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EXPECTATION-DRIVEN ASSET PRICE FLUCTUATIONS UNDER THE SPIRIT OF CAPITALISM HYPOTHESIS: THE ROLE OF HETEROGENEITY

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In this paper, I study how heterogeneity amongst agents affects the occurrence of expectation-driven asset price fluctuations in a pure exchange economy *à la* Lucas, with infinitely lived households, under the hypothesis of spirit of capitalism (SOC). I consider heterogeneous households in terms of preferences, endowments, and initial wealth, and capture the SOC through preferences for wealth. Preferences for wealth are the key element of this paper in a twofold aspect. First, they explain the occurrence of asset price fluctuations driven by self-fulfilling changes in expectations. Second, heterogeneity in endowments affects asset price level and dynamics only if preferences are heterogeneous. For instance, if agents with the strongest SOC are also the rich in terms of endowments, heterogeneity in endowments heightens the asset price level in the long run and destabilizes by enlarging the range of parameter values for which expectation-driven asset price fluctuations occur.

Keywords: Asset Pricing Model, Spirit of Capitalism, Heterogeneity, Expectation-Driven Fluctuations

1. INTRODUCTION

Since the Great Recession, which officially followed the financial crisis of 2007–2008 in the USA, topics on inequality have risen in the developed countries [see among others Rajan (2010), Atkinson, Piketty and Saez (2011), and Kumhof et al. (2015)]. For instance, Rajan (2010) argues that a rise in inequality was the cause of the financial crisis of 2007–2008 in the USA. Furthermore, it is well known in the finance literature that asset prices undergo an excessive volatility with regard to their underlying fundamentals as dividends [see Shiller (1981, 2015)]. Instead of explaining the relationship between inequality and financial crises, the question addressed in this paper is the following: What are the implications of heterogeneity amongst agents on the excessive asset price volatility?

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Aim of this paper. The main purpose of this theoretical paper is to provide new insights into the role of heterogeneity, in particular in terms of preferences and endowments, in asset price volatility. I attribute the excessive asset price volatility to the existence of multiple equilibria likely to occur in dynamic general equilibrium models. Therefore, fluctuations are the results of consecutive jumps from one equilibrium to another driven by self-fulfilling changes in agents' expectations without requiring any shock on fundamentals, that is, preferences and/or technology.

Framework. In this paper, I consider an extension of the Lucas (1978) pure exchange economy. First, I introduce heterogeneity and consider three sources of heterogeneity: preferences, endowments, and initial wealth.[1] Second, I argue that psychological motives for savings give rise to expectation-driven fluctuations in asset prices. I focus on the spirit of capitalism (henceforth, SOC) hypothesis developed by Weber (1905), which captures the intrinsic desire for wealth accumulation.[2] Following Zou (1994, 1995), I introduce SOC hypothesis through preferences for wealth, and consider a non-separable utility function between consumption and wealth holdings. Several empirical studies about the saving behavior of top income households in the USA support the existence of wealth preferences [see Carroll (2000), Dynan et al. (2004), and Kumhof et al. (2015)]. Moreover, some microfoundations can explain wealth preferences as underlined in Saez and Stantcheva (2016): bequests motives, services from wealth, and social status.[3]

Results. Preferences for wealth are the key element in this paper for three reasons. First, all agents accumulate wealth in the long run, and thus hold financial assets whatever the interest rate level and their endowments. This result contrasts with neoclassical optimal growth models *à la* Becker (1980) with borrowing constraints and heterogeneity in discount rates. Since the interest rate is given by the most patient, only the most patient agent holds the whole financial stock in the long run in these models.[4] Second, preferences for wealth also explain the occurrence of asset price fluctuations driven by self-fulfilling expectations. This result provides a theoretical evidence for the excessive asset price volatility unrelated to fundamentals observed in the data. More precisely, expectation-driven asset price fluctuations are likely to occur when wealth and consumption are Edgeworth-substitutes, that is, when the marginal utility of consumption is decreasing with wealth. Third, investigating the role of heterogeneity, I show that heterogeneity in endowments affects asset price level and dynamics only if preferences for wealth are heterogeneous. For instance, if agents with the strongest willingness for wealth accumulation are also the rich in terms of endowments, heterogeneity in endowments heightens the asset price level in the long run, and destabilizes by enlarging the range of parameter values for which expectation-driven asset price fluctuations occur. This result justifies the implementation of economic policy in a stabilizing perspective, like dividend taxation or redistribution policies.

Related literature. At first, this paper relates to the literature on asset pricing in macroeconomics, and in particular to the consumption-based asset pricing models, pioneered by Stiglitz (1970), Lucas (1978), and Breeden (1979). In order to improve the performance of Lucas (1978) with observed facts in asset prices, several papers propose to consider alternative assumptions on preferences and/or to include some heterogeneity. I first discuss the literature dealing with alternative assumptions on preferences, and then the literature on heterogeneity in asset pricing models. A first approach was to consider Epstein–Zin preferences to disentangle risk aversion from intertemporal substitution, while another approach was to include consumption externalities, as "keeping up with Joneses" [Gali (1994)] or external consumer habits [Campbell and Cochrane (1999)]. Several theoretical studies also claim that preferences for wealth are useful for understanding asset price behavior [see Bakshi and Chen (1996), Smith (2001), and Boileau and Braeu (2007)]. This paper is in line with this latter branch of the literature and is closely related to Kamihigashi (2008) and Airaudo (2017), which highlight the role of preferences for wealth in the occurrence of expectation-driven asset price fluctuations in a representative agent framework. The main contribution of this paper with respect to Kamihigashi (2008) and Airaudo (2017) is to investigate the role of heterogeneity in the occurrence of expectation-driven asset price fluctuations when agents derive preferences for wealth.

This paper is also related to the literature about heterogeneous agents and asset pricing. A large body of the literature considers models *à la* Bewley (1977) or *à la* Aiyagari (1994). Following these models, households are ex ante identical, but face different idiosyncratic shocks on endowments or on labor income. In this paper, households are ex ante different with respect to their preferences, their endowments, and their initial wealth. For this reason, this paper is close to Becker (1980), Kocherlakota (1992), Santos and Woodford (1997), Huang and Werner (2004), and more recently Le Van et al. (2015). In contrast, these papers consider a framework without SOC hypothesis and deal with a different topic, which is the existence of a rational bubble.

Finally, a number of theoretical papers have stressed that non-separable utility function between consumption and money is important to generate multiple equilibria in dynamic general equilibrium models, and thus fluctuations driven by the volatility of agents' expectations [see Obstfeld (1984) and Matsuyama (1990) for an overview]. As Airaudo (2017), I show that expectation-driven asset price fluctuations are likely to occur when wealth and consumption are Edgeworth-substitutes, that is, when the marginal utility of consumption is decreasing with wealth. A similar result appears in the literature about Money-in-the-Utility-Function (henceforth, MIUF). Assuming an Edgeworth substitutability between consumption and money is neither empirically plausible [Walsh (2010)] nor consistent with the idea that money serves as a medium exchange. However, a negative cross-derivative between wealth and consumption is coherent with the concept of frugality at the root of SOC hypothesis developed by Weber (1905). Furthermore, it is worth pointing out that housing wealth is a large component of households' wealth [see Survey of Consumer Finances (2013) for the US data]. Since several studies shed light on the fact that housing and consumption are

Edgeworth-substitutes [see Yogo (2006), Piazzesi et al. (2007), and Flavin and Nakagawa (2008)], considering a negative cross-derivative between consumption and wealth would be compatible with empirical evidences.

Layout. Section 2 presents the model. Section 3 is devoted to the intertemporal equilibrium. In Section 4, I describe the mean-preserving method. Section 5 analyses the existence and uniqueness of the steady state. In Section 6, I study local dynamics and the occurrence of expectation-driven asset price fluctuations. Section 7 is devoted to the role of heterogeneity in the existence of asset price fluctuations. Concluding remarks are provided in Section 8, while computational details are gathered in the Appendix.

2. THE MODEL

The starting point is a modified version of the pure exchange economy developed by Lucas (1978) with n infinitely lived heterogeneous households.[5] There are three sources of heterogeneity: preferences, initial wealth, and endowments. Without loss of generality, I consider two types of households, labeled with $i = 0, 1$. More precisely, there are $n_i > 1$ agents of type i with $n_0 + n_1 = n$.

Preferences. Agents derive utility both from consumption $c_i(t)$ and from financial wealth $w_i(t)$. Preferences for wealth capture the SOC hypothesis. The utility function of an agent i at time $t = 0$ is the discounted sum of instantaneous utilities:

$$\int_0^{+\infty} e^{-\rho t} u_i\left(c_i(t), w_i(t)\right) dt, \tag{1}$$

where $\rho > 0$ is the common subjective rate of time preference.

Following Smith (2001) and Airaudo (2017), preferences of an agent i are summarized by the following non-separable utility function in consumption and wealth:

$$u_i\left(c_i(t), w_i(t)\right) = \begin{cases} \dfrac{\left[c_i(t)^{\alpha} w_i(t)^{\gamma_i}\right]^{1-\varepsilon} - 1}{1-\varepsilon} & \text{if } \varepsilon > 0, \varepsilon \neq 1; \\ \alpha \log c_i(t) + \gamma_i \log w_i(t) & \text{if } \varepsilon = 1, \end{cases} \tag{2}$$

where $\varepsilon > 0$ and $\alpha \in (0, 1)$ are respectively the inverse of the intertemporal elasticity of substitution and the weight of consumption in the utility function.

Heterogeneity in preferences. Heterogeneity in preferences is captured by the parameter $\gamma_i > 0$, which measures the weight of wealth in the utility function, as long as $\gamma_1 \neq \gamma_0$. Without loss of generality, I rule out the case in which agents do not display preferences for wealth, and consider a particular distribution for γ_i[6]:

ASSUMPTION 1. $0 < \gamma_0 \leq \gamma_1$.

Under Assumption 1, agents 1 have a stronger willingness for wealth accumulation than agents 0. To ensure the concavity of the utility function, I make the following assumption:

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ASSUMPTION 2. $1-(\alpha+\gamma_i)(1-\epsilon)\geq 0$.

Consumption and wealth are normal goods under this utility function. Moreover, when $\varepsilon \leq 1$, the marginal utility of consumption is increasing with wealth (i.e., $u_{icw}(c_i, w_i) > 0$), meaning that consumption and wealth are Edgeworth-complements. When $\varepsilon \geq 1$, the marginal utility of consumption is decreasing with wealth (i.e., $u_{icw}(c_i, w_i) < 0$), then consumption and wealth are Edgeworth-substitutes.[7]

Budget constraint. At the initial period $t=0$, households are endowed with some shares of the initial stock $s_i(0)$. At time t, each household i receives a constant dividend d per share and an exogenous endowment of $y_i > 0$ units of final good, which can be interpreted as labor income. Each household trades and buys new shares $s_i(t)$ at price $q(t)$, and consumes $c_i(t)$ units of final good.

Heterogeneity in endowments. Heterogeneity in endowments (i.e., $y_1 \neq y_0$) could depict heterogeneity in skills. Even though there is no production side in this paper, y_i can be seen as earnings coming from a labor activity. If all agents face the same wage, a low-skilled agent has a lower labor income compared to a high-skilled. In contrast to heterogeneity in preferences, I do not impose restrictions on the distribution of endowments y_i. Along the paper, I consider two configurations: $y_0 < y_1$ and $y_0 > y_1$. In the first configuration (i.e., $\gamma_0 < \gamma_1$ and $y_0 < y_1$), agents 1 with the strongest willingness for wealth accumulation also have the highest labor income in the economy.[8] In the second configuration (i.e., $\gamma_0 < \gamma_1$ and $y_0 > y_1$), agents 1 with the strongest willingness for wealth accumulation have the lowest labor income in the economy. This last configuration could illustrate an economy in which agents 1 would be an annuitant, that is, a person mainly living on capital income.

Heterogeneity in initial wealth. Households are also heterogeneous with respect to their initial wealth, $s_i(0)$. As shown in Section 5, the initial distribution of wealth does not matter in the steady state because of the homothetic property of the utility function. Since I am interested in the occurrence of expectation-driven fluctuations in the neighborhood of a steady state, such a heterogeneity will not affect asset price dynamics. In the remainder of the paper, I restrict my attention to the role of heterogeneity in preferences and in endowments.

Optimal behavior. Given an initial level of wealth $w_i(0)$, a household i maximizes her utility function (1) with respect to $(c_i(t), w_i(t), s_i(t))$ under the following budget and stock constraints:

$$\dot{w}_i(t) = \left[\dot{q}(t) + d\right] s_i(t) + y_i - c_i(t), \tag{3}$$

$$w_i(t) = q(t)s_i(t). \tag{4}$$

Let $r(t)$ be the interest rate of the asset defined as follows:

$$r(t) = \frac{\dot{q}(t) + d}{q(t)}. \tag{5}$$

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Under Assumptions 1 and 2, the optimal behavior of a household i is summarized by the following Euler equation and transversality condition:

$$\frac{\dot{c}_i(t)}{c_i(t)} = \frac{1}{1+\alpha(\varepsilon-1)}\left[r(t) - \rho + \frac{\gamma_i}{\alpha}\frac{c_i(t)}{w_i(t)} + \gamma_i(1-\varepsilon)\frac{\dot{w}_i(t)}{w_i(t)}\right], \quad \textbf{(6)}$$

$$\lim_{t\to+\infty} e^{-\rho t} u_{ic}(c_i(t), w_i(t))w_i(t). = 0. \quad \textbf{(7)}$$

Since all agents have preferences for wealth, and Inada conditions are satisfied both for consumption and wealth, $s_i(t) > 0$ is the only solution satisfying the optimal behavior of the household i.

Without preferences for wealth, I get the standard Euler equation found in Lucas (1978) ($\gamma_i = 0$):

$$\frac{\dot{c}_i(t)}{c_i(t)} = \frac{r(t) - \rho}{1 + \alpha(\varepsilon - 1)}.$$

In this settings, there is no room for expectation-driven asset price fluctuations, and heterogeneity in endowments does not matter. When agents have preferences for wealth, the Euler equation given by equation (6) depicts two additional terms compared to Lucas (1978):

$$\frac{\gamma_i}{\alpha}\frac{c_i(t)}{w_i(t)} \text{ and } \gamma_i(1-\varepsilon)\frac{\dot{w}_i(t)}{w_i(t)}.$$

The first term corresponds to the marginal rate of substitution of consumption for wealth, through which the preferences for wealth increase the willingness to delay consumption for the future. It is worth noting that this term will have an impact on the wealth distribution in the long run. Because of it, every agent has an incentive to hold financial assets in the long run. Through the second term, the willingness to postpone consumption is reinforced if wealth and consumption are Edgeworth-complements (i.e., $\varepsilon < 1$), or dampened if substitutes (i.e., $\varepsilon > 1$). Interestingly, the second term is at the origin of expectation-driven fluctuations. Indeed, Kamihigashi (2008) considers a separable utility function between consumption and wealth (i.e., $\varepsilon = 1$), meaning that the second term does not appear in his framework. He shows that the equilibrium is unique, and thus there is no room for expectation-driven fluctuations. This result shows that non-separable utility function is a necessary condition for multiplicity of equilibria, and the second term $\gamma_i(1-\varepsilon)\dot{w}_i(t)/w_i(t)$ is the key ingredient through which expectation-driven fluctuations occur.

3. INTERTEMPORAL EQUILIBRIUM

An intertemporal equilibrium is defined as follows:

DEFINITION 1. *Under Assumptions 1 and 2, an equilibrium of the economy* $E = (n, \rho, d, (n_i, y_i, u_i, s_i(0))_{i=0}^1)$ *is an intertemporal path* $(q(t), (s_i(t), c_i(t))_{i=0}^1)_{t\geq 0}$ *satisfying the optimal behavior of agents (3)–(7) and the equilibrium condition on the asset market:*

$$n_0 s_0(t) + n_1 s_1(t) = 1. \quad \textbf{(8)}$$

Let $\psi = 1 + \alpha(\varepsilon - 1)$ and $\theta_i = \gamma_i(1 - \varepsilon)$. From Definition 1, an intertemporal equilibrium is a path $(q(t), c_1(t), s_1(t))_{t=0}^{+\infty}$ satisfying the following three-dimensional dynamic system:

$$
\begin{cases}
-\psi \dfrac{\dot{c_1}(t)}{c_1(t)} + (1+\theta_1)\dfrac{\dot{q}(t)}{q(t)} + \theta_1 \dfrac{\dot{s_1}(t)}{s_1(t)} = \rho - \dfrac{\gamma_1}{\alpha}\dfrac{c_1(t)}{q(t)s_1(t)} - \dfrac{d}{q(t)}, & \textbf{(9)} \\[2ex]
\psi \dfrac{n_1 c_1(t)}{d + ny - n_1 c_1(t)}\dfrac{\dot{c_1}(t)}{c_1(t)} + (1+\theta_0)\dfrac{\dot{q}(t)}{q(t)} - \theta_0 \dfrac{n_1 s_1(t)}{1 - n_1 s_1(t)}\dfrac{\dot{s_1}(t)}{s_1(t)} & \\
\qquad = \rho - \dfrac{\gamma_0}{\alpha}\dfrac{d + ny - n_1 c_1(t)}{q(t)(1 - n_1 s_1(t))} - \dfrac{d}{q(t)}, & \textbf{(10)} \\[2ex]
\dfrac{\dot{s_1}(t)}{s_1(t)} = \dfrac{d}{q(t)} + \dfrac{y_1}{q(t)s_1(t)} - \dfrac{c_1(t)}{q(t)s_1(t)}, & \textbf{(11)}
\end{cases}
$$

where $s_1(0) > 0$ is given. Note that there are one predetermined variable $s_1(t)$ and two non-predetermined variables $q(t)$ and $c_1(t)$.

I can now study the existence and the uniqueness of the steady state, and then local dynamic properties of the economy, while emphasizing the role of the heterogeneity both in preferences and in endowments. In order to evaluate the effect of the heterogeneity on the stationary asset price level and on the occurrence of expectation-driven fluctuations, I apply the mean-preserving method. Before starting the analysis of the steady state, I present this method in the next section.

4. MEAN-PRESERVING APPROACH TO HETEROGENEITY

To highlight and understand the role of heterogeneity in preferences and in endowments in the stationary asset price level and local dynamics, I impose a mean-preserving spread of distribution. In other words, I fix the midpoints γ and y defined as follows:

$$\gamma \equiv \frac{n_0\gamma_0 + n_1\gamma_1}{n} \quad \text{and} \quad y \equiv \frac{n_0 y_0 + n_1 y_1}{n}, \tag{12}$$

and I define a measure for each source of heterogeneity:

$$\sigma_\gamma \equiv \sqrt{\frac{n_0}{n}(\gamma_0 - \gamma)^2 + \frac{n_1}{n}(\gamma_1 - \gamma)^2}, \tag{13}$$

$$\sigma_y \equiv \sqrt{\frac{n_0}{n}(y_0 - y)^2 + \frac{n_1}{n}(y_1 - y)^2}, \tag{14}$$

where σ_γ and σ_y are respectively the standard deviations of the distribution of weight of wealth in preferences and the standard deviations of the distribution of endowments.

To keep the analysis as simple as possible, I define two heterogeneity parameters, x and z, given by $x = \gamma_1 - \gamma$ and $z = y_1 - y$. Under Assumption 1, x is defined on $(0, \gamma n_0/n_1)$. Since I do not impose any restrictions on the distribution of endowments, z is defined on $(-y, yn_0/n_1)$. When $y_1 < y_0$, one has $z < 0$, and conversely, $z > 0$ when $y_1 > y_0$. I can rewrite the standard deviations as functions of x and z:

$$\sigma_\gamma = x\sqrt{n_1/n_0},\ \forall\, x \in (0, \gamma n_0/n_1), \tag{15}$$

$$\sigma_y = \begin{cases} -z\sqrt{n_1/n_0}, & \text{if } y_1 < y_0, \\ z\sqrt{n_1/n_0}, & \text{if } y_1 > y_0. \end{cases} \tag{16}$$

Note for future reference that an increase in z in absolute value expresses a raise in the dispersion of endowments y_i, *ceteris paribus*.

5. STEADY-STATE ANALYSIS

A steady state is an equilibrium where $\dot{s}_1(t) = 0$, $\dot{c}_1(t) = 0$, $\dot{q}(t) = 0$, and $r(t) = r$ for all t. From equations (5) and (8)–(11), I deduce that a steady state satisfies the following equations:

$$r = \rho - \frac{x+\gamma}{\alpha}\frac{ds_1 + z + y}{qs_1}, \tag{17}$$

$$r = \rho - \frac{\gamma - xn_1/n_0}{\alpha}\frac{ds_0 + y - zn_1/n_0}{qs_0}, \tag{18}$$

$$r = \frac{d}{q}. \tag{19}$$

From equations (8), (17), and (18), I get the distribution of wealth (s_1^* and s_0^*), the asset price level (q^*), and combining with equation (19) the interest rate (r^*) in the steady state.

Therefore, a steady state is a solution (s_1^*, q^*) with $s_1^* \in (0, 1/n_1)$ and $q^* > 0$ satisfying the following system:

$$\begin{cases} (x+\gamma)\dfrac{ds_1 + z + y}{s_1} = (\gamma - xn_1/n_0)\dfrac{d(1 - n_1 s_1) + n_0\,(y - zn_1/n_0)}{1 - n_1 s_1}, & (20) \\[2ex] q = \dfrac{d}{r(q, s_1)}, & (21) \end{cases}$$

$$\text{with } r(q, s_1) = \rho - \frac{\gamma nc + xn\left[d\,(s_1 - 1/n) + z\right] n_1/n_0}{\alpha q} \tag{22}$$

$$\text{and} \qquad nc = d + ny. \tag{23}$$

The next proposition proves the existence of a unique steady state bringing the heterogeneity parameters (x and z) out.[9]

PROPOSITION 1. *Under Assumptions 1 and 2, there exists a unique steady state* (s_1^*, q^*) *such that* $s_1^* = s_1^*(x, z) \in (0, 1/n_1)$ *and* $q^* = q^*(x, z) > 0$.

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Proof. See Appendix A.1.

Proposition 1 indicates that the stationary asset price level q^* and the distribution of wealth given by s_1^* ($s_0^* = (1 - n_1 s_1^*)/n_0$) are affected both by the dispersion of endowments (z) and the dispersion of preferences (x). If two economies differ with respect to x and z, then they experience different levels of asset prices and different distributions of wealth.

Note that the stationary distribution of wealth and asset price level are unaffected by the initial distribution of wealth despite taking nonstandard preferences into account through the assumption of SOC. This result can be explained by the homothetic property of the utility function. Since I am interested in the occurrence of expectation-driven fluctuations in the neighborhood of a steady state, heterogeneity in initial wealth will also play no role in asset price dynamics. This result differs from Sorger (2000) and Chatterjee and Turnovsky (2012). In one-sector growth model with endogenous labor supply, Sorger (2000) shows that households, who only differ with respect to their initial wealth, can hold different wealth levels in the steady state. Considering a homothetic utility function in consumption and leisure, Chatterjee and Turnovsky (2012) obtain a similar result to Sorger (2000) in a endogenous growth model with endogenous labor supply. In these papers, the mechanism relies on the interplay between the household savings decision and her optimal labor supply.

In contrast, I show in the remainder of the paper that heterogeneity in endowments and in preferences matter. First, I characterize the steady-state wealth distribution, then I describe the stationary asset price level, and the role of heterogeneity in endowments and in preferences in this latter.

5.1. Stationary Wealth and Total Income Distributions

Financial wealth and total income are naturally used to rank individuals in a society, and thus provide some insights about social inequalities. I define the stationary wealth of an agent i as the real value of assets she holds, that is, $w_i^*(x,z) = q^*(x,z)s_i^*(x,z)$, whereas her total income is given by $R_i^*(x,z) = ds_i^*(x,z) + y_i$. Through the next proposition, this subsection aims to characterize the distribution of wealth and total income within the economy in the steady state.

PROPOSITION 2. *Let* $\underline{x} \equiv -\dfrac{\gamma nz}{d + n_0 y_1 + n_1 y_0}$ *and* $\bar{x} \equiv -\dfrac{\gamma nz}{d}$. *Under Assumptions 1 and 2, the following holds in the steady state:*

(1) The wealth distribution is nondegenerate meaning that all agents hold financial assets $s_i^*(x,z) > 0$.

(2) Agents 1 hold a greater amount of financial assets than agents 0 (i.e., $s_1^*(x,z) > s_0^*(x,z)$*), if and only if they have a sufficiently strong willingness for wealth accumulation compared to agents 0, that is,* $\gamma_1 > \gamma_0 + \underline{x}n/n_0$ *(or,* $x > \underline{x}$*).*

(3) Agents 1 are the richest in terms of total income in the economy, if and only if they have a sufficiently strong willingness for wealth accumulation compared to agents 0, that is, $\gamma_1 > \gamma_0 + \bar{x}n/n_0$ *(or,* $x > \bar{x}$*).*

Proof. See Appendix A.2.

Proposition 2 shows that all agents in the economy hold a positive share of stock $s_i(t) > 0$ in contrast to Becker (1980) and Kocherlakota (1992). In infinite-horizon general equilibrium models *à la* Becker (1980), agents differ with respect to their subjective discount rate and their initial wealth. They face borrowing constraints, but display no preferences for wealth. In such a framework, a household would like to dissave until $s_i(t) = 0$ as soon as her subjective discount rate ρ is greater than the interest rate $r(t)$. Since the interest rate is given by the most patient, the wealth distribution is degenerate: The most patient agent holds all the financial wealth of the economy. Because of preferences for wealth, all agents accumulate financial assets whatever the interest rate level in this framework.[10] In infinite-horizon models *à la* Kocherlakota (1992), agents differ in terms of endowments and initial wealth. They face short-sales constraints, but display no preferences for wealth. Under some conditions, the poor would like to short shell, but they cannot. Hence, they sell until $s_i(t) = 0$. In my framework, preferences for wealth encourage all agents in the economy to hold financial assets in the steady state. As markets are complete, agents are able to equalize their marginal rate of substitution between wealth and consumption in the steady state.

Note that $x > \underline{x} > \bar{x}$ is always satisfied when agents 1 with the strongest willingness for wealth accumulation in the economy are the rich in terms of endowments (i.e., $\gamma_1 > \gamma_0$ and $y_1 > y_0$). Proposition 2 provides a theoretical evidence of some stylized facts about the saving behavior of the rich people observed in the data, in particular why the rich save too much [see Carroll (2000) and Dynan et al. (2004)].[11]

In the rest of the paper, I focus only on the case when agents 1 are the richest in terms of total income in the economy. To do this, I make the following assumption:

ASSUMPTION 3. $\gamma_1 > \gamma_0 + \bar{x}n/n_0$.

5.2. Stationary Asset Price Level

Equation (19) indicates that the stationary asset price level is equal to dividends per share deflated by the stationary interest level r^*, which is equivalent to:

$$\frac{d}{r^*} = d \int_t^{+\infty} e^{-r^*(s-t)} ds. \tag{24}$$

The right-hand side of equation (24) corresponds to the present discounted value of future dividends, which is the definition of the fundamental value of an asset. The following proposition characterizes the asset price level in the steady state.

PROPOSITION 3. *Under Assumptions 1–3, the following holds in the steady state:*

(1) There is no bubble in the steady state.

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(2) When $\gamma_0 = \gamma_1$ (i.e., $x = 0$), the asset price q^* does not depend on heterogeneity in endowments, σ_y.

(3) When $\gamma_0 < \gamma_1$ (i.e., $x > 0$):

a. The asset price is decreasing with heterogeneity in endowments (i.e., a lower z) when $y_1 < y_0$, and is increasing with heterogeneity in endowments (i.e., a higher z) when $y_1 > y_0$.

b. The asset price q^* is increasing with heterogeneity in preferences, σ_γ (i.e., with x).

Proof. See Appendix A.4.

Proposition 3.(1) shows the nonexistence of bubbles in the steady state. As underlined by Kamihigashi (2008) and Airaudo (2017) in a representative agent framework with preferences for wealth and a financial asset providing dividends, the presence of positive dividends explains this result. In contrast, Zhou (2015) proves that a bubble on an asset providing no dividends can exist in the steady state. Since I restrict my attention on the occurrence of expectation-driven asset price fluctuations in the neighborhood of a steady sate, the economy does not exhibit bubbles.

When agents face same preferences (i.e., $\gamma_1 = \gamma_0$), heterogeneity in endowments does not affect the stationary asset price level [Proposition 3.(2)]. I can show from equations (17) and (19) that the asset price is equal to $\bar{q} = d/\rho + (d + ny)\gamma/(\alpha\rho)$, which corresponds to the stationary asset price level found in a representative agent framework. This result relies on the homothetic property of the preferences. Chatterjee (1994) shows that when preferences are homothetic or quasi-homothetic and agents are only heterogeneous in terms of initial wealth, the aggregate dynamics are exactly the same as in the standard optimal growth model with a representative agent. Caselli and Ventura (2000) extend this result to heterogeneity in skills and preferences for public goods.

Interestingly, Propositions 3.(2) and 3.(3).(a) show that heterogeneity in endowments matters only if preferences are heterogeneous. The explanation relies on the fact that I consider preferences for wealth and not for public goods. When preferences are heterogeneous (i.e., $\gamma_1 > \gamma_0$), the stationary asset price level decreases with the dispersion of endowment distribution when agents 1 have the smallest endowments in the economy, and increases with the dispersion of endowment distribution otherwise [Proposition 3.(3).(a)].

Economic intuition behind Proposition 3.(3).(a). Suppose that agents 1 with the strongest willingness for wealth accumulation in the economy are the rich in terms of endowments (i.e., $\gamma_1 > \gamma_0$ and $y_1 > y_0$). An increase in the dispersion of endowment distribution (i.e., y_0 decreases and y_1 increases in the same proportions) urges agents 1 to accumulate more wealth, and agents 0 less, since wealth is a normal good. The increase in asset demand of agents 1 is sufficient to counteract the decrease in asset demand of agents 0. Since the asset supply is fixed, the asset price level increases following an increase in σ_y. I can provide the same rationale for the case $y_1 < y_0$ by considering the reversed mechanism.

Proposition 3.(3).(b) also shows that the stationary asset price level increases with the dispersion of γ_i distribution if the willingness for wealth accumulation of agents 1 is sufficiently high compared to agents 0 (i.e., $\gamma_1 > \gamma_0 + \bar{x}n/n_0$).

Economic intuition behind Proposition 3.(3).(b). When $y_1 > y_0$, agents 1 have a higher income compared to agents 0. An increase in the dispersion of γ_i (i.e., γ_1 increases and γ_0 decreases in the same proportions) urges agents 1 to accumulate more wealth, and agents 0 less. The increase in asset demand of agents 1, sufficient to counteract the decrease in asset demand of agents 0, generates a rise in the asset price level. If agents 1 are the poor (i.e., $y_1 < y_0$), but their willingness for wealth accumulation is sufficiently high compared to agents 0 (i.e., $\gamma_1 > \gamma_0 + \bar{x}n/n_0$), the previous mechanism prevails as well.

5.3. Stationary Welfare of Agent 0

Proposition 3 highlights some opposite effects on the stationary welfare of agent 0 when the dispersion of endowment distribution increases. For instance, when the agent 0 is the poor in the economy, a decrease in her endowments y_0 combined with an increase in the agent 1's endowments y_1 raises the stationary asset price level, which positively affects the welfare of agent 0 through preferences for wealth. However, the decrease in y_0 negatively affects the welfare of agent 0 through the direct income effect. The aim of this subsection is to clarify the role of heterogeneity in the stationary welfare of agent 0. The steady-state welfare of agent 0 is given by:

$$v_0^* = \frac{u(c_0^*(x,z), w_0^*(x,z))}{\rho} \equiv v_0^*(x,z), \tag{25}$$

with $c_0^*(x,z) = d(1 - n_1 s_1^*(x,z))/n_0 + y - zn_1/n_0$ and $w_0^*(x,z) = q^*(x,z)(1 - n_1 s_1^*(x,z))/n_0$.

The next proposition summarizes the role of heterogeneity in endowments and preferences in the welfare of agent 0 in the steady state:

PROPOSITION 4. *Under Assumptions 1–3, the following holds in the steady state:*

(1) The stationary welfare of agent 0, v_0^*, is increasing with heterogeneity in endowments, σ_y, (i.e., a lower z) when $y_1 < y_0$, and is decreasing with heterogeneity in endowments (i.e., a higher z) when $y_1 > y_0$.

(2) The stationary welfare of agent 0, v_0^*, is decreasing with heterogeneity in preferences, σ_γ (i.e., with x).

Proof. See Appendix A.5.

Proposition 4.(1) shows that the stationary welfare of agent 0 is decreasing with the dispersion of endowment distribution when agent 0 has the smallest endowments in the economy, and is increasing with the dispersion of endowment distribution otherwise.

Economic intuition behind Proposition 4.(1). Suppose that the agent 0 is the poor in terms of endowments (i.e., $y_0 < y_1$). From Proposition 3.(3).(a), an increase in the dispersion of endowment distribution (i.e., y_0 decreases and y_1 increases in the same proportions) leads to a decrease in asset demand of agent 0. As her endowments and her stock of assets decrease, the consumption of agent 0 decreases as well. Furthermore, an increase in the dispersion of endowment distribution also leads to an increase in the stationary asset price level. However, since wealth is a normal good, the rise in asset price is not sufficient to counteract the decrease in assets of agent 0. Therefore, the wealth of agent 0 decreases. As consumption and wealth drop, the welfare of agent 0 decreases following an increase in σ_y. I can provide the same rationale for the case $y_1 < y_0$ by considering the reversed mechanism.

Proposition 4.(2) shows that the stationary welfare of agent 0 also decreases with the dispersion of γ_i distribution if the willingness for wealth accumulation of agent 0 is sufficiently low compared to agent 1 (i.e., $\gamma_0 < \gamma_1 - \bar{x}n/n_0$).

Economic intuition behind Proposition 4.(2). Suppose that the agent 0 is the poor in terms of endowments (i.e., $y_0 < y_1$). An increase in the dispersion of γ_i distribution (i.e., γ_1 increases and γ_0 decreases in the same proportions) encourages agent 0 to accumulate less assets. As the asset stock of agent 0 decreases, the consumption of agent 0 decreases as well. Furthermore, an increase in the dispersion of γ_i distribution also leads to an increase in the stationary asset price level. However, since wealth is a normal good, the increase in asset price is not sufficient to counteract the decrease in assets of agent 0. Therefore, the wealth of agent 0 drops. As consumption and wealth decrease, the welfare of agent 0 decreases following an increase in σ_γ. If the agent 0 is the rich (i.e., $y_1 < y_0$), but her willingness for wealth accumulation is sufficiently low compared to the agent 1 (i.e., $\gamma_0 < \gamma_1 - \bar{x}n/n_0$), the previous mechanism prevails as well.

6. EXPECTATION-DRIVEN ASSET PRICE FLUCTUATIONS

In this paper, I am interested in the existence of asset price fluctuations driven by self-fulfilling changes on agents' expectations without requiring any shock on the fundamentals, that is, preferences and/or dividends. To do this, I analyze the local dynamic properties of the model, and refer to local indeterminacy concept for the existence of expectation-driven fluctuations. Local indeterminacy means that there exist multiple equilibria with the same initial condition which converge to a steady state.

I log-linearize the three-dimensional dynamic system (9)–(11) around the steady state (q^*, s_1^*) to obtain the characteristic polynomial. As shown in Appendix A.6, I can derive the trace $T(\varepsilon)$, the sum of the second order principal minor $S(\varepsilon)$ and the determinant $D(\varepsilon)$ of the associated Jacobian matrix as functions of ε. The characteristic polynomial of this economy is given by:

$$P(\lambda) = \lambda^3 - T(\varepsilon)\lambda^2 + S(\varepsilon)\lambda - D(\varepsilon). \quad \textbf{(26)}$$

Local indeterminacy occurs when the stable manifold has a dimension greater than the number of predetermined variables. Since $s_1(t)$ is the only predetermined, the steady state is locally determinate when the Jacobian matrix has zero or one eigenvalue with negative real part, and locally indeterminate when it has at least two eigenvalues with negative real part. The next proposition provides the conditions on the inverse of the intertemporal elasticity of substitution ε for which local indeterminacy occurs.

PROPOSITION 5. *Let* $\underline{\varepsilon}(x,z) \equiv 1 + \dfrac{nc}{\gamma nc + x\left[d\left(s_1^* - 1/n\right) + z\right] nn_1/n_0} > 1.$ *Under Assumptions 1–3, the following holds:*

(1) If $\varepsilon \in (0, \underline{\varepsilon}(x,z))$*, the steady state is locally determinate.*
(3) If $\varepsilon > \underline{\varepsilon}(x,z)$*, the steady state is locally indeterminate.*

Proof. See Appendix A.6.

Proposition 5 shows that expectation-driven fluctuations are likely to occur when the inverse of the intertemporal elasticity of substitution ε is sufficiently high, in particular greater than 1. This implies that local indeterminacy occurs only if wealth and consumption are Edgeworth-substitutes (i.e., $u_{cw} < 0$). Otherwise, the steady state is always determinate.

The literature about MIUF provides a similar result. A necessary condition for local indeterminacy in MIUF model is a negative cross-derivative of the utility function between consumption and money. Nevertheless, a negative cross-derivative is neither empirically plausible [Walsh (2010)] nor consistent with the idea that money serves as a medium exchange. In contrast, assuming a negative cross-derivative between wealth and consumption is coherent with the concept of frugality at the root of SOC hypothesis developed by Weber (1905). Furthermore, several studies shed light on the fact that housing and consumption are Edgeworth-substitutes [see Yogo (2006), Piazzesi et al. (2007), and Flavin and Nakagawa (2008)]. Since housing wealth is a large component of households' wealth [see Survey of Consumer Finances (2013) for the US data], a negative cross-derivative between wealth and consumption would be empirically consistent.

Economic intuition. I provide the mechanisms through which fluctuations driven by self-fulfilling changes on expectations occur in the economy. Since aggregate consumption is constant along an equilibrium path, and the economy remains near the steady state, combining equations (9) and (10) gives:

$$\frac{n_0}{nc(t)}\left[\frac{\gamma_1}{\alpha}\frac{c_1(t)}{q(t)s_1(t)}c_1(t) + \frac{\gamma_0}{\alpha}\frac{c_0(t)}{q(t)s_0(t)}c_0(t)\right] = \rho - \frac{\dot{q}(t)+d}{q(t)}$$
$$-\frac{1}{nc(t)}\left\{n_1c_1(t)\left[\gamma_1(1-\varepsilon)\right] + n_0c_0(t)\left[\gamma_0(1-\varepsilon)\right]\right\}\frac{\dot{q}(t)}{q(t)}, \qquad \textbf{(27)}$$

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with $c(t)$ the average consumption, that is, $c(t) = [n_1 c_1(t) + n_0 c_0(t)]/n = (d + ny)/n$.

Equation (27) indicates that the aggregate marginal rate of substitution of consumption for wealth (left-hand side) is equal to the opportunity cost of wealth holdings (right-hand side) along an equilibrium path. It is worth noting that a change in asset price level has an ambiguous effect through the opportunity cost when $\varepsilon > 1$. When $\varepsilon > \underline{\varepsilon}(x, z)$, which is equivalent to $n_1 c_1(t) \left[1 + \gamma_1(1 - \varepsilon)\right] + n_0 c_0(t) \left[1 + \gamma_0(1 - \varepsilon)\right] < 0$, the opportunity cost of holding asset increases following a positive change in asset prices, and decreases otherwise.

Suppose that agents change their expectations at the same time and in the same manner, and they believe that a small drop in the asset price from the stationary level q^* occurs. As the consequence of their new beliefs, the opportunity cost of holding asset decreases when $\varepsilon > \underline{\varepsilon}(x, z)$. Equation (27) can be satisfied only if the marginal rate of substitution decreases. This occurs only if the asset price increases. After a small deviation of the asset price from its stationary level, the economy monotonically converges toward its steady state. Therefore, the resulting new path is an equilibrium. Because there are an infinite number of such paths, the steady state is locally indeterminate, and any change in expectations generates fluctuations.

More interestingly, Proposition 5 also indicates that the critical value $\underline{\varepsilon}$, for which fluctuations are likely to occur, is a function of both heterogeneity in preferences and heterogeneity in endowments (x and z). This means that heterogeneity plays a role in the emergence of expectation-driven asset price fluctuations.

7. THE ROLE OF HETEROGENEITY ON EXPECTATION-DRIVEN FLUCTUATIONS

In this section, I investigate the issue of heterogeneity in preferences and endowments on the existence of expectation-driven asset price fluctuations in the neighborhood of the steady state. Heterogeneity promotes asset price volatility as soon as it enlarges the range of parameter values for which local indeterminacy occurs. If heterogeneity reduces the range of parameter values, then it has stabilizing virtues. The results are twofold: First, heterogeneity in endowments plays a role in the asset price volatility only when agents face different preferences. Second, heterogeneity in preferences and endowments destabilize the economy under some parameter conditions.

First of all, I examine how $\underline{\varepsilon}$ varies in function of z when $\gamma_0 = \gamma_1$ (i.e., $x = 0$), then when $\gamma_0 < \gamma_1$ (i.e., $x > 0$). For a qualitative illustration, see Figure 1 when $\gamma_0 = \gamma_1$ and Figure 2 when $\gamma_0 < \gamma_1$. Gray areas correspond to local indeterminacy regions for which expectation-driven fluctuations occur.

COROLLARY 1. *Under Assumptions 1 and 2, when preferences for wealth are homogeneous ($x = 0$), heterogeneity in endowments has no impact on the conditions for the existence of local indeterminacy.*

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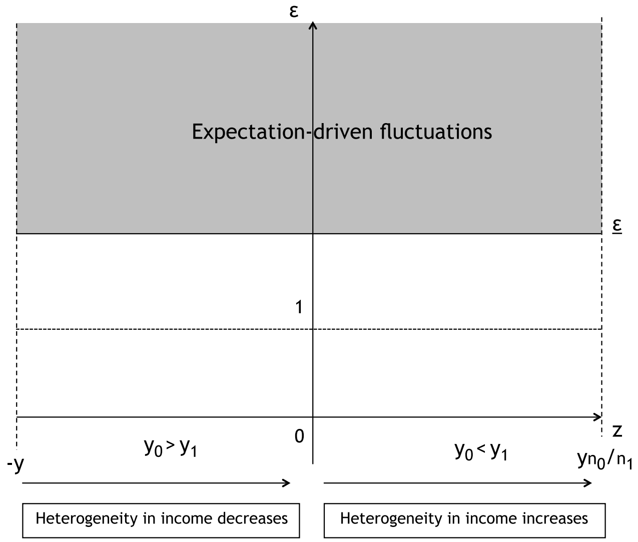

FIGURE 1. Stabilizing role of heterogeneity in endowments when $\gamma_0 = \gamma$.

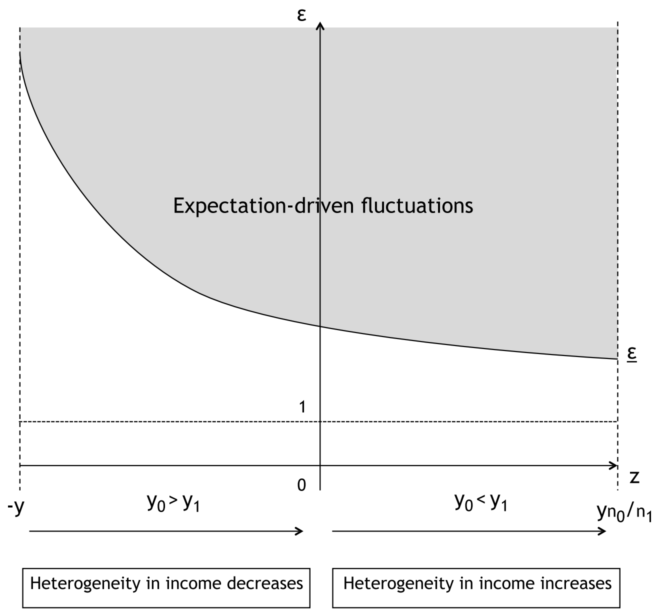

FIGURE 2. Stabilizing role of heterogeneity in endowments when $\gamma_0 < \gamma_1$.

When preferences are homogeneous, the critical value $\underline{\varepsilon}$ is given by $1 + 1/\gamma$, where γ is given by equation (12). Therefore, heterogeneity in endowments plays no role in the occurrence of expectation-driven asset price fluctuations. As discussed in Section 5.2, this is due to the homothetic properties of preferences.

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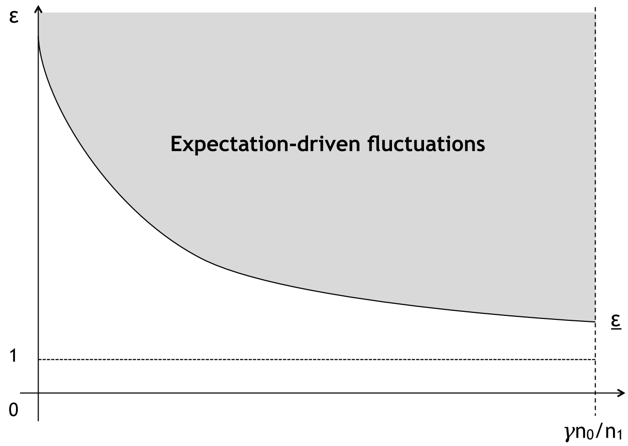

FIGURE 3. Destabilizing role of heterogeneity in preferences.

COROLLARY 2. *Under Assumptions 1 and 2, the following holds when $\gamma_0 < \gamma_1$ (i.e., $x > 0$):*

(1) When $y_0 < y_1$, an increase in endowment heterogeneity (i.e., a higher z) destabilizes, by enlarging the range of parameter value for which local indeterminacy occurs.
(2) When $y_0 > y_1$, an increase in endowment heterogeneity (i.e., a lower z) stabilizes, by reducing the range of parameter values for which local indeterminacy occurs.

Proof. See Appendix A.7.

Economic intuition. Recall that a change in asset price level has an ambiguous effect through the opportunity cost when $\varepsilon > 1$ [see equation (27)], and that local indeterminacy occurs only if $\varepsilon > \underline{\varepsilon}(x, z)$, which is equivalent to:

$$n_1 c_1(t)\left[1 + \gamma_1(1-\varepsilon)\right] + n_0 c_0(t)\left[1 + \gamma_0(1-\varepsilon)\right] < 0. \tag{28}$$

Under Assumption 2, Inequality (28) can be satisfied only if $\left[1 + \gamma_1(1-\varepsilon)\right] < 0$. This implies that local indeterminacy occurs if the effect on the opportunity cost stemming from a change in behavior of agents 1 dominates.

When $y_0 > y_1$, a rise in the dispersion of endowment distribution dampens the effect stemming from a change in the behavior of agents 1, and thus prevents the economy from the occurrence of expectation-driven asset price fluctuations. The reverse argument holds when $y_1 > y_0$.

I now examine how $\underline{\varepsilon}$ varies in function of x. For a qualitative illustration, see Figure 3.

COROLLARY 3. *Under Assumptions 1–3, an increase in preference heterogeneity (i.e., a higher x) destabilizes, by enlarging the range of parameter value for which local indeterminacy occurs.*

Proof. See Appendix A.7.

Economic intuition. When agents 1 have a sufficiently strong willingness for wealth accumulation compared to agents 0, that is, $\gamma_1 > \gamma_0 + \bar{x}n/n_0$, a rise in the dispersion of preference distribution γ_i strengthens the effect on the opportunity cost stemming from a change in the behavior of agents 1, and thus promotes the occurrence of expectation-driven asset price fluctuations in the economy.

8. CONCLUDING REMARKS

In this paper, I address the following question: What are the implications of heterogeneity in terms of preferences and endowments on the occurrence of asset price fluctuations driven by self-fulfilling changes on expectations under the SOC hypothesis?

I provide a theoretical approach by adding two ingredients to the pure exchange economy developed by Lucas (1978). The first ingredient is heterogeneity—in preferences, endowments, and initial wealth. The second ingredient is preferences for wealth, which captures the SOC hypothesis originated from Weber (1905).

These two novelties induce a nondegenerate wealth distribution in the steady state, meaning that all agents hold financial assets. The existence of preferences for wealth matters for the occurrence of expectation-driven fluctuations in asset prices. Investigating the role of heterogeneity, this paper shows that heterogeneity in preferences is a key element. Heterogeneity in endowments can destabilize by promoting the occurrence of expectation-driven asset price fluctuations in the economy only when preferences are heterogeneous.

By providing new insights into the role of heterogeneity in asset price volatility, this framework can be used to investigate how a redistribution policy, like capital income taxation, should be implemented to rule out excessive asset price fluctuations and stabilize the economy as a whole.

NOTES

1. In this paper, we only focus on heterogeneity in terms of fundamentals, and not in terms of expectations as recently in Gasteiger (2018).

2. Marx (1867), but also Keynes (1919) point out that the desire to accumulate wealth as end in itself by some agents is often the causes of financial and economic crises.

3. For more details, the reader can refer to Saez and Stantcheva (2016). They devote a section for the foundations of wealth preferences.

4. See Becker (2006) for a survey of the literature about neoclassical growth models with heterogeneous agents and borrowing constraints.

5. Since I want to focus on the role of wealth preferences and heterogeneity in asset price dynamics, I ignore physical capital accumulation.

6. Since Becker (1980), heterogeneity in preferences is a common assumption in infinite-horizon models with borrowing constraints, but mostly concerns the rate of time preference. Since I want to highlight the role of heterogeneity in wealth preferences in the occurrence of expectation-driven fluctuations, I ignore others forms of preference heterogeneity, that is, in the rate of time preference, in the intertemporal elasticity of substitution and in the weight of consumption in the utility function.

7. In this framework, I restrict attention to a class of utility functions which imposes a substitution elasticity between wealth and consumption equal to the unity. This functional form is quite standard

in the existing literature about SOC [see Bakshi and Chen (1996), Gong and Zou (2002), and Boileau and Braeu (2007)]. Considering this formulation allows me to provide a clear insight into the role of preference heterogeneity in dynamics by focusing only on a single parameter, namely γ_i.

8. This case seems consistent with empirical studies on the US data which show that top income earners have higher propensity to save [see Carroll (2000), Dynan et al. (2004), and Kumhof et al. (2015)]. In their theoretical model, Kumhof et al. (2015) suppose that only top earners display wealth preferences. They refer to Bakshi and Chen (1996) in which the preference for wealth is specific to a social-wealth index increasing with the income of the group. In my paper, all agents have preferences for wealth. Nevertheless, this configuration is also in line with Bakshi and Chen (1996).

9. In this paper, the uniqueness of steady state is a direct consequence of the class of preferences I consider, namely homothetic preferences. However, Greenwood–Hercowitz–Huffman preferences $u(c_i(t) + G_i(w_i(t))$, with $G_i'(.) > 0$ and $G''(.) < 0$, also ensure a unique steady state. Proofs are available upon requests.

10. Suen (2014) also obtains a nondegenerate distribution of capital in a model with time preference heterogeneity and preferences for wealth.

11. Admittedly, it partially replicates the US data in the sense that a part of American households do not hold financial wealth [see Survey of Consumer Finances (2013)], but I can overcome this issue by adding a third class of agents who do not express preferences for wealth. In the steady state, this class of agents holds no wealth (see Appendix A.3). Since the dynamics in the neighborhood of the steady state is driven by the behavior of agents holding wealth in the steady state, this third class of agents does not affect the dynamics of asset prices.

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APPENDIX

A.1: PROOF OF PROPOSITION 1

A steady state (s_1^*, q^*) is a solution of $q_1(s_1) = q_0(s_1)$, with:

$$\begin{cases} q_1(s_1) = \dfrac{x+\gamma}{\alpha\rho}\dfrac{ds_1 + y + z}{s_1} + \dfrac{d}{\rho}, & \text{(A1)} \\[2ex] q_0(s_1) = \dfrac{\gamma - xn_1/n_0}{\alpha\rho}\dfrac{d(1-n_1s_1) + n_0(y - zn_1/n_0)}{1-n_1s_1} + \dfrac{d}{\rho}. & \text{(A2)} \end{cases}$$

There exists at least one value $s_1^* \in (0, 1/n_1)$ such that $q_1(s_1^*) = q_0(s_1^*)$. Recall that $x + \gamma = \gamma_1 > 0$ and $y + z = y_1 > 0$. As $q_1(s_1) > 0 \;\forall\; s_1 \in (0, 1/n_1)$, I deduce that $q^* = q_1(s_1^*) > 0$.

Since $q_1(s_1)$ is strictly decreasing on $(0, 1/n_1)$ and $q_0(s_1)$ is strictly increasing on $(0, 1/n_1)$, the solution (q^*, s_1^*) is unique. ■

A.2: PROOF OF PROPOSITION 2

Proposition 2.(1) is a direct consequence of Proposition 1. Let us now prove Proposition 2.(2).[12] Note that $s_1^* > s_0^*$ is equivalent to $s_1^* > 1/n$. One has $q_1'(s_1) < 0$, $q_0'(s_1) > 0$ and $q_1(s_1^*) = q_0(s_1^*)$, where $q_1(s_1)$ and $q_0(s_1)$ are respectively given by equations (A1) and (A2). Hence, $s_1^* > 1/n$ if and only if $q_1(1/n) > q_0(1/n)$. This inequality is equivalent to $x > -\gamma nz/(d + n_0y_1 + n_1y_0) \equiv \underline{x}$. As $x = \gamma_1 - \gamma = (\gamma_1 - \gamma_0)n_0/n$, then $x > \underline{x}$ is equivalent to $\gamma_1 > \gamma_0 + \underline{x}n/n_0$. Proposition 2.(2). follows. I now prove Proposition 2.(3). Note that $ds_1^* + y_1 > ds_0^* + y_0$ is equivalent to $s_1^* > 1/n - z/d$. $s_1^* > 1/n - z/d$ if and only if $q_1(1/n - z/d) > q_0(1/n - z/d)$. This inequality is equivalent to $x > -\gamma nz/d \equiv \bar{x}$. As $x = \gamma_1 - \gamma = (\gamma_1 - \gamma_0)n_0/n$, then $x > \bar{x}$ is equivalent to $\gamma_1 > \gamma_0 + \bar{x}n/n_0$. Proposition 2.(3) follows. ■

A.3: THREE CLASSES OF AGENTS IN THE ECONOMY

Suppose that there are three classes of agents in the economy. Two classes display preferences for wealth ($\gamma_i > 0$) and the third one not ($\gamma_j = 0$ with $i \neq j$). The Euler equation associated with an agent without preferences for wealth is given by:

$$\frac{\dot{c}_j(t)}{c_j(t)} \geq \frac{r(t) - \rho}{1 + \alpha(\varepsilon - 1)},$$

$$s_j(t) \geq 0.$$

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The Euler equation associated with an agent displaying preferences for wealth is given by:

$$\frac{\dot{c}_i(t)}{c_i(t)} \geq \frac{r(t)-\rho}{1+\alpha(\varepsilon-1)} + \frac{\gamma_i c_i(t)/(w_i(t)) - \gamma_i(\varepsilon-1)\dot{w}_i(t)/w_i(t)}{1+\alpha(\varepsilon-1)},$$
$$s_i(t) \geq 0.$$

In the steady state, an equilibrium is such that:

$$r = \rho - \frac{\gamma_i c_i}{w_i} < \rho.$$

Therefore, agents without preferences for wealth hold no assets in the steady state.

A.4: PROOF OF PROPOSITION 3

I first prove Proposition 3.(1) claiming that the asset price is equal to its fundamental value in the steady state.

Let $r(t)$ be an interest rate, $q(t)$ the asset price in terms of consumption good at time t, and d the dividends in terms of consumption good generated by the asset. The no-arbitrage condition that governs the evolution of the asset price is given by:

$$q(t) = \frac{\dot{q}(t)+d}{r(t)}. \quad \textbf{(A3)}$$

Solving equation (A3) by iterating forward, I obtain:

$$q(t) = \int_t^{+\infty} e^{\int_t^s -r(i)di} d\text{d}s + e^{\int_t^{+\infty} -r(i)di} q(+\infty). \quad \textbf{(A4)}$$

The first term depicts the fundamental value of the asset $v(t)$, while the second term is the definition of a bubble $b(t)$:

$$v(t) = \int_t^{+\infty} e^{\int_t^s -r(i)di} d\text{d}s \quad \textbf{(A5)}$$

$$b(t) = e^{\int_t^{+\infty} -r(i)di} q(+\infty). \quad \textbf{(A6)}$$

In the steady state, $q(t) = q^*$ and $r(t) = r^*$. Therefore, from equation (A5), I have

$$v(t) = d\int_t^{+\infty} e^{-r^*(s-t)}\text{d}s = \frac{d}{r^*} \equiv v^*. \quad \textbf{(A7)}$$

Combining equation (A3) evaluated in the steady state with equation (A7), one has:

$$q^* = \frac{d}{r^*} = v^*. \quad \textbf{(A8)}$$

The asset price is equal to its fundamental component in the steady state.

I now prove Propositions 3.(2) and 3.(3) which address the effect of heterogeneity on the asset price level q^* in the steady state. In a first step, I study the effect of heterogeneity in endowments [Propositions 3.(2) and 3.(3).(a)]. Then in a second step, I study the effect of heterogeneity in preferences [Proposition 3.(3).(b)].

Recall that a steady state is a solution (s_1^*, q^*) with $s_1^* \in (0, 1/n_1)$ and $q^* > 0$ satisfying the following system:

$$\begin{cases} (x+\gamma)\dfrac{ds_1+z+y}{s_1} = (\gamma - xn_1/n_0)\dfrac{d(1-n_1s_1)+n_0\,(y-zn_1/n_0)}{1-n_1s_1} & \text{(A9)} \\ q = \dfrac{d}{r(q,s_1)}, & \text{(A10)} \end{cases}$$

$$\text{with } r(q,s_1) = \rho - n\frac{\gamma c + x\left[d\,(s_1-1/n)+z\right]n_1/n_0}{\alpha q} \tag{A11}$$

$$\text{and} \qquad nc = d + ny. \tag{A12}$$

Using equations (A10) and (A11), I get:

$$q^* = \frac{d}{\rho} + n\frac{\gamma c + x\left[d\left(s_1^* - 1/n\right)+z\right]n_1/n_0}{\rho\alpha}. \tag{A13}$$

Therefore,

$$\frac{\partial q^*}{\partial \sigma_y} = \frac{nn_1}{n_0\rho\alpha}x\left(1+d\frac{\partial s_1^*}{\partial z}\right)\frac{\partial z}{\partial \sigma_y}. \tag{A14}$$

Note that when $x=0$, one has $\partial q^*/\partial\sigma_y = 0$. Proposition 3.(2) follows. When $x>0$, I have to determine the signs of $\partial s_1^*/\partial z$ and $\partial z/\partial\sigma_y$.

Applying the implicit theorem function to equation (A9), one has under Assumptions 1 and 2:

$$\frac{\partial s_1^*}{\partial z} = \frac{n_1\gamma_0/(n_0s_0^*)+\gamma_1/s_1^*}{\gamma_1y_1/s_1^{*2}+n_1\gamma_0y_0/(n_0s_0^{*2})} > 0. \tag{A15}$$

Recall that $z = y_1 - y$ with $ny = n_1y_1 + n_0y_0$ and

$$\sigma_y = \begin{cases} -z\sqrt{n_1/n_0}, & \text{if } y_1 < y_0, \\ z\sqrt{n_1/n_0}, & \text{if } y_1 > y_0 \end{cases} \tag{A16}$$

Since $\partial z/\partial\sigma_y \le 0$ when $y_1 \le y_0$, and $\partial z/\partial\sigma_y \ge 0$ when $y_1 \ge y_0$, Proposition 3.(3).(a) follows. ■

I now study the effect of heterogeneity in preferences [Proposition 3.(3).(b)]. Using equation (A13), I get:

$$\frac{\partial q^*}{\partial \sigma_\gamma} = \frac{nn_1}{n_0\rho\alpha}\left[d\left(s_1^*-\frac{1}{n}\right)+z+xd\frac{\partial s_1^*}{\partial x}\right]\frac{\partial x}{\partial\sigma_\gamma}. \tag{A17}$$

Applying the implicit theorem function to equation (A9), one has under Assumptions 1 and 2:

$$\frac{\partial s_1^*}{\partial x} = \gamma\frac{nys_1^*}{x}\frac{s_1^*-y_1/ny}{s_1^{*2}xdnn_1/n_0+(x+\gamma)y_1}. \tag{A18}$$

Note that $\partial s_1^*/\partial x > 0$ is equivalent to $s_1^* > y_1/(ny)$. One has $q_1'(s_1) < 0$, $q_0'(s_1) > 0$ and $q_1(s_1^*) = q_0(s_1^*)$, where $q_1(s_1)$ and $q_0(s_1)$ are respectively given by equations (A1) and (A2). Hence, $s_1^* > y_1/(ny)$ if and only if $q_1(y_1/(ny)) > q_0(y_1/(ny))$. This inequality is equivalent to

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$$x\left[\frac{dy_1y_0}{ny^2}+\frac{y_1n_1}{yn_0}(y\frac{n_0}{n_1}-z)\right]>0 \tag{A19}$$

As $z<yn_0/n_1$, Inequality (A19) is always satisfied. Hence, $\partial s_1^*/\partial x>0$ is always satisfied. Furthermore, under Assumption 3, one has $d\left(s_1^*-\frac{1}{n}\right)+z>0$. Therefore, under Assumptions 1–3:

$$\frac{nn_1}{n_0\rho\alpha}\left[d\left(s_1^*-\frac{1}{n}\right)+z+xd\frac{\partial s_1^*}{\partial x}\right]>0. \tag{A20}$$

Recall that $x=\gamma_1-\gamma$ with $n\gamma=n_1\gamma_1+n_0\gamma_0$ and $\sigma_\gamma=x\sqrt{n_1/n_0}$. Since $\partial x/\partial\sigma_\gamma\geq 0$, under Assumption 1, Proposition 3.(3).(b) follows. ■

A.5: PROOF OF PROPOSITION 4

Consider the agent 0, her steady-state welfare is given by

$$v_0^*=\frac{u(c_0^*(x,z),w_0^*(x,z))}{\rho}\equiv v_0^*(x,z), \tag{A21}$$

with $c_0^*(x,z)=d(1-n_1s_1^*(x,z))/n_0+y-zn_1/n_0$ and $w_0^*(x,z)=q(x,z)^*(1-n_1s_1^*(x,z))/n_0$.

I start by analyzing the effect of heterogeneity in endowments on the welfare of agent 0. From equation (A21), one has:

$$\frac{\partial v_0^*(x,z)}{\partial\sigma_y}=\frac{1}{\rho}\left\{-\frac{\partial u(c_0^*(x,z),w_0^*(x,z))}{\partial c_0^*(x,z)}\left[d\frac{\partial s_1^*(x,z)}{\partial z}+1\right]\frac{n_1}{n_0}\right.$$
$$\left.+\frac{\partial u(c_0^*(x,z),w_0^*(x,z))}{\partial w_0^*(x,z)}\frac{\partial w_0^*(x,z)}{\partial z}\right\}\frac{\partial z}{\partial\sigma_y}. \tag{A22}$$

Because of Assumption 2, consumption and wealth are normal goods. Therefore, I can conjecture about the effect of heterogeneity in endowments in the welfare of agent 0. A higher z, meaning that y_0 decreases and y_1 increases in the same proportions, decreases the steady-state level of consumption and wealth of agent 0. This means that $\partial c_0^*(x,z)/\partial z\leq 0$ and $\partial w_0^*(x,z)/\partial z\leq 0$. Furthermore, note that $c_0^*(x,z)=d(1-n_1s_1^*(x,z))/n_0+y-zn_1/n_0$, it is clear that $\partial c_0^*(x,z)/\partial z<0$ from the proof of Proposition 3.(3).(a). To prove the negative sign of $\partial w_0^*(x,z)/\partial z$, I can use equation (18) which depicts the Euler equation of agent 0 in the steady state combined with equation (19). We obtain:

$$\frac{d}{q^*(x,z)}=\rho-\frac{\gamma-xn_1/n_0}{\alpha}\frac{c_0^*(x,z)}{w_0^*(x,z)}. \tag{A23}$$

Hence,

$$w_0^*(x,z)=\frac{\gamma-xn_1/n_0}{\alpha}\frac{c_0^*(x,z)}{\rho-d/q^*(x,z)}. \tag{A24}$$

We deduce that:

$$\frac{\partial w_0^*(x,z)}{\partial z}=\frac{\gamma-xn_1/n_0}{\alpha}\frac{\left\{\left[(\rho-d/q^*(x,z))\partial c_0^*(x,z)/\partial z-c_0^*(x,z)(d/q^*(x,z)^2)\partial q^*(x,z)/\partial z\right]\right\}}{\left[\rho-d/q^*(x,z)\right]^2}. \tag{A25}$$

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From the proof of Proposition 3.(3).(a), I know that $\partial q(x,z)^*/\partial z > 0$. As $\gamma - xn_1/n_0 > 0$, $\rho - d/q^*(x,z) > 0$, and $\partial c_0^*(x,z)/\partial z < 0$, we can deduce from equation (A25) that $\partial w_0^*(x,z)/\partial z$ has a negative sign. Therefore, from equation (A22), I can deduce that a higher z implies a lower welfare for agent 0 in the steady state. Since $\partial z/\partial \sigma_y \leq 0$ when $y_1 \leq y_0$, and $\partial z/\partial \sigma_y \geq 0$ when $y_1 \geq y_0$, Proposition 4.(1) follows. ■

Now, I analyze the effect of heterogeneity in preferences on the welfare of agent 0. From equation (A21), one has:

$$\frac{\partial v_0^*(x,z)}{\partial \sigma_\gamma} = \frac{1}{\rho}\left\{-\frac{\partial u(c_0^*(x,z), w_0^*(x,z))}{\partial c_0^*(x,z)}\frac{\partial s_1^*(x,z)}{\partial x}\frac{n_1 d}{n_0} + \frac{\partial u(c_0^*(x,z), w_0^*(x,z))}{\partial w_0^*(x,z)}\frac{\partial w_0^*(x,z)}{\partial x}\right\}\frac{\partial x}{\partial \sigma_\gamma}. \tag{A26}$$

From the proof of Proposition 3.(3).(b), I know that $\partial s_1^*(x,z)/\partial x > 0$ and $\partial q^*(x,z)/\partial x > 0$. As $c_0^*(x,z) = d(1 - n_1 s_1^*(x,z))/n_0 + y - zn_1/n_0$, it is clear that $\partial c_0^*(x,z)/\partial x < 0$. From equation (A24), I get:

$$\frac{\partial w_0^*(x,z)}{\partial x} = \frac{\gamma - xn_1/n_0}{\alpha}\frac{\left\{\left[(\rho - d/q^*(x,z))\partial c_0^*(x,z)/\partial x - c_0^*(x,z)(d/q^*(x,z)^2)\partial q^*(x,z)/\partial x\right]\right\}}{\left[\rho - d/q^*(x,z)\right]^2} - \frac{n_1/n_0}{\alpha}\frac{c_0^*(x,z)}{\rho - d/q^*(x,z)} \tag{A27}$$

As $\gamma - xn_1/n_0 > 0$, $\rho - d/q^*(x,z) > 0$, $\partial q^*(x,z)/\partial x > 0$, and $\partial c_0^*(x,z)/\partial x < 0$, I can deduce from equation (A27) that $\partial w_0^*(x,z)/\partial x$ has a negative sign. Therefore, from equation (A26), I can deduce that a higher x implies a lower welfare for agent 0 in the steady state. Since $\partial x/\partial \sigma_\gamma \geq 0$ under Assumption 1, Proposition 4.(1) follows. ■

A.6: PROOF OF PROPOSITION 5

Linearized dynamic system To conduct the analysis, I log-linearize the dynamic system (9)–(11) around the steady state (s_1^*, q^*) with respect to (s_{1t}, q_t, c_{1t}), and define $\hat{x} = log(x/x^*)$. Let $\psi = 1 + \alpha(\varepsilon - 1)$ and $\theta_i = \gamma_i(1-\varepsilon)$, I obtain[13]:

$$\begin{pmatrix} -\psi & 1+\theta_1 & \theta_1 \\ \psi n_1 c_1^*/(n_0 c_0^*) & 1+\theta_0 & -\theta_1 n_1 c_1^*/(n_0 c_0^*) \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} \dot{\hat{c}}_1 \\ \dot{\hat{q}} \\ \dot{\hat{s}}_1 \end{pmatrix}$$
$$= \begin{pmatrix} -(\rho - d/q^*) & \rho & (\rho - d/q^*) \\ (\rho - d/q^*)n_1 c_1^*/(n_0 c_0^*) & \rho & -(\rho - d/q^*)\gamma_1 n_1 c_1^*/(\gamma_0 n_0 c_0^*) \\ -(\rho - d/q^*)\alpha/\gamma_1 & 0 & d/q^* \end{pmatrix} \times \begin{pmatrix} \hat{c}_1 \\ \hat{q} \\ \hat{s}_1, \end{pmatrix}$$

with $c_1^* = ds_1^* + y + z$ and $c_0^* = d(1/n_0 - n_1 s_1^*/n_0) + y - zn_1/n_0$.

The characteristic polynomial $P(\lambda)$ The characteristic polynomial of this economy is given by:

$$P(\lambda) = \lambda^3 - T(\varepsilon)\lambda^2 + S(\varepsilon)\lambda - D(\varepsilon), \tag{A28}$$

where

$$T(\varepsilon) = \rho \frac{\varepsilon - \bar{\varepsilon}}{\varepsilon - \underline{\varepsilon}}, \quad \textbf{(A29)}$$

$$D(\varepsilon) = \frac{D_1(\varepsilon)}{\varepsilon - \underline{\varepsilon}}, \quad \textbf{(A30)}$$

$$D(\varepsilon) - S(\varepsilon)T(\varepsilon) = \frac{\delta_0(\varepsilon)}{(\varepsilon - \underline{\varepsilon})^2}\left[\delta_1(\bar{\varepsilon} - \underline{\varepsilon}) + \delta_2(1-\varepsilon)(\bar{\varepsilon} - \varepsilon)\right] - \delta_3 \frac{\bar{\varepsilon} - \varepsilon}{(\varepsilon - \underline{\varepsilon})^2}, \quad \textbf{(A31)}$$

where under Assumptions 1–2,

$$\underline{\varepsilon} = 1 + \frac{nc}{\gamma_1 n_1 c_1^* + \gamma_0 n_0 c_O^*} > 1, \quad \textbf{(A32)}$$

$$\bar{\varepsilon} = 1 + 2\frac{nc}{\gamma_1 n_1 c_1^* + \gamma_0 n_0 c_O^*} > \underline{\varepsilon}, \quad \textbf{(A33)}$$

$$D_1 = \frac{\rho}{1 - \alpha(1-\varepsilon)} \frac{1}{\alpha q^*}\left(n_1 c_1^* \frac{y_0}{q^* s_0^*} + n_0 c_0^* \frac{y_1}{q^* s_1^*}\right) > 0, \ \forall\, \varepsilon > 0, \quad \textbf{(A34)}$$

$$\delta_0(\varepsilon) = \rho \frac{\rho - d/q^*}{1 - \alpha(1-\varepsilon)} \frac{1}{\gamma_1\gamma_0} \frac{s_1^*}{2\gamma_1 c_1^*} > 0 \ \forall\, \varepsilon > 0, \quad \textbf{(A35)}$$

$$\delta_1 = \gamma_1\gamma_0\left(n_1 c_1^* \frac{y_0}{q^* s_0^*} + n_0 c_0^* \frac{y_1}{q^* s_1^*}\right) > 0, \quad \textbf{(A36)}$$

$$\delta_2 = \gamma_1\gamma_0\left(\gamma_1 n_1 c_1^* \frac{y_0}{q^* s_0^*} + \gamma_0 n_0 c_0^* \frac{y_1}{q^* s_1^*}\right) > 0, \quad \textbf{(A37)}$$

$$\delta_3 = \rho^3 \frac{nc}{c_1^*} \frac{s_1^*}{\gamma_1} > 0, \quad \textbf{(A38)}$$

with $c = (n_1 c_1^* + n_0 c_0^*)/n = d/n + y$.

Eigenvalues

- For $T(\varepsilon) < 0$, if $D(\varepsilon) < 0$ and $D(\varepsilon) > S(\varepsilon)T(\varepsilon)$, there are three eigenvalues with negative real parts;
- For $T(\varepsilon) < 0$ and $D(\varepsilon) > 0$ or for $T(\varepsilon) > 0$, $D(\varepsilon) > 0$ and $D(\varepsilon) > S(\varepsilon)T(\varepsilon)$, there are two eigenvalues with negative real parts;
- For $T(\varepsilon) < 0$, if $D(\varepsilon) < 0$ and $D(\varepsilon) < S(\varepsilon)T(\varepsilon)$ or, for $T(\varepsilon) > 0$, if $D(\varepsilon) < 0$, there is one eigenvalue with negative real part;
- For $T(\varepsilon) > 0$, if $D(\varepsilon) > 0$ and $D(\varepsilon) < S(\varepsilon)T(\varepsilon)$, there is no eigenvalue with negative real part.

By analyzing equations (A29)–(A31), one has:

- If $\varepsilon < \underline{\varepsilon}$, then $T(\varepsilon) > 0$ and $D(\varepsilon) < 0$, and thus there is one eigenvalue with negative real parts.
- If $\varepsilon \in (\underline{\varepsilon}, \bar{\varepsilon})$, then $T(\varepsilon) < 0$ and $D(\varepsilon) > 0$, and thus there are two eigenvalues with negative real parts.
- If $\varepsilon > \bar{\varepsilon}$, then $T(\varepsilon) > 0$, $D(\varepsilon) > 0$, and $D(\varepsilon) > S(\varepsilon)T(\varepsilon)$, and thus there are two eigenvalues with negative real parts.

Following Blanchard and Kahn (1980) conditions, I conclude that:

- Local determinacy when there are zero or one eigenvalue with negative real part;
- Local indeterminacy when there are at least two eigenvalues with negative real part.

Since $\gamma_1 = \gamma + x$, $\gamma_0 = \gamma - xn_1/n_0$, $nc = n_1 c_1^*(x,z) + n_0 c_0^*(x,z) = d + ny$ and $c_1^*(x,z) = ds_1^*(x,z) + y + z$, $\underline{\varepsilon}$ can be rewritten as follows:

$$\underline{\varepsilon} = 1 + \frac{nc}{\gamma nc + x\left[d\left(s_1^*(x,z) - 1/n\right) + z\right] nn_1/n_0}. \tag{A39}$$

Proposition 4 follows. ■

A.7: PROOFS OF COROLLARIES 2 and 3

Recall that:

$$q^* = \frac{dn}{\rho} + \frac{\gamma nc + x\left[d\left(s_1^*(x,z) - 1/n\right) + z\right] nn_1/n_0}{\rho\alpha}, \tag{A40}$$

$$\underline{\varepsilon} = 1 + \frac{nc}{\gamma nc + x\left[d\left(s_1^*(x,z) - 1/n\right) + z\right] nn_1/n_0}. \tag{A41}$$

From equations (A40) and (A41), I deduce that

$$\text{sign}\ \frac{\partial \underline{\varepsilon}}{\partial z} = \text{sign}\ -\frac{\partial q^*}{\partial z}, \tag{A42}$$

$$\text{sign}\ \frac{\partial \underline{\varepsilon}}{\partial x} = \text{sign}\ -\frac{\partial q^*}{\partial x}. \tag{A43}$$

Corollaries 2 and 3 follow Proposition 3. ■

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