# Operational cost minimization of heat pump for milk pasteurization in dairy

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A thermo economic optimization analysis is presented yielding simple algebraic formula for estimating the optimum operating conditions of interconnected heat pump assisted milk pasteurizing systems. The overall operational cost method including the cost of auxiliary heater is used in the present study, together with the thermal analyses of all system components, for thermo economic analysis of the system.

Keywords: Thermo economics, heat pump, dairy, milk pasteurizing, optimization.

Optimization of the operating temperatures and the sizes of system elements for the combination of both heating and refrigeration applications for dairy heat pump systems is extremely important in order to get minimum operational energy cost for these systems. Operation cost for milk pasteurizing system is a continuous type of cost lasting throughout the life cycle and constitutes the biggest portion of the total production cost. Although operating temperatures of the system has an effect on the initial cost of the system, it is assumed that annualized cost of the equipments has slight effect in optimization due to relatively high energy costs in recent times. Energy cost rate is becoming greater than market discount rate. This tends towards energy costs of systems being more important compared with the other costs. Original capital cost of the equipment may be treated as a constant value throughout the life of the system whereas the energy cost is becoming more and more important in all sectors in recent years. There are many parameters to consider in optimizing heat pump and refrigerating systems, as depicted in Fig. 1, in a thermo economical manner. Fixing and thus eliminating all these thermal and economical parameters, except the condensing temperature  $(T_C)$ , depending on the certainty of operating characteristics of applications and the most efficient operating condition of the system, can determine optimum operating temperatures. The importance of energy saving application is increasing continuously, and interconnected heat pump and refrigeration systems may be employed for this purpose in a similar way to cogeneration systems. It is known that the performance of these

types of systems is directly related to the operating temperatures and so the capacity of the auxiliary heaters has a significant portion of energy cost. A thermo economic feasibility study is necessary before installing the combination of heat pump assisted heating and refrigeration systems. The basic topic of the present work depends upon this idea. A new thermo economic optimization technique is realized and presented for this purpose. An original formula is developed for calculating the optimum operating condition of the condensing unit at which minimum total operation cost (i.e. energy cost) occurs. A thorough search of the current literature showed that there were no previous studies on optimizing the heat pump-refrigeration interconnected systems for obtaining maximum thermo economic performance from these systems as detailed as the presented one. A practical method is used for optimizing the operating temperatures of heat pump-refrigeration system yielding the best energy economy, and original results are presented. Milk inlet and outlet temperatures, temperature of hot milk after auxiliary heater, and design temperature differences for evaporating and condensing units are all assumed as fixed design parameters for the best milk quality in pasteurizing process. Variable parameters used in formulating the thermo economically optimum condensing temperature of the milk pasteurizing system are listed as present net price of energy for auxiliary heater and electricity, design temperatures for the milk at entry, outlet and after auxiliary heater due to the design limitations. Additionally, optimum total minimum operating cost of the system and optimal sizes or capacities of all system components are obtained algebraically in the present formulation method. Optimum condensing temperature can be calculated

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Fig. 1. Schematic figure of Milk Pasteurizing System.

easily in a few minutes with the help of practical formulae. A thorough search of the present literature showed that there were several studies about the HPS performance and optimization (Ubbels & Boumann 1979; NEI Projects Ltd 1983; Smith 1983; Wright & Steward 1985; Tyagi 1989; Kent 1997; Li 1999; Sahin & Kodal 1999; Salah El-Din 1999; Bourouis et al. 2000; Jung et al. 2000; Kodal et al. 2001; Sahin B et al. 2001; Kaushik et al. 2002). All of these studies are not directly related to the present work. Original formulae are developed and presented finally.

## Mathematical formulation

#### Nomenclature

CE	Unit cost of energy for the auxiliary heater, [\$/(kW.h)]							
C <sub>EL</sub>	Unit cost of electricity, [\$/(kW.h)]							
COP	Actual coefficient of performance of the heat pump,							
COP <sub>C</sub>	Theoretical coefficient of performance of the Carnot							
	cycle heat pump,							
CP	Specific heat capacity of milk, [kJ/(kg.K)]							
m	Mass flow rate of milk, (kg/s)							
OC <sub>C</sub>	Hourly operation cost of the compressor, (\$/hr)							
OCH	Hourly operation cost of the auxiliary heater, (\$/hr)							
Р	Power input to the compressor, (kW)							
Qc	Heat load of condenser, (kW)							
Q <sub>H</sub>	Heat load of auxiliary heater, (kW)							
TC	Total operation (energy) cost of the milk pasteurizing							
	system, (\$)							
T <sub>C</sub>	Condensing temperature, (K)							
T <sub>C,opt</sub>	Optimum condensing temperature, (K)							
TD	Evaporator and condenser design temperature							
	differential between refrigerant and milk, (K)							
T <sub>e</sub>	Design temperature of evaporator due to milk outlet							
	temperature, (K)							
T <sub>m,hot</sub>	Design pasteurizing temperature of heated milk, (K)							
T <sub>m,in</sub>	Inlet temperature of milk, (K)							
T <sub>m,out</sub>	Design outlet temperature of milk, (K)							

- $\begin{array}{ll} T_1 & & \text{Temperature of preheated milk at condenser outlet, (K)} \\ \Delta T & & \text{Temperature difference between condenser and} \\ & & \text{evaporator, (K),} \end{array}$
- ε Effectiveness of condenser.

Heat pumps are designed for heating a medium by absorbing heat from another medium. The operating characteristics and performances of them (coefficient of performance, COP) are strongly related to their operating temperatures. For that reason, the main parameter in heat pump design that must be taken into consideration is COP of the heat pump system. COP of a heat pump is defined by the following approximated formula for liquid-to-liquid type heat pumps referring to temperatures as in Fig. 1 (Zogou & Stamelos 1998).

$$COP = \frac{COP_C}{3} = \frac{T_C}{3.\Delta T} = \frac{T_C}{3.(T_C - T_e)}$$
 (1)

Evaporating temperature of the heat pump can be determined as a function of milk outlet temperature as:

$$T_e = T_{m,out} - TD \tag{2}$$

and the temperature of preheated milk after condensing unit of a heat pump is:

$$T_1 = T_C - TD \tag{3}$$

The heat rejection capacity of condenser is evaluated follows.

$$Q_{C} = m.C_{P}.(T_{1} - T_{m,in})$$
(4)

Power consumed by the compressor of the heat pump can be determined by combining eqns. (1) and (4) from the following.

$$P = \frac{Q_C}{COP} = \frac{3.m.C_{P.}(T_1 - T_{m,in}).(T_C - T_e)}{T_C}$$
(5)

The amount of heating capacity of auxiliary heater can be formulated as in the following form:

$$Q_{H} = m.C_{P}.(T_{m,hot} - T_{1}) = m.C_{P}.(T_{m,hot} + TD - T_{C})$$
(6)

Hourly operation cost of compressor can be determined by combining eqns. (4) and (5).

$$\frac{OC_C}{m.C_P} = \frac{3.C_{EL}.(T_C - T_{m,out} + TD).(T_C - TD - T_{m,in})}{T_C}$$
(7)

Eqn. (7) is expanded yielding:

$$\frac{OC_{C}}{m.C_{P}} = \frac{3.C_{EL}.[T_{C}^{2} - T_{C}.(T_{m,out} + T_{m,in}) - TD.(TD + T_{m,in} - T_{m,out}) + T_{m,out}.T_{m,in}]}{T_{C}}$$
(8)

Hourly operation cost of auxiliary heater can be obtained by means of the following eqn

$$\frac{OC_H}{m.C_P} = C_E \cdot (T_{m,hot} + TD - T_C) \tag{9}$$

**Table 1.** Total operating costs per unit milk capacity rate in ( $\frac{K}{W}$ ) versus condensing temperature T<sub>C</sub>

T <sub>C</sub> (K)	300	305	311.76	315	320	325	330	335	340
Cost	2.50	2.4221	2.3846	2.3929	2.4375	2.5192	2.6363	2.7833	2.9706

Eqns (8) and (9) are added to get the total cost of energy explicitly after arrangements yielding:

$$\frac{TC}{m.C_P} = (3.C_{EL} - C_E).T_C - 3.C_{EL}.(T_{m,out} + T_{m,in}) + \frac{3.C_{EL}.[T_{m,out}.T_{m,in} - TD.(TD + T_{m,in} - T_{m,out})]}{T_C} + C_E.(T_{m,hot} + TD)$$
(10)

The first derivative of the total cost function wrt. condensing temperature  $T_C$  is calculated as follows.

$$\frac{\partial (TC/m.C_P)}{\partial T_C} = (3.C_{EL} - C_E) - \frac{3.C_{EL} \cdot [T_{m,out} \cdot T_{m,in} - TD.(TD + T_{m,in} - T_{m,out})]}{T_C^2}$$
(11)

By setting eqn (11) to zero, optimum condensing temperature can be obtained, eqn (12).

$$T_{C, opt} = \sqrt{\frac{3.C_{EL} \cdot [T_{m, out}. T_{m, in} - TD. (TD + T_{m, in} - T_{m, out})]}{3.C_{EL} - C_E}}$$
(12)

The second derivative of the overall cost function with respect to  $T_C$  is calculated by using this specific optimum  $T_C$  value in this second derivative, and result is found to be always positive as illustrated in eqn (13), which indicates a local minimum point certainly.

$$\frac{\partial^2 (TC/m.C_P)}{\partial T_C^2} = \frac{6.C_{EL}.[T_{m,out}.T_{m,in} - TD.(TD + T_{m,in} - T_{m,out})]}{T_C^3} > 0$$
(13)

Effectiveness values for the condensing unit can be determined by using the following (Söylemez 2000).

$$\varepsilon = \frac{T_1 - T_{m,in}}{T_C - T_{m,in}} = \frac{T_C - T_D - T_{m,in}}{T_C - T_{m,in}}$$
(14)

## **Results and Discussion**

For a typical milk pasteurizing optimization problem (Stoecker 1989), it is given that,  $T_{m,hot}$ =343 K,  $T_{m,in}$ =293 K,  $T_{m,out}$ =277 K, TD=7 K,  $C_{EL}$ =0·1 \$/(kW.hr),  $C_{E}$ =0·05 \$/(kW.hr). This data is used in the present work since they are designed values for a specific milk pasteurizing plant. The optimum  $T_{C}$  value for the milk pasteurizing system is calculated by using eqn (12) as 311·76

Variation of hourly total operation cost as a function of T<sub>C</sub>



Fig. 2. Overall Operation Costs versus Condensing Temperature  $\mathsf{T}_\mathsf{C}.$ 

degrees K. The values of overall operation (energy) costs values are plotted in Fig. 2 and presented in Table 1. It can be deduced that there exists a local minimum condensing temperature value for heat pump assisted milk pasteurizing applications. Neither excessive nor deficient values of  $T_C$  of the system will be cost effective other than the local optimum point. Effectiveness of the condensing unit is calculated as 0.626 by the help of eqn (14) for this specific optimum point.

It is clear that there is good thermal performance at this optimum point for the heat pump assisted milk pasteurizing systems.

The optimum condensing temperature is calculated at which minimum total energy cost occurs for a typical the milk pasteurizing heat pump system. It is clear as in eqn (12) that optimum condensing temperature is directly related to cost of auxiliary energy. The validity of the optimization formulation is checked. These systems must be designed close to this optimum point. The present formulae may seem to be helpful for the designers and manufacturers of these systems.

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