The Bulletin of Symbolic Logic
Volume 23, Number 2, June 2017

2016 EUROPEAN SUMMER MEETING OF THE ASSOCIATION FOR SYMBOLIC LOGIC

LOGIC COLLOQUIUM '16

Leeds, UK

July 31–August 6, 2016

Logic Colloquium '16, the annual European Summer Meeting of the Association of Symbolic Logic, was hosted by the University of Leeds, the third time Leeds has hosted this event. The meeting took place from July 31 to August 6, 2016, at the campus of the university. The plenary lectures were held in the Business School Western Lecture Theatre and the other lectures in various rooms in the Maurice Keyword Building.

Major funding for the conference was provided by the Association for Symbolic Logic (ASL), the US National Science Foundation, Cambridge University Press, the School of Mathematics of the University of Leeds, and the British Logic Colloquium.

The success of the meeting was due largely to the excellent work of the Local Organizing Committee under the leadership of its Chair, Nicola Gambino (University of Leeds). The other members were Olaf Beyersdorff, Andrew Brooke-Taylor, Barry Cooper (deceased), Immi Halupczok, H. Dugald Macpherson, Vincenzo Mantova, Michael Rathjen, John Truss, and Stan Wainer.

The Program Committee consisted of Manuel Bodirsky (Technical University Dresden), Sam Buss (University of California, San Diego), Nicola Gambino (University of Leeds), Rosalie Iemhoff (Utrecht University, chair), Hannes Leitgeb (Munich Center for Mathematical Philosophy), Steffen Lempp (University of Wisconsin), Maryanthe Malliaris (University of Chicago), Ralf Schindler (University of Münster), and Yde Venema (University of Amsterdam).

The main topics of the conference were: Algebraic Logic, Computability Theory, Model Theory, Proof Theory, Philosophical Logic, and Set Theory. The program included two tutorial courses, twelve invited lectures, among which were the Twenty-seventh Annual Gödel Lecture and the British Logic Colloquium Lecture, twenty-four invited lectures in six special sessions, and 125 contributed talks. There were 238 participants, and ASL travel grants were awarded to thirty-five students and recent Ph.D's.

The following tutorial courses were given:

Thierry Coquand (University of Gothenburg), *Univalent type theory*.

Uri Andrews (University of Wisconsin), *Computable model theory*.

The following invited plenary lectures were presented:

Stevo Todorcevic (University of Toronto and CNRS Paris), (the Gödel Lecture), *Basis problems in set theory*.

Laurent Bienvenu (CNRS et Université Paris Diderot), (the British Logic Colloquium Lecture), *Randomized algorithms in computability theory*.

Richard Garner (Macquarie University), *Non-standard arities*.

Rob Goldblatt (Victoria University of Wellington), *Spatial logic of tangled closure and derivative operators*.

Itay Kaplan (The Hebrew University of Jerusalem), *Developments in unstable theories focusing on NIP and NTP_2*.

1079-8986/17/2302-0006
DOI:10.1017/bsl.2017.16

Toniann Pitassi (University of Toronto), *Connections between proof complexity, circuit complexity and polynomial identity testing*.

Farmer Schlutzenberg (University of Münster), *Ordinal definability in extender models*.

Dima Sinapova (University of Illinois at Chicago), *Compactness-type combinatorial principles*.

Henry Towsner (University of Pennsylvania), *A concrete view of ultraproducts*.

Benno van den Berg (University of Amsterdam), *Homotopy type theory via path categories*.

Timothy Williamson (University of Oxford), *Alternative logics and abductive methodology*.

Boris Zilber (University of Oxford), *On the semantics of algebraic quantum mechanics and the role of model theory*.

More information about the meeting can be found at the conference website, `http://www.lc2016.leeds.ac.uk/`.

Abstracts of invited and contributed talks given in person or by title by members of the Association follow.

For the Program Committee
ROSALIE IEMHOFF

Abstracts of Invited Tutorials

▸ URI ANDREWS, *Computable model theory*.
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In this tutorial series, I will talk about many instances where ideas from computability theory and ideas from model theory intertwine. The focus will be mostly on questions related to computation of models or theories, but I will also try to highlight how computability can help refine our understanding of model theoretic ideas.

▸ THIERRY COQUAND, *Univalent type theory*.
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This tutorial will be an introduction to dependent type theory, and to the univalence axiom (V. Voevodsky, 2010).

Simple type theory, as formulated by A. Church (1940), constitutes an elegant alternative of set theory for representing formally mathematics. The stratification of mathematical objects in a type of propositions, a type of individuals and a type of functions between two types is indeed quite natural. The axiom of extensionality (the first axiom of set theory) comes in two forms: the fact that two equivalent propositions are equal, and the fact that two pointwise equal functions are equal. Simple type theory as a formal system has however some unnatural limitations, in that we cannot express the notion of an *arbitrary* structure, for instance the type of an arbitrary group. Dependent type theory solves this issue by introducing in type theory the notion of *universe*. What was missing until the work of V. Voevodsky was a formulation of the extensionality axiom for universe. This is the *univalence axiom*, which generalizes propositional extensionality. When introducing universes, we also can use the principle of propositions-as-types and proofs-as-programs, so that the logical operations themselves, and their proofs, can be represented as type theoretic operations.

The lectures will roughly proceed as follows. The first lecture will be an introduction to simple type theory and dependent type theory, where we shall try to point out the connections and differences with set theory and end with a formulation of the univalence axiom. The second lecture will explore some consequences of this axiom, such as a proof of a strong form of the axiom of "unique choice", from which we can derive results that would require the full

https://doi.org/10.1017/bsl.2017.16 Published online by Cambridge University Press

axiom of choice in set theory. We will end with a presentation of a model of the univalence axiom.

Abstract of the 27th Annual Gödel Lecture

▸ STEVO TODORCEVIC, *Basis problems in set theory*.
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Given a class $\mathcal{C}$ of mathematical structures we are interested in finding a list $\mathcal{C}_0 \subseteq \mathcal{C}$ of *critical structures* in $\mathcal{C}$, a list with the property that every structure in $\mathcal{C}$ is in some way related to or built from some elements of $\mathcal{C}_0$. Such results are of course useful if $\mathcal{C}_0$ is small (typically finite) and when elements of $\mathcal{C}$ have strong relationship to the elements of $\mathcal{C}_0$. We give an overview of the results of this area concentrating on the more recent ones. We also list some open problems.

Abstracts of Invited Plenary talks

▸ LAURENT BIENVENU, *Randomized algorithms in computability theory*.
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Are randomized algorithms more powerful than deterministic ones? This is perhaps one of the most important general questions in computational complexity, the problem P ?= BPP perhaps being the best known instance of it. In computability theory, this question is typically less considered because of a theorem of De Leeuw et al., which states that if a given sequence/language can be probabilistically computed, it can in fact be deterministically computed. However the problem remains interesting if one considers classes of objects: there are some classes $\mathcal{C}$ containing no computable element but for which there is a probabilistic algorithm which produces an element of $\mathcal{C}$ with positive probability. We will discuss some some positive and negative examples and will explain how to get a quantitative analysis of such classes using Kolmogorov complexity, with numerous applications to algorithmic randomness. Finally, we will see how the theorem of De Leeuw et al. can be turned around to get the existence of computable objects from probabilistic algorithms.

[1] K. Allen, L. Bienvenu, and T. Slaman, *On zeros of Martin-Löf random Brownian motion*. ***Journal of Logic and Analysis***, vol. 6 (2014).

[2] L. Bienvenu and L. Patey, *Diagonally non-computable functions and fireworks*. Available at http://arxiv.org/abs/1411.6846.

[3] L. Bienvenu and C. Porter, *Deep Π^0_1 classes*, this Bulletin, to appear. Available at http://arxiv.org/abs/1403.0450.

[4] S. Epstein and L. A. Levin, *Sets have simple members*. Available at http://arxiv.org/abs/1107.1458.

[5] S. M. Kautz, ***Degrees of random sets***, Ph.D. dissertation, Cornell University, 1991.

[6] A. Rumyantsev and A. Shen, *Probabilistic constructions of computable objects and a computable version of Lovász local lemma*. ***Fundamenta Informaticae***, vol. 132 (2014), no. 1, pp. 1–14. Available at http://arxiv.org/abs/1305.1535.

▸ RICHARD GARNER, *Non-standard arities*.
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Finitary equational theories admit a presentation-independent realisation through the notion of *finitary monad* on the category of sets. It is well-understood that the monad-theoretic presentation of algebraic theories is apt for generalisation to other settings (= other base categories); it is less well-appreciated that, even without leaving the world of sets and functions, the monadic approach allows one to detect richer and more complex structure associated to a theory than just that of its derived n-ary operations and the equations they satisfy. This talk will investigate some of the *nonstandard arities* detectable in this way, and examine how these impinge on other parts of logic.

[1] A. Blass, *Exact functors and measurable cardinals*. ***Pacific Journal of Mathematics***, vol. 63 (1976), no. 2, pp. 335–346.

[2] M. Fiore, G. Plotkin, and D. Turi, *Abstract syntax and variable binding*, ***Logic in Computer Science, vol. 14***, IEEE Computer Society Press, Trento, 1999, pp. 193–202.

[3] M. Hyland, M. Nagayama, J. Power, and G. Rosolini, *A category theoretic formulation for Engeler-style models of the untyped λ-calculus*. ***Electronic Notes in Theoretical Computer Science***, vol. 161 (2006), pp. 43–57.

[4] A. Joyal, *Foncteurs analytiques et espèces de structures*. ***Springer Lecture Notes in Mathematics***, vol. 1234 (1986), pp. 126–159.

[5] M. Makkai, *The topos of types*. ***Springer Lecture Notes in Mathematics***, vol. 859 (1981), pp. 157–201.

▸ ROBERT GOLDBLATT, *Spatial logic of tangled closure and derivative operators*.
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The tangled closure of a collection of sets is the largest set in which each member of the collection is dense. This operation models a generalised modality that was introduced by Dawar and Otto [1], who showed that its addition to basic propositional modal logic produces a language that is expressively equivalent over certain classes of finite transitive structures to the bisimulation-invariant fragments of both first-order logic and monadic second-order logic. (By contrast, over arbitrary structures the bisimulation-invariant fragment of monadic second-order logic is equivalent to the more powerful modal mu-calculus.) The name 'tangle' and its spatial meaning are due to Fernández-Duque [2].

This talk surveys joint work [3, 4] with Ian Hodkinson on interpretations of the tangle modality, including a variant in which topological closure is replaced by the derivative (= set of limit points) operation. We prove the finite model property for, and provide complete axiomatisations of, the logics of a range of topological spaces in a number of languages, some with the universal modality. This includes results for all Euclidean spaces $\mathbb{R}^n$, and all zero-dimensional dense-in-themselves metric spaces. The methods used involve new kinds of 'dissections' of metric spaces in the sense of McKinsey and Tarski [5].

[1] A. Dawar and M. Otto, *Modal characterisation theorems over special classes of frames*. ***Annals of Pure and Applied Logic***, vol. 161 (2009), pp. 1–42.

[2] D. Fernández-Duque, *Tangled modal logic for spatial reasoning*, ***Proceedings of the Twenty-Second International Joint Conference on Artificial Intelligence (IJCAI)*** (T. Walsh, editor), AAAI Press/IJCAI, 2011, pp. 857–862.

[3] R. Goldblatt and I. Hodkinson, *Spatial logic of modal mu-calculus and tangled closure operators*, 2014, `arXiv.org/abs/1603.01766`.

[4] ———, *The tangled derivative logic of the real line and zero-dimensional spaces*, ***Advances in Modal Logic*** (L. Beklemishev, S. Demri, and A. Máté, editors), vol. 11, College Publications, Budapest, 2016.

[5] J. C. C. McKinsey and A. Tarski, *The algebra of topology*. ***Annals of Mathematics***, vol. 45 (1944), no. 1, pp. 141–191.

▸ ITAY KAPLAN, *Developments in unstable theories focusing on NIP and* NTP_2.
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For many years after Morley's celebrated categoricity theorem [2], and Shelah's discovery [4] of stable theories, abstract model theory *was* stability theory: the study of stable theories and related subclasses (totally transcendental, strongly minimal, etc.).

However, since the (re)discovery of simple theories [1, 6], and of o-minimal theories [3], there has been much research going into these classes. In recent years there was much focus in NIP and NTP_2 theories. NIP theories (introduced by Shelah in [5]) generalize both o-minimal and stable theories and NTP_2 theories (defined in [7]) generalize both NIP and simple theories.

In this talk I will review some of the progress done in recent years in the study of NIP and NTP_2. The talk will be aimed at a wide audience.

[1] B. Kim, ***Simple first order theories***, Ph.D. thesis, University of Notre Dame, 1996.

[2] M. Morley, *Categoricity in power*. ***Transaction of the American Mathematical Society***, vol. 114 (1965), pp. 514–538.

[3] A. Pillay and C. Steinhorn, *Definable sets in ordered structures. I*. ***Transactions of the American Mathematical Society***, vol. 295 (1986), no. 2, pp. 565–592.

[4] S. Shelah, *Stable theories*. ***Israel Journal of Mathematics***, vol. 7 (1969), pp. 187–202.

[5] ———, *Stability, the f.c.p., and superstability; model theoretic properties of formulas in first order theory*. ***Annals of Mathematical Logic***, vol. 3 (1971), no. 3, pp. 271–362.

[6] ———, *Simple unstable theories*. ***Annals of Mathematical Logic***, vol. 19 (1980), no. 3, pp. 177–203.

[7] ———, ***Classification Theory and the Number of Nonisomorphic Models***, Studies in Logic and the Foundations of Mathematics, vol. 92, second ed., North-Holland Publishing Co., Amsterdam, 1990.

► TONIANN PITASSI, *Connections between proof complexity, circuit complexity and polynomial identity testing*.
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In this talk we discuss new variants on algebraic proof systems, and establish tight connections to central questions in (algebraic) circuit complexity. In particular, we show that any super-polynomial lower bound on any Boolean tautology in our proof system implies that the permanent does not have polynomial-size algebraic circuits (VNP is not equal to VP). As a corollary to the proof, we also show that super-polynomial lower bounds on the number of lines in Polynomial Calculus proofs imply the Permanent versus Determinant Conjecture. Prior to our work, there was no proof system for which lower bounds on an arbitrary tautology implied any computational lower bound. Our proof system helps clarify the relationships between previous algebraic proof systems, and begins to shed light on why proof complexity lower bounds for various proof systems have been so much harder than lower bounds on the corresponding circuit classes. In doing so, we highlight the importance of the polynomial identity testing (PIT) problem for understanding proof complexity.

This is joint work with Joshua A. Grochow.

► FARMER SCHLUTZENBERG, *Ordinal definability in extender models*.
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Gödel's universe L of constructible sets admits a detailed analysis, and ZFC decides much of its first-order theory. This makes the study of L tractable and interesting.

However, many natural, desirable set theoretic principles—in particular, moderate strength large cardinal and determinacy principles—must fail in L. Extender models $L[\mathbb{E}]$ are generalizations of L, which still admit a detailed analysis, but can also satisfy large cardinal principles. The predicate $\mathbb{E}$ is a sequence of *extenders*, which witness large cardinals. The more canonical $L[\mathbb{E}]$ are called *iterable*; iterability is witnessed in V by an *iteration strategy*. It is known that if $L[\mathbb{E}]$ has Woodin cardinals then it cannot know too much of this iteration strategy.

One can ask whether $\mathbb{E}$ can be defined over $L[\mathbb{E}]$, possibly from some parameter. Related to this, one can ask about the structure of $\mathrm{HOD}^{L[\mathbb{E}]}$ (that is, the universe HOD of hereditarily ordinal definable sets, as computed in $L[\mathbb{E}]$). I will survey what is known to the author regarding these questions.

In simple cases, $L[\mathbb{E}]$ satisfies "$V = \mathrm{HOD}$". This holds in L, and, for example, in the minimal proper class $L[\mathbb{E}]$ with a measurable cardinal. But in the presence of Woodin cardinals, the question becomes harder to understand. I will cover the following recent results. Let $L[\mathbb{E}]$ be iterable and satisfy ZFC. Then (i) $\mathbb{E}$ is definable from the parameter $\mathbb{E} \upharpoonright \omega_1^{L[\mathbb{E}]}$ in $L[\mathbb{E}]$; and (ii) in many circumstances, $L[\mathbb{E}]$ is a small forcing extension of $\mathrm{HOD}^{L[\mathbb{E}]}$ and $\mathrm{HOD}^{L[\mathbb{E}]}$ admits a detailed analysis above $\omega_2^{L[\mathbb{E}]}$. However, the full structure of $\mathrm{HOD}^{L[\mathbb{E}]}$ is an open question, even in rather basic cases.

► DIMA SINAPOVA, *Compactness-type combinatorial principles*.
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Compactness-type combinatorial principles like the tree property and failure of square are remnants of large cardinals but can hold at successor cardinals. They test how much can be obtained from forcing and large cardinals versus how L-like the universe is. It is especially difficult to force these properties at small cardinals. We will introduce some background and then discuss some new results on obtaining the tree property and related combinatorial principles at smaller cardinals.

► HENRY TOWSNER, *A concrete view of ultraproducts*.
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Ultraproducts are one of the tools from logic most widely used in mathematics, playing a role in functional analysis, differential algebra, algebraic geometry, and recently combinatorics. We describe how to reinterpret proofs which use ultraproducts to reveal the underlying constructive calculations. In particular, this makes it possible to replace proofs which use ultraproducts with constructive, explicit proofs which avoid the use of the axiom of choice.

► BENNO VAN DEN BERG, *Homotopy type theory via path categories*.
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Homotopy type theory is based on the fact that similar categorical structures appear in both type theory and homotopy theory. Paths and higher homotopies give every topological space the structure of an ∞-groupoid, and so does the identity type in type theory. Quillen model categories are the most common abstract framework for homotopy theory, but also give rise to models of the identity type (modulo coherence problems relating to substitution).

This talk starts from another link: in homotopy theory a well-known weakening of the notion of a Quillen model structure is that of a category of fibrant objects, due to Kenneth Brown. A slight variation of this notion, which I will call a path category, corresponds to a natural weaking of the rule for the identity type, where we ask for the computation rule for J to hold only in a propositional form. Indeed, this weakening has been considered by Coquand and collaborators in their attempt to build constructive models of homotopy type theory. Ignoring the coherence problems again, we can say that path categories provide a sound and completeness semantics for these weak identity types.

In constructive mathematics one often avoids taking quotients; instead, one considers sets together with an arbitrary equivalence relation. In type theory such an object is called a setoid. In this talk I will show that the category of setoids can be seen as a two-step construction, where one first builds a new path category out of an old one and then takes the homotopy category. It turns out that the intermediate path category has interesting properties: for example, it satisfies functional extensionality even when the original one does not.

https://doi.org/10.1017/bsl.2017.16 Published online by Cambridge University Press

If time permits, I also plan to talk about algebraic set theory and models of Aczel's constructive set theory **CZF** from weak universes, also in the context of path categories.

This is joint work with Ieke Moerdijk and based on the preprints [1, 2].

[1] B. van den Berg, *Path categories and propositional identity types*, 2016, `arXiv:1605.02534`.

[2] B. van den Berg and I. Moerdijk, *Exact completion of path categories and algebraic set theory*, 2016, `arXiv:1603.02456`.

► TIMOTHY WILLIAMSON, *Alternative logics and abductive methodology*.
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Sometimes we have to make genuine choices between classical logic and various non-classical alternatives, e.g., as to which we rely on for establishing meta-logical results or for reasoning about extra-logical matters. Discussion is needed, of a not purely formal kind, of the methodology we should use to make such a choice between logics. It can be understood as a special case of theory choice in science, governed by similar broadly abductive criteria, such as elegance, simplicity, strength, explanatory power, and fit with evidence. There is no need to appeal to a pre-given relation of logical consequence, since a candidate logic's consistency with evidence can be understood as the closure of the set of evidence under that logic's consequence relation. There is also no requirement for the logic to be weak enough to be 'neutral' in some sense, because there is no useful standard of neutrality. The idea that sub-classical logics can recapture the strength of classical logic 'when they need it', e.g., by adding instances of excluded middle as auxiliary assumptions, will be criticized on the grounds that it degrades a wide range of scientific explanations by introducing extraneous ad hoc assumptions. The most promising rivals to classical logic are those preserving simple and strong principles that classical logic renders inconsistent, e.g., disquotational principles in the case of semantic paradoxes and tolerance principles in the case of sorites paradoxes. However, even these examples are arguably bad bargains in which local benefits are gained at the expense of global costs.

► BORIS ZILBER, *On the semantics of algebraic quantum mechanics and the role of model theory*.
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I will talk about the methods and results of my recent paper 'The semantics of the canonical commutation relation' (`arxiv.org/abs/1604.07745`). The particular emphasis in this talk will be on how the model-theoretic approach is leading to a novel interpretation of quantum mechanics.

Abstracts of invited talks in the Special Session on Computability Theory

► CHRIS CONIDIS, *New directions in reverse algebra*.
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We will survey some old results in the reverse mathematics of Artinian and Noetherian rings, showing how computability-theoretic insights can give rise to new algebraic techniques. Finally, we will conclude with some new results concerning the reverse mathematics of the (very general) class of Noetherian rings.

[1] C. J. Conidis, *Chain conditions in computable rings*. ***Transactions of the American Mathematical Society***, vol. 362 (2010), no. 12, pp. 6523–6550.

[2] ———, *The computability, definability, and proof theory of Artinian rings*, to appear.

[3] ———, *The meta-metamathematics of Neotherian rings*, in preparation.

https://doi.org/10.1017/bsl.2017.16 Published online by Cambridge University Press

▸ RUPERT HÖLZL AND CHRISTOPHER C. PORTER, *Randomness for computable measures and initial segment complexity*.
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According to the Levin–Schnorr theorem, a sequence $X \in 2^\omega$ is Martin-Löf random with respect to the Lebesgue measure if and only if $\mathrm{K}(X \restriction n) \geq n - O(1)$ for every n, where K denotes prefix-free Kolmogorov complexity. Roughly, this means that the Martin-Löf random sequences are precisely those sequences with high initial segment complexity. It is well-known that the Levin–Schnorr theorem can be extended to proper sequences, that is, sequences that are random with respect to some computable measure on 2^ω. However, in this more general setting the initial segment complexity of sequences that are random with respect to different computable measures can vary widely.

We study the various growth rates of proper sequences. In particular, we show the initial segment complexity of a proper sequence X is bounded from below by a computable function if and only if X is random with respect to some computable, continuous measure. We also identify a global computable lower bound for the initial segment complexity of all μ-random sequences for a computable, continuous measure μ. Furthermore we show that there are proper sequences with extremely slow-growing initial segment complexity, i.e., there is a proper sequence the initial segment complexity of which is infinitely often below every computable function, and even a proper sequence the initial segment complexity of which is dominated by all computable functions. Lastly, we prove various facts about the Turing degrees of such sequences and show that they are useful in the study of certain classes of pathological measures on 2^ω, namely diminutive measures and trivial measures.

[1] R. Hölzl and C. P. Porter, *Randomness for computable measures and initial segment complexity*, arXiv e-prints, October 2015, 1510.07202.

▸ ANDRÉ NIES, *Describing finite groups by first-order sentences of polylogarithmic length*.
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I discuss recent research with Katrin Tent [2] that connects group theory, logic, and the idea of Kolmogorov complexity.

We call a class of finite structures for a finite first-order signature R-compressible, for an unbounded function R on the natural numbers, if each structure G in the class has a first-order description of length at most $O(R(|G|))$. We show that the class of finite simple groups is log-compressible, and the class of all finite groups is $\log^3$-compressible.

The results rely on the classification of finite simple groups, the existence of profinite presentations with few relators for finite groups, and group cohomology. We also indicate why the bounds are close to optimal.

A much easier result that still conveys the flavour of our research is the following: for each n there is a first-order sentence of length $O(\log n)$ expressing that a group has size n (see [1]).

[1] A. Nies (ed.), Logic Blog 2016. Available at cs.auckland.ac.nz/∼nies.
[2] A. Nies and K. Tent, *Describing finite groups by short first-order sentences*. ***Israel Journal of Mathematics***, to appear. Available at arXiv:1409.8390.

▸ YANG YUE, *A computation model on real numbers*.
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The study of computation models on real numbers has a history almost as long as the one of recursion theory. Various models have been proposed, for instance, the type-two theory

https://doi.org/10.1017/bsl.2017.16 Published online by Cambridge University Press

of effectivity (TTE) based on oracle Turing machines and the Blum–Shub–Smale model of computation (BSS).

In this talk I will identify a class of functions over real numbers, which can be characterized by three equivalent ways: by functional schemes similar to the class of partial recursive functions; by master-slave machines which are generalizations of Turing machines; and by λ-calculus with extra δ-reductions. These equivalent characterizations seem to suggest something intrinsic behind this class of functions.

The talk is based on joint work with Keng Meng Ng from Nanyang Technological University, Singapore and Nazanin Tavana from Amirkabir University of Technology, Iran; Duccio Pianigiani and Andrea Sorbi from University of Siena, Italy and Jiangjie Qiu from Renming University, China.

Abstracts of invited talks in the Special Session on Formal Theories of Truth

► THEODORA ACHOURIOTI, *Truth, intensionality and paradox*.
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We know since Tarski that the truth of a sufficiently strong first-order theory cannot be captured by means of a truth predicate which satisfies the full T-schema. A number of formal theories of type-free truth have been developed that restrict the instances of the T-schema in some interesting way so that a consistent theory of truth can be obtained. We present an untyped satisfaction predicate restricted to the geometric fragment of the language and motivated by semantic structures that embody a model of falsification on which the truth of some theory resides. This is a predicate with intensional meaning that it inherits from its semantic environment where the existence of objects and interpretation of predicates depend on what can be seen as some decision process. We show how the consistency of this predicate follows from a certain form of groundedness conditions and we explain what these mean in our context. Finally, we turn to complexity issues and show that even though syntactically severely restricted, this truth predicate does not amount to a simple notion of truth.

► ROY T. COOK, *Embracing inference*: *Three revenge-free deductive systems for truth*.
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The Embracing Revenge account, which provides a logical and semantic treatment of the Liar paradox and the revenge phenomena, has been developed in a series of papers by Roy T. Cook (2008, 2009), Nicholas Tourville (under review), and Philippe Schlenker (2010). In this paper, after surveying the semantics developed in the most recent such work, I present three deductive systems for the Embracing Revenge view. Each of these three systems reflect distinct ways to "read" the semantics, modelled roughly on many-valued (or "gappy"), dialethic (or "glutty"), and degree-theoretic approaches to semantics.

[1] R. T. Cook, *Embracing revenge: On the indefinite extensibility of language*, ***Revenge of the Liar*** (J. C. Beall, editor), Oxford University Press, 2008, pp. 31–52.

[2] ———, *What is a truth value, and how many are there*? ***Studia Logica***, vol. 92 (2009), pp. 183–201.

[3] P. Schlenker, *Super-liars*. ***Review of Symbolic Logic***, vol. 3 (2010), no. 3, pp. 374–414.

[4] P. Schlenker, N. Tourville, and R. Cook, *Embracing the technicalities: Expressive completeness and revenge*. ***Review of Symbolic Logic***, vol. 9 (2016), no. 2, pp. 325–358.

► VOLKER HALBACH AND CARLO NICOLAI, *Axiomatizing Kripke's theory of truth in classical and nonclassical logic*.
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https://doi.org/10.1017/bsl.2017.16 Published online by Cambridge University Press

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We consider axiomatizations of Kripke's [3] theory of truth, more precisely, of all fixed points of Kripke's operator based on Strong Kleene logic over the standard model over arithmetic. We also consider analogous models with gluts as considered by [4]. Feferman [1] axiomatized Kripke's theory in classical logic. Feferman's axiomatization is sound in the sense that whenever $T\ulcorner\phi\urcorner$ is provable then ϕ holds in all of Kripke's fixed-point models. Halbach and Horsten [2] provided an axiomatization that is directly sound: ϕ holds in all of Kripke's fixed-point models, if ϕ is provable in their system. Halbach and Horsten [2] showed that their system is proof-theoretically much weaker than Feferman's system and lacks many arithmetical theorems provable in Feferman's system. There are many sentences such that $T\ulcorner\phi\urcorner$ is provable in Feferman's system, while ϕ isn't provable in the Halbach–Horsten system.

We pinpoint the source of the deductive weakness of the Halbach–Horsten system in the induction rule. Both systems contain the same rule of induction. If it is removed from both systems, then $T\ulcorner\phi\urcorner$ is provable in Feferman's system iff ϕ is provable in the Halbach–Horsten system. Thus the effect of switching from classical logic to the nonclassical logic of the Halbach–Horsten system limits the usability of the mathematical principle of induction. We take this as evidence that the often advocated strategy of restricting classical logic for semantic vocabulary doesn't necessarily affect our semantic reasoning, but it can cripple our mathematical reasoning.

[1] S. Feferman, *Reflecting on incompleteness*. ***The Journal of Symbolic Logic***, vol. 56 (1991), no. 1, pp. 1–49.

[2] V. Halbach and L. Horsten, *Axiomatizing Kripke's theory of truth*. ***The Journal of Symbolic Logic***, vol. 71 (2006), no. 2, pp. 677–712.

[3] S. A. Kripke, *Outline of a theory of truth*. ***Journal of Philosophy***, vol. 72 (1975), no. 19, pp. 690–716.

[4] A. Visser, *Four-valued semantics and the liar*. ***Journal of Philosophical Logic***, vol. 13 (1984), no. 2, pp. 181–212.

▸ JAN HUBIČKA AND JAROSLAV NEŠETŘIL, *All those Ramsey classes* (*Ramsey classes with closures and forbidden homomorphisms*).

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Class $\mathcal{K}$ of finite structures is *Ramsey class* if for every choice of $\mathbf{A}, \mathbf{B} \in K$ there exists $\mathbf{C} \in \mathcal{K}$ such that for every coloring of its substructures isomorphic to $\mathbf{A}$ with 2 colors there exists an isomorphic copy of $\mathbf{B}$ in $\mathbf{C}$ where all copies of $\mathbf{A}$ are monochromatic. It is a classical result that for every purely relational language L the class of all finite ordered L-structures is Ramsey [1, 5]. We extend this theorem for languages containing both relations and functions.

We also give a new sufficient condition for subclass of a Ramsey class to be Ramsey. By verifying this condition we prove Ramsey property of many classes such as convexly ordered S-metric spaces (solving an open problem [6]), totally ordered structures (structures with linear order on both vertices and relations), and ordered single constraint Cherlin Shelah Shi classes [2].

[1] F. G. Abramson and L. A. Harrington, *Models without indiscernibles*. ***The Journal of Symbolic Logic***, vol. 43 (1978), no. 3, pp. 572–600.

[2] G. Cherlin, S. Shelah, and N. Shi, *Universal graphs with forbidden subgraphs and algebraic closure*. ***Advances in Applied Mathematics***, vol. 22 (1999), no. 4, pp. 454–491.

[3] J. Hubička and J. Nešetřil, *Bowtie-free graphs have a Ramsey lift*, `arXiv:1402.2700`, arXiv preprint.

[4] ———, *All those Ramsey classes* (*Ramsey classes with closures and forbidden homomorphisms*), in preparation.

https://doi.org/10.1017/bsl.2017.16 Published online by Cambridge University Press

[5] J. NEŠETŘIL and V. RÖDL, *Partitions of finite relational and set systems*. ***Journal Combinatorial Theory, Series A***, vol. 22 (1977), no. 3, pp. 289–312.

[6] L. NGUYEN VAN THÉ, *Structural Ramsey theory of metric spaces and topological dynamics of isometry groups*, ***Memoirs of the American Mathematical Society***, American Mathematical Society, 2010.

► LAVINIA PICOLLO, *Truth, reference and disquotation*.
Munich Center for Mathematical Philosophy, Ludwig-Maximilians-Universität München, Fakultät für Philosophie, Wissenschaftstheorie und Religionswissenschaft, Lehrstuhl für Logik und Sprachphilosophie, Geschwister-Scholl-Platz 1, 80539 München, Germany.
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I first provide intuitively appealing notions of reference, self-reference, and well-foundedness of sentences of the language of first-order Peano arithmetic extended with a truth predicate. They are intended as a tool for studying reference patterns that underlie expressions leading to semantic paradox, and thus to shed light on the debate on whether every paradox formulated in a first-order language involves self-reference or some other vicious reference pattern.

I use the new notions to formulate sensible restrictions on the acceptable instances of the T-schema, to carry out the disquotationalist project. Since the concept of reference I put forward is proof-theoretic—i.e., it turns to the provability predicate rather than the truth predicate—and, therefore, arithmetically definable, it can be used to provide recursive axiomatizations of truth. I show the resulting systems are ω-consistent and as strong as Tarski's theory of ramified truth iterated up to ϵ_0.

Abstracts of invited talks in the Special Session on Homogeneous structures: Model theory meets universal algebra

► LIBOR BARTO, *The algebraic dichotomy conjecture for infinite domain constraint satisfaction problems*.
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We prove that an ω-categorical core structure primitively positively interprets all finite structures with parameters if and only if some stabilizer of its polymorphism clone has a homomorphism to the clone of projections, and that this happens if and only if its polymorphism clone does not contain operations α, β, s satisfying the identity $\alpha s(x, y, x, z, y, z) \approx \beta s(y, x, z, x, z, y)$.

This establishes an algebraic criterion equivalent to the conjectured borderline between P and NP-complete CSPs over reducts of finitely bounded homogenous structures, and accomplishes one of the steps of a proposed strategy for reducing the infinite domain CSP dichotomy conjecture to the finite case.

Our theorem is also of independent mathematical interest, characterizing a topological property of any ω-categorical core structure (the existence of a continuous homomorphism of a stabilizer of its polymorphism clone to the projections) in purely algebraic terms (the failure of an identity as above).

This is a joint work with Michael Pinsker.

► ROSS WILLARD, *The decidable discriminator variety problem*.
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This talk is an advertisement for an old, unsolved problem in which universal algebra meets homogeneous structures. An *equational class* is any class in an algebraic signature (i.e., constants and function symbols only) which is axiomatized by universally quantified

equations. Such a class is *locally finite* if every finitely generated substructure of a member is finite. The problem in question is that of describing all locally finite equational classes with finite signature whose first-order theory is decidable.

The work of Burris, McKenzie, and Valeriote [1, 3] in the 1980s reduced this problem to two special kinds of equational classes:

1. For a given finite ring R, the class ${}_R\mathcal{M}$ of all R-modules.
2. Locally finite "discriminator varieties" in a finite signature.

What are discriminator varieties? They are equational classes $\mathcal{E}$ which resemble the class of Boolean algebras in certain ways. In particular, (i) the class $\mathcal{S}$ of simple algebras in $\mathcal{E}$ is $\forall_1$-axiomatizable, and (ii) each algebra in $\mathcal{E}$ has a Stone-like representation via a sheaf over $\mathcal{S}$.

In practice [4, 2], the question of whether a locally finite discriminator variety $\mathcal{E}$ has a decidable first-order theory hinges on how well-structured are the members of $\mathcal{S}$, and in particular on the degree to which the countable members of $\mathcal{S}$ fail to be homogeneous. In this talk I will make this precise and explain the current state of the problem.

[1] S. Burris and R. McKenzie, *Decidability and Boolean representations*. ***Memoirs of the American Mathematical Society***, vol. 32 (1981), no. 246.

[2] D. Delić, *Decidable discriminator varieties arising from dihedral varieties of groups*. ***Journal of Pure and Applied Algebra***, vol. 198 (2005), no. 1–3, pp. 75–92.

[3] R. McKenzie and M. Valeriote, ***The Structure of Decidable Locally Finite Varieties***, Progress in Mathematics, Birkhäuser, Boston, 1989.

[4] R. Willard, *Decidable discriminator varieties from unary classes*. ***Transactions of the American Mathematical Society***, vol. 336 (1993), no. 1, pp. 311–333.

► JOSHUA WISCONS, *The status of Cherlin's conjecture for primitive structures of relational complexity* 2.
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The relational complexity of a structure $\mathbf{X}$ is the least $k < \omega$ for which the orbits of $\mathrm{Aut}(\mathbf{X})$ on X^k "determine" the orbits of $\mathrm{Aut}(\mathbf{X})$ on X^n for all $n < \omega$. This invariant originated in Lachlan's classification theory for homogeneous finite—and more generally, countable stable—relational structures, but not much was known about the complexities of specific structures until the work of Cherlin, Martin, and Saracino in the 1990's. In this talk, I will present some background on relational complexity and discuss recent work on Cherlin's conjecture regarding the classification of the finite primitive structures of complexity 2.

Abstracts of invited talks in the Special Session on Model Theory and Limit Structures

► OVE AHLMAN, *Simple structures axiomatized by almost sure theories*.
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Studying simple structures, one of the nicest examples of a simple yet not stable structure is the Rado graph. The Rado graph is the unique countable graph $\mathcal{G}$ which satisfies that for any finite disjoint subsets A and B of $\mathcal{G}$ there is a vertex c such that c is adjacent to all vertices in A and no vertices in B. Some of the properties of the Rado graph which makes it especially nice include ω-categoricity (even homogeneity), SU-rank 1 and trivial algebraic closure.

For each $n \in \mathbb{N}$ let $\mathbf{K}_n$ be a set of finite structures and let μ_n be the uniform probability measure on $\mathbf{K}_n$, assigning the same probability to each structure. This measure induce a probability measure on first order sentences φ by putting $\mu_n(\varphi) = \mu(\{\mathcal{M} \in \mathbf{K}_n : \mathcal{M} \models \varphi\})$. Put T to be the theory, called the almost sure theory, of all first order sentences φ such that $\lim_{n\to\infty} \mu_n(\varphi) = 1$. We directly see that T is complete if and only if each sentence has asymptotic probability 0 or 1, in which case we say that the pair $(\mathbf{K}_n, \mu_n)_{n\in\mathbb{N}}$

https://doi.org/10.1017/bsl.2017.16 Published online by Cambridge University Press

has a $0 - 1$ law. Sets which have a $0 - 1$ law under the uniform probability measure include all finite structures, all finite partial orders, all finite l-colorable graphs and many more.

One can easily show that the Rado graph is not only axiomatized by the vertex-extension property described above, but can also be seen as the unique countable structure satisfying the almost sure theory coming from $\mathbf{K}_n$ consisting of all graphs with universe $\{1, \ldots, n\}$. Further more many other almost sure theories, including all of the above mentioned examples, are ω-categorical and simple with SU-rank 1. In this talk we will discuss why this connection occurs and see that the binary simple ω-categorical structures with SU-rank 1 is in direct connection with almost sure theories.

▸ JAROSLAV NESETRIL AND PATRICE OSSONA DE MENDEZ, *Structural limits and clustering near infinity*.
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Structual limits (including FO-limits and X-limits for various fragments X) arise as a natural generalization of limits of both dense and bounded degree graphs, yet having a distinctive model theoretic flavour. It is in a way dual approach which allows to prove distributional limits in a full generality. More recently it leads to clustering which seems to be interesting from both analytic and model theoretic perspective. This can be also outlined as follows: The cluster analysis of very large objects is an important problem, which spans several theoretical as well as applied branches of mathematics and computer science. Here we suggest a novel approach: under assumption of local convergence of a sequence of finite structures we derive an asymptotic clustering. This is achieved by a blend of analytic and geometric techniques, and particularly by a new interpretation of the authors' representation theorem for limits of local convergent sequences, which serves as a guidance for the whole process. Our study may be seen as an effort to describe connectivity structure at the limit (without having a defined explicit limit structure) and to pull this connectivity structure back to the finite structures in the sequence in a continuous way.

▸ C. TERRY, *An application of model theoretic Ramsey theory*.
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Chudnovsky, Kim, Oum, and Seymour recently established that any prime graph contains one of a short list of induced prime subgraphs. In this talk we present joint work with Malliaris, in which we reprove their theorem using many of the same ideas, but with the key model theoretic ingredient of first determining the so-called amount of stability of the graph. This approach changes the applicable Ramsey theorem, improves the bounds, and offers a different structural perspective on the graphs in question.

Abstracts of invited talks in the Special Session on Proof Theory and Reverse Mathematics

▸ DAVID BELANGER AND KENG MENG NG, *A computable perfect-set theorem*.
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We gauge the difficulty of finding a perfect subtree in a tree of a given Cantor–Bendixson rank. To simplify the analysis we introduce *half-derivative*, and extend the definition of rank to include values of the form n-and-a-half; each increase of one-half in the rank corresponds to one added jump in the perfect-subtree problem.

▶ LEV GORDEEV, *On Harvey Friedman's finite phase transitions.*
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DEFINITION 1 (H. Friedman). The *proof theoretic integer* of formal system **T** (abbreviation: PTI (**T**)) is the least integer n such that every Σ_1^0 sentence

$$\exists x_1 \dots \exists x_m A\left(x_1, \dots, x_m\right)$$

that has a proof in **T** with at most 10, 000 symbols, has witnesses $x_1, \dots, x_m < n$. (Actually $m = 1$ would suffice.)

A good source of examples is in the area surrounding Kruskal's theorem. This talk is devoted to PTI (**T**)'s basic properties, examples, comparisons and related phase transitions.

▶ FLORIAN PELUPESSY, *Ramsey like principles and well-foundedness of d-height ω-towers.*
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Recently it has been highlighted by Kreuzer and Yokoyama [1] that, over RCA_0, there are many principles equivalent to the well foundedness of the ordinal ω^ω. We will observe that there are Ramsey-like principles which are equivalent to the well-foundedness of ω_d, where $\omega_0 = 1$ and $\omega_{n+1} = \omega^{\omega_n}$. One of these examples is based on the relativised Paris–Harrington principle as mentioned in [1], but for dimension d. The more interesting example is the restricton of Friedman's adjacent Ramsey theorem to fixed dimension d, which is equivalent to the well-foundedness of ω_{d+1}.

DEFINITION 1 (Adjacent Ramsey in dimension d). For every $C : \mathbb{N}^d \to \mathbb{N}^r$ there exist $x_0 < \cdots < x_{d+1}$ such that $C(x_1, \dots, x_d) \leq C(x_2, \dots, x_{d+1})$, where $\leq$ is the coordinate-wise ordering.

[1] A. P. KREUZER and K. YOKOYAMA, *On principles between Σ_1 and Σ_2 induction and monotone enumerations*, arXiv:1306.1936v5.
[2] S. G. SIMPSON, ***Subsystems of Second Order Arithmetic***, second ed., Perspectives in Logic, Cambridge University Press, 2009.

▶ SAM SANDERS, *The unreasonable effectiveness of Nonstandard Analysis.*
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As suggested by the title, we will uncover the vast *computational content* of *classical* Nonstandard Analysis. To this end, we formulate a template $\mathfrak{CI}$ which converts a theorem of 'pure' Nonstandard Analysis, i.e., formulated solely with the *nonstandard* definitions (of continuity, integration, differentiability, convergence, compactness, et cetera), into the associated *effective* theorem. The latter constitutes a theorem of computable mathematics *no longer involving Nonstandard Analysis*. The template often produces theorems of Bishop's *Constructive Analysis* ([1]).

To establish the vast scope of $\mathfrak{CI}$, we apply this template to representative theorems from the *Big Five* categories from *Reverse Mathematics* ([3, 5]). The latter foundational program provides a classification of the majority of theorems from 'ordinary', that is nonset theoretical, mathematics into the aforementioned five categories. The *Reverse Mathematics zoo* ([2]) gathers exceptions to this classification, and is studied in [4] using $\mathfrak{CI}$. Hence, the template $\mathfrak{CI}$ is seen to apply to essentially *all of ordinary mathematics*, thanks to the Big Five classification (and associated zoo) from Reverse Mathematics. Finally, we establish that certain 'highly constructive' theorems, called Herbrandisations, imply the original theorem of Nonstandard Analysis from which they were obtained via $\mathfrak{CI}$.

Acknowledgment. This research is generously sponsored by the John Templeton Foundation and the Alexander Von Humboldt Foundation.

[1] E. Bishop and D. Bridgess, *Constructive analysis*. ***Grundlehren der Mathematischen Wissenschaften***, vol. 279, Springer, 1985, xii + 477.

[2] D. Dzhafarov, *The reverse mathematics zoo*, `http://rmzoo.uconn.edu/`.

[3] U. Kohlenbach, *Higher-order reverse mathematics*, ***Reverse Mathematics 2001***, Lecture Notes in Logic, vol. 21, Association for Symbolic Logic, La Jolla, CA, 2005, pp. 281–295.

[4] S. Sanders, *The refining of the taming of the reverse mathematics zoo*. ***Notre Dame Journal for Formal Logic***, 2016, to appear.

[5] S. Simpson, *Subsystems of second-order arithmetic*, ***Perspectives in Logic***, second ed., Cambridge University Press, Cambridge, 2009.

Abstracts of invited talks in the Special Session on Set Theory

► ANDREW MARKS, *Borel and measurable matchings*.
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We discuss several results related to the question of when a Borel graph has a Borel matching. Here, the analogue of Hall's matching theorem fails, but there are positive results giving Borel matchings in several contexts if we are willing to discard null or meager sets, or restrict the types of graphs we consider. We also discuss some applications to geometrical paradoxes.

► BENJAMIN MILLER AND ANUSH TSERUNYAN, *Integer cost and ergodic actions*.
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A countable Borel equivalence relation E on a probability space can always be generated in two ways: as the orbit equivalence relation of a Borel action of a countable group and as the connectedness relation of a locally countable Borel graph, called a *graphing* of E. Assuming that E is measure-preserving, graphings provide a numerical invariant called *cost*, whose theory has been largely developed and used by Gaboriau and others in establishing rigidity results. A well-known theorem of Hjorth states that when E is ergodic, treeable (admits an acyclic graphing), and has cost $n \in \mathbb{N} \cup \{\infty\}$, then it is generated by an a.e. free measure-preserving action of the free group $\mathbf{F}_n$ on n generators. We give a simpler proof of this theorem and the technique of our proof, combined with a recent theorem of Tucker–Drob, yields a strengthening of Hjorth's theorem: the action of $\mathbf{F}_n$ can be arranged so that each of the n generators acts ergodically.

► DAVID SCHRITTESSER, *Definable discrete sets, forcing, and Ramsey theory*.
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Let $\mathcal{R}$ be a family of finitary relations on a set X. A set $A \subseteq X$ is called $\mathcal{R}$-discrete if no relation $R \in \mathcal{R}$ relates any elements of A; A is called maximal discrete if it is maximal with respect to subset-inclusion among $\mathcal{R}$-discrete subsets of X. For any family of relations $\mathcal{R}$, maximal $\mathcal{R}$-discrete sets exist by the axiom of choice; whether such sets can be *definable* is contentious.

Maximal discrete sets have been widely studied: instances are maximal co-finitary groups, maximal almost disjoint families, and maximal orthogonal families of measures. In many (but not all) cases one can show such objects cannot be analytic (i.e., projections of closed sets). On the contrary, certain definable (in fact, co-analytic) maximal discrete sets have been shown to exist under the assumption that every set is constructible.

We present some new results, exhibiting definable maximal discrete sets in forcing extensions, e.g., extensions where the continuum hypothesis fails. In most cases, these results rely on Ramsey theoretic considerations.

▸ NAM TRANG, *Large cardinals, determinacy, and forcing axioms*.
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We discuss some recent progress in descriptive inner model theory. In particular, we discuss some current results concerning connections of the three hierarchies of models: canonical models of large cardinals (pure extender models), canonical models of determinacy, and strategic hybrid models (e.g., HOD of determinacy models). These structural results can be used to improve (lower-bound) consistency strength of strong combinatorial principles such as The Proper Forcing Axiom (PFA), strong forms of the tree property etc. In particular, I proved that PFA implies the existence of a transitive model of "$\mathsf{AD}_{\mathbb{R}} + \Theta$ is regular". Building on this and structural results above, G. Sargsyan and I have constructed models of theory $\mathsf{LSA} =_{\text{def}}$ "$\mathsf{AD}^+ +$ there is an α such that $\Theta = \theta_{\alpha+1} + \theta_\alpha$ is the largest Suslin cardinal" from PFA. This result is the strongest of its kind and reflects our current understanding of HOD of models of determinacy.

Abstracts of Contributed talks

▸ BAHAREH AFSHARI, STEFAN HETZL, AND GRAHAM E. LEIGH, *Structural representation of Herbrand's theorem*.
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We present recent results on the deepening connection between proof theory and formal language theory. To each first-order proof with cuts of complexity at most Π_n/Σ_n, we associate a typed (nondeterministic) tree grammar of order n (equivalently, an order n recursion scheme) that abstracts the computation of Herbrand sets obtained through Gentzen-style cut elimination. Apart from offering a means to compute Herbrand expansions directly from proofs with cuts, these grammars provide a structural counterpart to Herbrand's theorem that opens the door to tackling a number of questions in proof-theory such as proof equivalence, proof compression and proof complexity, which will be discussed.

The grammars presented naturally generalise the rigid regular and context-free tree grammars introduced in [2] and [1] that correspond to (respectively) proofs with Π_1/Σ_1 and Π_2/Σ_2 cuts.

[1] B. Afshari, S. Hetzl, and G. E. Leigh, *Herbrand disjunctions, cut elimination and context-free tree grammars*, ***13th International Conference on Typed Lambda Calculi and Applications, vol. 38*** (T. Altenkirch, editor), Schloss Dagstuhl–Leibniz-Zentrum fuer Informatik, Warsaw, Poland, 2015, pp. 1–16.

[2] S. Hetzl, *Applying tree languages in proof theory*, ***Language and Automata Theory and Applications: 6th International Conference, vol. 7183*** (A.-H. Dediu and C. Martín-Vide, editors), Springer Berlin Heidelberg, A Coruña, Spain, 2012, pp. 301–312.

▸ SVETLANA ALEKSANDROVA AND NIKOLAY BAZHENOV, *Automatic and tree-automatic list structures*.
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https://doi.org/10.1017/bsl.2017.16 Published online by Cambridge University Press

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Moore and Russell introduced their formal theory of linear lists in [1]. Generalizing this theory, Goncharov [2] constructed the axiomatic theory of linear lists over the elements of a given data type.

In this talk we will discuss algorithmic complexity of models of this theory of lists. We exhibit automatic properties of different classes of list structures using the framework of automatic and tree-automatic structures. In particular, we show that list structures over certain sets do not have automatic copies but have tree-automatic presentations, while stronger hereditarily finite list superstructure is not tree-automatically presentable.

We will also explore properties of list structures with respect to ordinal-automatic structures introduced by Schlicht and Stephan [3].

Acknowledgment. Supported by the Grants Council (under RF President) for State Aid of Leading Scientific Schools (grant NSh-6848.2016.1).

[1] D. J. Moore and B. Russell, *Axiomatic data type specifications: a first order theory of linear lists*. ***Acta Informatica***, vol. 15 (1981), no. 3, pp. 193–207.

[2] S. S. Goncharov, *A theory of lists and its models*. ***Vychislitel'nye Sistemy***, vol. 114 (1986), pp. 84–95 (in Russian).

[3] P. Schlicht and F. Stephan, *Automata on ordinals and automaticity of linear orders*. ***Annals of Pure and Applied Logic***, vol. 164 (2013), no. 5, pp. 523–527.

► URI ANDREWS AND ANDREA SORBI, *Uniformly effectively inseparable equivalence relations*.

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A computably enumerable equivalence relation (ceer) R is called *uniformly effectively inseparable* (*u.e.i.*) if it is nontrivial and there is a computable function $p(a, b)$ such that the partial computable function $\varphi_{p(a,b)}$ witnesses effective inseparability of the pair $([a]_R, [b]_R)$, whenever a and b are non-R-equivalent. It is shown in [1] that any u.e.i. ceer R is *universal*, i.e., for every ceer S there exists a computable function f such that, for all x, y, $x \; R \; y$ if and only if $f(x) \; S \; f(y)$. Despite universality, and unlike Smullyan's classical theorem establishing computable isomorphism of any two pairs of effectively inseparable c.e. sets, the u.e.i. ceers do not fall into a unique computable isomorphism type. The previously known largest class of u.e.i. ceers was the class of the *uniformly finitely precomplete* (*u.f.p.*) ceers, which can be characterized as those ceers which are computably isomorphic to nontrivial c.e. extensions of the relation $\sim_{PA}$ of provable equivalence in Peano Arithmetic. Answering a question in [1], we show:

Theorem 1. *There exist u.e.i. ceers that are not u.f.p.*

Among the u.f.p. ceers, $\sim_{PA}$ itself can be characterized (up to computable isomorphim) as the unique ceer R which has a diagonal function, i.e., a computable function d such that, for all x, x and $d(x)$ are non-R-equivalent, [2], or equivalently (up to computable isomorphim) as the unique ceer R which has a strong diagonal function, i.e., a computable function d such that for every finite set D, $d(D)$ is non-R-equivalent to any element in D. We show:

Theorem 2. *Every u.e.i. ceer R with a strong diagonal function is computably isomorphic to $\sim_{PA}$.*

[1] U. Andrews, S. Lempp, J. S. Miller, K. M. Ng, L. San Mauro, and A. Sorbi, *Universal computably enumerable equivalence relations*. ***The Journal Symbolic Logic***, vol. 79 (2014), no. 1, pp. 60–88.

https://doi.org/10.1017/bsl.2017.16 Published online by Cambridge University Press

[2] C. Bernardi and F. Montagna, *Equivalence relations induced by extensional formulae: Classifications by means of a new fixed point property*. ***Fundamenta Mathematicae***, vol. 124 (1984), pp. 221–232.

► SYLVY ANSCOMBE, *Generalised measurable structures with the Tree Property*.
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In [1], Chatzidakis, van den Dries, and Macintyre gave an asymptotic description of the number of points in definable sets in finite fields, building on earlier celebrated work of Lang and Weil, [2], for varieties. Later, in [3], Macpherson and Steinhorn turned these results into the definition of a 'one-dimensional asymptotic class'. This is a class $\mathcal{C}$ of finite structures in which, given a definable set X, there is a real number r and a natural number d such that the number of points in X in a structure $M \in \mathcal{C}$ is approximately equal to $r\,|M|^d$. Moreover, the pair (r, d) may be chosen somewhat uniformly, if X is allowed to vary through a definable family.

Taking ultraproducts of such classes, yields a 'measurable structure': an infinite structure equipped with a function

$$\begin{aligned} \mathrm{Def}(M) &\longrightarrow \mathbb{R}_{\geq 0} \times \mathbb{N} \\ X &\longmapsto (r, d) \end{aligned}$$

satisfying certain natural axioms which correspond to the intuition that r is the 'measure' and d is the 'dimension' of X. In particular, the fact that dimension takes values in the natural numbers implies that any measurable structures is supersimple, of finite rank.

In joint work between the speaker and Macpherson, Steinhorn, and Wolf, we have broadened this framework to allow more exotic measures and dimensions. We call such structures 'generalised measurable'. In this talk we describe several key examples of generalised measurable structures, including the generic triangle-free graph which has the Tree Property of the First Kind.

[1] Z. Chatzidakis, L. van den Dries, and A. Macintyre, *Definable sets over finite fields*. ***Journal fur die Reine und Angewandte Mathematik***, vol. 427 (1992), pp. 107–136.

[2] S. Lang and A. Weil, *Number of points of varieties in Finite Fields*. ***American Journal of Mathematics***, vol. 76 (1954), pp. 819–827.

[3] D. Macpherson and C. Steinhorn, *One-dimensional asymptotic classes of finite structures*. ***Transactions of the American Mathematical Society***, vol. 360 (2008), pp. 411–448.

► STEVE AWODEY, *Natural models of type theory*.
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The notion of a *natural model* of type theory provides an entirely algebraic description of a system of dependent type theory with an operation of context extension, and can serve as a flexible notion of a model of type theory. We give the main definition, originally stated in [1], make the algebraic character explicit, and then sketch the proof that this notion is equivalent to that of a *category with families* in the sense of Dybjer [2].

[1] S. Awodey, *Natural models of homotopy type theory*, `arXiv:1406.3219v2`.

[2] P. Dybjer, *Internal type theory*. ***Lecture Notes in Computer Science***, vol. 1158 (1996), pp. 120–134.

► SERIKZHAN BADAEV, *A chain of weekly precomplete computably enumerable equivalence relations*.
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A computably enumerable equivalence relation (ceer) E on ω is weakly precomplete if and only if E has no computable diagonal function. The class of weakly precomplete ceers includes the important classes of precomplete ceers and uniformly finitely precomplete (u.f.p.) ceers but not e-complete.

In [1], it was shown that there are infinitely many computable isomorphism types of universal weakly precomplete (in fact u.f.p.) ceers; and there are infinitely many computable isomorphism types of nonuniversal weakly precomplete ceers.

We consider ceers relatively to the following well known reduction: a ceer R is said to be reducible to a ceer S, if there is a computable function f such that, for all x and y, $xRy \iff f(x)Sf(y)$. We construct an infinite ω-chain of nonequivalent weakly precomplete ceers under this reduction.

[1] S. Badaev and A. Sorbi, *Weakly precomplete computably enumerable equivalence relations*. ***Mathematical Logic Quarterly***, vol. 62 (2016), no. 1, pp. 111–127.

▸ FARSHAD BADIE AND HANS GÖTZSCHE, *Towards logical analysis of occurrence values in truth-functional independent occurrence logic*.
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The human beings never really understood how truth could be recognised as the centrepiece of philosophy. The idea of truth vs. falsity is based on the assumption that the truth-value of statements about things beyond actual settings can, indisputably, be determined (false statements about settings are just counterfactuals).

In this discussion, we will rely on our alternative kind of logic: *Occurrence Logic* (Occ Log), which is not based on truth functionality, see [1]. The Occ Log $z \,^\circ\!\!> y$ expression denotes the fact that y occurs in case and only in case z occurs. Note that '$z \,^\circ\!\!> y$' does not by itself express any kind of truth-value semantics. We will see that the *Occurrences* as the main building blocks of our approach are independent from truth-values, but they are strongly dependent on the occurrence values. The fact that 'y would only occur [and would only have an occurrence value] in case z occurs [and has an occurrence value]', has been represented by Occ Log expression $z \,^\circ\!\!> y$. We shall stress that what is in logic often called states of affairs (including events) of the real world could be called *Local Universes* that are made of Entities and Properties. Focusing on the events z and y we can justifiably say that in case, and only in case, the local universe of z differs from the local universe of y regarding at least one but not all Entities and Properties, one of them can, potentially, be said to be a change of the other.

[1] Hans Götzsche, ***Deviational Syntactic Structures***, Bloomsbury Academic, London/New Delhi/New York/Sydney, 2013.

▸ BEKTUR BAIZHANOV, OLZHAS UMBETBAYEV, AND TATYANA ZAMBARNAYA, *The properties of linear orders defined on the classes of convex equivalence of 1-formulas*.
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In the report we consider small countable theories with an $\emptyset$-definable binary relation of linear order. Let A be a finite subset of a countable saturated model N, and $H(x)$ and $\Theta(x)$ be A-definable 1-formulas such that $H(N) \subset \Theta(N)$.

Define $E_{H,\Theta}(x, y) := H(x) \wedge H(y) \wedge (x < y \to \forall z((x < z < y \wedge \Theta(z)) \to H(z))) \wedge (y < x \to \forall z((y < z < x \wedge \Theta(z)) \to H(z))$.

$E_{H,\Theta}(x, y)$ is an A-definable relation of equivalence on $H(N)$ such that any $E_{H,\Theta}$-class is convex in $\Theta(N)$.

We say that an ordered theory T has the *property of finiteness of discrete chains of convex equivalences* (*FDCCE*) if for every two one-formulas $H(x)$ and $\Theta(x)$ such that $H(N) \subset \Theta(N)$, for any k $(1 < k < \omega)$ every discrete chain of convex $E_{H,\Theta}$-classes is finite.

We say that the set of A-definable one-formulas $C \subset F_1(A)$ is a $BH - algebra$ if it is closed under the following logical operations: $\wedge$, $\neg$, $\vee$, $\lhd_k^i$ $(0 < i < k,\ 1 < k < \omega)$.

THEOREM 1. *Let T be a small ordered theory with FDCCE, A be a finite subset of a countable saturated model N of the theory T. Then for every finite set of A-definable one-formulas $\{\phi_1(x), \dots, \phi_n(x)\}$, $n < \omega$ the BH-algebra generated by this set is finite.*

An ordered theory T is a theory of a *pure order* if it is in a language $L = \{=, <\}$.

THEOREM 2. *Let T be a small theory of a pure order. Then T is ω-categorical if and only if it has FDCCE.*

COROLLARY 3. *Let T be a non-ω-categorical small theory of a pure order. Then there is $\emptyset$-definable 1-formula $\phi(x)$ such that for some elements $\alpha, \beta \in \phi(N)$ $(\alpha < \beta)$, $(\alpha, \beta) \cap \phi(N)$ is an infinite discrete chain.*

COROLLARY 4. *Let T be a countable complete ordered theory in a language L and $T_0 \subset T$ be a complete theory in a language $L_0 := \{=, <\} \subset L$. If T_0 is non-ω-categorical then $I(T, \omega) = 2^\omega$.*

[1] M. RUBIN, *Theories of linear order*. ***Israel Journal of Mathematics***, vol. 17 (1974), pp. 392–443.

[2] S. SHELAH, *End extensions and numbers of countable models*. ***The Journal of Symbolic Logic***, vol. 43 (1978), pp. 550–562.

► DANA BARTOŠOVÁ, *Combinatorics of ultrafilters on automorphism groups*.

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For a topological group G, an ambit is a compact pointed space (X, x_0) with a (jointly) continuous action of G on X with the orbit $Gx_0 = \{gx_0 : g \in G\}$ dense in X. The greatest ambit of G, denoted by $S(G)$, is an ambit that has every ambit as its quotient preserving the distringuished points. We will study $S(G)$ for G an automorphism group of a countable first order structure as a space of ultrafilters, describe how the multiplication in G extends to $S(G)$ and show a couple of results about combinatorics and algebra in $S(G)$.

This is partially a joint work with Andrew Zucker (CMU).

► MARTIN BAYS AND JONATHAN KIRBY, *Exponential-algebraic closedness and quasi-minimality*.

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It is well-known that the complex field $\mathbb{C}$, considered as a structure in the ring language, is strongly minimal: every definable subset of $\mathbb{C}$ itself is finite or co-finite. Zilber conjectured that the complex exponential field $\mathbb{C}_{\exp}$ is quasiminimal, that is, every subset of $\mathbb{C}$ definable in this structure is countable or co-countable.

He later showed that if Schanuel's conjecture of transcendental number theory is true and $\mathbb{C}_{\exp}$ is *strongly exponentially-algebraically closed* then his conjecture holds [1]. Schanuel's conjecture is considered out of reach, and proving strong-exponential algebraic closedness involves finding solutions of certain systems of equations and then showing they are generic, the latter step usually done using Schanuel's conjecture.

We show that if $\mathbb{C}_{\exp}$ is *exponentially-algebraically closed* then it is quasiminimal. Thus Schanuel's conjecture can be dropped as an assumption, and strong exponential-algebraic closedness can be weakened to exponential-algebraic closedness which requires certain systems of equations to have solutions, but says nothing about their genericity.

[1] B. ZILBER, *Pseudo-exponentiation on algebraically closed fields of characteristic zero*. ***Annals of Pure and Applied Logic***, vol. 132 (2005), no. 1, pp. 67–95.

▶ NIKOLAY BAZHENOV, *Effective categoricity for polymodal algebras*.
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Let $\mathbf{d}$ be a Turing degree. A computable structure $\mathcal{A}$ is $\mathbf{d}$-*computably categorical* if for every computable structure $\mathcal{B}$ isomorphic to $\mathcal{A}$, there is a $\mathbf{d}$-computable isomorphism from $\mathcal{A}$ onto $\mathcal{B}$. The *categoricity spectrum* of $\mathcal{A}$ is the set

$$CatSpec(\mathcal{A}) = \{\mathbf{d} \colon \mathcal{A} \text{ is } \mathbf{d}\text{-computably categorical}\}.$$

The results of [1] imply that not every categoricity spectrum is the categoricity spectrum of a Boolean algebra. A natural question that arises is the following: how does expanding the language of Boolean algebras affect categoricity spectra and other related properties?

Hirschfeldt, Khoussainov, Shore, and Slinko [2] introduced the notion of a class which is *complete with respect to degree spectra of nontrivial structures, effective dimensions, expansion by constants, and degree spectra of relations*. For brevity, we call such classes *HKSS-complete*. If a class K is HKSS-complete, then for every computable structure $\mathcal{S}$, there is a structure $\mathcal{A}_{\mathcal{S}} \in K$ with the property $CatSpec(\mathcal{A}_{\mathcal{S}}) = CatSpec(\mathcal{S})$.

Khoussainov and Kowalski [3] proved that the class of Boolean algebras with operators is HKSS-complete. They also asked whether the similar result is true for polymodal algebras. Here we give the positive answer to the question:

THEOREM 1. *The class of Boolean algebras with four distinguished modalities is HKSS-complete.*

In particular, this result implies that every categoricity spectrum is the categoricity spectrum of some polymodal algebra.

Acknowledgment. This work was supported by the Grants Council (under RF President) for State Aid of Leading Scientific Schools (grant NSh-6848.2016.1).

[1] N. A. BAZHENOV, Δ^0_2-*categoricity of Boolean algebras*. ***Journal of Mathematical Sciences***, vol. 203 (2014), no. 4, pp. 444–454.

[2] D. R. HIRSCHFELDT, B. KHOUSSAINOV, R. A. SHORE, and A. M. SLINKO, *Degree spectra and computable dimensions in algebraic structures*. ***Annals of Pure and Applied Logic***, vol. 115 (2002), no. 1–3, pp. 71–113.

[3] B. KHOUSSAINOV and T. KOWALSKI, *Computable isomorphisms of Boolean algebras with operators*. ***Studia Logica***, vol. 100 (2012), no. 3, pp. 481–496.

▶ JEFFREY BERGFALK, *Homological characterizations of "small" cardinals*.
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We consider a number of homological invariants of the "small" cardinals ω_n. We show, for example, that ω_n is the least ordinal whose nth sheaf cohomology group is nonzero. The best-understood case is that of $n = 1$: here the nonzero group is that of nontrivial coherent families of functions on ω_1. We consider what the (ZFC) existence of analogous, higher-dimensional families corresponding to higher nonzero homology groups begins to tell us about the combinatorics of higher ω_n.

▶ OLAF BEYERSDORFF, ILARIO BONACINA, AND LEROY CHEW, *Lower bounds: From circuits to QBF proof systems*.
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https://doi.org/10.1017/bsl.2017.16 Published online by Cambridge University Press

A general and long-standing belief in the proof complexity community asserts that there is a close connection between progress in lower bounds for Boolean circuits and progress in proof size lower bounds for strong propositional proof systems. Although there are famous examples where a transfer from ideas and techniques from circuit complexity to proof complexity has been effective [4], a formal connection between the two areas has never been established so far. Here we provide such a formal relation between lower bounds for circuit classes and lower bounds for Frege systems for quantified Boolean formulas (QBF).

Starting from a propositional proof system P we exhibit a general method how to obtain a QBF proof system $P + \forall\mathsf{red}$, which is inspired by the transition from resolution to Q-resolution. For us the most important case is a new and natural hierarchy of QBF $\mathcal{C}$-Frege systems $\mathcal{C}$-Frege+$\forall$red that parallels the well-studied propositional hierarchy of $\mathcal{C}$-Frege systems, where lines in proofs are restricted to a circuit class $\mathcal{C}$.

Building on earlier work for resolution [1] we establish a lower bound technique via strategy extraction that transfers arbitrary lower bounds for the circuit class $\mathcal{C}$ to lower bounds in $\mathcal{C}$-Frege+$\forall$red.

By using the full spectrum of state-of-the-art circuit lower bounds [2, 3, 5, 6], our new lower bound method leads to very strong lower bounds for QBF Fregesystems:

1. exponential lower bounds and separations for $\mathsf{AC}^0[p]$-Frege+$\forall$red for all primes p;
2. an exponential separation of $\mathsf{AC}^0[p]$-Frege+$\forall$red from $\mathsf{TC}^0[p]$-Frege+$\forall$red;
3. an exponential separation of the hierarchy of constant-depth systems AC^0_d-Frege+$\forall$red by formulas of depth independent of d.

In the propositional case, all these results correspond to major open problems.

[1] O. Beyersdorff, L. Chew, and M. Janota, *Proof complexity of resolution-based QBF calculi*, ***32nd International Symposium on Theoretical Aspects of Computer Science (STACS 2015)***, 2015, pp. 76–89.

[2] R. B. Boppana and M. Sipser, ***Handbook of Theoretical Computer Science***, vol. A, Chapter: The Complexity of Finite Functions, MIT Press, Cambridge, MA, 1990, pp. 757–804.

[3] J. Håstad, *Almost optimal lower bounds for small depth circuits*, ***Proceedings of the 18th Symposium on the Theory of Computing***, ACM Press, 1986, pp. 6–20.

[4] J. Krajíček, *Interpolation theorems, lower bounds for proof systems and independence results for bounded arithmetic*. ***The Journal of Symbolic Logic***, vol. 62 (1997), no. 2, pp. 457–486.

[5] A. A. Razborov, *Lower bounds for the size of circuits of bounded depth with basis* $\{\&, \oplus\}$. ***Mathematical Notes of the Academy of Sciences of the USSR***, vol. 41 (1987), no. 4, pp. 333–338.

[6] R. Smolensky, *Algebraic methods in the theory of lower bounds for Boolean circuit complexity*, ***Proceedings of the 19th Symposium on the Theory of Computing***, ACM Press, 1987, pp. 77–82.

▸ OLAF BEYERSDORFF AND JÁN PICH, *Understanding Gentzen and Frege systems for QBF*.

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Recently Beyersdorff, Bonacina, and Chew [1] introduced a natural class of Frege systems for quantified Boolean formulas (QBF) and showed strong lower bounds for restricted versions of these systems. Here we provide a comprehensive analysis of their new extended Frege system, denoted EF+$\forall$red, which is a natural extension of classical extended Frege EF.

Our main results are the following:

Firstly, we prove that the standard Gentzen-style system G_1^* p-simulates EF+$\forall$red and that G_1^* is strictly stronger under standard complexity-theoretic hardness assumptions.

Secondly, we show a correspondence of EF+$\forall$red to bounded arithmetic: EF+$\forall$red can be seen as the nonuniform propositional version of intuitionistic S^1_2. Specifically, intuitionistic

https://doi.org/10.1017/bsl.2017.16 Published online by Cambridge University Press

S_2^1 proofs of arbitrary statements in prenex form translate to polynomial-size EF+$\forall$red proofs, and EF+$\forall$red is in a sense the weakest system with this property.

Finally, we show that unconditional lower bounds for EF+$\forall$red would imply either a major breakthrough in circuit complexity or in classical proof complexity, and in fact the converse implications hold as well. Therefore, the system EF+$\forall$red naturally unites the central problems from circuit and proof complexity.

Technically, our results rest on a formalised strategy extraction theorem for EF+$\forall$red akin to witnessing in intuitionistic S_2^1 and a normal form for EF+$\forall$red proofs.

[1] O. Beyersdorff, I. Bonacina, and L. Chew, *Lower bounds: From circuits to QBF proof systems*, ***Proceedings of the ACM Conference on Innovations in Theoretical Computer Science (ITCS)***, ACM, 2016, pp. 249–260.

► JOSHUA BLINKHORN AND OLAF BEYERSDORFF, *Dependency schemes and soundness in QBF calculi*.
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The tremendous success of SAT solvers in recent years is motivating advances in quantified Boolean formula (QBF) solving. Afforded by its PSPACE-completeness, QBF allows natural and compact encodings of real-world problems [1], due to the addition of a quantifier prefix to the propositional formula.

The quantifier prefix of a QBF imposes a linear order on the variables, in which a given variable may depend upon any preceding variable. It need not, however, necessarily depend on *all* the preceding variables. A dependency scheme is an algorithm that attempts to identify cases of independence by appeal to the syntactic form of an instance, producing a partial order on the variables that describes the dependency structure more accurately. Harnessing independence in this way, a solver enjoys greater freedom to navigate the search space, and frequently solves the instance faster despite the computational overhead incurred in computing the dependency scheme [2].

Whereas there is potential to implement independence further in QBF solving, a major concern is whether the use of a given scheme preserves the correctness of the solving method [4]; that is, does the underlying proof system remain sound when parametrised by the dependency scheme?

To that end, we show how to implement dependency schemes in stronger 'long-distance' QBF calculi, and demonstrate that the notion of 'full exhibition', which is a property of dependency schemes, is sufficient for soundness in all the resulting systems. Further, we show that the reflexive resolution path dependency scheme is fully exhibited, thereby proving a conjecture of Slivovsky [3].

[1] M. Benedetti and H. Mangassarian, *QBF-based formal verification: Experience and perspectives*. ***Satisfiability, Boolean Modeling and Computation***, vol. 5 (2008), no. 1–4, pp. 133–191.

[2] F. Lonsing, ***Dependency shemes and search-based QBF solving: Theory and practice***, Ph.D. thesis, Johannes Kepler University, 2012.

[3] F. Slivovsky, ***Stucture in #SAT and QBF***, Ph.D. thesis, Vienna University of Technology, 2015.

[4] F. Slivovsky and S. Szeider, *Soundness of Q-resolution with dependency schemes*. ***Theoretical Computer Science***, vol. 612 (2016), pp. 83–101.

► DAVID BRADLEY-WILLIAMS, *Limits of betweenness relations*.
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Betweenness relations on semilinear orderings, among other tree-like relational stuctures, were studied extensively by Adeleke and Neumann in [3]. Such tree-like structures were also investigated though a host of examples constructed by Cameron in [6]. Adeleke and Macpherson [2] then built on this knowledge to determine that when a transitive permutation group is a Jordan group, it preserves one from a list of various kinds of relational structures.

https://doi.org/10.1017/bsl.2017.16 Published online by Cambridge University Press

Some of these structures are quite well understood as relational structures, but there are cases in which the invariant structure appears only as an exotic infinite 'limit' of more familiar structures. In this talk we will discuss what 'limits of betweenness relations' are and how they have been constructed (by Adeleke [1], Bhatacharjee and Macpherson [4] and myself [5]).

[1] S. A. ADELEKE, *On irregular infinite Jordan groups*. ***Communications in Algebra***, vol. 41 (2013), no. 4, pp. 1514–1546.

[2] S. A. ADELEKE and D. MACPHERSON, *Classification of infinite primitive Jordan permutation groups*. ***Proceedings of the London Mathematical Society***, vol. 72 (1996), no. 3, pp. 63–123.

[3] S. A. ADELEKE and P. M. NEUMANN, ***Relations Related to Betweenness: Their Structure and Automorphisms***, Memoirs of the American Mathematical Society, vol. 623, The American Mathematical Society, 1998.

[4] M. BHATTACHARJEE and D. MACPHERSON, *Jordan groups and limits of betweenness relations*. ***Journal of Group Theory***, vol. 9 (2006), no. 1, pp. 59–94.

[5] D. BRADLEY-WILLIAMS, ***Jordan groups and homogeneous structures***, Ph.D. thesis, University of Leeds, 2015.

[6] P. CAMERON, *Some treelike objects*. ***The Quarterly Journal of Mathematics***, vol. 38 (1987), no. 2, pp. 155–183.

▸ HAZEL BRICKHILL, *A generalisation of closed unbounded sets and square sequences*.
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I will introduce a generalisation of the notion closed unbounded (club) set which ties in with the generalisation of stationary set given in [1]. We can use these n-clubs to define a generalisation of Jensen's $\square$ sequence below a nonweakly compact (constructed in [2]). The main result is that in L we have such a $\square^n$ sequence below every cardinal that is Π^1_n-indescribable but not Π^1_{n+1}-indescribable. It is interesting to note that construction of the $\square^n$ sequence does not require fine-structure.

[1] J. BAGARIA, M. MAGIDOR, and H. SAKAI, *Reflection and indescribability in the constructible universe*. ***Israel Journal of Mathematics***, vol. 208 (2015), no. 1, pp. 1–11.

[2] R. B. JENSEN, *The fine structure of the constructible hierarchy*. ***Annals of Mathematical Logic***, vol. 4 (1972), no. 3, pp. 229–308.

▸ ULRIK BUCHHOLTZ AND EGBERT RIJKE, *The real projective spaces in HoTT*.
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We construct the real projective spaces as certain higher inductive types in Homotopy Type Theory. The classical definition of $\mathbb{RP}^n$, as the quotient space where the antipodal points of the n-sphere are identified, does not translate directly to Homotopy Type Theory. Instead, we define $\mathbb{RP}^n$ by induction on n simultaneously with its tautological bundle. As the base case, $\mathbb{RP}^{-1}$ is taken to be the empty type. In the inductive step, $\mathbb{RP}^{n+1}$ is taken to be the mapping cone of the projection map of the tautological bundle. It is then possible to define the tautological bundle on $\mathbb{RP}^{n+1}$ using the universal property of $\mathbb{RP}^{n+1}$ and the univalence axiom.

With this definition, one can use the descent theorem to show that the total space of the tautological bundle of $\mathbb{RP}^n$ is the n-sphere. Hence one can retrieve the description of $\mathbb{RP}^{n+1}$ as $\mathbb{RP}^n$ with an $(n+1)$-disk attached to it. The infinite dimensional real projective space $\mathbb{RP}^\infty$, defined as the sequential colimit of $\mathbb{RP}^n$ with the canonical inclusion maps, is equivalent to $K(\mathbb{Z}/2\mathbb{Z}, 1)$, and the tautological bundles of the finite dimensional real projective spaces factor through $\mathbb{RP}^\infty$ as expected. Indeed, the infinite dimensional projective space classifies the 0-sphere bundles—which one can think of as synthetic line bundles—and using the join connectivity theorem one can then show that the tautological bundle of $\mathbb{RP}^n$ is an $(n-1)$-connected map into $\mathbb{RP}^\infty$.

https://doi.org/10.1017/bsl.2017.16 Published online by Cambridge University Press

▸ JOAN CASAS-ROMA, ANTONIA HUERTAS, AND M. ELENA RODRÍGUEZ, *Towards a semantics for Dynamic Imagination Logic*.
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Imagining alternative ways the reality could be is something we do almost everyday in our lives: when we wonder how things would be if I was to win the lottery or if I were working in another company, when we read a book that describes a reality different than ours, when we "log in" to a virtual world and we incarnate our *alter-ego*, or even when we consider more recent technological developments like virtual reality: while wearing a pair of VR goggles, we start experiencing a new, different reality in which we accept some things to be different, in which we are someone else, and in which we embrace new rules that we know they would be impossible in our reality but which are, nonetheless, possible in that alternative one.

We define and present the syntax and semantics of the Dynamic Imagination Logic: this system models an epistemic agent who is able to deal with issues involving imagination in a dynamic way, and perform actions such as imagining "whether φ could be the case, provided everything else stays the same", or even imagining "how different things should be in order for φ to be the case". These actions are processed in a dynamic way that expands the model by creating new and different "spaces" that represent alternative realities the agent thinks about; moreover, the model is expanded in such a way that every imagination step can be fully traced to see what the agent imagined and from where, which alternative realities are generated by each imagination step, and what would the agent come to know or ignore in each of these alternatives.

Acknowledgments. This work has been supported by a doctoral grant from the Universitat Oberta de Catalunya (UOC) and the funding for the project "Hybrid Intensional Logic" (with reference FFI2013-47126-P) given by the Spanish Ministry of Economy and Competitiveness.

▸ FABIANA CASTIBLANCO AND PHILIPP SCHLICHT, *Tree forcings and sharps*.
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The Levy–Solovay theorem shows that any measurable cardinal κ is preserved by all forcings of size strictly less than κ, and there are similar results for many other large cardinals. Moreover, many global consequences of large cardinals are preserved by certain forcings. For instance, a *sharp* for a set of ordinals x states the existence of a nontrivial elementary embedding $j: L[x] \to L[x]$ with $\mathrm{crit}(j) > \sup(x)$. It is well-known that the existence of sharps for all sets of ordinals is preserved by all forcings. In this project, we study interesting consequences of large cardinals in $H(\omega_1)$ and prove that they are preserved under certain proper forcings. In particular, we study the existence of sharps for all sets of ordinals in $H(\omega_1)$, equivalently analytic determinacy. We show that this condition is preserved by various tree forcings, for instance Mathias forcing, Sacks forcing, and Laver forcing.

▸ JUAN DE VICENTE, *Locally $\mathbb{C}$-Nash groups*.
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Locally $\mathbb{C}$-Nash groups are analytic groups which also carry a semialgebraic structure—seing $\mathbb{C}$ as $\mathbb{R}^2$. For example, the universal coverings of algebraic groups are locally C-Nash groups. The definition of the latter is based on the concept of $\mathbb{C}$-Nash map, which has been studied—with may be other names—by different authors (see e.g., [1, 3]).

In this talk we give a classification of abelian locally $\mathbb{C}$-Nash groups of dimension one and two.

This is joint work with E. Baro and M. Otero.

[1] J. Adamus and S. Randriambololona, *Tameness of holomorphic closure dimension in a semialgebraic set*. ***Mathematische Annalen***, vol. 355 (2013), no. 3, pp. 985–1005.

[2] E. Baro, J. de Vicente, and M. Otero, *Locally $\mathbb{C}$-Nash groups*, in preparation.
[3] Y. Peterzil and S. Starchenko, *Complex analytic geometry in a nonstandard setting*, ***Model Theory with Applications to Algebra and Analysis, vol. 1*** (Z. Chatzidakis, D. Macpherson, A. Pillay, and A. Wilkie, editors), Cambridge University Press, 2008, pp. 117–165.

▸ JONATHAN DITTRICH, *The inadequacy of nontransitive solutions to paradox*.
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[1] has argued that a nontransitive substructural logic (NT) provides both a solution to semantic paradoxes and preserves full classical logic. Here we argue that NT fulfils these goals only inadequately.

We distinguish between weak and strong inconsistency: A language is weakly inconsistent for some formula ϕ iff it proves both $\vdash \phi$ and $\phi \vdash$. It is strongly inconsistent iff it proves the empty sequent. Although NT is strongly consistent, it remains weakly inconsistent. For even without Cut, NT derives both $\vdash \mathrm{T}(\ulcorner\lambda\urcorner)$ and $\mathrm{T}(\ulcorner\lambda\urcorner) \vdash$ for the Liar sentence λ.

According to Ripley, the characteristic of any paradoxical sentence ψ is that both $\vdash \psi$ and $\psi \vdash$ are provable. However, one can show that NT is weakly inconsistent for many central theorems of classical logic as well; including the law of noncontradiction, excluded middle and identity. This is straightforward in first-order logic when these principles are understood in terms of the Truth-predicate. With second-order logic, this can be extended to hold of these theorems understood with arbitrary predicates.

There are two problems. First, NT fails to provide an adequate distinction between paradoxical and nonparadoxical sentences. For classical theorems bear the characteristic of paradoxes. Second, it casts doubt on the classicality of NT. Intuitively, it is not sufficient to merely prove all theorems of classical logic. For even an inconsistent system with explosion will do this. One would further have to ensure that the system does not prove anything weakly inconsistent with these theorems—again, NT fails to do so.

[1] D. Ripley, *Conservatively extending classical logic with transparent truth*. ***Review of Symbolic Logic***, vol. 5 (2012), no. 2, pp. 354–378.

▸ NATASHA DOBRINEN, CLAUDE LAFLAMME, AND NORBERT SAUER, *Rainbow Ramsey simple structures*.
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We prove that the Rado graph $\mathcal{R}$ has the rainbow Ramsey property. This means that given any finite graph G, any finite number k, and any coloring of the copies of G in the Rado graph, $\mathcal{R}$, where each color appears no more than k times, there is a subgraph $\mathcal{R}' \leq \mathcal{R}$ which is also a Rado graph in which the copies of G use each color at most once. More generally, we show that a class of binary relational structures generalizing the Rado graph are rainbow Ramsey. By compactness, it follows that for all finite graphs B and C and any finite number k, there is a graph A so that for every coloring of the copies of C in A such that each color is used at most k times, there is a copy B' of B in A in which each copy of C has a different color.

This is joint work with Claude Laflamme and Norbert Sauer.

▸ PHILIP EHRLICH, *Number systems with simplicity hierarchies*: *A generalization of Conway's theory of surreal numbers II*.
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https://doi.org/10.1017/bsl.2017.16 Published online by Cambridge University Press

In [1], the algebraico-tree-theoretic simplicity hierarchical structure of J. H. Conway's ordered field **No** of surreal numbers was brought to the fore and employed to provide necessary and sufficient conditions for an ordered field to be isomorphic to an initial subfield of **No**, i.e., a subfield of **No** that is an initial subtree of **No**. In this sequel to [1], which is joint work with Elliot Kaplan, analogous results for ordered abelian groups and ordered domains are established which in turn are employed to characterize the convex subgroups and convex subdomains of initial subfields of **No** that are themselves initial. It is further shown that an initial subdomain of **No** is discrete if and only if it is an initial subdomain of **No**'s canonical integer part **Oz** of omnific integers. Finally, extending results of [1], the theories of nontrivial divisible ordered abelian groups and real-closed ordered fields are shown to be the sole theories of nontrivial densely ordered abelian groups and ordered fields all of whose models are isomorphic to initial subgroups and initial subfields of **No**.

[1] P. Ehrlich, *Number systems with simplicity hierarchies: A generalization of Conway's theory of surreal numbers*. ***The Journal of Symbolic Logic***, vol. 66 (2001), no. 3, pp. 1231–1258.

► JACOPO EMMENEGGER AND ERIK PALMGREN, *Exact completion and constructive theories of sets*.
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In [2] Palmgren proposed a constructive and predicative version of Lawvere's Elementary Theory of the Category of Sets (ETCS), called CETCS, which provides a structuralist foundation for constructive mathematics in the style of Bishop. As shown in [2], a CETCS category is precisely a well-pointed locally cartesian closed pretopos with a natural number object and enough projectives.

In both the intended models of CETCS categories, namely setoids in Martin-Löf type theory and sets in Aczel's CZF, sets can be seen as quotients of projective sets and can thus be regarded as exact completions. We generalise this approach to CETCS categories and characterise them in terms of weaker properties of their projective covers. In particular, it seems that Carboni and Rosolini's characterisation of local closure [1] in terms of weak closure of the maximal projective cover cannot be applied in this case, because splitting of idempotents is undecidable in Martin-Löf type theory. We solve this issue providing an alternative characterisation of local closure that applies to any projective cover.

We apply this characterisation to the category of small types in Martin-Löf type theory with a universe and basic type formers, obtaining as a consequence that the category of setoids is a CETCS category.

[1] A. Carboni and G. Rosolini, *Locally cartesian closed exact completions*. ***Journal of Pure and Applied Algebra***, vol. 154 (2000), no. 3, pp. 103–116.

[2] E. Palmgren, *Constructivist and structuralist foundations: Bishop's and Lawvere's theories of sets*. ***Annals of Pure and Applied Logic***, vol. 163 (2012), no. 10, pp. 1384–1399.

► CHRISTIAN ESPINDOLA, *Completeness of infinitary intuitionistic logics*.
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Completeness theorems for infinitary classical logics $\mathcal{L}_{\kappa,\kappa}$ (for, say, an inaccessible κ) have been known for decades. When removing excluded middle, however, the situation is more difficult to analyze even in the propositional case, as the main difficulty in studying infinitary intuitionistic logics is the huge variety of nonequivalent formulas that one can obtain. Completeness results for the propositional fragment $\mathcal{L}_{\omega_1,0}$ have been obtained, but the general case has not been addressed. The purpose of this talk is to outline set-theoretical and category-theoretical techniques that allow the study of completeness theorems for infinitary intuitionistic logics in the general case, both for propositional and first-order logics, in terms of an infinitary Kripke semantics. We will also analyze to what extent the use of large cardinal axioms (more precisely, the condition that κ be weakly compact) is necessary, and some applications of the completeness results will be presented.

https://doi.org/10.1017/bsl.2017.16 Published online by Cambridge University Press

[1] P. Johnstone, ***Sketches of an Elephant—A Topos Theory Compendium—vol. I and II***, Oxford University Press, 2002.

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[3] C. Karp, ***Languages with Expressions of Infinite Length***, North-Holland Publishing Co, 1964.

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[5] M. Makkai, *A theorem on Barr-exact categories, with an infinite generalization*. ***Annals of Pure and Applied Logic***, vol. 47 (1990), pp. 225–268.

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► EKATERINA FOKINA AND DINO ROSSEGGER, *Enumerable functors*.
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We propose a new notion of reducibility between structures, *enumerable functors*, inspired by the recently investigated notion of computable functors [2, 3]. An enumerable functor from a structure $\mathcal{A}$ to a structure $\mathcal{B}$ is a pair (Ψ, Φ) where Ψ is an enumeration operator transforming every presentation of $\mathcal{A}$ to a presentation of $\mathcal{B}$ and Φ is a Turing functional transforming every isomorphism between two presentations of $\mathcal{A}$ to an isomorphism of their image. Our main results are that enumerable functors preserve Σ_n–spectra [1] and that they are equivalent to a restricted version of effective interpretability. We also extend this equivalence to effective bi-interpretability and reducibility between classes of structures by effective bi-interpretability.

[1] E. Fokina, P. Semukhin, and D. Turetsky, *Degree spectra of sructures under equivalence relations*, in preparation.

[2] M. Harrison-Trainor, A. Melnikov, R. Miller, and A. Montalbán, *Computable functors and effective interpretability*, submitted.

[3] R. Miller, B. Poonen, H. Schoutens, and A. Shlapentokh, *A computable functor from graphs to fields*, submitted.

► JAYKOV FOUKZON, *Inconsistent countable set in second order ZFC and nonexistence of the strongly inaccessible cardinals*.
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We derived an important example of the inconsistent countable set in second order $ZFC(ZFC_2)$ with the full second-order semantics. Main results: (i) $\neg Con(ZFC_2)$, (ii) let k be an inaccessible cardinal and H_k is a set of all sets having hereditary size less then k, then $\neg Con(ZFC + (V = H_k))$.

[1] J. Foukzon, *Inconsistent countable set in second order ZFC and nonexistence of the strongly inaccessible cardinals*. ***British Journal of Mathematics & Computer Science***, vol. 9, no. 5, pp. 380–393.

[2] ———, *Strong reflection principles and large cardinal axioms*, ***Fall Southeastern Sectional Meeting University of Louisville***, Louisville, KY October 5–6, 2013 Meeting ♯ 1092 (1092-03-13) http://www.ams.org/meetings/sectional/2208_program_saturday.html♯2208:AMSCP1.

► ANTON FREUND, *The slow reflection hierarchy*.
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We generalize the notion of slow consistency, due to S.-D. Friedman, Rathjen and Weiermann [3], to obtain slow (uniform) reflection statements of arbitrary arithmetical complexity. The resulting hierarchy over Peano Arithmetic is incomparable with the hierarchy of usual reflection principles: No single one of the usual reflection statements implies all slow reflection statements. Interestingly, though, slow reflection is much weaker when it comes to Π_1-consequences: Any Π_1-formula that is provable in the slow reflection hierarchy already follows from the (usual) consistency of Peano Arithmetic. For detailed proofs we refer to [2]. Henk and Pakhomov [4] independently establish similar results. A more computational viewpoint is adopted in [1], where we determine the provably total functions of slow Σ_1-reflection.

[1] A. Freund, *Proof lengths for instances of the Paris–Harrington Principle*, preprint, `arXiv:1601.08185`.

[2] ———, *Slow reflection*, preprint, `arXiv:1601.08214`.

[3] S.-D. Friedman, M. Rathjen, and A. Weiermann, *Slow consistency*. ***Annals of Pure and Applied Logic***, vol. 164 (2013), no. 3, pp. 382–393.

[4] P. Henk and F. Pakhomov, *Slow and ordinary provability for Peano Arithmetic*, `arXiv:1602.01822`.

▸ SY-DAVID FRIEDMAN, RADEK HONZIK, AND ŠÁRKA STEJSKALOVÁ, *The tree property at the double successor of a singular cardinal with a larger gap*.

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Starting from a Laver-indestructible supercompact κ and a weakly compact λ above κ, we show there is a forcing extension where κ is a strong limit singular cardinal with cofinality ω, $2^\kappa = \kappa^{+3} = \lambda^+$, and the tree property holds at $\kappa^{++} = \lambda$. Next we generalize this result to an arbitrary cardinal μ such that $\kappa < \mathrm{cf}(\mu)$ and $\lambda^+ \leq \mu$. This result provides more information about possible relationships between the tree property and the continuum function.

Acknowledgment. All three authors were supported by FWF/GAČR grant I 1921-N25.

▸ MAKOTO FUJIWARA, *Effective computability and constructive provability for existence sentences*.

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Along the line of [1], we investigate the relationship between effective computability and constructive provability for existence sentences. We show the following. Here $\mathsf{E\text{-}PA}^\omega$ (resp. $\mathsf{E\text{-}HA}^\omega$) is the finite type extension of PA (resp. HA), EL is the system of elementary analysis, AC (resp. $\mathrm{AC}_{00}, \Pi^0_1\text{-AC}_{00}$) is the axiom of choice in all finite types (resp. the axiom of countable choice, that for purely universal formulas), $\mathrm{IP}^\omega_{\mathrm{ef}}$ is the independence of premise for $\exists$-free formulas, $\Sigma^0_2\text{-DNS}^0$ is the fragment of double negation shift principle:

$$\forall \alpha^1 (\forall x^0 \neg\neg \exists y^0 \forall z^0 \alpha(x,y,z) = 0 \to \neg\neg \forall x^0 \exists y^0 \forall z^0 \alpha(x,y,z) = 0).$$

Theorem 1. *For every sentence $\forall \xi^1 (A(\xi) \to \exists \zeta^1 B(\xi,\zeta))$ where $A(\xi)$ is $\exists$-free and $B(\xi,\zeta) \in \Gamma_1$, if*

$$\mathsf{E\text{-}HA}^\omega + \mathrm{AC} + \mathrm{IP}^\omega_{\mathrm{ef}} + \Sigma^0_2\text{-DNS}^0 \vdash \forall \xi (A(\xi) \to \exists \zeta B(\xi,\zeta)),$$

then there exists a term $t^{1\to 1}$ of $\mathsf{E\text{-}PA}^\omega$ such that

$$\mathsf{E\text{-}PA}^\omega + \Pi^0_1\text{-AC}_{00} \vdash \forall \xi (A(\xi) \to \exists \zeta B(\xi, t\xi)).$$

The classes $\mathcal{A}$, $\mathcal{B}$ of formulas in $\mathcal{L}(\mathsf{E\text{-}HA}^\omega)$ are defined simultaneously by

- P, $A_1 \wedge A_2$, $A_1 \vee A_2$, $\forall x A_1$, $\exists x A_1$, $B_1 \to A_1$ are in $\mathcal{A}$;
- P, $B_1 \wedge B_2$, $\forall x B_1$, $A_1 \to B_1$ are in $\mathcal{B}$;

where P, A_i, B_i range over prime formulas, formulas in $\mathcal{A}$, $\mathcal{B}$ respectively.

Theorem 2. *For every sentence $\forall \xi^1 (A(\xi) \to \exists \zeta^1 B(\xi,\zeta))$ where $A(\xi) \in \mathcal{A}$ and $B(\xi,\zeta) \in \mathcal{B}$, if there exists a term $t^{1\to 1}$ of $\mathsf{E\text{-}PA}^\omega$ such that*

$$\mathsf{E\text{-}PA}^\omega + \Pi^0_1\text{-AC}_{00} \vdash \forall \xi (A(\xi) \to \exists \zeta B(\xi, t\xi)),$$

https://doi.org/10.1017/bsl.2017.16 Published online by Cambridge University Press

then

$$\mathsf{EL} + \Pi^0_1\text{-AC}_{00} + \Sigma^0_2\text{-DNS}^0 \vdash \forall\xi(A(\xi) \to \exists\zeta B(\xi,\zeta)).$$

COROLLARY 3. *For every sentence* $\forall\xi^1(A(\xi) \to \exists\zeta^1 B(\xi,\zeta))$ *where* $A(\xi)$ *and* $B(\xi,\zeta)$ *are* $\exists$*-free, there exists a term* $t^{1\to 1}$ *of* $\mathsf{E\text{-}PA}^\omega$ *such that*

$$\mathsf{E\text{-}PA}^\omega + \Pi^0_1\text{-AC}_{00} \vdash \forall\xi(A(\xi) \to \exists\zeta B(\xi, t\xi))$$

if and only if

$$\mathsf{EL} + \text{AC}_{00} + \Sigma^0_2\text{-DNS}^0 \vdash \forall\xi(A(\xi) \to \exists\zeta B(\xi,\zeta)).$$

These also hold for $\widehat{\mathsf{E\text{-}PA}}^\omega\restriction$, $\widehat{\mathsf{E\text{-}HA}}^\omega\restriction$ and EL_0, which are the restrictions of corresponding systems to primitive recursion of type 0 and quantifier-free induction, instead of $\mathsf{E\text{-}PA}^\omega$, $\mathsf{E\text{-}HA}^\omega$ and EL. Note that $\widehat{\mathsf{E\text{-}PA}}^\omega\restriction + \Pi^0_1\text{-AC}_{00}$ contains arithmetical comprehension, which allows to develop most of ordinary mathematics. It is known that most of Bishop's constructive mathematics can be formalized in $\mathsf{EL} + \text{AC}_{00}$.

[1] M. FUJIWARA, *Intuitionistic provability versus uniform provability in RCA*. ***Lecture Notes in Computer Science***, vol. 9136 (2015), pp. 186–195.

▸ MURDOCH J. GABBAY, *Consistency of Quine's NF using nominal techniques*.
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Naive set theory has one rule; *naive sets comprehension*:

If ϕ is a predicate, then $\{a \mid \phi(a)\}$ is a set (the a such that ϕ).

This is inconsistent by Bertrand Russell's famous observation of 1901 that $\{a \mid a \notin a\} \in \{a \mid a \notin a\}$ if and only if $\{a \mid a \notin a\} \notin \{a \mid a \notin a\}$.

Solutions proposed included Zermelo–Fraenkel set theory, simple type theory, and Quine's New Foundations (NF). Zermelo–Fraenkel set theory restricts comprehension so we can only form comprehension within an existing set. Simple type theory imposes a type system.

Quine's NF weakens comprehension by restricting it to *stratifiable formulae*; formulae in which variables can be assigned 'levels', which are natural numbers, such that if $a \in b$ occurs in a formula and a has level n, then b must have level $n+1$. Russell's example is clearly ruled out because $a \in a$ cannot be stratified.

Consistency of NF has been an open problem since it was proposed by Quine in 1937. I will present a claimed proof of consistency of Quine's NF, based on ideas previously developed to extend duality theory to logics with quantifiers (the paper is available on my webpage and on arXiv).

In the paper:

- Stratifiability corresponds to a simple normalisability property on terms,
- substitution corresponds to both a nominal algebra axiomatisation and a renormalisation procedure,
- universal quantification corresponds to a colimit in nominal lattices, and to a form of the nominal new-quantifier (and existential quantification to its dual),
- sets comprehension corresponds to nominal atoms-abstraction, and
- extensionality corresponds to a nonevident transfinite construction which amounts to saturating an equality theory with all possible equalities to ensure that extensionality holds.

The end result is a points-based representation of NF in which predicates correspond to sets of points, and points are deductively closed sets of predicates subject to some interesting filter-style conditions.

▸ MURDOCH J. GABBAY AND MICHAEL J. GABBAY, *What are variables of first-order logic and the lambda-calculus?*

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Intuitively, conjunction corresponds to sets intersection; disjunction corresponds to sets union; and negation corresponds to sets complement. If we axiomatise these connectives in universal algebra then we obtain Boolean algebras.

Every Boolean algebra can be presented as a field of sets, and in this sense conjunction *is* sets intersection; disjunction *is* sets union; and negation *is* sets complement.

First-order logic has variables. The usual approach to variables is to assign them values with a *valuation*, which is just a lookup table.

An alternative is offered by *nominal algebra*, an extension of universal algebra with nominal-style names and binding. This admits a simple finite axiomatisation of variables and substitution as just another 'connective'. That is, substitution is a single logical connective, with an associated standard finite algebraic theory (no axiom-schemes).

Furthermore, we can finitely axiomatise binders in nominal algebra, including:

- the $\forall$ universal quantifier of first-order logic,
- the λ of the untyped lambda-calculus, and
- the sets comprehension binder in set theory.

Remarkably, we can extend the presentation as a field of sets to these algebraic theories, and even extend Stone duality to give full topological dualities for first-order logic and the untyped lambda-calculus (the treatment of set theory is the topic of a separate abstract).

By this account, substitution, quantification, and λ-abstraction become

- axiomatisations in nominal algebra alongside those for conjunction, disjunction, and negation; and dually, they become
- operations on sets of points of topological spaces—which turn out to be fairly elementary—existing alongside sets intersection, union, and complement.

This approach to semantics differs from what is usually found in the textbooks, and it offers some technical advantages: the axioms for $\forall$ and λ do not reference the axioms for substitution, or vice versa, and this *decoupling* of the algebraic theories is reflected in the concrete proofs and modular constructions. A general method seems to be emerging here.

The end results are clean, elegent, nonevident, and suggestive of future work.

▸ GUIDO GHERARDI, PAOLO MAFFEZIOLI, AND EUGENIO ORLANDELLI, *Interpolation theorem for first-order theories*.
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In this work we prove the interpolation theorem for some first-order theory. Previous results by Negri and von Plato [2] and Dyckhoff and Negri [1] have shown how to prove cut elimination in the presence of nonlogical rules. Often cut elimination allows a constructive proof of the interpolation theorem. Thus, the question as to whether and to what extent interpolation holds in first-order theories naturally arises. Our aim is to give an answer to this question. First, we show that the standard Takeuti–Maehara techinique fails when first-order axioms are formulated as rules. Then, using an alternative strategy based on negative normal forms, we identify a class of theories for which interpolation holds. The proof we give is entirely constructive. As a case study, we consider the theory of strict partial orders and we show how to extend the result to linear orders as well.

[1] R. Dyckhoff and S. Negri, *Geometrization of first-order logic*, this Bulletin, vol. 21 (2015), no. 2, pp. 123–163.

[2] S. Negri and J. von Plato, ***Proof Analysis***, Cambridge University Press, 2011.

▸ SERGEY GONCHAROV, NIKOLAY BAZHENOV, AND MARGARITA MARCHUK, *Autostability relative to strong constructivizations of computable 2-step nilpotent groups*.
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For a class K of structures, closed under isomorphism, the index set is the set I(K) of all indices for computable members of K in a universal computable numbering of all computable structures for a fixed computable language. We study the complexity of the index set of class of computable structures, which are autostable relative to strong constructivizations.

A computable model $\mathcal{M}$ is called strongly constructivizable if there exists a decidable model $\mathcal{N}$ such that $\mathcal{N}$ is isomorphic to $\mathcal{M}$. A strongly constructivizable model $\mathcal{M}$ is autostable relative to strong constructivizations if for any decidable copies $\mathcal{N}_0$ and $\mathcal{N}_1$ of the model $\mathcal{M}$, there is a computable isomorphism $f\colon \mathcal{N}_0 \to \mathcal{N}_1$.

Using the result of [1] for rings and a coding of rings into groups due to Mal'cev [2] we prove the following theorem.

Theorem 1. *The index set* $SCAut(Gr)$ *of computable* 2*-step nilpotent groups, which are autostable relative to strong constructivizations is* m*-complete* $\Sigma^0_3(\emptyset^\omega)$.

Acknowledgments. This work was supported by RFBR (grant 14-01-00376) and by the Grants Council (under RF President) for State Aid of Leading Scientific Schools (grant NSh-6848.2016.1). The second author was supported by RFBR (research project No. 16-31-60058 mol_a_dk).

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[2] A. Malcev, *On a correspondence between rings and groups*. ***American Mathematical Society Translations, Series 2***, vol. 45 (1965), pp. 221–232.

▸ SHERWOOD HACHTMAN, *Determinacy and admissible reflection*.
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For transitive sets X, let $\mathfrak{M}(X)$ denote the least admissible set containing X as an element. Say κ *reflects admissibly* if for any Π_1 formula φ and $A \subseteq V_\kappa$, if $(V_{\kappa+1}, \in, A) \models \varphi$, then for some $\alpha < \kappa$, $(V_{\alpha+1} \cap \mathfrak{M}(V_\alpha \cap A), \in, A \cap V_\alpha) \models \varphi$. Note that if we replaced $\mathfrak{M}(V_\alpha)$ with $V_{\alpha+1}$, this would simply be weak compactness of κ. But admissible reflection is much weaker.

Our interest in this principle is due to its use in calibrating the strength of determinacy hypotheses. We show that the minimal model of NBG + "ON reflects admissibly" satisfies clopen, but not open, determinacy for proper class games; this answers a question of Gitman and Hamkins.

▸ SIMON HEWITT, *Indefinite extensibility and set-theoretic relativity*.
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Following Dummett, several authors have responded to the set-theoretic paradoxes by claiming that the concept *set* is indefinitely extensible [1]. One way of understanding this is in terms of ontological extensibility: however many sets there are, it is always possible that there be more. Recent work by Gabriel Uzquiano provides an alternative characterisation of indefinite extensibility, laying out an account of set-formation within a fixed domain modal logic [2]. The modality is interpreted linguistically, in terms of possible expansions of the extensions of predicates. We review Uzquiano's proposal, and make clear the underlying philosophical picture of set-theoretic ontology. We argue that this picture sits uncomfortably with widespread convictions about the nature of sets. More seriously, we call into question whether the proponent of a linguistic account of indefinite extensibility is entitled to assume a sufficiently large cardinality of objects to recover set-theory.

[1] M. DUMMETT, ***Frege: Philosophy of Mathematics***, Duckworth, 1991.

[2] G. UZQUIANO, *Varieties of indefinite extensibility*. ***Notre Dame Journal of Formal Logic***, vol. 56 (2015), no. 1, pp. 147–166.

▸ TANMAY INAMDAR, *A fragment of PFA consistent with large continuum*.
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The side condition method of Todorčević is an important technique to build proper partial orders. It has been used to establish several consequences of PFA, the most important of which are the Open Graph Axiom as well as the P-Ideal Dichotomy. Many of these applications can be reformulated to assert that given a graph on an uncountable set, if there is a proper σ-ideal which is well behaved in a certain way with respect to this graph, then a certain partial order to add an uncountable clique to this graph is proper. On the other hand, in recent years Asperó and Mota have developed new techniques to iterate proper partial orders which have allowed them to establish the consistency of several consequences of PFA with the continuum large. In my talk I shall talk about how using the methods of Asperó and Mota, one can get models where the continuum is arbitrarily large, Martin's Axiom holds, and a certain 'side condition forcing axiom' holds for graphs on ω_1 and ω_1-generated σ-ideals. For example, such models have no S-spaces, $\omega_1 \to (\omega_1, \alpha)^2$ holds for any countable ordinal α, and certain restricted forms of OGA and PID hold.

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It is easy to show that

- For every family $\mathcal{S}$ on n-c.e. sets, if the Rogers semilattice $\mathcal{R}_m^{-1}(\mathcal{S})$ is infinite for some $m \geq n$ then $\mathcal{R}_m^{-1}(\mathcal{S})$ is infinite for all $k \geq m$.
- For every two-element family of n-c.e. sets $\mathcal{S} = \{A, B\}$, the Rogers samilattices $\mathcal{R}_m^{-1}(\mathcal{S})$ is infinite if $m > 2n$. If n is even, then this statement is true for $m = 2n$.

All known Rogers semilattices of the family in the hierarchy of Ershov are either one-element or infinite.

THEOREM ([1]). *For every nonzero $n \in \omega \cup \{\omega\}$, and for every ordinal notation a of a nonzero ordinal, there exists a Σ_a^{-1}-computable family $\mathcal{A}$ of exactly n sets such that $|\mathcal{R}_a^{-1}(\mathcal{A})| = 1$.*

Main result:

THEOREM. *For every nonzero $n \in \omega$, there exist n-c.e. sets A and B such that*

$$|\mathcal{R}_n^{-1}(A, B)| = |\mathcal{R}_{n+1}^{-1}(A, B)| = \cdots = |\mathcal{R}_{2n}^{-1}(A, B)| = 1 \textit{ if } n \textit{ is odd},$$
$$|\mathcal{R}_n^{-1}(A, B)| = |\mathcal{R}_{n+1}^{-1}(A, B)| = \cdots = |\mathcal{R}_{2n-1}^{-1}(A, B)| = 1 \textit{ if } n \textit{ is even}.$$

COROLLARY. *For every nonzero $n \in \omega$ and for every $n < m \leq 2n$, there exist n-c.e. sets A and B such that*

$$1 = |\mathcal{R}_n^{-1}(A, B)| = \cdots = |\mathcal{R}_{m-1}^{-1}(A, B)| < |\mathcal{R}_m^{-1}(A, B)|.$$

QUESTION. *Does there exist a family of sets in some level of the hierarchy of Ershov whose Rogers semilattice consists of* 2, 3, ... *elements*?

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https://doi.org/10.1017/bsl.2017.16 Published online by Cambridge University Press

► MALTE S. KLIESS, *Strong predicate exchangeability in inductive logic*.
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In Pure Inductive Logic we study the consequences for an agent's rational belief function given that she assumes a list of rational principles [4], in the framework following Carnap's Inductive Logic [1].

The Principle of Strong Predicate Exchangeability, or SPx, is a rational principle stating that an agent's belief in a sentence ϕ should be the same as that in the statement ψ, where ψ is obtained from ϕ by permuting the atoms occurring in ϕ freely as long as the number of negations remains fixed. The principle was discovered during work on Predicate Exchangeability [2, 3].

The principle is in strength between Predicate Exchangeability and Atom Exchangeability, in the sense that each probability function satisfying a stronger principle also satisfies the weaker one, but not vice versa. Representation Theorems for SPx show a remarkable similarity to those for Atom Exchangeability. In terms of justification, we can obtain SPx both through a generalization of Atom Exchangeability and a weakening of Johnson's Sufficientness Postulate.

We will give an outline of the representation theorems and show the relation between the rational principles.

[1] R. Carnap, *A basic system of inductive logic*, ***Studies in Inductive Logic and Probability*** (R. Carnap and R. C. Jeffrey, editors), University of California Press, 1971, pp. 33–165.

[2] M. S. Kließ, ***The principle of predicate exchangeability in pure inductive logic***, Ph.D. thesis, University of Manchester, 2013.

[3] M. Kließ and J. Paris, *Predicate Exchangeability and Language Invariance in Pure Inductive Logic*. ***Logique et Analyse***, vol. 57 (2014), no. 228, pp. 513–540.

[4] J. Paris and A. Vencovská, ***Pure Inductive Logic***, Perspectives in Logic, Cambridge University Press, 2015.

► NURLAN KOGABAEV, *Freely generated projective planes with finite computable dimension*.
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In [2] it was proved that every countable free projective plane has computable dimension either 1 or ω. Futhermore, such a plane is computably categorical if and only if it has finite rank.

Hence the natural question arises: can we extend the above results to the case of freely generated projective planes? In particular, we are interested in the existence question of computably categorical freely generated projective plane of infinite rank. We also investigate the realizability of finite computable dimension $n > 1$ in the class of freely generated projective planes.

In [1] it was shown that the class of symmetric irreflexive graphs is *complete* in the following computable-model-theoretic sense: for every countable structure $\mathcal{A}$, there exists a countable symmetric irreflexive graph $\mathcal{G}$ which has the same degree spectrum as $\mathcal{A}$, the same $\mathbf{d}$-computable dimension as $\mathcal{A}$ (for each degree $\mathbf{d}$), the same computable dimension as $\mathcal{A}$ under expansion by a constant, and which realizes every degree spectrum $\mathrm{DgSp}_{\mathcal{A}}(R)$ (for every relation R on $\mathcal{A}$) as the degree spectrum of some relation on $\mathcal{G}$.

In the present paper we construct an effective coding of symmetric irreflexive graphs into freely generated projective planes preserving most computable-model-theoretic properties and obtain the following result.

Theorem. *The class of freely generated projective planes is complete with respect to degree spectra of nontrivial structures, **d**-computable dimensions, expansion by constants, and degree spectra of relations.*

In particular, for every natural $n \geqslant 1$ there exists a freely generated projective plane of infinite rank with computable dimension n.

Acknowledgments. This work was supported by RFBR (grant 14-01-00376-a) and by the Grants Council under RF President for State Aid of Leading Scientific Schools (grant NSh-6848.2016.1).

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► RUSLAN KORNEV, *Reducibilities of computable metrics on the real line*.

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We are concerned with a natural computable-model-theoretic question for uncountable metric spaces: do there exist computable metric spaces, equivalent in some standard sense but not computably equivalent? Pour-El and Richards [3] considered different countable dense substructures of a given Banach space up to computable isometries; see also more recent papers [1] and [2]. In contrast, our goal is to construct computably inequivalent metrics on a separable space with *fixed* dense subset. In this talk, two notions of computable reducibility of metrics are discussed.

One of them is induced by reducibility of Cauchy representations. Namely, let ρ and ρ' be complete metrics on a separable space X, let W be a countable dense subset of X, enumerated by integers. We say that the metric ρ is computably reducible to the metric ρ' (and denote this as $\rho \leq_c \rho'$) if the Cauchy representation δ_ρ of effective metric space (X, ρ, W) is computably reducible to the representation $\delta_{\rho'}$ of the space (X, ρ', W); precise definitions can be found in [4].

It is possible to characterize $\leq_c$ in the following terms: $\rho \leq_c \rho'$ iff the identity homeomorphism id_X is $(\delta_\rho, \delta_{\rho'})$-computable. Based on this, we introduce weak reducibility of metrics: we say $\rho \leq_{ch} \rho'$ if there exists a $(\delta_\rho, \delta_{\rho'})$-computable autohomeomorphism of X. Clearly, c-reducibility implies ch-reducibility.

The results obtained are mainly related to the case $X = \mathbb{R}$ with the standard real line topology and $W = \mathbb{Q}$.

Theorem 1. *All convex (i.e., admitting midpoints) computable metrics on the reals are c-equivalent.*

Theorem 2. *Any countable tree T can be isomorphically embedded into the ordering of computable metrics on the reals under c-reducibility.*

Theorem 3. *There exists a countable sequence of computable metrics on $\mathbb{R}$ which are not ch-reducible to each other. Informally, copies of the real line, equipped with these metrics, are pairwise homeomorphic, but not computably homeomorphic.*

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► GRAHAM E. LEIGH, *The simple truth*.

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What is implicit in the acceptance of the Tarskian truth biconditionals? In this talk I will present recent results that characterise the proof- and truth-theoretic content of iterated reflection principles over disquotational theories of truth. In particular, I confirm the conjecture that, modulo reflection, all there is to compositional and Kripke–Feferman truth is captured by simple and natural collections of local truth and falsity biconditionals.

▸ MICHAEL LIEBERMAN AND JIŘÍ ROSICKÝ, *Abstract tameness from large cardinals, via accessible categories*.

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Tameness of types in an abstract elementary class—the principle that distinct types over large models can be distinguished by restrictions to a submodel of fixed small size—appears as a necessary condition in nearly all of the major classification-theoretic results for AECs. By a theorem of [1], tameness of AECs is known to follow from the existence of a proper class of (almost) strongly compact cardinals. The authors reprove this result in [3] by entirely different methods, namely by reducing tameness to the accessibility of the (powerful) image of a certain accessible functor. We show that this method extends naturally to prove that the metric analogue of tameness holds for metric AECs (analyzed as in [4]) under the same cardinal assumption. While this fact is already known, having been proven by different means in [2], we note that our analysis provides a template for the analysis of tameness more broadly. As in the metric case, we may consider notions of tameness intrinsic to the ambient category of objects over which an abstract class' structures are built—the characterization via accessible images provides a natural, unified framework in which to address these generalized notions.

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[2] W. Boney and P. Zambrano, *Around the set-theoretical consistency of d-tameness of abstract elementary classes*, arXiv:1508.05529.

[3] M. Lieberman and J. Rosický, *Classification theory for accessible categories*. ***The Journal of Symbolic Logic***, vol. 81 (2016), no. 1, pp. 151–165.

[4] ———, *Metric abstract elementary classes as accessible categories*, arXiv:1504.02660.

▸ ROBERT LUBARSKY, *D-Fan and c-Fan*.

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Brouwer's Fan Theorem—that every binary tree with no infinite path is finite—is an important principle in constructive mathematics. Various fragments of the Fan Theorem, which limit the trees to which it applies, have been of interest over the years, because they are equivalent to basic principles of analysis. These weakenings of Fan are easily seen to be implicationally linear:

$$\mathrm{FAN}_{\mathrm{full}} \Rightarrow \mathrm{FAN}_{\Pi^0_1} \Rightarrow \mathrm{FAN}_c \Rightarrow \mathrm{FAN}_\Delta.$$

The obvious question is whether those implications are strict. Some of them were shown over the years to be strict, by a variety of methods; in [4], they were all shown to be strict, by a fairly uniform method. Here I provide a new proof that Decidable Fan (FAN_Δ) does not imply c-Fan (FAN_c). The argument is a mixture of realizability, Heyting-valued models, and Kripke models. It remains possible, yet still unknown, that the same method will separate the other implications.

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[2] J. Berger, *A separation result for varieties of Brouwer's Fan Theorem*, ***Proceedings of the 10th Asian Logic Conference (ALC 10), Kobe University in Kobe, Hyogo, Japan, September 1–6, 2008*** (Arai et al., editors), World Scientific, 2010, pp. 85–92.

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[4] R. Lubarsky and H. Diener, *Separating the Fan Theorem and its weakenings*. ***The Journal of Symbolic Logic***, vol. 79 (2014), pp. 792–813.

► PHILIPP LÜCKE, *The infinite productivity of Knaster properties*.
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Given an uncountable regular cardinal κ, we say that a partial order $\mathbb{P}$ is κ-*Knaster* if every set of κ-many conditions in $\mathbb{P}$ contains a subset of cardinality κ consisting of pairwise compatible conditions. This strengthening of the κ-chain condition is typically used because of its nice product behavior: finite support products of κ-Knaster partial orders are κ-Knaster, and the product of a κ-Knaster partial order with a partial order satisfying the κ-chain condition satisfies the κ-chain condition. Moreover, if κ is weakly compact, then the class of κ-Knaster partial orders is closed under ν-support products for every $\nu < \kappa$. This raises the question whether it is possible that the class of κ-Knaster partial orders is closed under countable support products and κ is not weakly compact. I will present results that show that the axioms of ZFC do not answer this question.

This is partially joint work with Sean Cox (VCU Richmond).

► RYSZARD MIREK, *Renaissance geometry*.
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Renaissance mathematicians and geometers like Piero della Francesca and Luca Pacioli refers directly or indirectly to Euclidean geometry. For instance, Piero della Francesca in his proofs refers to the similarity of the triangles. In *Elements* discussion of these issues is in the Book VI, Propositions 4 to 8. There is also no doubt that Piero is familiar with Book XIII of *Elements*. Piero constructs each of the regular solids from its circumsphere, but unlike Euclid, he gives a numerical value for the diameter of the sphere. In turn, in Proposition I.8 he shows that the perspective images of orthogonals converge to a *centric point* what follows from Euclid's theorem from the Book VI, Proposition 21. On the other hand, Luca Pacioli provides a direct reference to Elements and the precise information are given to where the result is proved by Euclid.

My goal here is to provide a detailed analysis of the methods of inference that are employed in the Renaissance treatises. For this purpose one can use a formal system *EF* that seems to present in a precise and visually readable way the geometrical systems.

Acknowledgment. This research is supported by the NCN research grant 2012/07/B/HS1/01986.

► ALIREZA MOFIDI, *Some symbolic dynamical views in model theory*.
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The topological dynamical aspects of model theory in particular stability theory has been recently investigated in several papers such as [2, 3] and [4]. We will have a symbolic dynamical and ergodic theoretical point of view, two other essential point of views in the theory of dynamical systems, to the action of automorphisms and definable groups on certain model theoretic objects, such as stone spaces, models, etc., in particular in the presence of invariant measures. Note that applications of measures as a technique in stability theory are extensively studied in several papers such as [1]. We borrow the notion of symbolic representation from dynamical systems theory and develop it in the context of model theory. In particular, the symbolic representations of actions on spaces of types will be under consideration. We generalize this notion in a way that deals with some standard definition of products of types (studied in for example [3]) as well as product of measures. In the particular case of the action of the group $\mathbb{Z}$ (which includes the case of action of single automorphisms), we characterize some stability theoretic hierarchies in particular NIP. Moreover, we make connections between the symbolic representations of spaces of types and certain mathematical notions such as the rotation number of circle homeomorphisms, Bohr sets and some additive number theoretic definitions.

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▶ CLIVE NEWSTEAD, *Categories of natural models of type theory*.
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Natural models of type theory (see [1]) provide an algebraic setting for the interpretation of dependent type theory. First, we define homomorphisms of natural models of type theory with a basic type and context extension, and prove that the syntactic category of contexts is initial in this category. We then extend our construction to allow for dependent sums and products. Time permitting, we prove that, in this latter case, the polynomial functor associated with a given natural model is a polynomial monad.

This is joint work with Steve Awodey.

[1] S. Awodey, *Natural models of homotopy type theory*, 2015, eprint `arXiv:1406.3219v2`.

▶ CARLO NICOLAI, *Transfinite induction and reflection for some systems of truth in basic De Morgan logic*.
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Semantic paradoxes force us to question our naïve intuition about truth, encompassed in the schema 'ϕ' *is true if and only if* ϕ. They may be approached by giving up the naïve truth schema and retain classical logic; alternatively, one may retain our naïve intuition and abandon classical logic. It has been argued that the latter approach severely cripples our capability of employing mathematical—or more generally extra-semantic – patterns of reasoning in semantics [2]. This is usually motivated by the different amount of transfinite induction that is provable in the classical axiomatization of the fixed point construction of [4]—due to [1] and known as KF—and the corresponding nonclassical system PKF formulated in a logic B featuring only introduction and elimination rules for positive connectives (and their negations) [3]. In the talk we consider several subsystems or extensions of PKF: we focus on (i) a basic disquotational theory extending B with arithmetical axioms and the basic principles 'if ϕ, then 'ϕ' is true', and 'if 'ϕ' is true, then ϕ' for all sentences in the language with the truth predicate; (ii) a variant of PKF with no semantic induction; (iii) the extension of PKF with a rule of transfinite induction up to ε_0 for the entire vocabulary. We first measure the strength of the systems by comparing them to the sentences provably true in KF and variants thereof. We then consider a strategy for employing—suitably adjusted—reflection principles to climb up, in a rather natural way, from (i) to (iii) via (ii). We conclude by examining the role of reflection and induction in the debate between advocates of the classical and the nonclassical approaches sketched above.

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[2] V. Halbach, ***Axiomatic Theories of Truth***, revised ed., Cambridge University Press, 2014.

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https://doi.org/10.1017/bsl.2017.16 Published online by Cambridge University Press

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▸ SERGEY OSPICHEV, *Minimal numberings of partial computable functionals*.
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One of the main questions in numbering theory is studying extremal elements of Rogers semilattices of different families. Here we concentrate our interest on partial computable functionals of finite types.

Let's define *functional type*. Let T will be the set of all types.

1. $0 \in T$;
2. if σ, τ are types, then $(\sigma \times \tau)$ and $(\sigma|\tau)$ are also types;
3. T - minimal set, satisfying 1 and 2.

Now we define *partial computable functionals*. Let C_σ be family of all partial computable functionals of type σ. Let C_0 be the family of all partial computable functions or the family of all computable enumerable sets. If C_σ and C_τ are already defined, then $C_{(\sigma\times\tau)} \rightleftharpoons C_\sigma \times C_\tau$ and $C_{(\sigma|\tau)} \rightleftharpoons \mathfrak{Mor}(C_\sigma, C_\tau)$.

Any C_σ has natural universal numbering ν_σ by definition. So we call numbering μ σ-computable if μ is reducible to ν_σ.

In work are proven

Theorem. *For any $\sigma \in T$ there are infinitely many nonequivalent σ-computable friedberg numberings of family C_σ.*

Theorem. *For any $\sigma \in T$ there are infinitely many nonequivalent σ-computable positive undecidable numberings of family C_σ.*

Theorem. *For any $\sigma \in T$ there is infinite σ-computable family $\mathcal{S} \subset C_\sigma$ without σ-computable friedberg numberings.*

Acknowledgments. The reported study was partially supported by RFBR, research project No. 14-01-00376 and by the Grants Council (under RF President) for State Aid of Leading Scientific Schools (grant NSh-6848.2016.1).

▸ FRANCO PARLAMENTO AND FLAVIO PREVIALE, *The cut elimination and nonlengthening property for Gentzen's sequent calculus for first order logic with equality*.
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Leibniz's indiscernibility principle, in the framework of second order logic, leads to a sequent calculus for first order logic with equality, that satisfies the cut elimination theorem, but cut free derivations may not satisfy the subformula property. We note that instead, the path described by von Plato in his historical reconstruction of Gentzen's discovery of the sequent calculus in [2], leads to a calculus that is fully satisfactory. In addition to the reflexivity axiom $\Rightarrow t = t$, it has the following two left introduction rules for $=$:

$$\frac{\Gamma \Rightarrow \Delta, F\{v/r\}}{\Gamma, r = s \Rightarrow \Delta, F\{v/s\}} =_1 \quad \frac{\Gamma \Rightarrow \Delta, F\{v/s\}}{\Gamma, r = s \Rightarrow \Delta, F\{v/r\}} =_2.$$

Other satisfactory calculi can be obtained by taking into account the following other rules:

$$\frac{\Gamma, F\{v/r\} \Rightarrow \Delta}{\Gamma, F\{v/s\}, r = s \Rightarrow \Delta} =_1^l \quad \frac{\Gamma, F\{v/s\} \Rightarrow \Delta}{\Gamma, F\{v/r\}, r = s \Rightarrow \Delta} =_2^l.$$

We give a very simple proof that cut elimination holds for a calculus obtained by adding to the reflexivity axiom some of the above four rules if and only if it contains (at least) $=_1$ and $=_2$ or $=_1$ and $=_1^l$ or $=_2$ and $=_2^l$. The admissibility results that are used, can be refined and extended in order to show that if (and only if) all the above four rules are added, then every derivation can be trasformed into one that is cut-free and satisfies the nonlenthening property of [1].

[1] A. V. LIFSCHITZ, *Specialization of the form of deduction in the predicate calculus with equality and function symbols*, ***The Calculi of Symbolic Logic I*** (V. P. Orevkov, editor), Proceedings of the Steklov Institute of Mathematics ,vol. 98, 1971, pp. 1–23.

[2] J. VON PLATO, *Gentzen's proof systems: Byproducts in a work of genius*, this BULLETIN, vol. 18 (2012), no. 3, pp. 317–367.

▸ ARNO PAULY, *Computability on the space of countable ordinals*.
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While there is a well-established notion of what a computable ordinal is, the question which functions on the countable ordinals ought to be computable has received less attention so far (but cf. [1]). In order to remedy this, we explore various potential representations (in the sense of computable analysis [3, 6]) of the set of countable ordinals. An equivalence class of representations is then suggested as a standard, as it offers the desired closure properties. This class is characterized exactly by the computability of four specific operations.

We show that the supremum of a continuous function from Baire space into the countable ordinals can be computed from a name of such function. The binary infimum is also a computable operation, and with some caveat regarding finite values, even countable infima are computable.

With a decent notion of computability on the space of countable ordinals in place, we can then state and prove a computable uniform version of the Hausdorff–Kuratowski theorem. We can also show yet another proof of the computable Lusin separation theorem (cf. [2]).

A preprint is available on the arXiv [5], and a preliminary version appeared as [4].

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[2] Y. MOSCHOVAKIS, *Classical descriptive set theory as a refinement of effective descriptive set theory*. ***Annals of Pure and Applied Logic***, vol. 162 (2010), pp. 243–255.

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[4] ———, *Computability on the Countable Ordinals and the Hausdorff–Kuratowski Theorem (Extended Abstract)*, ***Mathematical Foundations of Computer Science 2015*** (G. Italiano, G. Pighizzini, and D. Sannella, editors), Lecture Notes in Computer Science, vol. 9234, Springer, 2015.

[5] ———, *Computability on the countable ordinals and the Hausdorff–Kuratowski theorem*, 2015, `arXiv 1501.00386`.

[6] K. WEIHRAUCH, ***Computable Analysis***, Springer, 2000.

▸ EDOARDO RIVELLO, *A revision-theoretic general theory of definitions*.
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The classical theory of definitions bans so-called *circular* definitions, namely, definitions of, say, a unary predicate P based on stipulations of the form

$$P(x) =_{\mathrm{Df}} \Phi(P, x),$$

where Φ is a formula of a fixed first-order language and the *definiendum* P occurs into the *definiens* Φ.

In their seminal book *The Revision Theory of Truth* [1], Gupta and Belnap claim that "*General* theories of definitions are possible within which circular definitions [...] make logical and semantic sense" [p. IX]. In order to sustain their claim, they develop in this book one general theory of definitions (in some variants) based on *revision sequences*, namely, ordinal-length iterations of the operator which is induced by the (possibly circular) definition of the predicate.

Gupta–Belnap's approach to circular definitions has been criticised, among others, by Martin [2] and McGee [3]. Their criticisms point on the logical complexity of revision sequences, on their relations with ordinary mathematical practice, and on their merits relative

https://doi.org/10.1017/bsl.2017.16 Published online by Cambridge University Press

to alternative approaches. In my talk I will present an alternative general theory of definitions, based on a combination of supervaluation and ω-length revision, which aims to address some issues raised by Martin and McGee while preserving the philosophical and mathematical core of revision.

[1] A. Gupta and N. Belnap, ***The Revision Theory of Truth***, A Bradford Book, MIT Press, 1993.

[2] D. A. Martin, *Revision and its rivals*. ***Philosophical Issues***, vol. 8 (1997), pp. 387–406.

[3] V. McGee, *Revision*. ***Philosophical Issues***, vol. 8 (1997), pp. 407–418.

▶ LUCA SAN MAURO, *Complexity of relations via computable reducibility*.
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Computable reducibility provides a natural way of ranking binary relations on ω according to their complexity. Let R and S be two binary relations, we say that R is *computably reducible* to S iff there is a computable function f such that, for all $x, y \in \omega$, the following holds:

$$x\ R\ y \Leftrightarrow f(x)\ S\ f(y).$$

Computable reducibility has been object of study for decades, being mostly applied to the case of equivalence relations. In particular, a prominent problem in the area has been that of characterizing *universal* equivalence relations, i.e., relations to which all others relations, of a given complexity, can be computably reduced.

In this talk, we address the problem of universality for a more general context than that of equivalence relations. First, we prove that, contrary to the case of equivalence relations and preorders, for each level of the arithmetical hierarchy there is a universal binary relation. Then, we define natural examples of universal Σ^0_n binary relations and of universal Π^0_n binary relations. More precisely, let $U_n^{\in}$ be the following Σ^0_n binary relation,

$$x\ U_n^{\in}\ y \Leftrightarrow x \in W_y^{\emptyset^{(n-1)}},$$

and, for $n > 2$, let $U_n^{\subseteq}$ be the following binary relation

$$x\ U_n^{\subseteq}\ y \Leftrightarrow W_x \subseteq W_y^{(n-2)}.$$

We show that:

1. For all n, $U_n^{\in}$ is a universal Σ^0_n binary relation;
2. There exists a total computable function f such that the following binary relation
$$x\ U_2^{\subseteq}\ y \Leftrightarrow W_x \subseteq W_{f(y)}$$
is a universal Π^0_2 binary relation;
3. For $n > 2$, $U_n^{\subseteq}$ is a universal Π^0_n binary relation.

[1] U. Andrews, S. Lempp, J. S. Miller, K. M. Ng, L. San Mauro, and A. Sorbi, *Universal computably enumerable equivalence relations*. ***The Journal of Symbolic Logic***, vol. 79 (2014), no. 1, pp. 60–88.

[2] E. Ianovski, R. Miller, K. M. Ng, and A. Nies, *Complexity of equivalence relations and preorders from computability theory*. ***The Journal of Symbolic Logic***, vol. 79 (2015), no. 3, pp. 859–881.

▶ CHRISTOPH-SIMON SENJAK, *A theory of parsers*.
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Parsing is an essential problem in Computer Science, and especially since streaming of data has become popular, parsing data "online" and getting partial results as fast as possible with a reasonably small amount of memory is a main objective. There are several formal approaches to this problem (e.g., the pi calculus). We present an approach that uses relations between input and output of a parser, and properties that, when proved about those relations,

https://doi.org/10.1017/bsl.2017.16 Published online by Cambridge University Press

make them well-suited for parsing, and usually lead to efficient parsers by program extraction. We present some monad-style combinators that we use to build up more complex relations from simpler ones. We used our approach to write an implementation of Deflate (compression standard) in Coq, which shows that it is not only theoretically beautiful but also practically usable. While our own research only uses Coq, it should be easily adaptable to any dependently typed programming language.

▸ IBRAHIM SENTURK, TAHSIN ONER, AND URFAT NURIYEV, *Completeness of categorical syllogisms by means of diagrammatic method.*
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In this study, our goal is to show the completeness of categorical syllogisms by means of diagrammatic method. For this, we firstly construct a formal system SLCD (Syllogistic Logic with Caroll Diagrams), which gives us a formal approach to logical reasoning with diagrams, for representations of the fundamental Aristotelian categorical propositions and show that they are closed under the syllogistic criterion of inference which is the deletion of middle term. Therefore, it is implemented to let the formalism comprise synchronically bilateral and trilateral diagrammatical appearance and a naive algorithmic nature. And also, there is no need specific knowledge or exclusive ability to understand as well as to use it.

In other respects, we scrutinize algebraic properties of categorical syllogisms together with a representation of syllogistic arguments by using sets in SLCD. To this end, we explain quantitative relation between two terms by means of bilateral diagrams. Thereupon, we enter the data, which are taken from bilateral diagrams, on the trilateral diagram. With the help of elemination method, we obtain a conclusion which is transformed from trilateral to bilateral diagram. A categorical syllogistic system consists of 256 syllogistic moods, 15 of which are unconditionally and 9 are conditionally; in total 24 of them are valid. Those syllogisms in the conditional group are also said to be *strengthened*, or valid under *existential import*, which is an explicit assumption of existence of some S, M or P. So, we add a rule, which is *Some* X *is* X when X exists, to SLCD. Therefore, we obtain the formal system SLCD† from SLCD.

Finally, we show that syllogism is valid if and only if it is provable in SLCD and strengthened syllogism is valid if and only if it is provable in SLCD†. This means that SLCD is sound and complete.

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[2] A. E. Kulinkovich, *Algorithmization of resoning in solving geological problems*, ***Proceedings of the Methodology of Geographical Sciences***, Naukova Dumka, 1979, pp. 145–161.

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▸ ANDREI SIPOŞ, *Proof mining and positive-bounded logic*.
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Proof mining is a research program introduced by U. Kohlenbach in the 1990s ([3] is a comprehensive reference), which aims to obtain explicit quantitative information (witnesses and bounds) from proofs of an apparently ineffective nature. This paradigm in applied logic has successfully led so far to obtaining some previously unknown effective bounds, primarily in nonlinear analysis and ergodic theory. A large number of these are guaranteed to exist by a series of logical metatheorems which cover general classes of bounded or unbounded metric structures.

In order to apply these metatheorems, the structures are typically formalized in higher-order systems of arithmetic and analysis, using appropriate codings of real numbers and related operations. The classes for which metatheorems have already been proven include normed spaces and hyperbolic spaces. Recently, Günzel and Kohlenbach [1] have shown that, in principle, one could obtain metatheorems for a plethora of classes of structures, provided that they are formalized in positive-bounded logic (in the sense of Henson and Iovino [2]) and that some preparation of the axioms is undertaken beforehand. We aim to show how this process may be carried out in some additional classes suggested by the two authors above. We illustrate it with some concrete applications.

[1] D. Günzel and U. Kohlenbach, *Logical metatheorems for abstract spaces axiomatized in positive bounded logic*. ***Advances in Mathematics***, vol. 290 (2016), pp. 503–551.

[2] C. Ward Henson and J. Iovino, *Ultraproducts in analysis*, ***Analysis and Logic***, London Mathematical Society Lecture Note Series, vol. 262, Cambridge University Press, 2002, pp. 1–115.

[3] U. Kohlenbach, ***Applied Proof Theory: Proof Interpretations and their use in Mathematics***, Springer Monographs in Mathematics, Springer-Verlag, Berlin-Heidelberg, 2008.

▸ OLGA ULBRIKHT AND AIBAT YESHKEYEV, *Cosemanticness and JSB-property for Abelian Groups*.
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We will give some model-theoretic results of Abelian groups in the frame of Jonsson theories. In the [1] John Goodrick gave necessary and sufficient conditions of the Schroder–Bernstein (SB) property for complete theories of Abelian groups. We consider Jonsson analogue of Theorem 1 from [1].

Jonsson theory T has the Schroder–Bernstein (JSB) property if for any two models $A, B \in E_T$ from the fact that they are mutually isomorphically embeddable each other follows that they are isomorphic.

We know that theory of Abelian groups will be a Jonsson theory and also perfect.

In connection with this concept, we got the result that the following theorem is true:

Theorem 1. *Let T be a Jonsson theory of Abelian groups, then the following conditions are equivalent*:

(1) *T is $J-\omega-$stable*;
(2) *T^* is $\omega-$stable*;
(3) *T has JSB property.*

Let $A \in Mod\ \sigma_{AG}$, where $\sigma_{AG} = \langle +, -, 0\rangle$, i.e., our considered theories are universal. Denote through $JSp(A)$ Jonsson spectrum of Abelian group A, where $JSp(A) = \{T | T$ is a Jonsson theory in language σ_{AG} and $A \in ModT\}$.

We say that T_1 is cosemantic to T_2 ($T_1 \bowtie T_2$) if $C_{T_1} = C_{T_2}$, where C_{T_i} is semantic model of T_i, $i = 1, 2$. Then it is easy to notice that $JSp(A)/_{\bowtie}$ is a factor set by relation $\bowtie$ and let its power equals μ, i.e., $|JSp(A)/_{\bowtie}| = \mu$.

Theorem 2. *Let T is Jonsson theory of Abelian groups then $C_T \in E_T$ and C_T is divisible group and its a Shmelev's standart group is $\bigoplus\limits_{\kappa}\mathbb{Z}_{p^\infty} \bigoplus\limits_{\kappa} \mathbb{Q}$, where $\kappa = |C|$.*

Let's call a pair $(\alpha, \beta)^A_C$ as Jonsson invariant of Abelian group A if a Shmelev's standart group of a group A is a group of the following form $\bigoplus\limits_{\alpha}\mathbb{Z}_{p^\infty} \bigoplus\limits_{\beta} \mathbb{Q}$, where C is semantic model of $[T] \in JSp(A)/_{\bowtie}$.

The following result is a Jonsson analogue of the well-known Shmelev's theorem about the elementary classification of Abelian groups.

THEOREM 3. *Let* $A, B \in Mod\ \sigma_{AG}$ *then* $A \bowtie B \Leftrightarrow (\alpha,\beta)^A_{C_i} = (\alpha,\beta)^B_{C_i}$, $i \in I$, $|I| = \mu$.

All additional information regarding Jonsson theories can be found in [2].

[1] J. GOODRICK, *The Schroder–Bernstein property for theories of abelian groups*, 2007, arXiv.org > math > `arXiv:0705.1850v1`.

[2] A. R. YESHKEYEV, ***Jonsson Theories***, Publisher of the Karaganda State University, 2009.

▸ IMME VAN DEN BERG, *Complete arithmetical solids and nonstandard analysis*.
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A *neutrix* is a convex additive subgroup of a nonstandard model of the real numbers. Obvious neutrices are £, the external set of of limited numbers and ⊘, the external set of infinitesimals, as groups they are not isomorphic. The set of nonisomorphic neutrices is at least countable [4]. An *external number* α is the sum $\alpha = a + A$ of a nonstandard real number a and a neutrix A. Due to the stability by some shifts, external numbers may be seen as mathematical models of vague transitions of Sorites type, orders of magnitude, or errors of measurement [2].

The external numbers form a completely regular commutative semigroup (union of groups) for addition and multiplication. The distributive law holds up to a neutrix. The order relation is total and respects the operations [1, 2].

Dedekind completeness is valid in a model which is sufficiently saturated for Nelson's reduction algorithm [3] to hold: definable halflines either are cofinal with an external number, or there is an external number just beyond.

The structure is Archimedean for nonstandard natural numbers.

In a joint work with B. Dinis, University of Lisbon, we present a first-order axiomatics for the external numbers in the language $\{+, \cdot, \leq\}$, using algebraic, analytic and arithmetical axioms. A model is called a *complete arithmetical solid*. Its precise numbers ($A = \{0\}$) must be contained in a nonstandard model for the real number system.

[1] B. DINIS and I. P. VAN DEN BERG, *Algebraic properties of external numbers*. ***Journal of Logic and Analysis***, vol. 3 (2011), no. 9, pp. 1–30.

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[4] I. P. VAN DEN BERG, *A decomposition theorem for neutrices*. ***Annals of Pure and Applied Logic***, vol. 161 (2010), no. 7, pp. 851–865.

▸ SEBASTIEN VASEY, *A proof of Shelah's eventual categoricity conjecture in universal classes*.
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Abstract elementary classes (AECs) are an axiomatic framework encompassing classes of models of an $\mathbb{L}_{\infty,\omega}$ theory, as well as numerous algebraic examples. They were introduced by Saharon Shelah forty years ago. Shelah focused on generalizations of Morley's categoricity theorem and conjectured the following eventual version: An AEC categorical in a high-enough cardinal is categorical on a tail of cardinals. I will present my proof of the conjecture for *universal classes*. They are a special case of AECs (studied by Shelah in a milestone 1987 paper [1]) corresponding to classes of models of a universal $\mathbb{L}_{\infty,\omega}$ theory.

An instance of our result is:

THEOREM. *If* ψ *is a universal* $\mathbb{L}_{\omega_1,\omega}$ *sentence that is categorical in* some $\lambda \geq \beth_{\beth_{\omega_1}}$, *then* ψ *is categorical in* all $\lambda' \geq \beth_{\beth_{\omega_1}}$.

https://doi.org/10.1017/bsl.2017.16 Published online by Cambridge University Press

The proof combines Shelah's earlier work on universal classes with a study of AECs that have amalgamation and are *tame* (a locality property isolated by Grossberg and VanDieren which says roughly that orbital types are determined by their small restrictions).

[1] S. SHELAH, *Universal classes*, ***Classification Theory*** (Proceedings of the U.S.–Israel Workshop on Model Theory in Mathematical Logic held in Chicago, Dec. 15–19, 1985), (J. T. Baldwin, editor), vol. 1292, Springer Berlin Heidelberg, 1987, pp. 264–418.

▸ ROGER VILLEMAIRE, *$\aleph_0$-categorical structures for which Forth suffices*.
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Cantor's original proof that countable dense linear orders are isomorphic maps elements in a single direction. This method has been named *Forth* by P. J. Cameron, who furthermore showed, answering a question of A. Mathias, that Forth fails to yield an onto mapping for some $\aleph_0$-categorical structures. In Cameron's terminology, *Forth does not suffice* for those structures.

In [1] Cameron gave a sufficient condition for Forth to suffice that was later generalized by McLeish [2]. However, none of these conditions are necessary.

A necessary and sufficient condition for Forth to suffice has been given in [3] in terms of an countable ordinal rank on types. While [3] gives, for any countable ordinal α, a homogeneous structure with the property that the ranks of its types form α, the considered languages are infinite for $\alpha \geq \omega$. These structures are unfortunately not $\aleph_0$-categorical.

This talk will present results on the ranks that occur in $\aleph_0$-categorical structures, with an emphasis on the combinatorial constraints that the existence of a rank imposes on types.

[1] P. J. CAMERON, ***Oligomorphic Permutation Groups***, London Mathematical Society Lecture Note Series, Cambridge University Press, 1990.

[2] S. J. MCLEISH, *The forth part of the back and forth map in countable homogeneous structures*. ***The Journal of Symbolic Logic***, vol. 62 (1997), no. 3, pp. 873–890.

[3] R. VILLEMAIRE, *Homogeneity and fix-points: Going Forth!* ***The Journal of Symbolic Logic***, vol. 80 (2015), no. 2, pp. 636–660.

▸ ANDREAS WEIERMANN, *On generalized Goodstein sequences*.
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The termination property of the classical Goodstein sequences provides a simple number-theoretic assertion which is true but independent of first order Peano arithmetic PA.

In this talk we consider Goodstein sequences which are defined relative to Ackermannian functions. We discuss the following two results.

THEOREM A. *When the zero-th branch of the Ackermann function is the successor function then the induced Goodstein principle will be equivalent to the one consistency of PA.*

THEOREM B. *When the zero-th branch of the Ackermann function is an exponential function $k \mapsto k^b$ then the induced Goodstein principle will be equivalent to the one consistency of* ATR_0.

Theorem B is joint work with Tosiyasu Arai and Stan Wainer.

▸ SUSUMU YAMASAKI, *A semantics for multi-modal mu-calculus with interaction on Heyting algebra*.
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As regards a human interaction to implementation in programming languages, abstraction of interactive states should be formulated in computer science logic. Communications (for interaction) and (λ-)terms motivate multi-modality in logic of action ([2]). The syntax of the logical formulas is now given by *BNF* (Backus Naur Form) into some modification of modal

https://doi.org/10.1017/bsl.2017.16 Published online by Cambridge University Press

mu-calculus:

$$\varphi ::= \mathrm{tt} \mid p \mid \neg\varphi \mid \ \sim\varphi \mid \varphi \vee \varphi \mid \langle c\rangle\varphi \mid \mu x.\varphi \mid \varphi\rangle t\rangle,$$

where a prefix modality $\langle c\rangle$ (for communications), a postfix one $\rangle t\rangle$ (for terms), and a negation $\sim$ (denoting incapability of interaction) are taken, in addition to truth tt, propositions p, the logical negation $\neg$ and a least fixed point operator μ.

To represent the meaning of (a formula) φ at a state, receiving communication c (requirement), and being followed by term t (effect), we here have the state sets $[\![\varphi]\!]_{pos}$, $[\![\varphi]\!]_{inter}$ and $[\![\varphi]\!]_{neg}$ for the formula (condition) to be positively, interactively, and negatively modeled, respectively, in a *transition system*. This is an extended semantics for logic of action, from the version of [3]. The state sets are applicable to a triplet of $\langle c\rangle\varphi$, φ and $\varphi\rangle t\rangle$, concerned with communication requirement and term effect. We then adopt a Heyting algebra $H = (\{0, 1/2, 1\}, \leq, \bigvee, \bigwedge, 0, 1)$, equipped with a binary operation $\longrightarrow$ such that $c \bigwedge a \leq b$ iff $c \leq a \longrightarrow b$. With H, we define a semantic function *Deno*: $\Phi \to S \to \{0, 1/2, 1\}$, to see the positive, interactive, or negative states for the formula (condition) to be at, where Φ and S are the set of formulas and the set of states, respectively.

A *semiring* structure ([1]) is related to, with multiplicative inverse, if the alternation of applying postfix modal operators is regarded as addition, and the composition is interpreted as multiplication. *Kleene star* can be included as in *star semiring*, with relevance to a state constraint system ([4]).

[1] M. Droste, W. Kuich, and H. Vogler (eds.), ***Handbook of Weighted Automata***, Springer, 2009.

[2] M. Hennessy and R. Milner, *Algebraic laws for nondeterminism and concurrency*. ***Journal of the ACM***, vol. 32 (1985), no. 1, pp. 137–161.

[3] A. Kucera and J. Esparza, *A logical viewpoint on process-algebraic quotients*. ***Journal of Logic and Computation***, vol. 13 (2003), no. 6, pp. 863–880.

[4] S. Yamasaki, *State constraint system applicable to adjusting*, ***CLMPS***, Book of Abstracts (I. Nevalainen, M. Virtanen, and P. Seppala, editors), printed at University of Helsinki, 2015, pp. 409–410.

► NOAM ZEILBERGER, *Linear lambda terms as invariants of rooted trivalent maps*.
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Recent work on the combinatorics of linear lambda calculus (also known as BCI combinatory logic) has uncovered a variety of surprising connections to the theory of graphs on surfaces (also known as "maps"). The main purpose of the talk will be to convey a simple and conceptual account for one of these connections, namely the correspondence (originally described by Bodini, Gardy, and Jacquot) between α-equivalence classes of closed linear lambda terms and isomorphism classes of rooted trivalent maps on compact oriented surfaces without boundary, as an instance of a more general correspondence between linear lambda terms with a context of free variables and rooted trivalent maps with a boundary of free edges. After recalling some basic definitions as well as a familiar diagrammatic representation for linear lambda terms, I'll explain how the "easy" direction of the correspondence is a simple forgetful operation which erases annotations on the diagram of a linear lambda term to produce a rooted trivalent map. The other more surprising direction views linear lambda terms as topological invariants of their underlying rooted trivalent maps, reconstructing the missing information through a Tutte-style recurrence on maps with free edges. As an application in combinatorics, I'll show how to use this analysis to enumerate bridgeless rooted trivalent maps as linear lambda terms containing no closed subterms, and conclude by giving a natural reformulation of the Four Color Theorem as a statement about typing in lambda calculus.

[1] H. P. Barendregt, ***The Lambda Calculus: Its Syntax and Semantics***, Studies in Logic 103, second, revised ed., North-Holland, Amsterdam, 1984.

[2] O. Bodini, D. Gardy, and A. Jacquot, *Asymptotics and random sampling for BCI and BCK lambda terms*. ***Theoretical Computer Science***, vol. 502 (2013), pp. 227–238.

https://doi.org/10.1017/bsl.2017.16 Published online by Cambridge University Press

[3] S. K. Lando and A. K. Zvonkin, *Graphs on surfaces and their applications*, ***Encyclopaedia of Mathematical Sciences***, vol. 141, Springer-Verlag, 2004.

[4] N. Zeilberger, *Counting isomorphism classes of β-normal linear lambda terms*, September 25, 2015, `arXiv:1509.07596`.

[5] ———, *Linear lambda terms as invariants of rooted trivalent maps*, December 21, 2015, `arXiv:1512.06751`.

[6] N. Zeilberger and A. Giorgetti, *A correspondence between rooted planar maps and normal planar lambda terms*. ***Logical Methods in Computer Science***, vol. 11 (2015), no. 3, pp. 1–39.

Abstracts of talks presented by title

▸ MARIJA BORIČIĆ, *Natural deduction probabilized*.
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By combining Gentzen's and Prawitz's approach to deductive systems and Carnap–Popper–type probability logic semantics, we introduce a probabilistic version of inference rules of natural deduction **NK**, denoted by **NKprob**. Probabilized natural deduction systems have already been considered (see [2], [3], and [4]). For each propositional formula A and each $a, b \in I$, where I is a finite subset of reals $[0, 1]$ containing 0 and 1, closed under addition, the expression $A[a, b]$ is probabilized formula in **NKprob**. The meaning of $A[a, b]$ is that 'the probability c of truthfulness of a sentence A belongs to the interval $[a, b]$'. Our system contains at least two inference rules for each connective, one introducing, and the other one eliminating the connective. For example, the following rules are treating the introduction and elimination of disjunction:

$$\frac{A[a,b] \quad B[c,d]}{(A \vee B)[\max(a,c), b+d]}(I\vee) \qquad \frac{A[a,b] \quad (A \vee B)[c,d]}{B[c-b,d]}(E\vee).$$

Also, there are specific rules treating inconsistency:

$$\frac{\begin{array}{c}[A[c_1,c_1]]\\ \hline\hline A\emptyset\end{array} \; \begin{array}{c}[A[c_2,c_2]]\\ \hline\hline A\emptyset\end{array} \; \dots \; \begin{array}{c}[A[c_m,c_m]]\\ \hline\hline A\emptyset\end{array}}{A\emptyset}(I\emptyset) \qquad \frac{A\emptyset}{B[a,b]}(E\emptyset)$$

for any propositional formulae A and B, and any $a, b \in I = \{c_1, c_2, \dots c_m\}$, where $A\emptyset$ is $A[a, b]$, for $a > b$.

Let For be the set of all propositional formulae. Then any mapping $p\colon \text{For} \to I$ will be an **NKprob**–model if it satisfies the following conditions: (i) $p(\top) = 1$ and $p(\bot) = 0$; (ii) if $p(A \wedge B) = 0$, then $p(A \vee B) = p(A) + p(B)$; (iii) if $A \leftrightarrow B$ in classical logic, then $p(A) = p(B)$. We prove that our probabilistic natural deduction system **NKprob** is sound and complete with respect to this kind of models (see [1]).

[1] M. Boričić, *Inference rules for probability logic*, ***Publications de l'Institut Mathématique***, to appear.

[2] A. M. Frisch and P. Haddawy, *Anytime deduction for probabilistic logic*. ***Artificial Intelligence***, vol. 69 (1993), pp. 93–122.

[3] T. Hailperin, *Probability logic*. ***Notre Dame Journal of Formal Logic***, vol. 25 (1984), pp. 198–212.

[4] C. G. Wagner, *Modus tollens probabilized*. ***British Journal for the Philosophy of Science***, vol. 54 (2004), no. 4, pp. 747–753.

▸ IRAKLI CHITAIA, *Density and non-density in the Q_1-degrees of computably enumerable sets*.
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Given sets A, B of natural numbers we say that A is Q_1-reducible to B, denoted by $A \leq_{Q_1} B$, if there exists a computable function f such that, for all x and y, if $x \neq y$ then

$W_{f(x)} \cap W_{f(y)} = \emptyset$, and $x \in A$ if and only if $W_{f(x)} \subseteq B$. We prove some density and nondensity results relative to the structure of the Q_1-degrees of the c.e. sets.

THEOREM 1. *There exist c.e. sets A, B such that $A <_{Q_1} B$, but there is no c.e. set C such that $A <_{Q_1} C <_{Q_1} B$.*

Building on [1], we show in fact that the previous result holds of any pair A, B where B is hyperhypersimple, and $A = B \cup \{m\}$ with $m \notin B$.

THEOREM 2. *If A is hyperhypersimple then no c.e. $B \leq_{Q_1} A$ is of minimal Q_1-degree, unless B is decidable.*

THEOREM 3. *If A is a maximal set and B is a nonmaximal hyperhypersimple set, then either $A|_{Q_1} B$, or there exist a nonmaximal hyperhypersimple set C and a maximal set D such that*

$$A <_{Q_1} D <_{Q_1} C <_{Q_1} B.$$

[1] R. S. OMANADZE and I. O. CHITAIA, *Q_1-degrees of c.e. sets*. ***Archive for Mathematical Logic***, vol. 51 (2012), pp. 503–515.

▸ ANAHIT CHUBARYAN AND GARIK PETROSYAN, *The proof complexities relations for strongly equal classical tautologies in Frege systems.*
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The traditional assumption that all tautologies as Boolean functions are equal to each other is not fine-grained enough to support a sharp distinction among tautologies. The authors of [1] have introduced the notion of determinative conjunct, on the basis of which the notion of strong equality of classical tautologies was suggested. The idea to revise the notion of equivalence between tautologies in such way that is takes into account an appropriate measure of their complexity. The relations between the proof complexities of strongly equal classical tautologies in some weak proof systems are investigated in [2]. It was proved that in these proof systems the strongly equal tautologies have the same proof complexities.

In this paper the relations between the four main measures of proof complexities (length, size, space, and width) for strongly equal tautologies are investigated in the most traditional proof systems of Classical Logic—Frege systems. We show that there is the sequence of tautology pairs φ_n and ψ_n such, that for every n φ_n and ψ_n are strongly equal, the main measures of proof complexities in Frege systems for φ_n are bounded by polynomial function in size of φ_n just as the lower bounds for the same measures of ψ_n are exponential function in size of φ_n.

[1] A. CHUBARYAN and A. CHUBARYAN, *A new conception of equality of tautologies*. ***L&PS, Triest, Italy***, vol. V (2007), no. 1, pp. 3–8.

[2] A. CHUBARYAN, A. CHUBARYAN, and A. MNATSAKANYAN, *Proof complexities of strong equal classical tautologies in some proof systems*. ***Nauka i studia, Poland***, vol. 42 (2013), no. 110, pp. 91–98.

▸ ANAHIT CHUBARYAN, ARTUR KHAMISYAN, AND ARMAN TSHITOYAN, *Some new proof systems for a version of many-valued logics and proof complexities in it*.
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Some method for construction of a deductive full propositional calculi for some version of k-valued ($k \geq 3$) logic is described in the paper. The propositional connectives are defined as follows: *conjunction* is *min*, *disjunction* is *max*, *negation* is defined by permuting the truthvalues cyclically. We use as literals the propositional variables, variables with negation, with double negations, with triple negations etc. We generalize the notions of determinative conjunct and determinative disjunctive normal form (dDNF), introduced by first coauthor for two-valued

Boolean functions in [1], and on the base of it construct the systems E_k, axioms of which are not fixed. Each conjunct from some dDNF of given formula can be considered as an axiom. The elimination rule (e-rule) infers conjunct $K' \cup K'' \cup K''' \cup \dots$ from conjuncts $K' \cup \{p\}$, $K'' \cup \{{}^{\sim}p\}$, $K''' \cup \{{}^{\sim\sim}p\}$ etc. for a propositional variable p. E_k-proof is defined as usually. It is obvious that some DNF $D = \{K_1, K_2, \dots, K_i\}$ is k-valued tautology iff using e-rule we can derive the empty conjnct from axioms $\{K_1, K_2, \dots, K_i\}$. We prove also that for every k there is some sequence of k-valued tautologies, which have in described systems the same by order upper and lower bounds for the main proof complexity characteristics: exponential for lines and size, polynomial for space and width.

[1] A. Chubaryan and A. Chubaryan, *A new conception of equality of tautologies*. ***L& PS, Triest, Italy***, vol. V (2007), no. 1, pp. 3–8.

► JOHN CORCORAN, *Weakening and strengthening conditionals*: *Thirty-six theorems*.
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We speak informally of "weakening" propositions (axioms or conjectures): e.g., axioms with undesirable consequences or conjectures with counterexamples. We also speak informally of "strengthening" propositions (theorems or hypotheses): e.g., if the theorem's proof reveals unnecessary restrictions or our evidence for the hypothesis suggests something stronger.

This paper makes such language more formal. We consider certain weakenings and strengthenings of conditionals. Using the alternative constituent format [this ***Bulletin***, vol. 15 (2009), p. 133], thirty-six hypotheses are presented, explained, and then finally settled—some are proved; some disproved. The result: thirty-six interrelated theorems about weakenings and strengthenings of conditionals.

A *weakening* of a proposition is implied by but doesn't imply the proposition. A *strengthening* of a proposition implies but isn't implied by the proposition. Thus, a certain proposition is a weakening of a given proposition iff the latter is a strengthening of the former. A *conserving* of a given proposition implies and is implied by the given proposition.

Other terminology has been used: weakenings were called *superimplications*; strengthenings *superimplicants*; conservings *logical-equivalents* [1].

Our first theorem is easy: Strengthening the antecedent of a conditional never strengthens the conditional.

The second theorem isn't as easy: Strengthening the antecedent of a conditional sometimes but not always weakens the conditional.

Our thirty-four remaining hypotheses arise from the first theorem by replacing with certain alternatives one or more of the following four expressions: *Strengthening*, *antecedent*, *never*, and *weakens*. The alternatives are as follows: for *Strengthening* just *Weakening*; for *antecedent* just *consequent*; for *never* two: *always* and *sometimes but not always*; and for *weakens* two: *strengthens* and *conserves*.

Using the alternative constituent format, the thirty-six hypotheses are presented as follows.

(Weakening * Strengthening) the (antecedent * consequent) of a conditional

(always * sometimes but not always * never)

(weakens * strengthens * conserves) the conditional.

[1] M. Cohen and E. Nagel, ***Introduction to Logic***, second ed. (J. Corcoran, editor), Hackett, 1993.

► JOHN CORCORAN AND JOSÉ MIGUEL SAGÜILLO, *Teaching paradoxes, antinomies, and contradictions*.
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Contradiction has several widely-used senses but *paradox* has essentially only one.

https://doi.org/10.1017/bsl.2017.16 Published online by Cambridge University Press

A *paradox* is an argumentation believed to deduce a conclusion believed false from premises believed true [1, pp. 21f]. Since 'believed' refers to participants, paradoxes are participant-relative. Mathematical writing often presumes as participant a community of mathematicians [2]. Ordinarily a paradox is *resolved* by a participant through *either* discovering a logical fallacy in the argumentation's reasoning *or* discovering the conclusion isn't false *or* discovering a premise isn't true.

Resolving is thus also participant-relative: only a change in a participant's beliefs can resolve that participant's paradox.

In some cases, the premises are accepted axioms and the conclusion is a *contradiction*, or *contradictory* proposition: one whose negation is logically-true. A paradox whose conclusion is believed to be a contradiction is an *antinomy*. In other cases, the premises and the conclusion contain a *contradiction*, or *contradictory pair*: two propositions, one logically equivalent to the other's negation. "No set contains itself", "Some set contains itself". Elsewhere, a *contradiction*, or *contradictory relationship*, holds between premises and conclusion: premises and conclusion together imply a contradictory proposition.

Examples illustrate the obvious fact that the presence of contradiction—in whatever above sense—isn't essential to paradox.

Calling contradictions paradoxes and speaking of resolving contradictions are category mistakes—as is calling paradoxes contradictions. Moreover, saying that a certain paradox "doesn't arise" in a certain situation—if this makes sense—is no resolution and no consolation to a participant troubled by the paradox.

[1] J. Corcoran, *Argumentations and logic*. ***Argumentation***, vol. 3 (1989), pp. 17–43.

[2] A. Garciadiego, *The emergence of some of the nonlogical paradoxes of the theory of sets, 1903–1908*. ***Historia Mathematica***, vol. 12 (1985), pp. 337–351.

▸ JOHN CORCORAN AND KEVIN TRACY, *"Every square is a rhombus": categorical, hypothetical, conditional, implicational, equational, or what?*

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The five-word English sentence (1) can express a true proposition if the word 'rhombus' is taken in the broad sense coextensive with "equilateral quadrangle".

(1) Every square is a rhombus.

Aristotle took a Greek equivalent to express a *universal-affirmative categorical* proposition: one composed of three parts: predicate, copula, and subject [2]. For emphasis Aristotle used an artificial formula never before used in Greek—rendered into equally artificial "English" below:

(2) Rhombus belongs-to-every square.

Aristotle's copula "belongs-to-every" is logical (or form-determining); the subject and predicate—which he appropriately called 'horoi' (boundaries, limits, extremities)—are non-logical (or content-determining).

After Aristotle such sentences have been "paraphrased" in several ways. Some older logics such as [1, pp. 41f] take such sentences to express "hypothetical" [sic] propositions involving more than three parts and rendered with 'if-then' as:

(3) If anything is a square, then it is a rhombus.

Other logics, e.g., [4], construe them as universalized conditionals of various forms as:

(4) For every object x, if x is a square, then x is a rhombus.

(5) For every object x, if x is square, then x is rhombic.

Building on [3], this descriptive, historical, and analytic lecture considers these and other construals, their logical interrelations, claims made about them, and reasons given for the claims. No attempt is made to reconcile conflicting claims or to decide which if any are correct—or even how such a decision could be made objectively.

https://doi.org/10.1017/bsl.2017.16 Published online by Cambridge University Press

[1] M. Cohen and E. Nagel, ***Introduction to Logic***, second ed., Hackett, 1993.

[2] J. Corcoran, *Aristotle's demonstrative logic*. ***History and Philosophy of Logic***, vol. 30 (2009), pp. 1–20.

[3] ———, *Meanings of implication*. ***Diálogos***, vol. 9 (1973), pp. 59–76.

[4] W. Quine, ***Methods of Logic***, Holt-Rinehart-Winston, 1959.

► MARY LEAH KARKER, *Products of metric structures*.

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This talk will focus on analogues, for continuous logic [1], of results of Feferman and Vaught on model-theoretic properties of various kinds of product [2]. For example: a natural continuous-logic version of direct product preserves elementary equivalence and elementary inclusion; and if a sentence θ is true in $\prod_{i=0}^{k} \mathcal{M}_i$ for every $k \in \mathbb{N}$, then θ is true in $\prod_{i \in N} \mathcal{M}_i$.

[1] I. Ben Yaacov, A. Berenstein, C. W. Henson, and A. Usvyatsov, *Model theory for metric structures*, ***Model Theory with Applications to Algebra and Analysis, vol. 2*** (Z. Chatzidakis, D. Macpherson, A. Pillay, and A. Wilkie, editors), London Mathematical Society Lecture Note Series, 350, Cambridge University Press, Cambridge, 2008, pp. 315–427.

[2] S. Feferman and R. L. Vaught, *The first order properties of products of algebraic systems*. ***Fundamenta Mathematicae***, vol. 47 (1951), pp. 57–103.

► BEIBUT KULPESHOV AND SERGEY SUDOPLATOV, *On Vaught's problem for quite o-minimal theories*.

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We study Vaught's problem for quite o-minimal theories. Quite o-minimal theories (introduced in [3]) form a subclass of the class of weakly o-minimal theories preserving a series of properties of o-minimal theories.

In the following definitions M is a weakly o-minimal structure, $A \subseteq M$, M is $|A|^+$-saturated, and $p, q \in S_1(A)$ are nonalgebraic.

Definition 1 ([2]). We say that p is not *weakly orthogonal* to q ($p \not\perp^w q$) if there are an A-definable formula $H(x, y)$, $\alpha \in p(M)$, and $\beta_1, \beta_2 \in q(M)$ such that $\beta_1 \in H(M, \alpha)$ and $\beta_2 \notin H(M, \alpha)$.

Definition 2 ([3]). We say that p is not *quite orthogonal* to q ($p \not\perp^q q$) if there is an A-definable bijection $f\colon p(M) \to q(M)$. We say that a weakly o-minimal theory is *quite o-minimal* if the relations of weak and quite orthogonality for 1-types coincide.

Lemma 3. *Any o-minimal theory is quite o-minimal.*

The Vaught problem has been solved for o-minimal theories by Laura Mayer in [4]. B. S. Baizhanov and A. Alibek [1] have constructed for each natural $n \geq 4$ examples of weakly o-minimal theories with exactly n countable models, and also an example of a weakly o-minimal theory with exactly ω countable models. We present the following theorem which is a solution of the Vaught problem for quite o-minimal theories:

Theorem 4. *Let T be a quite o-minimal theory in a countable language. Then either T has 2^ω countable models or T has exactly $6^a 3^b$ countable models, where a and b are natural numbers. Moreover, for any $a, b \in \omega$ there is a quite o-minimal theory T with exactly $6^a 3^b$ countable models.*

[1] A. Alibek and B. S. Baizhanov, *Examples of countable models of a weakly o-minimal theory*. ***International Journal of Mathematics and Physics***, vol. 3, no. 2 (2012), pp. 1–8.

[2] B. S. Baizhanov, *Expansion of a model of a weakly o-minimal theory by a family of unary predicates*. ***The Journal of Symbolic Logic***, vol. 66 (2001), pp. 1382–1414.

[3] B. S. Kulpeshov, *Convexity rank and orthogonality in weakly o-minimal theories*, ***News of the National Academy of Sciences of the Republic of Kazakhstan***, Physical and Mathematical Series, vol. 227, 2003, pp. 26–31.

[4] L. L. Mayer, *Vaught's conjecture for o-minimal theories*. ***The Journal of Symbolic Logic***, vol. 53 (1988), pp. 146–159.

▶ FABIO LAMPERT, *Fitch-style natural deduction with diagonal operators and their eliminability*.
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We present a sound and complete Fitch-style natural deduction system for first-order **S5** modal logic containing an actuality operator, a "strongly diagonal" operator, and a "weakly diagonal" operator. The logic is two-dimensional, where we evaluate sentences with respect to an actual world (first dimension) and a world of evaluation (second dimension). Strongly diagonal works as a quantifier over every point on the diagonal between actual worlds and worlds of evaluation, whereas weakly diagonal quantifies only at some point on the diagonal. Thus, they behave just like the epistemic operators for apriority and its dual. We take this extension of Fitch's familiar derivation system to be a very natural one, since the new rules and labeled lines hereby introduced preserve the structure of Fitch's own rules for the modal case. We also present a modified version of this system containing a two-dimensional analogous of the actuality operator, which we call "strongly actual." It will be shown that the diagonal operators—as well as the strongly actual operator—are eliminable in the propositional portion of the resulting logic, thereby occasioning no increase in the expressive power of **S5** modal logic despite all appearances.

▶ ANDREA SERENI AND MARIA PAOLA SFORZA FOGLIANI, *How to water a thousand flowers*: *On the logic of logical pluralism*.
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Our aim is to investigate some fundamental, though underexplored, metatheoretical issues regarding logical pluralism (LP); we focus on the following target question:

(Q) How many logics is the logical pluralist using when arguing for LP?

After showing how (Q) is prompted by a well-known argument in the philosophy of logic—the Centrality Argument (e.g., [3])—we discuss three strategies for reply: (*a*) no logic, (*b*) one single logic, and (*c*) more than one logic.

We argue that neither (*a*)—what Beall and Restall ([1]) (cf. also [2]) opted for—nor (*b*) are defendable. As a way out, we explore a form of modest pluralism; this requires clarifying how logics should be in order to be acceptable for a pluralist (*correct/legitimate* or *true*).

Shapiro ([4]) suggests that arguments for LP may not employ deductions, being rather Inferences to the Best Explanation (IBE); but, we show, since IBE can collide with Bayesian Confirmation Theory, LP for inductive logics must be either turned down, or further defended.

Finally, (*c*) amounts to:

(LP') We can argue for LP using different logics.

But then, how is one to defend (LP')? The same options open up again; assuming the third strategy is chosen, an infinite regress threatens.

We thus reach a skeptical conclusion. If LP is to be a stable position, pluralists must offer suitable answers to our methodological questions. However, none of the options presently available seems to be viable.

[1] J. C. BEALL and G. RESTALL, *Defending logical pluralism*, ***Logical Consequence: Rival Approaches*** (J. Woods and B. Brown, editors), Hermes Science, Middlesex, England, 2001, pp. 1–22.

[2] ———, ***Logical Pluralism***, Oxford University Press, 2006.

[3] H. PUTNAM, *There is at least one a priori truth*. ***Erkenntnis***, vol. 13 (1978), no. 1, pp. 153–170.

[4] S. SHAPIRO, ***Varieties of Logic***, Oxford University Press, 2014.

▸ VLADIMIR STEPANOV, *Hypercomplex numbers for the semantics of self-reference statements*.

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In [1] it is described the dynamic model for the semantics of self-referential statements for $\underline{\leftrightarrow\ \neg}$ language. In this model, the truth table positive estimates for connection of biconditional ($\underline{\leftrightarrow}$) represents the Cayley table for the Klein four group:

$\underline{\leftrightarrow}$	**T**	**V**	**A**	**K**
T	**T**	**V**	**A**	**K**
V	**V**	**T**	**K**	**A**
A	**A**	**K**	**T**	**V**
K	**K**	**A**	**V**	**T**

Here $\mathbf{T}$ = True, $\mathbf{V}$ = TruthTeller, $\mathbf{A}$ = Liar, $\mathbf{K} = (\mathbf{V}\underline{\leftrightarrow}\mathbf{A})$. $\mathbf{V}^2 = \mathbf{A}^2 = \mathbf{K}^2 = \mathbf{VAK} = \mathbf{T}$. Elements of Klein group remind properties of vector product. Example: $(\mathbf{V}\underline{\leftrightarrow}\mathbf{A}) = \mathbf{K}$, etc. It allows to formulate the following hypothesis:

The hypercomplex hypothesis: We postulate that truth space of self-reference statements is a hypercomplex structure, so that the units $\{\mathbf{V}, \mathbf{A}, \mathbf{K}\}$ represent dimensions of truth space of properly self-reference statements, while the scalar $\mathbf{T}$ represents a classical statements.

This property we try to use for recording estimates of logical formulas in the form of a hypercomplex numbers: $\mathbf{Q} = a_0\mathbf{T} + a_1\mathbf{V} + a_2\mathbf{A} + a_3\mathbf{K}$. Here $a_0 \div a_3$ take the values $1, \sim, 0$, which means that the component may be positive or negative occurrence, or may not have it all. As the multiplication table for components of hypercomplex numbers the Cayley table for the Klein four group is used. Example:

$$\mathbf{P}\underline{\leftrightarrow}\mathbf{Q} = (a_0\mathbf{T} + a_1\mathbf{V} + a_2\mathbf{A} + a_3\mathbf{K})\underline{\leftrightarrow}(b_0\mathbf{T} + b_1\mathbf{V} + b_2\mathbf{A} + b_3\mathbf{K}) =$$

$$\begin{array}{c} (a_0b_0\mathbf{T} + a_0b_1\mathbf{V} + a_0b_2\mathbf{A} + a_0b_3\mathbf{K}) + \\ (a_1b_0\mathbf{V} + a_1b_1\mathbf{T} + a_1b_2\mathbf{K} + a_1b_3\mathbf{A}) + \\ (a_2b_0\mathbf{A} + a_2b_1\mathbf{K} + a_2b_2\mathbf{T} + a_2b_3\mathbf{V}) + \\ (a_3b_0\mathbf{K} + a_3b_1\mathbf{A} + a_3b_2\mathbf{V} + a_3b_3\mathbf{T}) \end{array} = \begin{array}{l} (a_0b_0 + a_1b_1 + a_2b_2 + a_3b_3)\mathbf{T} + \\ (a_1b_0 + a_0b_1 + a_2b_3 + a_3b_2)\mathbf{V} + \\ (a_2b_0 + a_0b_2 + a_1b_3 + a_3b_1)\mathbf{A} + \\ (a_3b_0 + a_0b_3 + a_2b_1 + a_1b_2)\mathbf{K} \end{array}.$$

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▸ DAMIAN SZMUC, *The logic of track-down operations on bilattices*.

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As was shown in [2] and [1], some nonclassical logics have close connections with a particular kind of lattices, namely *bilattices*. These kind of lattices are equipped with two partial orders $\leq_t$ and $\leq_k$, usually called the truth-order and the knowledge order, whose meet and join operations are $\wedge$, $\vee$, and $\otimes$, $\oplus$, respectively. More remarkably, in [1] it was proved that Belnap and Dunn's four-valued logic **FDE** could be rightfully called *the* logic of bilattices.

In this paper we discuss the system **FDEpwk**, a modification of **FDE**, where the *paraconsistent* semantic value is an absorbing element with regard to all the *logical* operations.

https://doi.org/10.1017/bsl.2017.16 Published online by Cambridge University Press

To connect this system with bilattices, we first define—with the aid of $\otimes$ and negation—a track-down operator (i.e., an operator that tracks down inconsistent values) and, later, we define proper track-down variants of the usual lattice-theoretic operations on the truth-order. We also modify correspondingly the clauses of the valuation functions, which renders a family of functions that we will call Halldén valuations (due to [3]). As a result, it is shown that **FDEpwk** could be rightfully called *the* logic of track-down operations on bilattices.

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A subset G of the natural numbers is n-generic $(n > 0)$ if every Σ^0_n sentence of G is decided by an initial segment of G in the sense of Cohen forcing. Jockusch proved that for positive $n < 3$ every n-generic G is properly Σ^0_n in some G-recursive set. As a corollary of his proofs Jockusch also proved that for positive $n < 3$ every $(n+1)$-generic computes a set X which is GL_{n+1} but not GL_n. He conjectured that the above results can be generalized to all positive integers n. Recently we confirm Jockusch's conjecture. Sketches of proofs shall be presented in this talk.

Acknowledgment. This research was partially supported by NSF Grant 11471342 of China.