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GROWTH AND IRREVERSIBLE POLLUTION: ARE EMISSION PERMITS A MEANS OF AVOIDING ENVIRONMENTAL AND POVERTY TRAPS?

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We consider an OLG model with emissions arising from production and potentially irreversible pollution. Pollution control consists of the assignment of permits to firms; private agents also can abate pollution. In this setting, we prove that multiple equilibria exist. Due to the possible irreversibility of pollution, the economy can be dragged into both environmental and poverty traps. First, we show that choosing an emission quota at the lowest level beyond a critical threshold is a means to avoid these two types of traps. We also prove that when the agents do not engage in maintenance, a reduction of the quota leads to a reduction in pollution but also to slower capital accumulation. In contrast, when agents do engage in maintenance, a reduction of the quota provides a double dividend.

Keywords: Overlapping Generations, Irreversible Pollution, Environmental and Poverty Traps, Emission Permits

1. INTRODUCTION

This paper raises the question of the performance of environmental policy in situations where economic activities may translate into irreversible degradation

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of the environment. Irreversibility means that ecosystems' natural regeneration capacity is limited and may fail once a critical level of pollution is exceeded. The notion of irreversibility is linked to the multiplicity of ecosystems' equilibria. It implies that when submitted to strong perturbations, typically pollution, an ecosystem may fail to recover its initial safe equilibrium. Rather, it may be caught in a new, highly polluted, equilibrium.[1] Therefore, in this paper, the performance of environmental policy will mainly be understood as its ability to protect the environment against irreversible degradation. Nevertheless, the impact of public policy on economic growth will also be examined.

Two different approaches to assessing the impact of environmental policy on growth have emerged in the literature. Works studying growth models with infinitely lived agents have concentrated on environmental taxation [see Bovenberg and Smulders (1995, 1996) or Bovenberg and de Mooij (1997)]. A feature common to all of these papers is the introduction of an environmental externality in production, which conveys the idea that environmental quality enhances the productivity of private inputs. Within this framework, the main conclusion is that a more stringent environmental policy may provide a double dividend consisting of a simultaneous increase in environmental quality and economic growth, provided the environment has a strong positive impact on technology. Ono (2003) addresses the same issue but considers overlapping generations (OLG) in place of infinitely lived agents. He extends John and Pecchenino's model (1994) with a process of innovation and distinguishes two phases of growth: the "no-innovation growth regime" and the "innovation-led growth regime." He shows that a critical level of environmental tax exists that determines whether, by raising the tax, the economy obtains a higher or lower growth rate and better or worse environmental quality.

In sum, most contributions addressing the impact of environmental policy on growth focus on environmental taxation. A notable exception is Ono (2002), who examines the issue using an OLG model where emission permits are the policy instrument. His aim is to measure the macroeconomic consequences that follow a tightening of environmental policy when emissions are modeled as an input. The most striking result is that, contrary to what is expected, a reduction of the allocation of permits to firms (the quota) may lower both economic growth and environmental quality.

The present paper contributes to this literature by examining the design of environmental policy under the potential irreversibility of pollution. As mentioned above, we are interested in the irreversibility at play in the pollution accumulation process. Accounting for irreversibility consists of recognizing that, on one hand, an ecosystem's natural capacity to assimilate pollution depends on the concentration of the pollutant and that, on the other, a critical threshold exists above which the assimilation capacity becomes permanently exhausted. In a partial equilibrium model, Tahvonen and Withagen (1996) have assessed the implications of irreversibility on the optimal control of pollution. In the same vein, Prieur (2009) introduces this kind of irreversibility into a general equilibrium model of growth and gives new insights into the relationship between growth and the environment.

He shows notably that a nonregulated growth process may lead a polluting economy into an economic and ecological poverty trap even when pollution abatement operates. The existence of such a long-term state justifies the intervention of public authorities in the management of pollution problems. This knowledge has led us to study the means and consequences of such intervention more closely. To do so, we further develop his model, making the assumption that emissions are a production by-product and can be controlled by an emission permits market. In that sense, our analysis also extends Ono (2002)'s study by considering irreversible pollution.

Within this framework, our aim is to address the issue of the impact of environmental policy on growth prospects. This issue may be broken into two different but related questions: Are emission permits a means for avoiding a drift toward poverty traps? Is environmental regulation necessarily unfavorable to economic growth?

Starting with the equilibrium analysis, we emphasize the existence of multiple long-run equilibria of different natures. Among possible equilibria, the stable steady state with the lowest level of capital and irreversible pollution corresponds to an environmental and poverty trap. In opposition to poverty traps, steady states exhibiting reversible pollution and higher wealth are called "desirable" equilibria. This feature of multiple equilibria with environmental and poverty traps has already been shown by Prieur (2009). It is used as a starting point for policy analysis. Our contribution is twofold.

First, we prove that the environmental and poverty trap may be avoided by setting the emission quota above a critical level. In other words, the preliminary recommendation is to allow firms to pollute sufficiently. Nevertheless, this does not mean that firms can pollute as much as they want. In fact, excluding the environmental and poverty trap gives rise to a new development trajectory that is worse than the one leading to the trap. This equilibrium trajectory has the features of an asymptotic environmental and poverty trap, in the sense that it is accompanied by a perpetual erosion in economic and/or environmental resources. Based on dynamic analysis in terms of attraction basins, further investigations reveal that the quota must be fixed at the lowest level above this critical value to avoid the asymptotic trap. Environmental regulation can thereby efficiently protect an economy from converging toward poverty traps caused by irreversible pollution.

Second, we emphasize the impact of environmental regulation on desirable equilibria. It appears that the impact of a change in the quota at equilibrium depends on whether agents engage in pollution abatement, which we also refer to in this paper as environmental maintenance. When agents do not maintain (zero maintenance equilibrium, ZME), there exists a trade-off between pollution control and economic growth. Although lowering the quota is a means of reducing pollution, it also generates a negative effect on capital accumulation. When agents do invest in environmental maintenance (positive maintenance equilibrium, PME), the key element is the evolution of the balance between financial and environmental constraints imposed on the agents. Under plausible conditions, in the locally stable equilibrium the economy enjoys a double dividend: a reduction of the quota allows the economy to reach a long-term state that is both richer and less polluted. Therefore, reducing the quota improves the welfare of living generations over the long term. Ono's analysis (2002) then is taken further by not only considering the impact of a change in the quota on ZME but also stressing the opportunity to obtain a double dividend in the PME. In contrast with the literature on tax reform reviewed above, the derivation of this result does not rely on the controversial assumption of a positive environmental externality on production.

The paper is organized as follows. Section 2 presents the model. Section 3 provides a detailed analysis of the equilibrium. Section 4 analyzes the impact of a political reform on equilibrium properties. Section 5 performs certain numerical simulations to outline the implications of a change in policy on global dynamics and, notably, on the possibility of reaching a safe and wealthy steady state. Section 6 concludes.

2. THE MODEL

In a perfectly competitive world, firms produce a single homogeneous good used for both consumption and investment. The production process generates harmful polluting emissions. The environmental policy consists of defining an emission quota, $\bar{E}_t$, for each period and of creating an exchange market for emission permits. The quota is imposed upon the economy in an exogenous manner. For example, the level of emissions to be respected may be decided during international negotiations (like the Kyoto protocol of 1997), each participating country being endowed with a quota for polluting emissions. In order to execute the agreement, a government sells a volume of permits corresponding to $\bar{E}_t$ to polluting firms. It also is responsible for the distribution of income obtained from the sale of permits (the environmental allowance) to households. In addition, in accordance with Ono (2002), we assume that households also can engage in environmental maintenance.

In this section, private agents' tradeoffs and pollution dynamics are set out. The result is a discrete-time dynamical system, which will be analyzed in Section 3.

2.1. Production

Under perfect competition, firms produce the final good Y_t with a constant–returns to scale technology using labor L_t, capital K_t, and emissions E_t:

$$Y_t = AK_t^{\alpha} L_t^{\beta} E_t^{1-\alpha-\beta}. \tag{1}$$

Jouvet et al. (2005) show that an auction system is more efficient than the allocation of permits to firms free of cost because the latter is a source of economic distortions. On the basis of their results, we assume that the total number of permits are auctioned. Firms thus are obliged to purchase at the market price q_t the number of emission permits E_t needed to produce. Note that in contrast, Ono (2002) assumes that part of the quota is allocated free of charge to firms, which may then participate in market transactions. Nonetheless, his approach is in fine rigorously identical to ours because in his model's equilibrium it is still an income $q_t E_t$, the revenue from the sale of permits, that is taxed entirely and paid back to young households.

Capital depreciates fully in one period. Firms maximize profits, taking the prices of inputs as given. This yields the expressions for input prices, expressed in terms of per capita variables with $k_t = K_t/L_t$ and $e_t = E_t/L_t$:

$$w_t = \beta A k_t^{\alpha} e_t^{1-\alpha-\beta}, \tag{2}$$

$$r_t = \alpha A k_t^{\alpha-1} e_t^{1-\alpha-\beta} - 1, \tag{3}$$

$$q_t = (1-\alpha-\beta) A k_t^{\alpha} e_t^{-\alpha-\beta}, \tag{4}$$

where w_t represents the wage rate, r_t is the interest rate, and q_t the price of permits.

2.2. Pollution Dynamics

Pollution accumulation for nonnegative levels of the stock P_t is described by the equation

$$P_{t+1} = P_t - \Gamma(P_t) + \tilde{E}_t, \tag{5}$$

where $\tilde{E}_t$ represents emissions, net of abatement, and $\Gamma(P_t)$ corresponds to the natural decay function that defines the amount of pollution nature can assimilate each period. Nature's ability to absorb pollution depends on the concentration level of the pollutant. Our aim is to express the idea that excessive levels of pollution irreversibly alter the environment's recovery process. Therefore, like Tahvonen and Withagen (1996) and Prieur (2009), we assume that the decay function is inverted U-shaped. Its properties, summarized in the assumption below, give an account of the potential irreversibility of environmental damage caused by pollution:[2]

Assumption 1. $\Gamma(P) : \mathbf{R}_+ \to \mathbf{R}_+$ is continuous and satisfies $\Gamma(0) = 0$; $\exists! \overline{P} > 0$ such that $\Gamma(P) = 0\ \forall P \geq \overline{P}$, and $\Gamma(P) > 0$, $\Gamma''(P) \leq 0\ \forall P \in [0, \overline{P})$.

Note that above the critical threshold $\overline{P}$, the recovery process of nature is completely and permanently overwhelmed.

2.3. The Households

We consider an infinite horizon economy composed of finite-lived agents. A new generation is born in each period $t = 1, 2, \ldots$, and lives for two periods: youth and old age. There is no population growth and the size of a generation is normalized to one. The young agent born at period t is endowed with one unit of labor that he or she inelastically supplies to firms for a real wage w_t. Her first-period income also includes of the revenue from the sale of a quantity E_t of permits, at the price q_t. This revenue corresponds to an environmental allowance distributed by the government. She divides this total income between savings s_t and environmental maintenance m_t. When retired, the agent supplies her savings to firms and earns the return from savings $R_{t+1}s_t$, with $R_{t+1} = 1 + r_{t+1}$, the interest factor. Her income is entirely devoted to consumption c_{t+1}.[3] The two budget constraints are

$$w_t + q_t E_t = s_t + m_t, \tag{6}$$

$$c_{t+1} = R_{t+1}s_t. \tag{7}$$

The economy thus may rely on two distinct instruments for fighting pollution: environmental policy in the form of emission permits and environmental maintenance in the form of household pollution abatement.

The preferences of an agent born at date t are defined over old age consumption and environmental quality. They are described by the following utility function $U(c_{t+1}, P_{t+1})$:[4]

Assumption 2. $U(c, P) : \mathbf{R}^+ \times \mathbf{R}^+ \to \mathbf{R}$ is $\mathcal{C}^2$ with $U_1 \geq 0$, $U_2 \leq 0$, $U_{11}, U_{22} \leq 0$. The cross derivative is negative, $U_{12} \leq 0$.[5] We further assume that $\lim_{c\to 0} U_1(c, P) = +\infty$.

Emissions contribute to the accumulation of the pollutant stock. It also is possible to control the periodic flow of emissions by investing in environmental maintenance m_t. Pollution dynamics (5) can be rewritten as[6]

$$P_{t+1} = P_t - \Gamma(P_t) + E_t - \gamma m_t, \tag{8}$$

where $\gamma > 0$ is the efficiency of environmental maintenance.

The agent born at date t divides her first-period income between savings (which determine the consumption of the final good) and maintenance (which influences the "consumption" of the environmental public good) in order to maximize her lifetime utility. Taking as given prices and pollution at the beginning of period t, the representative agent's problem is written as[7]

$$\max_{s_t, m_t, c_{t+1}} U(c_{t+1}, P_{t+1}),$$

subject to constraints (6), (7), (8), and $m_t \geq 0$.

Direct calculations yield the first-order condition

$$\begin{aligned} &R_{t+1}U_1(c_{t+1}, P_{t+1}) + \gamma U_2(c_{t+1}, P_{t+1}) \geq 0, m_t(R_{t+1}U_1(c_{t+1}, P_{t+1}) \\ &\quad + \gamma U_2(c_{t+1}, P_{t+1})) = 0, m_t \geq 0. \end{aligned} \tag{9}$$

To conclude this section, the distinction between the decision makers involved in this economy is discussed. In the same vein as the related literature on the impact of the environmental policy on growth prospects [John and Pecchenino (1994), John et al. (1995), and Ono (2002, 2003)], the decision maker who undertakes the abatement decision can be seen as a *short-lived* government that seeks to maximize the utility of generation t individuals taking the prices (w_t and R_{t+1}), the pollution, and the amount of emissions (P_t and E_t), at the beginning of period t as given.

So it is possible to reinterpret m_t as a tax levied by the short-lived government in order to finance abatement, for the benefit of (young) agents living during its period of office. In addition, there exists another policy maker, say the *long-lived* government, in charge of the assignment of emission permits to firms. It is also responsible for the distribution of the income generated by the sale of permits to households. The existence of a short-lived is not at variance with the other, long-lived policy maker. In fact, consider the climate change issue. The short-lived government is a local or national government that can set an instrument, say for instance a carbon tax as in Finland, in Sweden, and in the Netherlands, to implement its own environmental policy. But, in addition to the local policy, there also exists an international policy, aimed at controlling world emissions of greenhouse gases. Since the Kyoto protocol (1997), countries have been engaged in emission reduction and the central instrument designed for reaching emission targets is the cap and trade system. In other words, (international) emission permits coexist with (local) environmental taxes.

Regarding the long-lived government's objective, if this government is seen as the outcome of international negotiations then there is evidence that it fails to assign efficient amounts of emissions (both in environmental and economic terms) to states. In other words, assuming that the long-lived government pursues the objective of maximizing social welfare is not realistic. However, in the spirit of discussions in Copenhagen (2009), the minimum requirement for this government is to succeed in protecting the economy against undesirable long-run outcomes. These outcomes are those where the future will be left with irreversible pollution. This concern is the central point that justifies our analysis.

3. THE COMPETITIVE EQUILIBRIUM

The intertemporal competitive equilibrium is the dynamical system resulting from the combination of (8) with solutions of the private agents' optimization problem. The nonnegativity constraint on m_t requires one to distinguish the case where abatement is active, i.e., $m_t \geq 0$, from the case where agents do not maintain the environment, i.e., $m_t = 0$. We therefore have a dynamical system where the dynamics are defined "piecewise" on two domains. The usual analysis of dynamical systems (determination of steady states, local stability, basins of attraction) is complicated by the issue of admissibility. Indeed, it may be that the dynamics defined in one domain admit some stable point but that this equilibrium does not belong to the domain. Under such circumstances, the point is not, when the overall system is considered as a whole, a stable point of the system. We shall call admissible the equilibria that are consistent with their domain of definition. A related issue is the possibility that during the convergence toward some admissible steady state the equilibrium path changes domains. In that case, the system becomes governed by the different dynamics of this new region and no longer has a reason to converge to the equilibrium to which it originally aimed. Basins of attraction therefore also are more complex to determine.

We shall analyze the dynamics of the two domains separately and then discuss admissibility.

3.1. Definitions

Before starting the analysis, we shall define three important concepts.

The intertemporal equilibrium. Let us call a trajectory of the economy a sequence of per capita variables $\{c_t, m_t, s_t\}$, aggregate variables $\{L_t, K_t, E_t, P_t\}$, and prices $\{R_t, w_t, q_t\}$. The intertemporal equilibrium with perfect foresight then is defined as follows:

DEFINITION 1 (Intertemporal Equilibrium). *Given the environmental policy $\{\bar{E}_t\}$, a competitive equilibrium is a trajectory of the economy such that*

(i) *households and firms are at their optimum: the condition (9) and the three conditions (2), (3), and (4), for profit maximization, are satisfied;*
(ii) *all markets clear: $L_t = 1$, $K_{t+1} = s_t$ $(= k_{t+1})$, and $E_t = \bar{E}_t$ $(= e_t)$ on the permits market;*
(iii) *budget constraints (6) and (7) are satisfied;*
(iv) *the dynamics of pollution is given by (8).*

The frontier case. In our general setting, two distinct frontiers exist.

DEFINITION 2 (Frontiers). *In the k–P space, the first frontier, delimiting irreversible pollution levels from reversible ones, corresponds to the irreversibility threshold: $P_t = \overline{P}$. The second frontier, hereafter called the indifference frontier, represents the set of points (k_t, P_t) where the agents are indifferent to whether they abate pollution. When the system is located in the region above the frontier (resp. below), maintenance is nonnegative: $m_t > 0$ (resp. there is no maintenance, $m_t \equiv 0$).*

The indifference frontier is implicitly given by the first-order condition (9), in which we set $m_t = 0$ and replace $\geq$ by $=$. It defines the pollution as a monotonically decreasing function of both the capital stock and the quota: $P_t = f(k_t, \bar{E}_t)$.[8] Because of this frontier, the dynamics is defined piecewise. Indeed, it separates the positive maintenance (PM) domain ($m_t > 0$) from the zero maintenance (ZM) space ($m_t \equiv 0$). The other frontier separates the irreversible pollution space from the reversible zone.

In the remaining sections, equilibrium dynamics will be derived from Definition 1 in both the ZM and the PM region. Among possible equilibria, some may have the features of environmental and/or poverty traps.

Environmental and poverty traps. Four kinds of equilibrium outcomes will be considered. In all of these definitions, the environmental policy $\{\bar{E}_t\}$ is taken as given.

DEFINITION 3 [Environmental Trap (ET)]. *An environmental trap is a trajectory of the economy such that*

$$\lim_{t \to \infty} P_t = P_\infty,$$

with $\overline{P} < P_\infty < +\infty$.

In other words, an environmental trap is a steady state that exhibits an irreversible level of pollution.

DEFINITION 4 [Poverty Trap (PT)]. *Suppose multiple equilibria exist. A trajectory that leads to the steady state with the lowest level of capital is called a poverty trap.*

The nature of the problem leads us to make a distinction between this kind of equilibria and what we shall call asymptotic traps. The asymptotic traps correspond to equilibrium trajectories accompanied by a perpetual erosion in environmental and/or economic resources:

DEFINITION 5 [Asymptotic Environmental Trap (AET)]. *An asymptotic environmental trap is a trajectory of the economy such that*

$$\lim_{t \to \infty} P_t = +\infty.$$

DEFINITION 6 [Asymptotic Poverty Trap (APT)]. *An asymptotic poverty trap is a trajectory of the economy such that*

$$\lim_{t \to \infty} K_t = 0.$$

In the analysis that follows, we shall exclude the case where the variable K_t takes the value 0, because this case is not economically relevant.

3.2. Zero Maintenance Equilibrium (ZME)

The first region is the one where the constraint $m_t \equiv 0$ holds. It corresponds to the situation where the weight of environmental and financial constraints is such that households do not have enough incentive to abate pollution. They thus devote all of their income to savings:

$$w_t + q_t \bar{E}_t = s_t. \tag{10}$$

Equilibrium dynamics directly follows from the combination of (2), (4), (8), (10), and market clearing conditions,

$$k_{t+1} = (1-\alpha) A k_t^\alpha \bar{E}_t^{1-\alpha-\beta}, \tag{11}$$

$$P_{t+1} = P_t - \Gamma(P_t) + \bar{E}_t,$$

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and we shall note that stock variables dynamics are independent from each other. In addition, pollution accumulation is solely determined by the exogenous quota. Denote the pollution level at which the assimilation capacity is maximum by $\widetilde{P}$.

Existence conditions are summarized in the following proposition:[9]

PROPOSITION 1. *For the dynamics* (11), *assume that there exists a limit* $\bar{E} = \lim_{t\to\infty} \bar{E}_t$. *Then*

(i) *There is no environmental trap, but an asymptotic environmental trap always exists.*

(ii) *There exists a steady state with a reversible level of pollution if and only if*

$$\max_{P\in[0,\overline{P}]} \{\Gamma(P)\} = \Gamma_{\max} \geq \bar{E}. \qquad \textbf{(12)}$$

If $\Gamma_{\max} > \bar{E}$, *then there exist two distinct steady states,* (k^*_{zr}, P^{*-}_{zr}) *and* (k^*_{zr}, P^{*+}_{zr}), *that are ordered as*

$$P^{*-}_{zr} \leq \widetilde{P} \leq P^{*+}_{zr} < \overline{P}.$$

The steady state (k^*_{zr}, P^{*-}_{zr}) *is locally stable; the other state is unstable.*

Proof. See Appendixes A and D.1. ■

Condition (12) was already used in Tahvonen and Withagen (1996) and Prieur (2009) and conveys the idea that the maximum potential assimilation by nature is higher than the stationary emissions level. The latter precisely corresponds, in the zero maintenance space, to the total number of emission permits allocated to the economy. It is worth noting that the necessary and sufficient condition (12) imposes an upper boundary on the domain of values of $\bar{E}$.

In the ZM region, the economy has only one instrument to control pollution—emission permits. When the quota is set below the upper bound $\Gamma_{\max}$, two scenarios may occur depending on the initial location in the space $k - P$. A relatively little polluted economy will reach the desirable zero maintenance steady state (k^*_{zr}, P^{*-}_{zr}). Even if production generates emissions, due to increasing assimilation, the pollution stock remains below the threshold $\bar{P}$ until the ZM steady state is hit. Conversely, the economy would likely suffer from irreversible pollution if its point of departure were a highly polluted state. Indeed, the higher the initial pollution, the sooner the assimilation capacity is exhausted. After the threshold is exceeded, the logic of the asymptotic environmental trap (AET) applies because pollution accumulates indefinitely. When permits are the only instruments at the economy's disposal, the situation may be even worse if the quota is fixed above the boundary $\Gamma_{\max}$. In this case, there are no wealthy steady states with reversible pollution and the economy is doomed to follow the trajectory of the AET.

This observation highlights the importance of introducing a second instrument, namely environmental maintenance. With maintenance, the AET no longer matters, because this trajectory will necessarily cross the indifference frontier and then reach the positive maintenance subspace. In other words, once maintenance is taken into account, the AET appears to be an inadmissible equilibrium trajectory.

The purpose of the next section is to study the dynamics of the positive maintenance region.

3.3. Positive Maintenance Equilibrium (PME)

The region with $m_t > 0$ refers to situations where the economy is relatively wealthy and/or suffers from harmful pollution. Consequently, agents are willing to engage in maintenance. In this case, the maintenance decision is given by

$$R_{t+1}U_1(c_{t+1}, P_{t+1}) + \gamma U_2(c_{t+1}, P_{t+1}) = 0. \qquad \textbf{(13)}$$

The equilibrium analysis consists of considering the system of equations (2)–(4), (8), (13), and the market clearing conditions. Combining these equations yields the expression of consumption and maintenance decisions as a function of the capital stock and the quota

$$c_t = c(k_t, \bar{E}_t) = \alpha A k_t^{\alpha} \bar{E}_t^{1-\alpha-\beta}, \qquad \textbf{(14)}$$

$$m_t = m(k_t, \bar{E}_t, k_{t+1}) = (1-\alpha) A k_t^{\alpha} \bar{E}_t^{1-\alpha-\beta} - k_{t+1}. \qquad \textbf{(15)}$$

Substituting expressions (3) and (14) into (13) yields

$$R(k_{t+1}, \bar{E}_{t+1})U_1[c(k_{t+1}, \bar{E}_{t+1}), P_{t+1}] + \gamma U_2[c(k_{t+1}, \bar{E}_{t+1}), P_{t+1}] = 0, \qquad \textbf{(16)}$$

which implicitly defines an equilibrium relation, valid for any $t \geq 1$, between P_t, k_t, and $\bar{E}_t$,

$$P_t = \Phi(k_t, \bar{E}_t), \qquad \textbf{(17)}$$

which governs the dynamics in the whole positive maintenance space.

Let us first examine the properties of this relationship. Under Assumption 2, $\Phi_1 < 0$.[10] A rise in k_t tends to reduce the cost of maintenance because it lowers the interest factor and increases consumption ($R_1 < 0$, $c_1 > 0$, $U_{11} < 0$). In addition, due to the distaste effect exerted by pollution and the rise in consumption, it also means that the benefits from maintenance rise ($c_1 > 0$, $U_{12} \leq 0$). Therefore, the higher the capital, the higher is the incentive to maintain the environment and reduce pollution. According to this relation, in each period, capital stock is inversely linked to the pollutant concentration. The sign of Φ_2 is *a priori* indeterminate. If a rise in $\bar{E}_t$ increases the benefits of maintainance (through the increase in c), it is associated with two opposite effects on the cost of maintenance. In fact, a rise in $\bar{E}_t$ raises both the interest factor and consumption. Now, if we assume that the intertemporal elasticity of substitution $\sigma = -U_1/(cU_{11})$ is less than one, which means that savings is decreasing in the interest factor, the overall effect on the cost is negative and $\Phi_2 < 0$.[11]

We shall now proceed with the dynamics, which is described by the system of equations

$$P_{t+1} = \Phi(k_{t+1}, \bar{E}_{t+1}), \qquad \textbf{(18)}$$

$$P_{t+1} = P_t - \Gamma(P_t) + \Theta(k_t, \bar{E}_t, k_{t+1}),$$

where $\Theta(k_t, \bar{E}_t, k_{t+1})$ represents real emissions:

$$\Theta(k_t, \bar{E}_t, k_{t+1}) = \bar{E}_t - \gamma\left[(1-\alpha) A k_t^{\alpha} \bar{E}_t^{1-\alpha-\beta} - k_{t+1}\right].$$

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Studying the existence of positive maintenance steady states (PMSS) requires one to consider a specific interval $[0, \bar{k}(\bar{E})]$, where $\bar{k}(\bar{E})$ is defined in Appendix B.1, on which steady state maintenance is necessarily nonnegative. This analysis being done for a given $\bar{E}$, for simplification we rewrite $\Phi(k, \bar{E}) = \varphi(k)$.

PROPOSITION 2. *For the dynamics* (18)*, assume that there exists a limit* $\bar{E} = \lim_{t\to\infty} \bar{E}_t$. *Then*

(i) *A necessary condition for an environmental and poverty trap to exist is that the quota be below the limit value* $\bar{E}_L$*, with*

$$\bar{E}_L = [\gamma A(1-\alpha)^2]^{\frac{1-\alpha}{\beta}} [\alpha(1-\alpha)A]^{\frac{\alpha}{\beta}}. \quad \textbf{(19)}$$

The dynamics (18) *admits at most two steady states (but only one locally stable) with irreversible pollution.*

(ii) $\bar{E}$ *being given, assume that the sequence* $\{E_t\}_{t=0}^{\infty}$ *is decreasing, and that*

$$\lim_{k\to 0} \varphi(k) = +\infty.$$

Then there exists an asymptotic poverty and environmental trap for the dynamics (18)*.*

(iii) *If* (12) *holds, that is,*

$$\max_{P\in[0,\overline{P}]} \{\Gamma(P)\} = \Gamma_{\max} \geq \bar{E},$$

and if

$$\bar{k}(\bar{E}) \geq \varphi^{-1}(P_{zr}^{*-}), \quad \textbf{(20)}$$

then there exists a locally stable steady state $(k_{pr}^{*+}(\bar{E}), P_{pr}^{*-}(\bar{E}))$ *with reversible pollution.*

Proof. See the analyses in Appendixes B.3, B.4 (existence), and D.2 (stability). ■

This statement is, on purpose, relatively imprecise with respect to the exact number of equilibria. Studying the existence of PME turns out to be a tedious exercise. Therefore, in Proposition 2 we chose to focus on the most relevant equilibria from the point of view of our analysis. They include, on one hand, asymptotic and stationary traps (our first aim being to identify conditions that enable such undesirable states to be avoided) and, on the other, the stable steady state with reversible pollution (because we shall subsequently focus on the impact of a change in policy on desirable equilibria properties).[12]

An economy may reach a steady state with irreversible pollution even when there is an environmental policy. The environmental trap also corresponds to a poverty trap because, according to (17), it exhibits a level of capital lower than the one reached at any PM steady state with reversible pollution. The important point is that environmental poverty generates economic poverty. The mechanism behind the emergence of this long-term state is discussed in detail in Prieur (2009) and can be summarized as follows. The economy is located in a region with a pollution level above the threshold $\overline{P}$. At the same time, when $\bar{E} \leq \bar{E}_L$ the policy is so stringent that the level of wealth is also very low. In such a context, the following logic applies. Environmental pressure forces agents to devote a sizable share of resources to maintenance. However, this effort is undertaken at the expense of productive savings and causes a break in capital accumulation. Coupled with the low level of wealth, this in turn implies that the opportunity cost of maintenance becomes increasingly severe. Finally, the economy fails to artificially bring pollution back to a reversible level. In fact, the reaction of agents to environmental damage only allows the economy to stabilize pollution at a constant—but higher than $\overline{P}$—level. However, this steady state is reached at the expense of wealth.

In contrast with the scenario discussed in Prieur (2009), there exists in our framework a means to prevent the occurrence of such a long-term state.

COROLLARY 1. *A sufficient condition to exclude the existence of environmental and poverty traps is to fix the quota on emissions to a sufficiently high level:* $\bar{E} > \bar{E}_L$.

Environmental policy should authorize firms to emit a sufficient amount of pollution to avoid having the economy stabilize in a trap. Indeed, in the positive maintenance space the economy has two instruments that affect the level of polluting emissions. The existence of a steady state requires real emissions to be nil. In other words, the amount of pollution emitted by firms must be exactly compensated for by households' abatement. This situation occurs when the exogenous quota is set below the critical value $\bar{E}_L$. By fixing $\bar{E} > \bar{E}_L$, one mechanically ensures the avoidance of environmental and poverty traps, $E\&PT$. However, the scope of this result must be put into context.

Two possible outcomes remain: depending on its initial location in the positive maintenance space, the economy may follow two opposite development trajectories. The first one leads to the desirable PM steady state, defined in Proposition 2 (iii), that exhibits the lowest level of pollution. Because pollution and capital are inversely linked at equilibrium, it also is associated with the highest wealth.[13] Again, existence involves (12), which is now a sufficient condition. The additional sufficient condition (20) ensures some correspondence between the domains of variation of the stock variables k and P. This equilibrium outcome is clearly better than the E&PT, whereas the second is not. Indeed, the second trajectory has the features of both an asymptotic environmental and an asymptotic poverty trap, AE&PT. Such an undesirable outcome exists under a very general condition involving preferences.[14] It is worth noting that the problem posed by the AE&PT only arises when a stable E&PT does not exist. In case where a stable E&PT exists ($\bar{E} \leq \bar{E}_L$), the region with irreversible pollution basically coincides with its attraction basin. The AE&PT thus does not really matter: if the economy were located in this region, it would reach the E&PT. Conversely, when the quota is fixed above $\bar{E}_L$, the irreversible pollution region may no longer be an attraction basin for a steady state, and this may open the path to perpetual increase in pollution associated with continuous erosion of the level of wealth that entails the AE&PT.

This raises the following question: from the point of view of a policy maker, what is the best option? Should one exclude stationary traps by imposing $\bar{E} > \bar{E}_L$ even though the economy then is more likely to be trapped into the AE&PT? Or should one keep the E&PT, which appears to be a lesser evil than the AE&PT, because when located in this region the economy would have a greater opportunity to reach E&PT than to diverge?

Answering this question[15] requires two additional but related questions to be addressed: Can an economy that initially does not belong to the irreversible region reach such a region and diverge? To what extent does the possible divergence depend on the choice of the quota? Assuming $\bar{E} > \bar{E}_L$, the answer involves assessing the impact of a change in the quota $\bar{E}$ on the frontier delimiting the basin of attraction of the PM steady state $(k_{pr}^{*+}(\bar{E}), P_{pr}^{*-}(\bar{E}))$ from the set of points where the economy follows the AE&PT. If a rule does exist concerning the choice of $\bar{E}$ that could prevent an economy with initial reversible pollution from diverging, then the recommendation would be to set $\bar{E} > \bar{E}_L$. This important point will be discussed in Section 5, which is devoted to the dynamic analysis.

3.4. Admissibility

This section provides insight into the issue of admissibility of equilibria. Admissibility analysis consists of checking whether the steady states found for each of the two dynamical systems are indeed located within the relevant subspace. We shall establish a convention that a potential equilibrium is admissible if and only if it lies in the interior of the domain. It proves difficult to state simple conditions for admissibility or nonadmissibility in the general case. There is, however, an important property that can be proved under quite general assumptions.

PROPOSITION 3 (Admissibility). *Assume that there is one stable ZME, (k_{zr}^*, P_{zr}^*). Then*

(i) *There exists a stable PME that satisfies the sufficient condition* (20) *of Proposition 2 (iii) if and only if the ZME is not admissible.*
(ii) *If there exists a stable PME that satisfies the sufficient condition* (20) *of Proposition 2 (iii), then this equilibrium is admissible.*

Proof. See Appendix C. ■

Note that some cases are left open by Proposition 3. Actually, the proposition does not preclude the existence of stable and admissible PME, which would coexist with the stable and admissible ZME. Naturally, those PME would not satisfy the sufficient condition (20).

The next section deals with the second issue raised by the paper. It examines the implications of a change in the policy on desirable steady state properties.

4. STEADY STATE RESPONSE TO A CHANGE IN THE POLICY

This analysis is restricted to (locally) stable equilibria: the ones with the lowest level of pollution, according to Proposition 1, (ii) and to Proposition 2, (iii).

First, consider the repercussions of a tightening of policy on the stable ZME.

PROPOSITION 4. *A decrease in the quota $\bar{E}$ implies a fall in the levels of both pollution and capital at the stable ZMSS.*

Proof. It is straightforward to see that $k_{zr}^{*'}(\bar{E}) > 0$. Now, if we refer to the inverted-U shape of the assimilation function $\Gamma(P)$, then it is clear that the fall in the quota $\bar{E}$ causes a fall in the level of stationary pollution at the low stable steady state: $P_{zr}^{*'}(\bar{E}) > 0$. ■

When the emission quota is the only available instrument, we detect a trade-off between economic growth and pollution control: a stricter policy means a lower level of stationary pollution, but this is achieved at the expense of capital accumulation and long-term wealth. A reduction of $\bar{E}$ causes a drop in both emissions and the pollution accumulated during each period. However, it also generates a negative income effect (see the budget constraint (6)). A lower quota causes a reduction in wages and the environmental allowance, which implies that an agent has relatively less resources to devote to savings and maintenance (tightening of the financial constraint). This income effect translates into slower capital accumulation and a lower stock of capital at the ZMSS.

The same analysis then is conducted for the PM steady state. It yields the following result:

PROPOSITION 5. *At the stable PM steady state $(k_{pr}^{*+}(\bar{E}), P_{pr}^{*-}(\bar{E}))$,*

(i) *if*

$$\bar{E} \geq [\gamma(1-\alpha-\beta)]^{(1-\alpha)/\beta}[A(1-\alpha)]^{1/\beta}, \tag{21}$$

then $k_{pr}^{*+'}(\bar{E}) < 0$;

(ii) *if, in addition,*[16]

$$\frac{\Theta_k(k_{pr}^{*+}(\bar{E}), \bar{E}, k_{pr}^{*+}(\bar{E}))}{\Theta_2(k_{pr}^{*+}(\bar{E}), \bar{E}, k_{pr}^{*+}(\bar{E}))} < \frac{c_1(k_{pr}^{*+}(\bar{E}), \bar{E})}{c_2(k_{pr}^{*+}(\bar{E}), \bar{E})}, \tag{22}$$

then $P_{pr}^{*-'}(\bar{E}) > 0$.

Proof. The proof is provided in Appendix E. ■

Increasing $\bar{E}$ has two opposite effects on real emissions. First, it entails a rise in polluting emissions by firms. However, through the positive income effect, it also stimulates maintenance, which in turn tends to reduce emissions. Under condition (21), the net effect now is positive; namely, real emissions are increasing in $\bar{E}$ at equilibrium.

This condition is sufficient to show that a reduction of the quota causes a rise in the stock of capital at the stable steady state. Let us break down the effect of

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a fall in the quota on stationary variables. This decrease still is associated with the negative income effect described above. However, there now is an additional substitution effect. This effect is mainly due to the fall in emissions and the pollution accumulated at each period (see the dynamics given by (8)). Ceteris paribus, with the reduction of the quota, the affected generation can allocate a smaller amount of resources to maintain environmental quality, which will be enjoyed in their second period of life (slackening of environmental constraint). It also allows households to save a larger share of their income, which favors capital accumulation.

Why does the latter effect dominate at the stable PM steady state? In this equilibrium, the economy is endowed with an important capital stock before the policy reform. Moreover, the pollution level is less than the value $\widetilde{P}$ and consequently natural assimilation is increasing in the stock of pollutant. The fall in the quota causes a decrease in income, which is a priori unfavorable to both savings and maintenance expenditures. However, this tightening of the financial constraint remains quite moderate, because the economy owns a sizable level of wealth. The reduction in the number of permits sold to firms also implies a slackening of the environmental constraint, which already was not very stringent. Therefore, agents have some latitude to absorb the repercussions of the income decrease for capital accumulation. The substitution effect is entirely applicable here: maintenance serves as an adjustment variable in such a way that the fall in income does not penalize savings. Finally, the level of capital rises.

The second condition (22) concerns the direct and indirect effects of a change in $\bar{E}$ on both real emissions and consumption. A decrease in $\bar{E}$ lowers consumption through its (direct) negative effect on the interest factor. At the same time, however, capital increases (because $k_{pr}^{*\prime}(\bar{E}) < 0$), which tends, in contrast, to raise consumption (indirect positive effect). The same logic applies for real emissions. If a lower quota means lower emissions, it also is associated with higher capital and, consequently, higher emissions. This condition finally states that the ratio of the effects on consumption exceeds the corresponding ratio for emissions. This inequality holds if, for instance, the global impact of a fall $\bar{E}$ on consumption is positive, whereas it is negative for emissions (see Appendix E) and we have $P_{pr}^{*\prime}(\bar{E}) > 0$.[17]

In sum, a reduction of the quota procures a double dividend, because this reform allows the economy to enjoy higher long-run welfare. In this situation, reducing the quota first implies a decrease in emissions, which tends to reduce the level of pollution in the stable steady state. Second, it results in an increase in equilibrium consumption, which also requires pollution to decrease because of the distaste effect exerted by pollution. Last, according to Assumption 2, utility increases.

Our results share some similarities with the conclusions found in the literature on tax reform [see, among others, Bovenberg and Smulders (1995, 1996) or Bovenberg and de Mooij (1997)]. However, in contrast with these studies, obtaining a double dividend does not rely on the controversial assumption of the existence of a positive environmental externality in production. In addition, by using more general functional forms for assimilation and utility than Ono (2002), we could fill out his analysis by showing the existence of a ZME and examining its sensitivity to a change in $\bar{E}$. Furthermore, whereas Ono mainly seeks to provide a sufficient condition for obtaining his striking result in the PME,[18] here we show that the opposite outcome may emerge under plausible conditions. A policy reform consisting of the assignment of a lower quota efficiently reduces pollution and promotes economic growth.

The next section expands the dynamic analysis. In particular, it deals with the issue of convergence toward the AE&PT and displays numerical simulations that facilitate understanding of how global dynamics evolve when the quota is modified.

5. DYNAMICS UNDER ENVIRONMENTAL REGULATION

In our model, the dynamics is defined piecewise. There exist two specific regions that are separated by the indifference frontier. We shall first study how this frontier moves with respect to a change in capital or in the quota.

5.1. The Indifference Frontier

To analyze the frontiers' sensitivity to a change in the quota, it is sufficient to assume a constant environmental policy, i.e., $\bar{E}_t = \bar{E}\ \forall t$.

Assuming that $m_t = 0$ and replacing $\geq$ by $=$, (9) can be rewritten as

$$R(\Omega, \bar{E})U_1[R(\Omega, \bar{E})\Omega, P_t - \Gamma(P_t) + \bar{E}] + \gamma U_2[R(\Omega, \bar{E})\Omega, P_t - \Gamma(P_t) + \bar{E}] = 0, \quad (23)$$

with $\Omega \equiv \Omega(k_t, \bar{E}) = (1-\alpha)Ak_t^{\alpha}\bar{E}^{1-\alpha-\beta}$. This implicitly defines the indifference frontier $f(k_t, \bar{E})$. Under our assumptions on preferences, the derivatives are negative: $P_t = f(\overset{-}{k_t}, \overset{-}{\bar{E}})$.[19]The richer the economy, the lower the level of pollution from which the decision to abate pollution is taken. Moreover, in the space of state variables, the frontier shifts downward when the quota increases. Starting from an initial state in the ZM domain, the economy will engage in maintenance even faster because the global quota is high. This feature has significant implications for agents' behavior. In fact, using a large number of permits will hasten the moment when agents will be willing to devote resources to environmental maintenance. This property is due to the tightening of the environmental constraint: when more permits are allocated to firms, there are more harmful emissions during each period. The explanation is also based on the role played by the redistributive facet of environmental policy: a rise in $\bar{E}$ tends to increase agents' incomes and implies that the weight of the financial constraint diminishes in their tradeoffs. They thus have a greater incentive to reduce pollution. In contrast, fixing the quota to a lower level provokes a shifting of the frontier toward the top of the k–P space and delays the moment when maintenance becomes operative.

Thereafter, we illustrate our analysis in the particular case where the assimilation and utility functions are

$$\Gamma(P_t) = \begin{cases} \theta P_t(\overline{P} - P_t) & \forall P_t < \overline{P} \\ 0 & P_t \geq \overline{P} \end{cases}$$

and

$$U(c, P) = \log c - \phi \frac{P^2}{2}.$$

The assimilation function defined above satisfies Assumption 1 if $\theta < 1$, with $\widetilde{P} = \overline{P}/2$ and $\Gamma_{\max} = \theta \overline{P}^2/4$. The utility function satisfies Assumption 2.

5.2. Dynamics and Basins of Attraction

The dynamics of the model for the specifications above are computed in Appendix F.1.

In the region of positive maintenance, the dynamics given by the system of equations (18) has the effect that every initial state is mapped onto some curve of equation $P = \varphi(k)$, and the state then remains on this curve. Under appropriate conditions, there are two fixed-point states that are located on this curve. The set of initial positions (k_0, P_0), which are mapped by the dynamics to these two equilibria, form two curves that delimit three zones that are illustrated schematically in Figure 1. The equations for these curves are described in Appendix F.2.

The fixed point with smaller capital and larger pollution is unstable. The other fixed point is a stable equilibrium. However, this point may or may not be located inside the PM region—or, in other words, it may or may not be admissible. These two situations are represented respectively on the left and on the right of Figure 1.

The zone at the top is the divergence zone in both cases.[20] A trajectory that originates there goes to an asymptotic trap, as illustrated in Figure 2, in which specific numerical values are used. When the equilibrium $(k_{pr}^{*+}(\bar{E}), P_{pr}^{*-}(\bar{E}))$ is admissible, a trajectory originating in the central zone approaches this equilibrium from above: the sequence k_t is increasing and the sequence P_t is decreasing, except possibly for the first step. In the bottom zone, trajectories approach the stable equilibrium from below. When the $(k_{pr}^{*+}(\bar{E}), P_{pr}^{*-}(\bar{E}))$ is not admissible, trajectories may follow the curve $P = \varphi(k)$, and then leave it to converge to the ZME, which is admissible in that case.

5.3. Effect of Choosing the Quota

Corollary 1 states that it is possible to exclude environmental and poverty traps provided the exogenous quota is higher than the critical level $\bar{E}_L$ (see Section 3.3). However, nothing guarantees that the growth path of an economy located in the region with irreversible pollution is not an asymptotic environmental and poverty trap, AE&PT. Indeed, Figure 2 provides an illustration of this kind of trajectory.

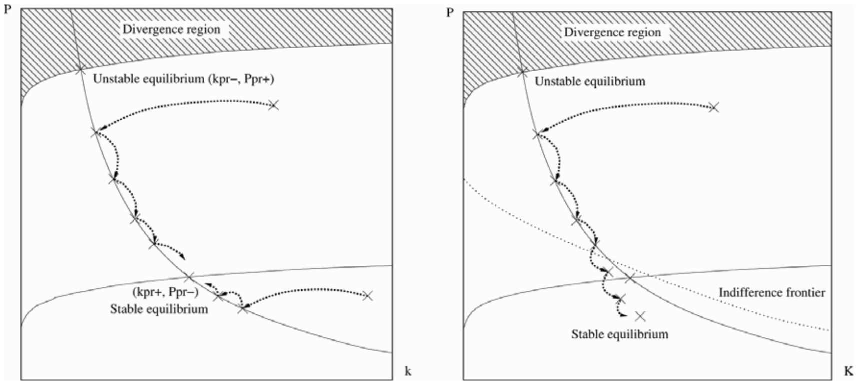

FIGURE 1. Convergence patterns. Left: without admissible zero-maintenance equilibrium. Right: without admissible positive-maintenance equilibrium.

The rationale behind the existence of an AE&PT is the following. In this region, the concentration of pollution is such that, on one hand, nature does not assimilate pollution any more and, on the other, households suffer from environmental damage. In order to remedy this damage, households have no other option but to devote a sizable share of their resources to maintenance. This decision is made at the expense of savings and consumption, therefore causing a break in capital accumulation. Moreover, over the long term this effort is not enough to compensate for emissions by firms or to stop the rise in pollution. Even if pollution decreases between the first and second periods, the trajectory finally meets the equilibrium

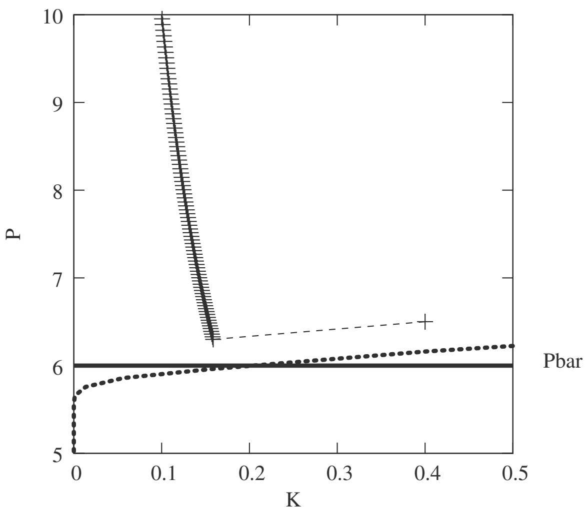

FIGURE 2. A divergence trajectory. The dashed curve is the limit of the divergence region. Parameters are listed in the text, and $\bar{E} = 0.6$.

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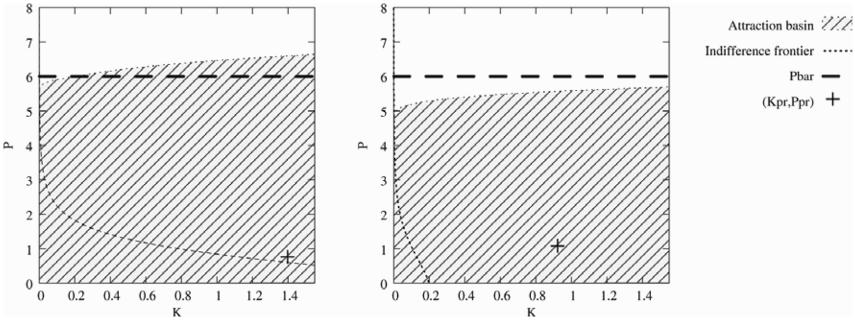

FIGURE 3. Basin of attraction and divergence region when $\bar{E} = 0.6$ (left) and $\bar{E} = 1.2$ (right).

relation, $P = \Phi(k, \bar{E})$. We then observe a decrease in capital stock accompanied by an increase in pollution. This impoverishment process inexorably reoccurs from period to period.

Starting the analysis by assuming that the quota equals a level $\bar{E} > \bar{E}_L$, we wonder what impact the choice of $\bar{E}$ may have on the economy's possibility of reaching the irreversibility space, given that it does not originally belong to it. Indeed, belonging to this space is the key factor explaining the divergence.

More precisely, we focus on the dynamics in the PM region and compare the evolution of the stable reversible solution's basin of attraction[21] with that of the divergence region. We chose for our model the following set of parameters: $(A, \alpha, \beta, \theta, \gamma, \phi, \overline{P}) = (1.9, 0.3, 0.6, 0.15, 1, 1, 6)$. In this case, the irreversibility threshold equals $\bar{E}_L = 0.58$. Graphic representations of the basins of attraction, for values $\bar{E}_1 = 0.6$ and $\bar{E}_2 = 1.2$, are displayed in Figure 3.

The comparison between these two graphics clearly reveals that the "frontier" delimiting the basin of attraction from the divergence region shifts down, in the *k–P* space, when the quota is raised. Diverging requires the economy to be situated initially in the irreversible pollution region when the quota is low (except for very low capital levels). However, once the quota is relatively important, we see that the set of initial conditions from which the economy experiences (asymptotic) divergence exhibits pollution levels less than the irreversibility threshold $\overline{P}$. In other words, for an economy that does not initially suffer from irreversible environmental damage, choosing the strictest quota minimizes, and indeed rules out, the risk that the economy will follow a development trajectory characterized by impoverishment in environmental and physical capital. This property seems quite natural because a high quota tends to strengthen the environmental constraint. The rhythm of pollution accumulation is more sustained and consequently the recovery process of nature is saturated faster. In turn, agents react by giving priority to maintenance expenditures at the expense of wealth accumulation. The impoverishment mechanism described above finally will arise for lower pollution levels (and less than the irreversibility threshold).

As far as PM solutions are concerned, this numerical example thus confirms the results obtained during the stationary analysis, because the observations tend to recommend assigning the lowest quota, provided that it is greater than the critical threshold $\bar{E}_L$.

Before ending this discussion, note that when pollution is reversible in the initial period, assigning a quota $\bar{E} = \bar{E}_L + \varepsilon$, with $\varepsilon > 0$ infinitesimal, protects the economy not only from convergence toward an environmental and poverty trap but also from a process of divergence.[22] However, the hypothetical case where initial pollution already is irreversible remains. If we considered this extreme situation, we would logically be induced to appreciably review our conclusions. In such a situation, one can expect that public authorities would have to set the quota to a very low level, and less than $\bar{E}_L$, in order to allow the economy to stabilize at a stationary trap. The convergence toward these states may finally constitute a lesser evil with regard to the perpetual impoverishment that goes with the asymptotic trap.

6. CONCLUSION

Using an OLG model with irreversible pollution, this paper first addresses the question of how to determine whether environmental regulation consisting of the assignment of an emission quota is a means to prevent the economy from reaching an environmental and/or poverty trap. In our framework, the economy potentially can face two kinds of traps. The first trap is a steady state, with an irreversible level of pollution and a low level of wealth, in which the economy can stabilize in the long run. The second is an asymptotic trap, in the sense that it corresponds to a growth path associated with perpetual erosion of both economic and environmental resources. The analysis reveals the existence of a critical threshold for polluting emissions. Choosing an emission quota above this level is a means of avoiding the "stationary" trap. Moreover, fixing the quota at the lowest level beyond this threshold is also sufficient to protect an economy that is not initially endowed with an irreversible level of pollution from falling into the asymptotic trap.

In the context of the absence of traps, we next analyze the impact of a reduction of the quota on desirable equilibrium properties. The repercussions are widely dependent on the type of equilibrium considered. In fact, equilibria with reversible pollution are only distinguished by the fact that private agents may or may not engage in maintenance. In the zero maintenance equilibrium, reducing the quota effectively causes a decrease in the level of pollution. However, it also implies a break in capital accumulation. There thus is a dilemma between pollution control and economic growth. In the positive maintenance equilibrium, on the other hand, we show that a tightening of environmental policy is accompanied by both a fall in pollution and a rise in capital at a steady state. Thus, a reform of permits that has ambitious environmental objectives brings a double dividend to the economy.

NOTES

1. Irreversibility is involved in local pollutant problems such as the eutrophication of lakes, the salinification of soils, or the loss of biodiversity because of land use [Dasgupta and Mäler (2003)]. There is also now more and more evidence that global environmental threats, such as global warming, are associated with irreversible pollution. Indeed, experts of the second working group of the Intergovernmental Panel on Climate Change (2007) have identified positive climate feedbacks due to emissions of greenhouse gases (GHG). Consequences of increasing emission levels and concentrations of GHG for the regeneration capacity of natural ecosystems can be summarized as follows. Oceans, which form the most important carbon sink, display a buffering capacity that begins saturating. At the same time, the assimilation capacity of terrestrial ecosystems (lands, forests, the other important carbon sink) will likely peak by midcentury and then decline to become a net source of carbon by the end of the present century.

2. See the discussion in Prieur (2009) on the relevance of this kind of shape.

3. We do not consider any first period consumption. This simplification allows us to focus on the crucial trade-off between final good and environmental good consumption. Adding first period consumption would not change our qualitative results.

4. The notation U_i stands for the derivative of function U with respect to its ith variable.

5. Pollution exerts a "distaste" effect on consumption [Michel and Rotillon (1995)].

6. Irreversibility should be understood as ecological irreversibility (due to the existence of a threshold in the decay function). It has nothing to do with irreversibility in the decision making process, á la Arrow and Fisher (1974), which would imforce real emissions $E_t - \gamma m_t$ to be nonnegative. In our setting, if in a period the threshold were to be exceeded, it would be possible—but not systematic—to bring pollution back to a level where natural assimilation becomes operative again by devoting a sufficient amount of resources to maintenance.

7. In this framework, households typically face an intergenerational externality. When a young agent chooses the amount of resources to devote to maintenance, she only cares about the environment she will enjoy in old age and ignores the benefits of her investment for future generations.

8. See Section 5.1 for a complete examination of its properties.

9. The subscript "z" (resp. "p") stands for the zero maintenance (resp. positive maintenance) solution. In the remainder of the paper, the second index "r" (resp. "i") will refer to a reversible (resp. irreversible) level of pollution.

10. Total differentiation of (16) with respect to k yields

$$\frac{dP_t}{dk_t} = -\frac{R_1 U_1 + R c_1 U_{11} + \gamma c_1 U_{12}}{R U_{12} + \gamma U_{22}}.$$

This derivative is negative under the assumptions on preferences and the Cobb–Douglas production function.

11. The expression of Φ_2 is given by

$$\frac{dP_t}{d\bar{E}_t} = -\frac{R_2 U_1 + R c_2 U_{11} + \gamma c_2 U_{12}}{R U_{12} + \gamma U_{22}}.$$

The sign of the numerator in Φ_2 is unknown. But we can rewrite

$$R_2 U_1 + R c_2 U_{11} = R_2 U_1 \left(1 - \frac{1}{\sigma}\right).$$

Now, imposing $\sigma < 1$ implies $\Phi_2 < 0$.

12. It is possible to provide a more general characterization of PMSS by referring to a simple graphical analysis (see Appendix B.4).

13. That is why we use the superscript "$-$" to reflect the level of pollution and "$+$" for the level of capital.

14. Which is fulfilled, for example, by the following utility function: $U(c, P) = \log c - \frac{\phi}{2} P^2$.

15. That is influenced by our aim of providing sufficient conditions to avoid traps.

16. With a slight abuse of notation, we have denoted

$$\Theta_k = \frac{d}{dk}\Theta(k, \bar{E}, k).$$

17. Note that if, following Ono (2002), we consider the specific class of separable utility functions that are logarithmic in consumption, then pollution is positively related to capital but independent of the quota. Thus, $P_{pr}^{*\prime}(\bar{E}) > 0$ under condition (21) alone.

18. That is, a decrease in the quota causes pollution to increase and capital to decrease in steady state.

19. They are given by

$$\frac{dP_t}{dk_t} = -\Omega_1 \frac{R_1 U_1 + (RU_{11} + \gamma U_{12})(R_1\Omega + R)}{(1-\Gamma')(RU_{12} + \gamma U_{22})},$$

$$\frac{dP_t}{d\bar{E}} = -\frac{(R_1\Omega_2 + R_2)U_1 + (RU_{11} + \gamma U_{12})((R_1\Omega_2 + R_2)\Omega + R\Omega_2)}{(1-\Gamma')(RU_{12} + \gamma U_{22})}.$$

20. In the rest of the paper, we will speak of "convergence toward an asymptotic trap" and "divergence" interchangeably, because the sequence of pollution stocks diverges along this type of trajectory.

21. The stable reversible solution is unique here and corresponds to the PM steady state defined in Proposition 2.

22. This is the case of study that a priori makes the most sense. In fact, the role of environmental policy is to intervene before facing an irreparable situation.

23. Because we restrict the analysis to quotas that are greater than the threshold $\bar{E}_L$, imposing $\bar{E}_L \geq \bar{E}_c$ is sufficient to conclude. More precisely, $\bar{E}_L \geq \bar{E}_c$ holds when and only when $\beta \geq (1-\alpha)(1-\alpha^{\alpha/(1-\alpha)})$. This bound is not very restrictive. If we suppose that the share of labor in production $1-\nu$ belongs the range (0.6, 0.7) (which is the common range for the estimations of this parameter), then this inequality is satisfied, for instance, for $\zeta = 1$.

24. Note that this function satisfies the assumption (ii) in Proposition 2.

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APPENDIX A: EXISTENCE CONDITIONS FOR ZME (PROPOSITION 1)

In the ZM region, a steady state solves

$$k = (1-\alpha)Ak^{\alpha}\bar{E}^{1-\alpha-\beta}, \tag{A.1}$$

$$\Gamma(P) = \bar{E},$$

where $\bar{E} = \lim_{t\to\infty}\bar{E}_t$ is assumed to exist.

The first equation has a unique solution different from 0:

$$k^*_{zr}(\bar{E}) = \left[(1-\alpha)A\bar{E}^{1-\alpha-\beta}\right]^{\frac{1}{1-\alpha}}.$$

When pollution is irreversible, the second equation in (A.1) imposes $\bar{E} = 0$. But, under Assumption 2, this limit case is excluded. As for the existence of an AET, it is enough to see that if, for some t_0, $P_{t_0} > \overline{P}$, then the dynamics of pollution (11) takes the form $P_{t+1} = P_t + \bar{E}_t$, for all $t \geq t_0$, and therefore if $\bar{E}_t \to \bar{E}$, the sequence P_t diverges to infinity.

If pollution is reversible, then steady state pollution must solve, for a given quota $\bar{E}$, $\Gamma(P) = \bar{E}$. According to the inverted U shape of $\Gamma(P)$, it is clear that $\Gamma(P) = \bar{E}$ admits a solution $P^*_{zr}(\bar{E})$ iff $\Gamma_{\max} \geq \bar{E}$, where $\Gamma_{\max}$ is the maximal absorption level reached for a given $\widetilde{P}$. Moreover, if condition (12) holds with strict inequality, then there are two positive steady state values for pollution, and $\widetilde{P}$ lies in between. The proof of local stability is provided in Appendix D.1.

APPENDIX B: EXISTENCE CONDITIONS FOR PME (PROPOSITION 2)

B.1. PROPERTIES OF $m(k, \bar{E})$

Equilibrium maintenance is defined as

$$m(k, \bar{E}) = (1-\alpha)A\bar{E}^{1-\alpha-\beta}k^{\alpha} - k$$

with, for $k > 0$ and $\bar{E} > 0$,

$$m_1(k, \bar{E}) = \alpha(1-\alpha)Ak^{\alpha-1}\bar{E}^{1-\alpha-\beta} - 1$$

and $m_{11}(k, \bar{E}) < 0$, $m_2(k, \bar{E}) > 0$, $m_{22}(k, \bar{E}) < 0$, $m_{12}(k, \bar{E}) > 0$.

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For a given $\bar{E} > 0$, $\exists! k \in [0, +\infty)$ such that $m(k, \bar{E}) = 0$. Let $\bar{k}(\bar{E})$ be this value:

$$\bar{k}(\bar{E}) = \left[(1-\alpha)A\bar{E}^{1-\alpha-\beta}\right]^{\frac{1}{1-\alpha}}. \tag{B.1}$$

Then $m(k, \bar{E}) \geq 0$ for $k \in [0, \bar{k}(\bar{E})]$, $\bar{k}'(\bar{E}) > 0$. Note that $\bar{k}(\bar{E})$ is equal to $k_z^*(\bar{E})$, the level of capital in the zero maintenance steady state.

In addition, $\exists! k \in [0, \bar{k}(\bar{E})]$ such that $m_1(k, \bar{E}) = 0$. Let $\widetilde{k}(\bar{E})$ be this value:

$$\widetilde{k}(\bar{E}) = \left[\alpha(1-\alpha)A\bar{E}^{1-\alpha-\beta}\right]^{\frac{1}{1-\alpha}}. \tag{B.2}$$

Then $m_1(k, \bar{E}) \geq 0$ for $k \in [0, \widetilde{k}(\bar{E})]$, the value reached in $\widetilde{k}(\bar{E})$, is

$$m(\widetilde{k}(\bar{E}), \bar{E}) = A(1-\alpha)^2 [\alpha(1-\alpha)A]^{\frac{\alpha}{1-\alpha}} \bar{E}^{\frac{1-\alpha-\beta}{1-\alpha}},$$

and $m(\widetilde{k}(\bar{E}), \bar{E}) > 0$ for all $\bar{E} > 0$. Note that $\widetilde{k}'(\bar{E}) > 0$, for all $\bar{E} > 0$.

B.2. EQUILIBRIUM DYNAMICS

The analysis focuses on the existence of both asymptotic traps and positive steady states.

As for steady states, we restrict the study to the interval $[0, \bar{k}(\bar{E})]$, with $\bar{k}(\bar{E})$ defined in (B.1), on which maintenance is necessarily nonnegative (see Appendix B.1). The system to solve is

$$P = \Phi(k, \bar{E}),$$

$$\Gamma(P) = \Theta(k, \bar{E}, k),$$

where $\Gamma(P)$ may be zero if P happens to be larger than $\overline{P}$. Note that the study of PM steady state will be performed taking $\bar{E}$ as given. This allows us to rewrite functions $\Phi(k, \bar{E})$ and $\Theta(k, \bar{E}, k)$ as follows:

$$\Phi(k, \bar{E}) = \varphi(k) \tag{B.3}$$

with $\varphi'(k) < 0$ and

$$\Theta(k, \bar{E}, k) = \theta(k).$$

Therefore, the problem of finding equilibria is reduced to the analysis of the equation

$$\Gamma[\varphi(k)] = \theta(k). \tag{B.4}$$

B.3. EXISTENCE OF AN ASYMPTOTIC TRAP (PROPOSITION 2 (II))

First, observe that as $\varphi(k)$ is decreasing, so is $\varphi^{-1}(P)$. Assume $\bar{E}$ is fixed, and pick a value $\eta \in (0, \bar{E})$.

Using a recurrence, we prove that if (k_t, P_t) is such that

$$P_t \geq \widetilde{P}, \quad P_t = \varphi(k_t), \quad \text{and} \quad \Gamma(P_t) + \gamma(1-\alpha)A\bar{E}_t^{1-\alpha-\beta}k_t^{\alpha} \leq \bar{E} - \eta, \tag{B.5}$$

for $t = t_0$, then for all $t \geq t_0$, Property (B.5) holds, and $P_{t+1} \geq P_t + \eta$. This will imply that $P_t \to \infty$; hence the asymptotic environmental trap. Because $\lim_{k\to 0} \varphi(k) = +\infty$, we have $\lim_{P\to\infty} \varphi^{-1}(P) = 0$. This will imply that $k_t \to 0$, hence the asymptotic poverty trap.

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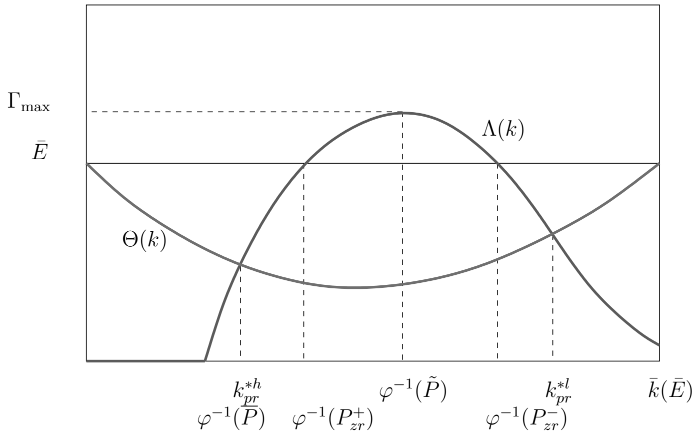

FIGURE B.1. Behavior of $\theta(k)$ and $\Lambda(k)$.

Assume therefore that (B.5) holds. The dynamics (18) for P_t implies that

$$\begin{aligned} P_{t+1} &= P_t - \Gamma(P_t) + \bar{E}_t - \gamma(1-\alpha)A\bar{E}_t^{1-\alpha-\beta}k_t^{\alpha} + \gamma k_{t+1} \\ &\geq P_t + \bar{E}_t - \left[\Gamma(P_t) + \gamma(1-\alpha)A\bar{E}_t^{1-\alpha-\beta}k_t^{\alpha}\right] \\ &\geq P_t + \bar{E}_t - \bar{E} + \eta \;\geq\; P_t + \eta. \end{aligned}$$

In the last inequality, we have used the fact that $\bar{E}_t$ decreases to $\bar{E}$. But because $\varphi^{-1}(P)$ is decreasing and $P_{t+1} > P_t$, we have $k_{t+1} = \varphi^{-1}(P_{t+1}) \leq \varphi^{-1}(P_t) = k_t$. Using the facts that $\Gamma(P)$ is also decreasing for $\Gamma(P)$ for $P \geq \widetilde{P}$, and that $\bar{E}_{t+1} \leq \bar{E}_t$ by assumption, we finally have

$$\Gamma(P_{t+1}) + \gamma(1-\alpha)A\bar{E}_{t+1}^{1-\alpha-\beta}k_{t+1}^{\alpha} \leq \Gamma(P_t) + \gamma(1-\alpha)A\bar{E}_t^{1-\alpha-\beta}k_t^{\alpha},$$

and property (B.5) holds at $t+1$.

To conclude with the existence of an asymptotic poverty and environmental trap, observe that there indeed exist values (k_t, P_t) such that (B.5) holds. It is sufficient to pick k_t small enough so that $\gamma(1-\alpha)A\bar{E}^{1-\alpha-\beta}k_t^{\alpha} \leq \bar{E} - \eta$, and $P_t = \max(\varphi(k_t), \overline{P})$.

B.4. EXISTENCE OF POSITIVE MAINTENANCE STEADY STATES

This section provides existence conditions for both reversible and irreversible steady states (Propositions 2 (i) and (iii)).

According to (B.4), studying existence boils down to comparing the behavior of two functions of k, $\bar{E}$ being given. The general form of these two functions is displayed in Figure B.1.

The first function, $\theta(k) := \Theta(k, \bar{E}, k) = \bar{E} - \gamma m(k, \bar{E})$, is such that $\theta'(k) = -\gamma m_1(k, \bar{E})$, $\theta(0) = \theta' s[\bar{k}(\bar{E})] = \bar{E}$. It is first decreasing until $\widetilde{k}(\bar{E})$, then increasing until $\bar{k}(\bar{E})$. It is convex: $\theta''(k) > 0$. Thus, $\theta(k)$ has a U shape.

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The second function, $\Lambda(k) = \Gamma[\varphi(k)]$, has a first derivative, $\Lambda'(k) = \varphi'(k)\Gamma'[\varphi(k)]$, whose sign follows from the properties of $\Gamma()$ because $\varphi'(k) < 0$: it is negative when $\varphi(k) \in [0, \widetilde{P}] \leftrightarrow k \in [\varphi^{-1}(\widetilde{P}), \varphi^{-1}(0)]$, whereas it is positive for any $k \in [\varphi^{-1}(\overline{P}), \varphi^{-1}(\widetilde{P})]$. Thus, $\Lambda(k)$ is also inverted U-shaped. In addition, $\Lambda[\varphi^{-1}(\overline{P})] = 0$ and $\Lambda[\varphi^{-1}(\tilde{P})] = \Gamma(\widetilde{P}) = \Gamma_{\max}$.

The main difficulty we encounter when comparing $\theta(k)$ and $\Lambda(k)$ is that the ranking between values of k determining the properties of $\theta(k)$ ($\tilde{k}(\bar{E})$, $\bar{k}(\bar{E})$, etc.) and those related to the behavior of $\Lambda(k)$ ($\varphi^{-1}(\overline{P})$, $\varphi^{-1}(\widetilde{P})$, etc.) is a priori unknown. Thus, there is no other option but to impose technical conditions in order to ensure some correspondence between the domains of definition of these two functions.

Assume $\bar{k}(\bar{E}) > \varphi^{-1}(\widetilde{P})$; then two kinds of equilibria may exist. Their corresponding levels of capital belong to three different subintervals of $[0, \bar{k}(\bar{E})]$.

Environmental traps: A SS in $[0, \varphi^{-1}(\overline{P})]$ is an ET. Referring to Figure B.1, such a SS exists if and only if the curves $\Lambda(k)$ and $\theta(k)$ intersect for some $k < \varphi^{-1}(\overline{P})$, that is to say, $\theta(k)$ crosses the horizontal axis. A necessary condition for this is that the minimal value of $\theta(k)$ be negative. This minimal value is $\bar{E} - \gamma m(\widetilde{k}(\bar{E}), \bar{E})$, where the value of $\widetilde{k}$ has been established in (B.2). A direct computation reveals that the relation $\gamma m(\widetilde{k}(\bar{E}), \bar{E}) \geq \bar{E}$ holds if and only if $\bar{E} \leq \bar{E}_L$ with

$$\bar{E}_L = \left[\gamma A(1-\alpha)^2\right]^{\frac{1-\alpha}{\beta}} [\alpha(1-\alpha)A]^{\frac{\alpha}{\beta}}.$$

Hence the necessary condition. This condition is not sufficient, however, because it remains possible that the two curves cross each other for some $k > \varphi^{-1}(\overline{P})$. If the curve $\theta(k)$ crosses the horizontal axis, it can do it at most twice, because it is convex.

According to the sufficient conditions for local stability/instability (see Appendix D.2), the SS with the lowest capital is unstable (because it verifies $\theta'(k) < 0$ and $\Lambda'(k) = 0$) whereas the second (with $\theta'(k) > 0$) is locally stable. Thus, the latter is the poverty trap.

Steady states with reversible pollution: Consider first the interval $[\varphi^{-1}(\overline{P}), \varphi^{-1}(\widetilde{P})]$:

- Assume first that $\widetilde{k}(\bar{E}) > \varphi^{-1}(\overline{P})$. If $\bar{E} > \bar{E}_L$ and (12) holds that is, if $\Gamma_{\max} \geq \bar{E}$, then there exists a unique SS when the curves cross each other, $\theta(k)$ being decreasing and $\Lambda(k)$ being increasing.
  A steady state may also exist at the intersection between the two curves when both are increasing.
  These SS alternatively satisfy $\theta'(k) < \Lambda'(k)$ and the reverse, these inequalities respectively being the sufficient condition for instability and for local stability.
- Consider now the case $\widetilde{k}(\bar{E}) < \varphi^{-1}(\overline{P})$: only the second type of equilibrium exists if $\bar{E} > \bar{E}_L$ and (12) holds. Under these conditions, the curves cross each other at least once. Because $\Lambda[\varphi^{-1}(\overline{P})] < \theta[\varphi^{-1}(\overline{P})]$ and $\Lambda[\varphi^{-1}(\widetilde{P})] \geq \bar{E} > \theta(\varphi^{-1}(\widetilde{P}))$, one intersection lies in the interval $[\varphi^{-1}(\overline{P}), \varphi^{-1}(\widetilde{P})]$. Note that it cannot be deduced, without further assumptions, that this intersection is unique, because $\Lambda(k)$ is not convex.

Next, on the interval $[\varphi^{-1}(\widetilde{P}), \bar{k}(\bar{E})]$, there is a unique steady state if $\bar{k}(\bar{E}) > \varphi^{-1}(P_{zr}^{-})$ and (12) holds. The technical condition enables to rank the two functions at the upper bound $\bar{k}(\bar{E})$ because $\theta[\bar{k}(\bar{E})] = \bar{E} > \Lambda[\bar{k}(\bar{E})]$. In addition, with (12), it is sufficient to ensure the existence of an intersection on the interval. Because it is more restrictive than $\bar{k}(\bar{E}) > \varphi^{-1}(\widetilde{P})$, it is the one that appears in Proposition 2.

This steady state $(k_{pr}^{*+}, P_{pr}^{*-})$, with $\theta'(k) > 0$ and $\Lambda'(k) < 0$, is locally stable (see Appendix D.2). Among the PM steady state with reversible pollution, it is the one associated with the highest wealth and, according to the equilibrium relation (17), the lowest pollutant concentration.

APPENDIX C: ADMISSIBILITY ANALYSIS (PROOF OF PROPOSITION 3)

First observe that according to Proposition 1 and the discussion on stability in Appendix D.1, inequality (12) in the strict sense is a necessary condition for a stable ZMSS to exist. From the analysis of Appendix D.1, this ZMSS (k_{zr}^*, P_{zr}^*) is a solution of the fixed-point system:

$$k = \Omega(k), \qquad P = g(P) := P - \Gamma(P) + \bar{E}$$

with $\Omega(k, \bar{E}) = (1-\alpha)Ak^{\alpha}\bar{E}^{1-\alpha-\beta}$. According to (15) and (18), the indifference frontier is the set of points (k_t, P_t) that satisfy $m_t = 0$; that is,

$$P_{t+1} = \varphi(k_{t+1}) \qquad P_{t+1} = g(P_t) \qquad k_{t+1} = \Omega(k_t).$$

In other words, this is the set of points that obey the two dynamics simultaneously. The equation of the frontier is therefore

$$g(P) = \varphi[\Omega(k)].$$

More precisely, the ZM region is the set of points where $\varphi[\Omega(k)] \geq g(P)$. The interior of this region is characterized by the inequality $\varphi[\Omega(k)] > g(P)$, and the positive maintenance region is where $m_t > 0$, which is equivalent to $\varphi[\Omega(k)] < g(P)$. We have already seen that under Assumption 2, the function $\varphi(k)$ is decreasing.

Assume now that the ZMSS (k_{zr}^*, P_{zr}^*) is admissible, that is, in the ZM region. Necessarily,

$$\varphi[\Omega(k_{zr}^*)] > g(P_{zr}^*).$$

But $k_{zr}^* = \Omega(k_{zr}^*)$ and $P_{zr}^* = g(P_{zr}^*)$, so the condition is simply

$$\varphi(k_{zr}^*) > P_{zr}^*. \tag{C.1}$$

Condition (20) of Proposition 2 (iii) for the existence of a locally stable PM steady state is

$$\varphi^{-1}(P_{zr}^*) \leq k_{zr}^*,$$

which is equivalent to

$$P_{zr}^* \geq \varphi(k_{zr}^*).$$

This is in contradiction to condition (C.1). This PM steady state does not exist. Conversely, if the inequality (20) holds, the PM steady state exists, but the ZMSS is not admissible. This proves statement (i).

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Assume now that (20) holds, and that the stable PM steady state exists. The analysis of Appendix B.4 explains that the capital level k_{pr}^{*+} solves the equation

$$\Gamma[\varphi(k_{pr})] = \bar{E} - \gamma m(k_{pr}). \tag{C.2}$$

On one hand, because the equilibrium maintenance is positive, we must have $m(k_{pr}) = \Omega(k_{pr}) - k_{pr} > 0$. Because φ is decreasing, this implies that

$$\varphi[\Omega(k_{pr})] < \varphi(k_{pr}). \tag{C.3}$$

On the other hand, introducing the functions $g(\cdot)$ and $\Omega(\cdot)$ into (C.2), we find

$$\varphi(k_{pr}) = g[\varphi(k_{pr})] - \gamma m(k_{pr}) < g[\varphi(k_{pr})]. \tag{C.4}$$

Joining (C.3) to (C.4) and using the fact that $P_{pr} = \varphi(k_{pr})$, we find

$$\varphi[\Omega(k_{pr})] < g(P_{pr}),$$

which means precisely that the PM steady state is admissible.

APPENDIX D: LOCAL DYNAMICS AND STABILITY

D.1. THE ZMSS

In the dynamics system (11), the two variables are independent. The dynamics for $\{k_t\}$ takes the form

$$k_{t+1} = \Omega(k_t) = m(k_t, \bar{E}) + k_t.$$

It is possible to deduce from the analysis of the function m (see Appendix B.1) that at the fixed point $\bar{k}(\bar{E})$, $0 < \Omega'(\bar{k}(\bar{E})) < 1$. This implies local stability for $k_{zr}^*(\bar{E})$.

Next, consider the dynamics for P. In the case $\Gamma_{\max} = \bar{E}$, there is a unique fixed point $P_{zr}^*(\bar{E}) = \widetilde{P}$. In that case, the sequence $\{P_t\}$ is increasing if $P_0 \leq \widetilde{P}$, and it converges to $\widetilde{P}$. However, if $P_0 > \widetilde{P}$, the sequence diverges to $+\infty$.

In the case $\Gamma_{\max} > \bar{E}$, there are two fixed points located on each side of $\widetilde{P}$: $P_{zr}^{*-}(\bar{E}) < \widetilde{P} < P_{zr}^{*+}(\bar{E})$. Moreover, from Assumption 1, $\Gamma'[P_{zr}^{*+}(\bar{E})] < 0 < \Gamma'[P_{zr}^{*-}(\bar{E})] < 1$. On the other hand, by linearizing the system (11), we get

$$dP_{t+1} = \{1 - \Gamma'[P_{zr}^*(\bar{E})]\} dP_t.$$

It follows that the SS $P_{zr}^{*-}(\bar{E})$ is stable, but the SS $P_{zr}^{*+}(\bar{E})$ is unstable.

D.2. THE PMSS

For a reversible SS $(k_{pr}^*(\bar{E}), P_{pr}^*(\bar{E}))$, the linearization of (18) gives the Jacobian matrix

$$J = \frac{1}{\varphi'[k_{pr}^*(\bar{E})] - \gamma} \begin{pmatrix} -\gamma\Omega_1(k_{pr}^*(\bar{E}), \bar{E}) & 1 - \Gamma'[P_{pr}^*(\bar{E})] \\ -\gamma\Omega_1(k_{pr}^*(\bar{E}), \bar{E})\varphi'(k_{pr}^*(\bar{E})) & \{1 - \Gamma'[P_{pr}^*(\bar{E})]\}\varphi'(k_{pr}^*(\bar{E})) \end{pmatrix}.$$

Now, it is clear that $\det(J) = 0$. In fact, due to the equilibrium relation (17), the system reduces to a one-dimensional dynamics. Thus, the two eigenvalues are 0 and

$$\text{tra}(J) = \frac{-\gamma\Omega_1(k^*_{pr}(\bar{E}), \bar{E}) + \{1 - \Gamma'[P^*_{pr}(\bar{E})]\}\varphi'[k^*_{pr}(\bar{E})]}{\varphi'[k^*_{pr}(\bar{E})] - \gamma}.$$

According to Assumption 1, $\Gamma'(P) < 1\ \forall P$. Following the same reasoning as in Appendix D.1, we have $\Omega_1() > 0$. Because $\varphi'() < 0$, the denominator is negative, and the expression above is positive. Therefore, there will be local stability if $\text{tr}(J) < 1$ and instability if $\text{tra}(J) > 1$.

The following condition is sufficient for local stability:

$$\gamma(1 - \Omega_1(k^*_{pr}(\bar{E}), \bar{E})) > \Gamma'[P^*_{pr}(\bar{E})]\varphi'[k^*_{pr}(\bar{E})]. \tag{D.1}$$

It is equivalent to $\Theta'[k^*_{pr}(\bar{E})] > \Lambda'[k^*_{pr}(\bar{E})]$, in the notation of Appendix B.4.

Under condition (20) in Proposition 2, the equilibrium k^{*+}_{pr} satisfies

$$\tilde{k}(\bar{E}) < k^{*+}_{pr} < \bar{k}(\bar{E})$$

and

$$P^{*-}_{pr}(\bar{E}) < P^{*-}_{zr}(\bar{E}) < \widetilde{P},$$

thus it is such that $\Omega_1(k^{*+}_{pr}(\bar{E}), \bar{E}) < 1$ and $\Gamma'[P^{*-}_{pr}(\bar{E})] > 0$, and the inequality (D.1) holds. The equilibrium is therefore stable.

Finally, the following condition is sufficient for local instability:

$$\gamma(1 - \Omega_1(k^*_{pr}(\bar{E}), \bar{E})) < \Gamma'[P^*_{pr}(\bar{E})]\varphi'[k^*_{pr}(\bar{E})]. \tag{D.2}$$

In the notation of Appendix B.4, this reads as $\Theta'[k^*_{pr}(\bar{E})] < \Lambda'[k^*_{pr}(\bar{E})]$.

Any PME with k^*_{pr} (if it exists), satisfying $\Omega_1(k^*_{pr}(\bar{E}), \bar{E}) > 1$ and $\Gamma'[P^*_{pr}(\bar{E})] < 0$, is therefore unstable.

APPENDIX E: PROOF OF PROPOSITION 5

We consider the impact of a change in $\bar{E}$ on the equilibrium outcome $(k^*_{pr}(\bar{E}), P^*_{pr}(\bar{E}))$.

The steady state solves the system

$$P^*_{pr} = \Phi(k^*_{pr}, \bar{E}),$$

$$\Gamma(P^*_{pr}) = \Theta(k^*_{pr}, \bar{E}, k^*_{pr}),$$

where $\Phi()$ and $\Theta()$ are deduced from equations (17) and (18). By substituting the equilibrium relation into the second equation, we get

$$\Gamma(\Phi(k^*_{pr}, \bar{E})) = \Theta(k^*_{pr}, \bar{E}, k^*_{pr}).$$

This equation implicitly defines k^*_{pr} as a function of $\bar{E}$: $k^*_{pr} = k^*_{pr}(\bar{E})$ with

$$\frac{dk^*_{pr}}{d\bar{E}} = \frac{\Theta_2 - \Phi_2\Gamma'}{\Phi_1\Gamma' - \Theta_k}.$$

With a slight abuse of notation, we have denoted

$$\Theta_k = \frac{d}{dk}\Theta(k, \bar{E}, k).$$

Our analysis only makes sense for the stable SS; hence we refer to the two sufficient conditions for local stability: $\Omega_1(k^*_{pr}(\bar{E}), \bar{E}) < 1$ and $\Gamma'[P^*_{pr}(\bar{E})] > 0$. The sign of the partial derivatives Φ_1 and Φ_2 being known, it remains to determine the sign of Θ_2. The emissions function takes the form $\Theta(k, \bar{E}, k) = \bar{E} - \gamma m(k, \bar{E})$. For any k, its derivative with respect to $\bar{E}$ is $\Theta_2(k, \bar{E}, k) = 1 - \gamma\Omega_2(k, \bar{E})$. Because $\Omega_{12}(k, \bar{E}) > 0$, Ω_2 is increasing in k. Computing its value at the upper bound $\bar{k}(\bar{E})$ yields

$$\Omega_2(\bar{k}(\bar{E}), \bar{E}) = (1 - \alpha - \beta)[A(1 - \alpha)]^{1/(1-\alpha)}\bar{E}^{-\beta/(1-\alpha)}.$$

Now, it appears that $\Omega_2(\bar{k}(\bar{E}), \bar{E}) \leq 1/\gamma$ is equivalent to

$$\bar{E} \geq \bar{E}_c := [\gamma(1 - \alpha - \beta)]^{(1-\alpha)/\beta}[A(1 - \alpha)]^{1/\beta}.$$

For any $\bar{E} \geq \bar{E}_c$, we have $\Theta_k(k, \bar{E}, k) \geq 0 \; \forall k \in (\varphi^{-1}(\overline{P}), \bar{k}(\bar{E})]$.[23] Therefore, if this inequality holds (condition (21) in Proposition 5), then it appears that $k^{*\prime}_{pr}(\bar{E}) < 0$.

Next, we replace k^*_{pr} with $k^*_{pr}(\bar{E})$ in the equilibrium relation to compute the derivative of $P^*_{pr}(\bar{E})$. We get

$$P^{*\prime}_{pr}(\bar{E}) = \frac{\Phi_1\Theta_2 - \Phi_2\Theta_k}{\Phi_1\Gamma' - \Theta_k},$$

and this expression is equivalent to

$$P^{*\prime}_{pr}(\bar{E}) = \frac{U_1(\Theta_k R_2 - \Theta_2 R_1) + (RU_{11} + \gamma U_{12})(\Theta_k c_2 - \Theta_2 c_1)}{(\Phi_1\Gamma' - \Theta_k)(RU_{12} + \gamma U_{22})}.$$

The denominator and the first term in the numerator are positive. Because $RU_{11} + \gamma U_{12} < 0$, imposing $\Theta_k c_2 - \Theta_2 c_1 < 0$ (second condition in Proposition 5) ensures that $P^{*\prime}_{pr}(\bar{E}) > 0$. This condition can be rewritten

$$\frac{\Theta_k(k^*_{pr}(\bar{E}), \bar{E}, k^*_{pr}(\bar{E}))}{\Theta_2(k^*_{pr}(\bar{E}), \bar{E}, k^*_{pr}(\bar{E}))} < \frac{c_1(k^*_{pr}(\bar{E}), \bar{E})}{c_2(k^*_{pr}(\bar{E}), \bar{E})}.$$

Now, note that SS consumption and emissions can be expressed as

$$c(k^*_{pr}(\bar{E}), \bar{E}) = R(k^*_{pr}(\bar{E}), \bar{E}) = c^*_{pr}(\bar{E}),$$

$$\Theta(k^*_{pr}(\bar{E}), \bar{E}, k^*_{pr}(\bar{E})) = \bar{E} - \gamma m(k^*_{pr}(\bar{E}), \bar{E}) =: \Theta^*_{pr}(\bar{E}),$$

and their derivatives with respect to $\bar{E}$ read respectively

$$c^{*\prime}_{pr}(\bar{E}) = c_1 k^{*\prime}_{pr}(\bar{E}) + c_2,$$

$$\Theta^{*\prime}_{pr}(\bar{E}) = \Theta_k k^{*\prime}_{pr}(\bar{E}) + \Theta_2.$$

Thus, imposing $c_{pr}^{*\prime}(\bar{E}) < 0$ and $\Theta_{pr}^{*\prime}(\bar{E}) > 0$ implies

$$-\frac{\Theta_k(k_{pr}^*(\bar{E}), \bar{E}, k_{pr}^*(\bar{E}))k_{pr}^{*\prime}(\bar{E})}{\Theta_2(k_{pr}^*(\bar{E}), \bar{E}, k_{pr}^*(\bar{E}))} < 1 < -\frac{c_1(k_{pr}^*(\bar{E}), \bar{E})k_{pr}^{*\prime}(\bar{E})}{c_2(k_{pr}^*(\bar{E}), \bar{E})}$$

and the condition (22) follows from this ranking.

APPENDIX F: DYNAMIC ANALYSIS: AN EXAMPLE

F.1. GLOBAL DYNAMICS

In the ZM region, dynamics is given by

$$k_{t+1} = (1-\alpha)Ak_t^{\alpha}\bar{E}^{1-\alpha-\beta},$$

$$P_{t+1} = P_t[1-\theta(\overline{P} - P_t)] + \bar{E}$$

if pollution is reversible. Otherwise, the dynamics for pollution is

$$P_{t+1} = P_t + \bar{E}.$$

For the specific functional forms considered in Section 5, the equilibrium relation $\Phi()$ defined by (17) simplifies to[24]

$$\Phi(k, \bar{E}) = \frac{1}{\gamma\phi k}.$$

Thus, the dynamics, when maintenance is positive, becomes

$$k_{t+1} = \frac{1}{\gamma\phi P_{t+1}},$$

$$P_{t+1} = \frac{x(k_t, P_t) + \sqrt{x(k_t, P_t)^2 + 4/\phi}}{2} \geq 0,$$

with

$$x(k_t, P_t) = \begin{cases} P_t + \bar{E} - \gamma(1-\alpha)Ak_t^{\alpha}\bar{E}^{1-\alpha-\beta} & \text{if} \quad P_t \geq \bar{P} \\ P_t[1-\theta(\overline{P} - P_t)] + \bar{E} - \gamma(1-\alpha)Ak_t^{\alpha}\bar{E}^{1-\alpha-\beta} & \text{else.} \end{cases}$$

F.2. BASINS OF ATTRACTIONS

According to the dynamics of the positive maintenance region, every initial state of the system (k_0, P_0) is mapped to the curve of the equation $P = \varphi(k) = 1/(\gamma\phi k)$. The set of initial positions that are mapped to a particular point $(k_1, P_1) = (k, P)$ of this curve is obtained by solving the second equation of (3.3) (written with $t = 0$) for (k_0, P_0). This gives

$$P = P_0 - \Gamma(P_0) + \Theta(k_0, \bar{E}, k);$$

that is,

$$k_0 = \left[\frac{P_0 - \Gamma(P_0) - P + \bar{E} + \gamma k}{\gamma(1-\alpha)A\bar{E}^{1-\alpha-\beta}} \right]^{\frac{1}{\alpha}}.$$

Restricted to the curve $P = \varphi(k)$, the dynamics (3.3) has steady states and corresponding basins of attraction. The relationship above extends these basins of attraction to the rest of the positive maintenance region. In the standard case depicted in Figure 1, the curve $P = \varphi(k)$ is split into three basins: the part above the unstable steady state, this steady state itself, and the part below it. The attraction basin of the stable equilibrium can be further refined into regions where the convergence is "from the top" and "from the bottom," separated by a line where convergence occurs in one step.