tailed tests by using two complementary asymptotic frameworks. The first approach follows the same lines and assumes that the AR coefficients shrink to one at a fast order $\alpha_T = 1 + \frac{c}{T}$ and $\rho_{i,T} = 1 + \frac{c_i}{T}$, where c and c_i are constants. This LTU asymptotic framework is expected to capture the finite sample properties of the tests when explosiveness is weak. Appendix I of the Supplementary Material shows that the common and the idiosyncratic tests have the standard local asymptotic power for both the left-tailed and right-tailed versions.

It is well known that the results under the LTU framework may not adequately approximate the finite sample behaviors of the test statistics. In the context of structural change tests, a certain type of test statistic may have good power when the magnitude of change is assumed to shrink to zero at a fast rate of $T^{-1/2}$, but

[9]The rate also depends on N because they consider the pooled tests.

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it loses power when the magnitude is fixed. This class of tests typically draws a concave-shaped power function, called the nonmonotonic power problem.[10] One reason for this phenomenon is that, under the alternative hypothesis, a change in the conditional mean and a change in the persistence parameter are not separately identified. Yamamoto and Tanaka (2015) further investigate this problem in the factor model, pointing out that the factor loading structural change and appearance of extra factors may not be separately identified under the alternative hypothesis when the structural changes occur at common dates. In such a case, the standard tests of Breitung and Eickmeier (2011) suffer from the nonmonotonic power problem.

We provide an analytic explanation for why the PANIC tests may have size distortions and nonmonotonic power. We claim that an identification problem between the common and explosive idiosyncratic components occurs under the alternative hypothesis. To this end, we take an approach that assumes the explosive root shrinking to one at a slower rate. In particular, we use the MLTU framework developed by Phillips and Magdalinos (2007).

Assumption M. (a) The AR coefficients satisfy $\alpha_T = 1 + \frac{c}{k_T}$ and $\rho_{i,T} = 1 + \frac{c_i}{k_T}$, where $c \geq 0$, $c_i \geq 0$, and k_T is a deterministic sequence such that $k_T \to \infty$ and $k_T = o(T)$. (b) $C(L) = 1$ and $D_i(L) = 1$, for all i.

The quantities c and c_i $(i = 1, \ldots, N)$ are localizing coefficients and take nonnegative values to focus on the explosive case. The scaling factor k_T is an arbitrary deterministic function of T that satisfies $k_T \to \infty$ strictly slower than T to consider stronger explosiveness than that in the local assumption. A typical formulation is $k_T = T^{\kappa}$, where $0 < \kappa < 1$.

In this setting, the principal component estimate cannot only consistently estimate the common factors, but also misidentify the common components. We illustrate this fact in the following theorem by considering two cases. The first case assumes that $c > 0$ but $c_i = 0$, for all i, such that only the common factor is explosive. The second case is $c_i > 0$ for some or all i but $c = 0$. Hence, only the idiosyncratic components are explosive.

THEOREM 1. *Let Assumptions 1–4 and M hold. If k_T grows sufficiently slowly such that $\alpha_T^T T^{-1/2}$ and $\rho_{i,T}^T T^{-1/2}$ go to infinity as $T \to \infty$, then the following equation holds for the factor estimate:*

$$\hat{f}_t = A f_t + N^{-1} \sum_{i=1}^{N} a_i u_{i,t}, \tag{3}$$

where $A \equiv V^{-1}N^{-1}T^{-1}\hat{f}'f\Lambda'\Lambda + V^{-1}N^{-1}T^{-1}\hat{f}'u\Lambda$ and $a_i \equiv V^{-1}T^{-1}\hat{f}'f\lambda_i' + V^{-1}T^{-1}\hat{f}'u_i$ with V being the largest eigenvalue of $N^{-1}T^{-1}xx'$. Then, the following hold:

[10] As far as the authors know, Perron (1991) was the first study to point out this problem in structural change tests. See Vogelsang (1999), Perron and Yamamoto (2016), and the references therein.

(i) *If* $c > 0$ *and* $c_i = 0$*, for all* i*, then* $V = O_p(\alpha_T^T T^{-1/2})$*. Furthermore, if the stochastic order of* V *is* $\alpha_T^T T^{-1/2}$*,* $A = O_p(1)$ *and* $a_i = O_p(1)$*.*

(ii) *If* $c = 0$ *and* $c_i > 0$*, for all* i*, then* $V = O_p(\rho_{i,T}^T T^{-1/2})$*. Furthermore, if the stochastic order of* V *is* $\rho_{i,T}^T T^{-1/2}$*,* $A = O_p(1)$ *and* $a_i = O_p(1)$*.*

A proof is provided in Appendix II of the Supplementary Material. From part (i), we can deduce that the common test behaves well under the alternative hypothesis. This is because, when the true common component is explosive, the estimation errors of the factor space consist of the second term of (3). Since the explosive common component dominates the factor estimation errors,[11] the common component estimate continues to be explosive and the power remains.[12]

Part (ii) yields a more interesting case. When the idiosyncratic components are explosive, the second term dominates the first term, because $u_{i,t}$ are explosive, whereas f_t is not. Hence, $\hat{f}_t$ is dominated by the explosive idiosyncratic components $u_{i,t}$. Therefore, even when the true common component is not explosive, its estimate may be so when some idiosyncratic components are explosive. More intuitively, because the principal component estimator is based on the eigenvectors associated with the largest eigenvalues of the covariance matrix of the panel data, when the idiosyncratic components are explosive, one of the eigenvalues diverges. This causes the idiosyncratic time series to be misidentified as a common component.

To provide intuition of the condition that $\alpha_T^T T^{-1/2}$ (and $\rho_{i,T}^T T^{-1/2}$) tends to infinity, let us consider the case of $k_T = T^\kappa$. In this case, α_T^T is approximated by $\exp(cT^{1-\kappa})$ and the condition requires it grows faster than $T^{1/2}$. Apparently, the LTU ($\kappa = 1$) does not satisfy this condition, because $\exp(cT^{1-\kappa}) = \exp(c)$ is a flat function of T. On the contrary, if $\kappa = 0$, then $\alpha_T^T = \exp(cT)$, and this always diverges faster than $T^{1/2}$; hence, $\alpha_T^T T^{-1/2}$ diverges to infinity.[13] Therefore, this condition is more relevant, as κ is smaller (or α_T is larger). In our unreported numerical exercise, $\alpha_T^T T^{-1/2}$ increases as T when κ is 0.80 or smaller when $c = 1.0$. The value of α_T that corresponds to $\kappa = 0.80$ when $T = 100$ is $\alpha_T = 1.03$. This gives us a rough guide for when the condition $\alpha_T^T T^{-1/2} \to \infty$ holds.[14]

We can derive clear implications of part (ii): when $u_{i,t}$ are explosive, the size of the common test is distorted because $\hat{f}_t$ is dominated by $u_{i,t}$ that are explosive. More interestingly, because $\hat{f}_t$ is dominated by the explosive $u_{i,t}$, the idiosyncratic component estimate $\hat{u}_{i,t}$ becomes nonexplosive for the reason in the following remark. This explains the nonmonotonic power of the idiosyncratic test.

[11] In Appendix III of the Supplementary Material, we show that, in case (i), the factor estimation errors in the differenced factor are $o_p(1)$, but those in the level factor are $O_p(T^{1/2}N^{-1/2})$.

[12] Things are different for the idiosyncratic tests, because the true idiosyncratic components are not explosive and do not dominate the factor estimation errors. Therefore, the idiosyncratic test could suffer from size distortions.

[13] Here, how long it takes for the divergence to be evident depends on c. We appreciate this advice from Professor Peter C.B. Phillips.

[14] As shown in the Monte Carlo simulation, the finite sample power of the idiosyncratic test starts to decrease when $T = 100$ and $\rho_i = 1.03$.

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Remark 2. We illustrate the power loss of the idiosyncratic test by taking the special case of (ii), where only the first cross-sectional unit has an explosive idiosyncratic component. We have $\hat{f}_t \approx a_1 u_{1,t}$. By plugging this into the factor loading estimate $\hat{\lambda}_1 = \left(\sum_{t=1}^{T}\hat{f}_t^2\right)^{-1}\left(\sum_{t=1}^{T}\hat{f}_t x_{1,t}\right)$,
we obtain

$$\begin{aligned}\hat{\lambda}_1 &= \left(\sum_{t=1}^{T}\hat{f}_t^2\right)^{-1}\left(\sum_{t=1}^{T}\hat{f}_t x_{1,t}\right) \approx \left(a_1^2\sum_{t=1}^{T}u_{1,t}^2\right)^{-1}\left(a_1\sum_{t=1}^{T}u_{1,t}x_{1,t}\right),\\ &= \left(a_1^2 T^{-1}\sum_{t=1}^{T}u_{1,t}^2\right)^{-1}\left(a_1\lambda_1 T^{-1}\sum_{t=1}^{T}u_{1,t}f_t + a_1 T^{-1}\sum_{t=1}^{T}u_{1,t}^2\right) \approx a_1^{-1},\end{aligned} \tag{4}$$

because the numerator of the second line is dominated by the second term. By plugging $\hat{f}_t \approx a_1 u_{1,t}$ and (4) into the idiosyncratic component estimate $\hat{u}_{1,t} = x_{1,t} - \hat{\lambda}_1\hat{f}_t$, we obtain

$$\begin{aligned}\hat{u}_{1,t} &= x_{1,t} - \hat{\lambda}_1\hat{f}_t,\\ &\approx u_{1,t} + \lambda_1 f_t - (a_1^{-1})(a_1 u_{1,t}),\\ &= u_{1,t} + \lambda_1 f_t - u_{1,t} = \lambda_1 f_t.\end{aligned} \tag{5}$$

Therefore, equations $\hat{f}_t \approx a_1 u_{1,t}$ and (4) imply $\hat{\lambda}_1\hat{f}_t \approx u_{1,t}$, and equation (5) implies $\hat{u}_{1,t} \approx \lambda_1 f_t$. These mean that the common and idiosyncratic components are reversely identified by their estimates. Hence, the idiosyncratic test loses power.

We validate this identification problem by investigating the correlation coefficient between $\hat{f}_t$ and f_t and the correlation coefficient between $\hat{f}_t$ and $u_{1,t}$. If the misidentification occurs, the former decreases, but the latter increases as $u_{1,t}$ becomes more explosive. To this end, we generate the same data as in Section 3.1 and compute the average of the absolute correlation coefficients between the estimated and true common components $\left|Corr(\hat{f}_t, f_t)\right|$ over 5,000 replications. We also compute the average of the absolute correlation coefficients between the estimated common component and the true idiosyncratic component $\left|Corr(\hat{f}_t, u_{1,t})\right|$. The left panel of Figure 3 shows that, as $u_{1,t}$ becomes more explosive, $\hat{f}_t$ becomes less correlated with f_t, but more correlated with $u_{1,t}$. This finding is consistent with Theorem 1(ii). Next, as equation (5) suggests, we compute the average of the absolute correlation coefficients between the estimated and true idiosyncratic components $\left|Corr(\hat{u}_{1,t}, u_{1,t})\right|$ and the average of the absolute correlation coefficients between the estimated idiosyncratic component and the true common component $\left|Corr(\hat{u}_{1,t}, f_t)\right|$. The right panel of Figure 3 shows that, as $u_{1,t}$ becomes more explosive, $\hat{u}_{1,t}$ becomes less correlated with $u_{1,t}$, but more correlated with f_t, because $\hat{u}_{1,t}$ inherits the time-series properties of f_t.

Remark 3. To make Theorem 1 more comprehensive, we may consider the case of $c > 0$ and $c_i > 0$, for some i. Then, we have $V = O_p(\max\{\alpha_T^T T^{-1/2}, \rho_{i,T}^T T^{-1/2}\})$,

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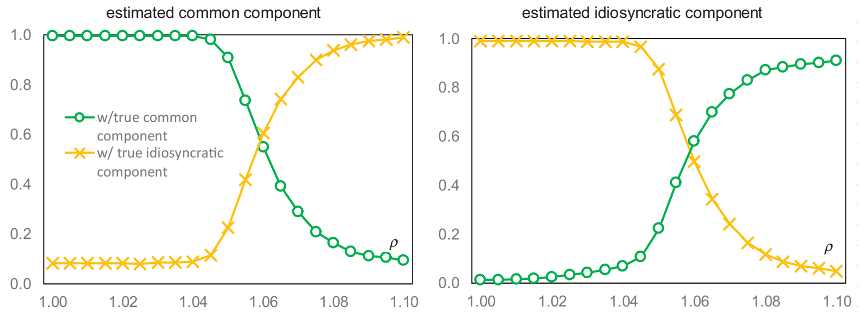

FIGURE 3. Absolute values of the correlation coefficients of the estimated components (with the true common and idiosyncratic errors).

$A = O_p(1)$, and $a_i = O_p(1)$. The explosive idiosyncratic components become dominating components in the estimated factors if they are more strongly explosive than the common components. This case yields the same implication as part (ii), because what matters is the explosive behavior in the idiosyncratic components. In addition, we may also consider the case of $c < 0$ and/or $c_i < 0$, for some i. Then, F_t and/or $U_{i,t}$ are $I(0)$ for some i, and they would not contaminate the factor estimation as shown in Bai and Ng (2004). Therefore, this case merely gives the same implication as when $c = 0$ and/or $c_i = 0$.

4. CROSS-SECTIONAL APPROACH

This section provides a new method of testing explosive behavior in the common and idiosyncratic components. It is based on the following two key ingredients. First, it takes advantage of the fact that explosive behaviors appear only in a certain subperiod and the series are not explosive in the rest of the sample period. If this is the case, we can timewise localize the explosive behaviors by considering model (1) with $F_t = \alpha F_{t-1} + e_t$ and $U_{i,t} = \rho_i U_{i,t-1} + z_{i,t}$, where $\alpha = \rho_i = 1$ for $t = 1, \ldots, T$ and $\alpha, \rho_i \geq 1$ for $t = T+1, \ldots, T+h$, for any i, with h being the length of the window, such that the data are assumed to have a certain period $t \in [1, T]$ in which no explosive behaviors exist in either the common or the idiosyncratic components. We call this the training sample.[15] On the contrary, the period of interest $t \in [T+1, T+h]$ is called the explosive window.

4.1. Algorithm

The second key element is using cross-sectional regressions to estimate the common factors in the explosive window instead of using the principal component

[15] We can easily show that weak (local) explosive processes with AR coefficients $1 + \frac{c}{T}$ and $1 + \frac{c_i}{T}$ can exist in the training sample.

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estimation of the first-differenced series. This is because the first-differenced series of the explosive process remains explosive and, thus, violates Assumption 3. Hence, the common factors are not consistently estimated. To address this problem, we estimate the factor loadings in the training sample in a nonexplosive environment. We then use these loadings as the regressors of the cross-sectional regressions in the explosive window to estimate the common factors as the coefficients attached to the factor loadings. In this way, we can avoid the identification problem between the common and idiosyncratic components investigated in Section 3.2. An important model assumption is that the factor loadings are constant for the training sample and explosive window. We also keep the assumption that the number of factors remains the same. We call this approach the CS method, and it involves the following steps:

Algorithm:

Step 1. Use the first-differenced data $x_{i,t}$, for $t = 1, \ldots, T$, to estimate the factor loadings λ_i by using the principal component method (2). Denote the factor loadings estimated in the training sample by $\hat{\lambda}_i^*$.

Step 2. At $t = T+1$, estimate the level of the common factors by the CS regression of $\{X_{i,t}\}_{i=1}^N$ on $\{\hat{\lambda}_i^*\}_{i=1}^N$ so that $\widetilde{F}_t = \left(\sum_{i=1}^N \hat{\lambda}_i^* \hat{\lambda}_i^{*\prime}\right)^{-1} \left(\sum_{i=1}^N \hat{\lambda}_i^* X_{i,t}\right)$ and the idiosyncratic components by $\widetilde{U}_{i,t} = X_{i,t} - \hat{\lambda}_i^{*\prime} \widetilde{F}_t$. Then, repeat this for $t = T+2, \ldots, T+h$.

Step 3. Construct the common test $t^*_{\widetilde{F}}$ by using $\widetilde{F}_t$ and $\widetilde{f}_t = \widetilde{F}_t - \widetilde{F}_{t-1}$ in the regression $\widetilde{f}_t = \delta \widetilde{F}_{t-1} + error$ and the idiosyncratic test $t^*_{\widetilde{U}}(i)$ by using $\widetilde{U}_{i,t}$ and $\widetilde{u}_{i,t} = \widetilde{U}_{i,t} - \widetilde{U}_{i,t-1}$ in the regression $\widetilde{u}_{i,t} = \delta_i \widetilde{U}_{i,t-1} + error$ for $t = T+1, \ldots, T+h$. In both regressions, lags of the dependent variable can be included if serial correlations in the errors are concerned. We denote the tests using the regression with an intercept by $\bar{t}^*_{\widetilde{F}}$ and $\bar{t}^*_{\widetilde{U}}(i)$.

Remark 4. Although we set $t \in [1, T]$ and $t \in [T+1, T+h]$ as the training sample and the explosive window, respectively, this does not mean that the origination dates of explosive behaviors have to be known in practice for the following reasons. First, the explosive behaviors can start later than $T+1$. If so, we are merely implementing the right-tailed unit-root tests for the sample that includes a nonexplosive subsample. Second, explosive behaviors can start before T as long as they are as weak as the LTU. This is because, even in the presence of the explosive behavior, the common and idiosyncratic components are identified as we see in Theorem SA-1 of Appendix I of the Supplementary Material and so are the factor loadings. Third, the origination dates of explosive behaviors in the common components and in any idiosyncratic components are allowed to be heterogeneous because we implement the tests series by series. One method of selecting the training sample is to use an existing time-stamping method, such as in Phillips et al. (2011), to the cross-sectionally averaged series $\bar{X}_t = N^{-1} \sum_{i=1}^N X_{i,t}$. In Appendix IV of the Supplementary Material, we show that the size and power of the CS tests are rarely distorted even when the training sample is selected using this method.

4.2. Theoretical Results

We next provide an asymptotic justification of the CS method. Note that the time dimension of the testing period is now h instead of T; hence, we consider the following Assumption 5 in place of Assumption M.

Assumption 5. (a) The AR coefficients satisfy $\alpha_h = 1 + \frac{c}{k_h}$ and $\rho_{i,h} = 1 + \frac{c_i}{k_h}$, where $c \geq 0$, $c_i \geq 0$, and k_h is a deterministic sequence such that $k_h \to \infty$ and $k_h = o(h)$. (b) $C(L) = 1$ and $D_i(L) = 1$, for all i.

We obtain the following results. For brevity, again, we provide a proof under i.i.d. assumptions, setting $C(L) = 1$ and $D_i(L) = 1$ in Appendix V of the Supplementary Material. The case with an intercept is shown, whereas the case with no intercept can be similarly given.

THEOREM 2. *Let Assumptions 1–5 hold. With $c \geq 0$ and $c_j \geq 0$, for any $j = 1, \ldots, N$, the following hold as $N, T, h \to \infty$.*
(a: common tests) If $c = 0$,

$$\bar{t}^*_{\widetilde{F}} \Rightarrow \left(\int_0^1 \bar{W}(r)dW(r)\right) \Big/ \left[\int_0^1 \bar{W}(r)^2 dr\right]^{1/2},$$

if $\frac{\rho_{j,h}^h h k_h^{-1}}{\min\{N^{1/2}, T^{1/2}\}} \to 0$*, for any j. If $c > 0$, with $\pi \in (0,\infty)$ and $\Theta \equiv N(0, \sigma^2/2c)$,*

$$\alpha_h^{-h} \bar{t}^*_{\widetilde{F}} \approx \begin{cases} \sqrt{\frac{c}{2\sigma^2}}\,|\Theta|, & \text{if } T/k_h \to 0, \\ \sqrt{\frac{c}{2\sigma^2}}\left|\frac{F_T}{\sqrt{T}}\sqrt{\pi} + \Theta\right|, & \text{if } T/k_h \to \pi, \end{cases}$$

$$\alpha_h^{-h} k_h^{1/2} T^{-1/2} \bar{t}^*_{\widetilde{F}} \approx \sqrt{\frac{c}{2\sigma^2}}\left|\frac{F_T}{\sqrt{T}}\right| \qquad \text{if } T/k_h \to \infty,$$

if $\frac{\alpha_h^h h k_h^{-1}}{\min\{N^{1/2}, T^{1/2}\}} \to 0$ *and* $\frac{\rho_{j,h}^h h k_h^{-1}}{\min\{N^{1/2}, T^{1/2}\}} \to 0$*, for any j.*
(b: idiosyncratic tests) If $c_i = 0$,

$$\bar{t}^*_{\widetilde{U}}(i) \Rightarrow \left(\int_0^1 \bar{W}_i(r)dW_i(r)\right) \Big/ \left[\int_0^1 \bar{W}_i(r)^2 dr\right]^{1/2}.$$

if $\frac{\alpha_h^h h k_h^{-1}}{\min\{N, T^{1/2}\}} \to 0$ *and* $\frac{\rho_{j,h}^h h k_h^{-1}}{\min\{N^{1/2}, T^{1/2}\}} \to 0$*, for any j. If $c_i > 0$, with $\pi \in (0,\infty)$ and $\Theta_i \equiv N(0, \sigma_i^2/2c_i)$,*

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$$\rho_{i,h}^{-h}\bar{t}_{\tilde{U}}^{*}(i) \approx \begin{cases} \sqrt{\frac{c_i}{2\sigma_i^2}}\,|\Theta_i|, & \textit{if } T/k_h \to 0, \\ \sqrt{\frac{c_i}{2\sigma_i^2}}\left|\frac{U_{i,T}}{\sqrt{T}}\sqrt{\pi}+\Theta_i\right|, & \textit{if } T/k_h \to \pi, \end{cases}$$

$$\rho_{i,h}^{-h}k_h^{1/2}T^{-1/2}\bar{t}_{\tilde{U}}^{*}(i) \approx \sqrt{\frac{c_i}{2\sigma_i^2}}\left|\frac{U_{i,T}}{\sqrt{T}}\right| \qquad \textit{if } T/k_h \to \infty,$$

if $\frac{\alpha_h^h h k_h^{-1}}{\min\{N^{1/2},T^{1/2}\}} \to 0$ *and* $\frac{\rho_{j,h}^h h k_h^{-1}}{\min\{N^{1/2},T^{1/2}\}} \to 0$, *for any j.*

Theorem 2 shows that the CS method provides the test statistics that have the correct asymptotic size under the MLTU framework if the stated conditions on the relative rate among N, T, and h hold. Under the alternative hypothesis, the tests are consistent and behave as follows. If k_h is faster than T so that $T/k_h \to 0$, then we obtain the limit involving the explosive sample thus $|\Theta|$ and the test diverges to positive infinity at a rate of α_h^h with probability one. If k_h is slower than T so that $T/k_h \to \infty$, then the test statistic scaled by $\alpha_h^{-h}k_h^{1/2}T^{-1/2}$ is asymptotically dominated by the absolute value of the initial value term. The test diverges to positive infinity at a rate of $\alpha_h^h k_h^{-1/2}T^{1/2}$ with probability one. If T and k_h grow at the same rate ($T/k_h \to \pi$), then both effects are dominant and the test diverges to positive infinity at a rate of α_h^h with probability one. More importantly, the divergence becomes faster as the localizing coefficient c increases, which ensures the monotonic power property of the CS test.

The added condition $\frac{\alpha_h^h h k_h^{-1}}{\min\{N^{1/2},T^{1/2}\}} \to 0$ (and $\frac{\rho_{j,h}^h h k_h^{-1}}{\min\{N^{1/2},T^{1/2}\}} \to 0$) requires h not to grow very fast to eliminate the effects of factor estimation error. To obtain an intuition behind the condition, we give a parametric example of $k_h = h^{\kappa}$ where κ lies on $(0,1)$ and we let $N = T$ for simplicity. Then, the condition reduces to $\frac{\alpha_h^h h^{1-\kappa}}{T^{1/2}} \to 0$. Taking its logarithm and using $\log(\alpha_h) = ch^{-\kappa} + o(1)$ yield

$$ch^{1-\kappa} + (1-\kappa)\log(h) - \frac{1}{2}\log(T) \to -\infty.$$

Suppose $c = 0.5$. Then, we can show that this is satisfied when h grows at a rate of $\log(T)$. As another example, we set $k_h = h/\log(h)$. Then, the condition becomes

$$c\log(h) + \log(\log(h)) - \frac{1}{2}\log(T) \to -\infty. \tag{6}$$

With $c = 0.5$, h is required to grow slower than T only slightly. Because the condition $\frac{\alpha_h^h h k_h^{-1}}{\min\{N^{1/2},T^{1/2}\}} \to 0$ (and $\frac{\rho_{j,h}^h h k_h^{-1}}{\min\{N^{1/2},T^{1/2}\}} \to 0$) requires h not to grow very fast, only the case of $T/k_h \to \infty$ applies in most examples. However, $T/k_h \to \pi$ and $T/k_h \to 0$ are also relevant when h is relatively fast. To see this in the second example, we let $T = h^{1-\epsilon}/\log(h)$ with $\epsilon \geq 0$. Then, $T/k_h \to 1$ when $\epsilon = 0$ and $T/k_h \to 0$ when $\epsilon > 0$. In either case, (6) holds when c is sufficiently smaller than $\frac{1}{2}$. In the next subsection, we consider the finite sample performance of the test via

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Monte Carlo simulation using realistic values for the parameters and the sample size.

Remark 5. The key element of the CS method is estimation errors in the levels and the first differences of the common factors as shown in Lemma B1 (or Lemma B6 for the demeaned version) in Appendix V:

$$\widetilde{F}_t - HF_t = O_p\left(\frac{\alpha_h^h k_h^{1/2}}{\min\{N, T^{1/2}\}}\right) + O_p\left(\frac{\rho_h^h k_h^{1/2}}{\min\{N^{1/2}, T^{1/2}\}}\right),$$
$$\widetilde{f}_t - Hf_t = O_p\left(\frac{\alpha_h^h k_h^{-1/2}}{\min\{N, T^{1/2}\}}\right) + O_p\left(\frac{\rho_h^h k_h^{-1/2}}{\min\{N^{1/2}, T^{1/2}\}}\right),$$

where $\rho_h = \max_i \rho_{i,h}$. Hence, we extensively use them to prove Theorem 2.

4.3. Finite Sample Properties

This subsection investigates the finite sample property of the CS method via Monte Carlo simulations. The data are generated by the same model as in Section 3.1. To investigate the validity of our theoretical results more directly, we set the AR coefficients to be $\alpha = 1 + \frac{c}{h^\kappa}$ and $\rho_i = 1 + \frac{c_i}{h^\kappa}$, where $c = c_i = 0$ for $t = 1, \ldots, T$ and c, $c_i \geq 0$ for $t = T+1, \ldots, T+h$, for any i. We use the sample size $N = 100$ and $T = 50$ and two lengths of the explosive window $h = 50$ and 100. All λ_i, $u_{i,t}$, $z_{i,t}$, F_0, and $U_{0,i}$ are independently drawn from the standard normal quasi-random variables in each replication. The size and power of the CS and PANIC tests in the explosive window are computed at the 5% nominal level using 5,000 replications.

Table 3 presents the size of the common and idiosyncratic tests in the upper and lower panels, respectively. The left and right panels correspond to the $h = 50$ and 100 cases, respectively. Consistent with our findings in Section 3.1, the PANIC common test shows serious size distortions when the idiosyncratic components are explosive and the PANIC idiosyncratic test becomes undersized when the common component is explosive when $\kappa = 0.8$ and c is large. Although the CS common test also shows size distortions, these are considerably smaller than those in the PANIC common tests. As for the CS idiosyncratic test, we now see over-rejections, especially when $\kappa = 0.8$ and c is large. This is consistent with the conditions provided in Theorem 2. In our unreported results, we observe that the size of the CS test slightly improves as both N and T increase, although the effect is not discernible. Figure 4 reports the power of both tests. Most importantly, the bottom panels of Figure 4 suggest that the CS idiosyncratic test is free of the nonmonotonic power problem. The power functions of the CS and PANIC common tests are similar because the tests are asymptotically equivalent. In summary, the CS tests display size distortions when the explosiveness is strong, but it performs well in general with a moderately explosive process with κ being 0.85 or lower. The CS tests outweigh the PANIC tests with respect to the power of idiosyncratic tests.

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TABLE 3. Size of the CS and PANIC tests.

Common tests								
h	50	50	50	50	100	100	100	100
κ	0.80	0.85	0.90	0.95	0.80	0.85	0.90	0.95
CS								
c_i=0.0	0.053	0.048	0.047	0.050	0.055	0.056	0.044	0.057
0.2	0.053	0.052	0.053	0.057	0.052	0.066	0.046	0.044
0.4	0.059	0.057	0.053	0.058	0.054	0.053	0.046	0.063
0.6	0.075	0.066	0.060	0.054	0.067	0.043	0.048	0.049
0.8	0.087	0.070	0.072	0.057	0.094	0.079	0.055	0.056
1.0	0.142	0.095	0.075	0.058	0.190	0.089	0.063	0.059
PANIC								
c_i=0.0	0.048	0.047	0.050	0.056	0.054	0.052	0.043	0.055
0.2	0.056	0.054	0.053	0.056	0.054	0.073	0.044	0.043
0.4	0.063	0.059	0.050	0.059	0.061	0.060	0.045	0.063
0.6	0.097	0.069	0.067	0.053	0.095	0.050	0.050	0.053
0.8	0.238	0.114	0.092	0.064	0.209	0.108	0.067	0.054
1.0	0.872	0.280	0.125	0.085	0.814	0.192	0.086	0.068
Idiosyncratic tests								
h	50	50	50	50	100	100	100	100
κ	0.80	0.85	0.90	0.95	0.80	0.85	0.90	0.95
CS								
c=0.0	0.067	0.063	0.056	0.043	0.050	0.055	0.054	0.048
0.2	0.068	0.046	0.057	0.043	0.051	0.062	0.041	0.054
0.4	0.049	0.048	0.049	0.052	0.057	0.053	0.055	0.045
0.6	0.079	0.069	0.074	0.063	0.055	0.060	0.052	0.062
0.8	0.108	0.066	0.076	0.064	0.116	0.090	0.065	0.058
1.0	0.206	0.122	0.069	0.064	0.219	0.103	0.078	0.065
PANIC								
c=0.0	0.068	0.062	0.058	0.037	0.049	0.057	0.054	0.042
0.2	0.064	0.047	0.050	0.042	0.048	0.064	0.044	0.043
0.4	0.044	0.044	0.055	0.057	0.048	0.043	0.055	0.047
0.6	0.059	0.059	0.052	0.054	0.046	0.053	0.041	0.058
0.8	0.038	0.040	0.052	0.046	0.027	0.046	0.051	0.051
1.0	0.031	0.039	0.042	0.044	0.025	0.037	0.052	0.048

Finally, the CS method relies on the fundamental model assumptions that the factor loadings and the number of factors are constant even when the explosive regime starts. We investigate the consequences of instabilities pertaining to them.

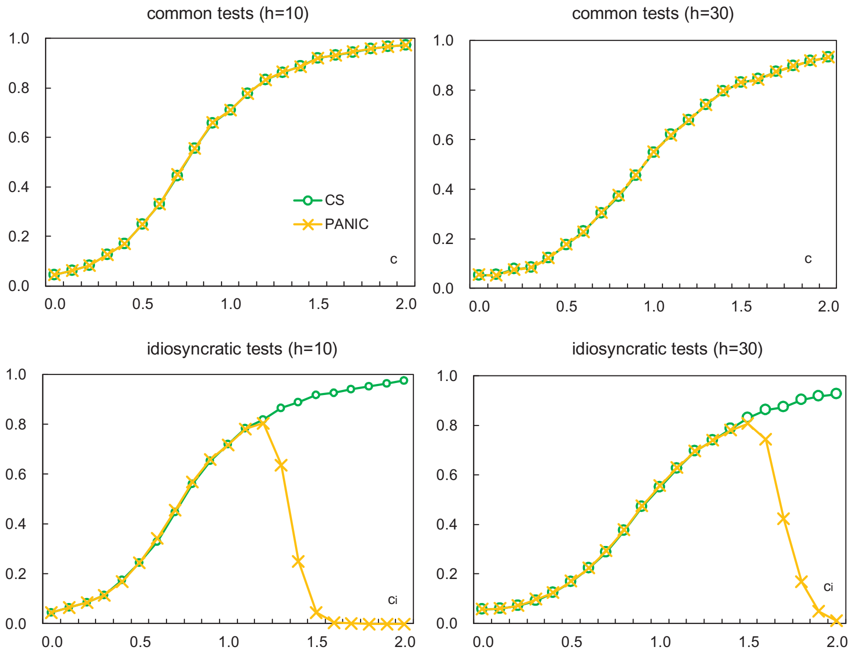

FIGURE 4. Power of the CS and PANIC tests.

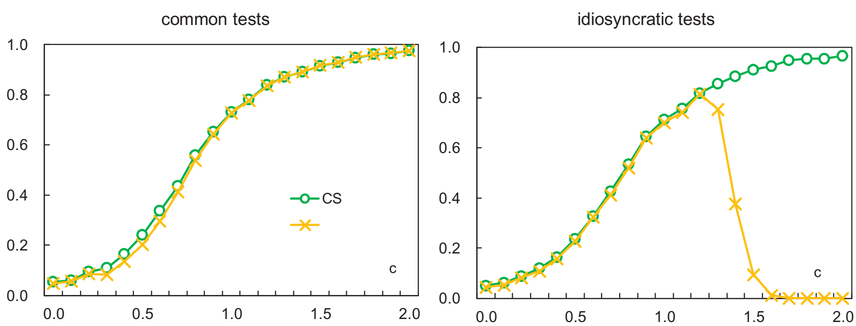

FIGURE 5. Power of the CS and PANIC tests when the factor loadings have structural changes.

First, the factor loadings have structural changes such that $X_{i,t} = \lambda_i F_t + U_{i,t}$, for $t = 1, \ldots, T$, and $X_{i,t} = (\lambda_i + \Delta_i)F_t + U_{i,t}$, for $t = T+1, \ldots, T+h$, where the change $\Delta_i \sim i.i.d.U[0,1]$. Second, we generate F_t and $U_{i,t}$ via the same processes, but with an additional common factor $G_t = 0$ for $t \in [1,T]$ and $G_t = \alpha G_{t-1} + v_t$ for $t \in [T+1, T+h]$, where v_t follows $i.i.d.N(0,1)$. Then, $X_{i,t} = \lambda_i F_t + U_{i,t}$, for $t = 1, \ldots, T$, and $X_{i,t} = \lambda_i F_t + \gamma_i G_t + U_{i,t}$, for $t = T+1, \ldots, T+h$, where the new factor loadings are generated by $\gamma_i \sim i.i.d.N(0,1)$. In both cases, we implement the PANIC and

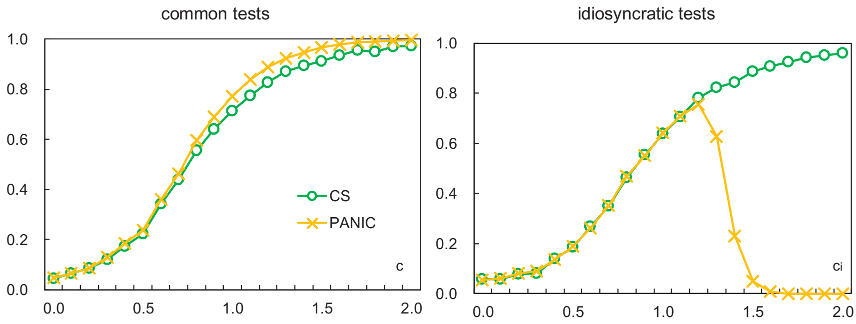

FIGURE 6. Power of the CS and PANIC tests when a new factor appears.

CS tests in the same manner as the previous case without accounting for such instabilities. Since the structural changes in the factor loadings and the presence of the additional factor are most likely to occur simultaneously when F_t switches to the explosive regime, we focus on the power of the tests. Figure 5 reports the power of the common and idiosyncratic tests in the case of structural changes, and Figure 6 presents them in the case of the new factor for $N = 100$, $T = 50$, $h = 100$, and $\kappa = 0.85$. They show that the nonmonotonic power of the idiosyncratic tests is still found in the PANIC tests, but it is resolved in the CS tests. The power of the common tests is rarely affected by the structural changes, although we see some power loss in the PANIC tests in Figure 5 and in the CS tests in Figure 6.

5. CONCLUSIONS

In this study, we showed that, when the PANIC tests are applied to the explosive alternative hypothesis, both the common and the idiosyncratic tests may exhibit serious size distortions. More importantly, the idiosyncratic tests suffer from the nonmonotonic power problem. We then provide a new CS method to disentangle the common and idiosyncratic components to obtain standard monotonic power function. The proposed tests achieve the correct asymptotic size and are consistent under the MLTU framework. A Monte Carlo simulation shows that the CS test for common components considerably reduces size distortions and the CS test for idiosyncratic components is robust to the nonmonotonic power problem.

Our study has several implications. First, the nonmonotonic power problem can occur not only in certain structural change tests, as shown in Perron and Yamamoto (2016), but also in more general circumstances in which important model parameters are not correctly identified under the alternative hypothesis. Earlier studies such as Müller and Elliott (2003) argued that Elliott, Rothenberg, and Stock's (1996) efficient unit-root tests may have power that drops to zero when the initial value is moderately large. Our study uncovers another possibility of the nonmonotonic power problem in unit-root testing when unobserved common and idiosyncratic components are misidentified. Second, asymptotic frameworks that

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allow general deviations from the null hypothesis, such as the MLTU of Phillips and Magdalinos (2007), are extremely useful in approximating such phenomena. Third, the proposed method can potentially extend the right-tailed PANIC tests to various empirical analyses, including testing financial bubbles (see Phillips et al., 2011) in large panel data and factor-augmented regressions (see Stock and Watson, 2016). A caveat is that the proposed method is not free from size distortions when the other nuisance components are strongly explosive. In addition, the relevance of the constant factor loading assumption must be assessed in particular empirical settings. These issues should be carefully incorporated in future studies.

SUPPLEMENTARY MATERIAL

To view the supplementary material for this article, please visit: https://doi.org/10.1017/S0266466622000044

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