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Short Paper Effect of sea-level lowering on ELA depression during the LGM

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ABSTRACT

Decreases in equilibrium-line altitudes (ELAs) varied geographically during the last glacial maximum (LGM), with a mid-range value of ~900 m commonly deduced from altitude ratio and accumulation–area ratio calculations. Sea level, however, was 120 m lower during the LGM, so the ELA lowering relative to sea level would only be 780 m for a 900-m absolute lowering. With a lapse rate of 0.006° C m⁻¹, this implies a 4.7° C lowering of global temperature. It has been argued that this correction for sea-level change is unnecessary, but the logic on which this is based requires adiabatic compression to apply over much longer time scales than is typically invoked. We find that the correction is necessary. In addition, geometric changes in the atmosphere during the LGM, pointed out by Osmaston (2006), could lead to 0.4° C decrease in the average temperature of the troposphere. Additionally, orographic effects could significantly change the snow distribution on mountain masses near sea level.

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Introduction

A simple way of estimating the temperature depression during a glacial period is to use field observations of changes in equilibriumline altitude (Δ ELA) and to assume an environmental lapse rate, $\Gamma_e \equiv - dT/dz$, which, when multiplied by Δ ELA, yields a temperature change. Osmaston (2006) raised the question of whether the temperature change implied by decreases in ELA during the late glacial maximum (LGM) should be estimated using the altitude above a lowered sea level or simply using the altitude change relative to today's sea level.

Studies of LGM moraines and cirque elevations suggest that equilibrium lines then were lower than at present by amounts that vary geographically. Mark et al. (2005) surveyed over 350 glacier valley locations within the tropics and subtropics, finding both intraregional and inter-regional differences. Although values vary widely, 900 m serves as a rough midpoint for the tropics and subtropics, and is useful for discussion of the implied temperature changes. As temperature lapse rates typically average about 0.006° C/m, this implies a cooling of ~5.4°C relative to temperatures today. However, sea level was then ~120 m lower; Gillespie and Molnar (1995) suggested that the actual ELA depression should be measured relative to this lowered sea level. Thus, for example, a 900 m lowering of an ELA results in a ~780 m lowering relative to present sea level, implying only a 4.7°C cooling if temperature is considered to be the only controlling factor.

Other estimates of temperature change during the LGM, based on proxy measures, are commonly somewhat less than this. Data assembled by CLIMAP (1976) suggested a decrease in sea-surface temperature of only 2.3°C. Rostek et al. (1993), using ¹⁸O and alkenone records from foraminifera, came up with ~2.5°C. Greene et al. (2002) found that a decrease in sea-surface temperature of 2.8°C could account for the observed snowline depression, given no change in precipitation, while allowing for such a change raised the value to ~3.0°C. Broecker (1997), combining ¹⁸O records from snow on Mt.Huascarán and calculations based on heat released by condensation as water vapor rises, suggested 4°C. Guilderson et al. (1994) used Sr/Cr ratios in corals to calculate a cooling of 5°C. Further discussions on this issue will benefit from having an unambiguous interpretation of how ELA changes should be translated, via lapse rates, into temperature changes.

Gillespie and Molnar's "correction" was picked up by Broecker (1997) and adopted by Porter (2001) and by Greene et al. (2002). The implication of this correction is that the entire troposphere sank 120 m, and that the lapse rate remained unchanged, resulting in a cooling of $0.006 \times 120 = 0.7^{\circ}$ C at all elevations.

Osmaston (2006) argued that the troposphere would not have undergone the overall 120-m downward shift because water taken out of the oceans would have been stacked up on land as ice. Thus the tropopause (the top of the troposphere) would have remained at a constant elevation, but the pressure at the lowered sea level would be slightly higher owing to the greater thickness of the atmosphere over the ocean basins. In this argument, adiabatic compression would lead to slightly higher sea-surface temperatures: using, as Osmaston did, a dry adiabatic lapse rate of 10°C/km, the increase would be about 1.2°C.

As Osmaston (2006) noted, the "shape" of the atmosphere is changed by the presence of a continental ice sheet, and this change

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does, indeed, have a small effect on some climatically interesting characteristics, as we will discuss. However, his argument for ignoring this effect in calculating the SST depression that would result in a 900 m drop in ELA was based on applying the dry adiabatic lapse rate in a situation that does not fit the physical assumptions behind that lapse rate. Osmaston's argument against using the sea-level correction has led to various responses, including not correcting for sea-level change (e.g., Stansell et al., 2007; Roy and Lachniet, 2010; Xu et al., 2010) or calculating implied temperature changes with and without the sea-level correction (Pigati et al., 2008).

Lapse Rates

Adiabatic lapse rates are introduced in meteorological textbooks at every level (e.g., Salby, 1996, §2.4; Wallace and Hobbs, 2006, §3.4). The dry adiabatic lapse rate, Γ_d , is calculated from $\Gamma_d = g/c_p$, where g is the acceleration of gravity and c_p is the specific heat at constant pressure for air. While variations in both g and c_p can be accounted for, they are small for cold conditions in the troposphere. We thus typically see Γ_d given as a constant 0.0098°C m⁻¹, rounded to 10°C/km.

More important than the value of Γ_d are the circumstances under which it applies. The dry adiabatic lapse rate describes the temperature change of an unsaturated parcel of air undergoing compression or expansion under circumstances where (diabatic) heating or cooling is negligible. In the atmosphere, such heating or cooling results from radiative processes as well as conductive heat transfers near the surface (the boundary condition). Adiabatic lapse rates are typically assumed for vertical motions in the atmosphere from the order of the synoptic scale, 10^{-1} m s⁻¹, to thunderstorm updrafts, 10 m s^{-1} . Thus, the time required to change a parcel temperature by 1°C under dry adiabatic compression or expansion varies from 1000 s (17 min) to 10 s under these rates of vertical motion. Diabatic heating rates associated with the diurnal cycle require on the order of 10⁴ s to produce a 1°C temperature change above the near-surface layer, so the dry adiabatic lapse rate makes a useful first approximation to estimating temperature changes in vertically moving air at a subdiurnal time scale.

In the paleoclimatic case, air follows sea level downward approximately 120 m over a period of millennia while being in contact with its direct heat source: the ocean surface. Osmaston (2006, p. 248) asserted that "the slight compression of the air due to its small downwards transfer is negligible from a volumetric balance aspect but is effective thermodynamically" so the air gets warmer without any heat being transferred to it. However, because of the time scale involved, significant heat transfer from the ocean to the air will take place, so that the temperature of the air remains in equilibrium with that of the ocean (for the moment, held constant). The thermodynamic situation more closely resembles an isothermal segment of a typical Carnot cycle rather than an adiabatic segment. Osmaston's approach would require that the slightly (1.2°C) warmed atmosphere is effective in warming the surface ocean, despite the small thermal mass of the atmosphere compared with that of the mixed layer of the ocean.

The effect of adiabatic compression also should not be directly calculated from a lapse rate under the circumstances being modeled. The fundamental physical cause of adiabatic warming is compression, not descent. The commonly used lapse rate ($\Gamma_d = g/c_p$) only applies if the compression is generated by descent through a hydrostatic atmosphere. The paleoclimatic situation is not one in which a parcel descends through an atmosphere. Rather it is one in which the majority of the atmosphere gets a little closer to the center of the earth as sea level falls. The associated pressure change is smaller than experienced by a parcel descending through a hydrostatic atmosphere. For example, in the scenario described in the next section, the temperature change for a 120-m sea-level fall would be slightly less than 1°C rather than the 1.2°C used by Osmaston (2006).

Displacement of air by ice

Osmaston (2006) presented his ideas using a model in which the atmosphere is a big, flat pan of air. The essential elements are two conservation of mass calculations. For water, the sea-level drop must equal the (assumed) water-equivalent volume of land ice (Fig. 1). The height of the land ice, relative to initial sea level, depends on the land fraction covered by ice and on the isostatic depression. For air, the volume of the troposphere remains constant but the level of the tropopause increases slightly because the space occupied by the ice is greater than that occupied by the water used to make the ice. The isostatic effect is neutral because the mass of mantle displaced under the ice sheet equals the mass gained under the surrounding land and ocean.

In our simple model, the initial (present) condition is that all surfaces are at the same assumed height of zero, and the LGM condition has unglacierized land remaining at zero, with sea level below zero and a glacier surface height above zero. The isostatic adjustment is assumed to occur uniformly across the unglacierized land and the ocean basins, resulting in a slight rise of the zero datum with respect to the center of the earth.

Pressure varies with height according to an altimeter equation calculated from hydrostatic balance in an ideal fluid with a constant environmental lapse rate (Appendix A, Eq. (5)). Combining this altimeter equation with global mass conservation for air provides a surface pressure for each of the three different surface heights. Following the logic in the previous section, the sea-surface temperature is fixed by the presence of the ocean, the temperature of which is held constant for purposes of this illustrative calculation, and all other temperatures must adjust. The main effect assumed by Porter (2001) and questioned by Osmaston (2006) is that, at any given level in the atmosphere, the temperature decreases 0.7°C purely due to the change in geometry. Looking at the results in more detail (Table 1), the redistribution of mass produces a 2-hPa drop in land surface pressure corresponding to a 39-m drop in the center-of-mass height of the atmosphere. Because glaciers and ice sheets now occupy some of the low (hence formerly warmer) space, the mean temperature of the troposphere has decreased 0.5°C. Finally, because all glacierized and unglacierized lands are now higher in elevation than the ocean, the average temperature of the surface (land, ice, and ocean combined) has dropped 0.6°C, while dropping 0.7°C over the land only.



Figure 1. Schematic and notation for a three-surface Earth consisting of land, sea, and glacier. Total area is 1, and g and I represent glacierized and nonglacierized land-surface fractions, respectively. For present-day conditions, *S*, *G*, and *h* are zero; they become positive in LGM conditions.

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Table 1

Comparison of present conditions, with a planetary-wide uniform surface height, and LGM conditions with oceans, continents, and glacier surface at three different levels. Numbers in boldface are specified for this scenario; all others are calculated. Additional specified constants include an environmental lapse rate of 6°C/km, an ice/water density ratio of 0.9, and a bedrock/ice density ratio of 3.6.

	Present	Last Glacial Maximum				LGM—
	Global	Land	Ocean	Ice sheet	Global	Present
Fraction of planet surfaces	1	0.24	0.7	0.06	1	
Surfaces height, m	0	0	-120	1123		
Surfaces pressure, hPa	986	984	998	859	986	0
Surfaces temperature, °C	15	14.3	15.0	7.5	14.4	-0.6
Ambient temperature at 4000 m, °C	-9				-9.7	-0.7
Pressure at 4000 m, hPa	601				599	-2
Troposphere thickness, m	11,518				11,491	-27
Mean tropospheric temperature, °C	-27.8				-28.3	-0.5
Tropopause temperature, °C	-54.1				-54.6	-0.5
Height of atmospheric center of mass, m	4573	4559	4432	5812	4534	-39

The minor changes in pressure have no direct paleoclimatic effect. An interesting corollary to this discussion is that climate change ultimately results from changes in temperature of the mixed layer of the oceans. The causes of changes in ocean temperature are, of course, still a matter of intense discussion.

However, the fact that air has risen farther to get to the snowline of a glacier may have a noticeable effect on accumulation, as we discuss next, and may give some support to Osmaston's (2006) suggestion that the change in the vertical mass distribution of the atmosphere under full-glacial conditions would have an effect on climate, albeit on precipitation more than on temperature.

Orographic rising from a tropical ocean

Some arguments regarding interpretations of ELA changes during the LGM arise from tropical and subtropical volcano sites (surveyed in Mark et al., 2005). For many peaks and ridges, orographic effects may be more important in determining the typical climate variation with altitude than the standard environmental lapse rate for a static atmosphere. For example, a Hawaiian volcano is subjected to a nearly constant trade-wind air flow, consisting of air that has long been in near contact with warm ocean water.

Once the initial temperature and relative humidity of a parcel of air are established, the effect of raising the parcel adiabatically to 4000 m involves standard calculations (Bolton, 1980). As an example, for purposes of illustration we assumed a surface temperature of 27°C and recalculated the surface pressure of the lowered LGM sea level accordingly. We then calculated several characteristics of the atmosphere using three different initial relative humidities: 40%, 60%, and 80% (Table 2).

Wherever the effects of orographic lifting control the climate of a mountain slope, the effects of elevation are strongly modified by humidity. Comparing LGM conditions with present ones for the low humidity case (40%), the average (effective) lapse rate between the surface and 4000 m elevation is slightly (0.2°C/km) higher than the assumed environmental lapse rate of 6°C/km and the temperature difference between LGM and present sea level is 0.8°C. However, when the humidity of the initial parcel is higher, the elevation of the lifting condensation level (LCL-the pressure in the atmosphere at which condensation first occurs in a parcel rising from the surface) is lowered sufficiently that the wet adiabatic lapse rate controls nearly all of the change in temperature of the rising air. This is because most of the vertical elevation through which the parcel rises is above the LCL, so the parcel is saturated. The direct effect on the temperature at 4000 m is very small, but doubling the humidity drops the LGM cloud base over 1200 m, putting nearly all of the mountain inside the cloud layer, and increases the moisture available for potential precipitation by a factor of 2.5. This effect is generally overlooked.

Conclusions

Estimates of the lowering of LGM temperatures implied by changes in ELA should be corrected for the change in sea level, because the change in sea level takes place over time scales of order 10^3 yr or more, during which time the atmospheric temperature remains in equilibrium with that in the ocean. In contrast, the adiabatic processes invoked by Osmaston (2006) normally act over

Table 2

Conditions of a parcel of air rising from a warm ocean surface with various initial relative humidity assumptions, all starting from a surface temperature of 27°C (300 K). Boldface numbers are specified; all others are calculated. "Potential Rainout" is just the drop in mixing ratio between the LCL and 4000 m. It cannot be directly interpreted as a precipitation amount but should be reasonably proportional to precipitation.

	Present (Sea	level 0 m)		LGM (Sea level—120 m)		
Relative humidity, %	40	60	80	40	60	80
Surface elevation, m	0	0	0	-120	-120	-120
Pressure, hPa	986	986	986	998	998	998
Lifting condensation level (LCL), hPa	804	878	936	813	888	948
LCL Temperature, °C	-1.3	5.6	10.8	-1.3	5.6	10.8
LCL elevation, m	1665	958	429	1545	838	309
Temperature of parcel rising to 4000 m, °C	-14.6	-11.5	-9.0	15.5	-12.3	-9.8
Average lapse rate of rising parcel, °C/km	7.4	6.6	6.0	7.6	6.8	6.2
Mixing ratio at surface, g/kg	4.3	6.5	8.7	4.3	6.4	8.6
Mixing ratio at 4000 m, g/kg	2.0	2.6	3.2	1.9	2.5	3.0
Potential rainout, g/kg	2.3	3.9	5.5	2.4	4.0	5.6
Effect of sea-level lowering on conditions at 4000 m	(Present minus LGM)				
Temperature change, °C				-0.84	-0.80	-0.76
Rainout change, g/kg				0.079	0.078	0.072

time scales of $<10^3$ s. However, his consideration of the effects of lowered sea level on atmospheric geometry and adiabatic lapse rates leads to two interesting results. First, the tropospheric average temperature is decreased slightly (0.5°C) by the change in geometry, because the ice sheets lift a significant fraction of the atmosphere farther away from the ocean surface (which is the controlling basal boundary condition). Second, for isolated mountain chains or tropical volcanic islands, advected orographic flows are more important than the static atmospheric layering. In this situation, the initial humidity of an advected parcel can have a dramatic effect on the amount and placement of precipitation on the mountain. This could, in turn, affect the snowline, thus creating a further source of uncertainty. Thus, in such situations, care should be exercised in comparing the LGM lowering of snowline with that in continental areas.

Acknowledgments

This note was inspired by a discussion in a class taught by Brenda Hall and benefited from discussions with Michael A. O'Neal. Henry Osmaston passed away shortly after his 2006 paper was published. Those who wrote in his honor uniformly referenced his long and distinguished career in a variety of fields, and we sincerely regret being unable to discuss these issues with him directly.

Appendix A

Geometric effects at the LGM

As described above, consider the planet as a pan of air with walls high enough to. The bottom of the pan has a flat "continent" of area, and the rest of the pan is filled with water to the height of the continent. For glacial conditions, assume that some fraction of the continents is covered with ice to a thickness h (Fig. 1). Define γ , λ , and χ as area fractions of ice, land, and continent, respectively, such that if A is the total planetary area, A_G is the glacierized land area, and A_L is unglacierized land area, then

$$\gamma \equiv \frac{A_G}{A}; \qquad \lambda \equiv \frac{A_L}{A}; \qquad \chi \equiv \gamma + \lambda \tag{1}$$

and the area-fraction of the ocean will be $(1 - \chi)$.

Water displaced to ice causes a sea-level depression *S* that can be calculated by setting the area-times-height of the ice sheet equal to area-times-depression of the sea level, modified for a ratio of ice density to water density, $r_w = \rho_{ice}/\rho_{water}$, yielding

$$S = \frac{h\gamma r_w}{1-\chi} \tag{2}$$

Full-glacial conditions also take into account isostatic adjustment under the ice sheet, so that the height of the ice-sheet surface *G* is less than *h* by an amount relative to the ratio $r_b = \rho_{ice}/\rho_{bedrock}$ leading to

$$G = h(1 - r_h) \tag{3}$$

The atmospheric mass overlying an area is directly proportional to surface pressure multiplied by area. Conservation of atmospheric mass thus requires

$$p_0 = p_G \gamma + p_L \lambda + p_S (1 - \chi) \tag{4}$$

where p_0 is the average surface pressure before glaciation, and the subscripts *G*, *L*, and *S* will be used to indicate glacierized land, unglacierized land, and sea-surface conditions of the LGM for pressure

and for various other quantities to follow. Finding these pressures requires an altimeter equation relating pressure to height. For an atmosphere with a constant lapse rate, $T(z) = T_0 - \Gamma z$ where *z* is height above a starting point, T_0 is temperature at that starting point, and Γ is the environmental lapse rate, the altimeter equation (in reverse) is

$$p(z) = p_0 \left(1 - \frac{\Gamma z}{T_0} \right)^{(1/\xi)}$$
(5)

for which temperature, as in all the calculations in this Appendix, must be expressed in the absolute Kelvin scale. The exponent term ξ is defined as $R\Gamma/g$, where *R* is the gas constant for dry air, 287 J kg⁻¹ K⁻¹. Applying this to the LGM case using a constant specified sea-surface temperature T_S as a starting point gives

$$\frac{p_G}{p_S} = \left(1 - \frac{\Gamma(G+S)}{T_S}\right)^{1/\xi}$$

$$\frac{p_L}{p_S} = \left(1 - \frac{\Gamma S}{T_S}\right)^{1/\xi}$$

$$p_S = p_0 \left[(1 - \chi) + \frac{\lambda p_L}{p_S} + \frac{\gamma p_G}{p_S}\right]^{-1}$$
(6)

Conditions of the static atmosphere

Conditions of the static atmosphere (i.e., not following a rising parcel) can only reasonably be calculated through the troposphere, which is the layer for which the constant lapse rate assumption is reasonable. We assume a constant tropopause pressure p_{τ} and assume that the stratosphere forms a mass blanket of no great consequence to these calculations.

The average temperature of a layer $\langle T \rangle$ is defined as the vertical integral of temperature weight by logarithm of pressure (e.g., Salby, 1996, §6.3). For a troposphere extending from a surface pressure p_0 to a top pressure p_{τ} the layer temperature becomes

$$\langle T \rangle = \frac{T_0}{\xi \ln(p_0 / p_\tau)} \left[1 - \left(\frac{p_\tau}{p_0}\right)^{\xi} \right]$$
(7)

With an atmosphere that has multiple surface levels (sea level *S* and an ice sheet at *G*), the atmospheric height indexes must be calculated for each segment and then combined as a weighting by mass, not by surface area. Define mass weight factors ϕ for the area over the glacier and ψ for the area over unglacierized land.

$$\phi = \frac{\gamma p_G}{p_0}; \quad \psi = \frac{\lambda p_L}{p_0} \tag{8}$$

in which case

$$\langle T \rangle_{\text{LGM}} = \phi \langle T \rangle_{G} + \psi \langle T \rangle_{L} + (1 - \phi - \psi) \langle T \rangle_{S}$$
(9)

for which $\langle T \rangle_G$, $\langle T \rangle_L$, and $\langle T \rangle_S$ are calculated by replacing T_0 and p_0 with appropriate surface values in (7).

Another potentially useful index is the center of mass of the troposphere, calculated as the density-weighted vertical integral of height

$$CM = \frac{\int_{z_0}^{\tau} \rho(z) z \, dz}{\int_{z_0}^{\tau} \rho(z) dz} = \frac{(T_o - \Gamma \tau)^{1/\xi} (T_o \xi + \Gamma \tau) - (T_o - \Gamma z_0)^{1/\xi} (T_o \xi + \Gamma z_0)}{[\Gamma(1 + \xi)] \left[(T_o - \Gamma \tau)^{1/\xi} - (T_o - \Gamma z_0)^{1/\xi} \right]}$$
(10)

and surface pressures and then weighted as in Eq. (9).

Temperature in a rising parcel

For a parcel of air rising by free convection or being orographically forced over a mountain by prevailing winds, the adiabatic assumption may be applied to a high degree of accuracy. The dry adiabatic lapse rate is easily applied as a constant lapse rate with height or via Poisson Equations as a function of pressure. However, it only applies while air remains unsaturated with respect to water, below the LCL. For these calculations, empirical equations were taking from Bolton (1980) to calculate the LCL (Bolton's Eq. 21) and then to raise the parcel following a wet-adiabatic process by maintaining a constant equivalent potential temperature (Bolton's Eq. 43).

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