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A QUANTITATIVE ANALYSIS OF TAX ENFORCEMENT AND OPTIMAL MONETARY POLICY

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This paper explores the consequences of tax enforcement policies for monetary policy. Agents may evade taxes by working in the informal sector, but they are detected with positive probability. Workers are rewarded with government benefits that are proportional to formal (taxed) work. When enforcement is imperfect and collecting taxes is costly, the optimal inflation rate is positive and inflation becomes a second-best tax. Deviations from the Friedman rule are optimal and depend on the tax enforcement policies. Using U.S. data, we compute the quantitative effect of different tax structures on inflation and interest rate. We show that different tax enforcement and government spending (benefits) policies induce different optimal outcomes for inflation and interest rates.

Keywords: Optimal Interest Rate, Tax Enforcement, Informal Economy, Tax Evasion

1. INTRODUCTION

The classic Friedman rule states that optimal monetary policy is characterized by a nominal interest rate equal to zero [Friedman (1969)]. This paper focuses on deviations from the Friedman rule when agents can evade taxes by working in an informal sector, where detection by a tax authority is imperfect.[1] Although the existing literature on optimal monetary policy is large, it has not considered environments where informal activities are within reach of a tax authority.[2] A key feature of our study is to observe that governments have tools to deal with tax evasion and informal activities.[3] We focus on the implications of three policy tools, namely benefits paid only to those who work in the formal sector, a penalty for tax evasion, and a detection probability. Thus, the government has a carrot and two sticks in its policy arsenal.

The first policy tool, benefits, includes things such as group insurance (health or dental), retirement benefits, and social security. Such benefits are available only

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to workers engaged in formal activities, and they create incentives for agents to work in the formal sector. The second tool, penalties, is also common. The OECD (2004c) reports that fines are usually in the range of 10–30% of the amount of tax evaded, with more serious offences involving deliberate evasion involving fines in the range of 40–75%. The third tool is the probability of detecting an agent working underground or evading taxes, and is related to a country's auditing procedures and monitoring technology. Overall, detection probabilities, which are based on the percentage of tax returns audited (frequency of audit), are very low in most countries.

We study optimal monetary policy in a cash-and-credit model where the economy is populated by a large number of identical, infinitely lived consumers. Agents choose to work in either the formal or informal sector.[4] Labor is the only factor of production and the final consumption good is a composite produced by both sectors. There are no information problems or aggregate uncertainty. The government incurs costs to collect taxes, e.g., to run the Internal Revenue Service in the United States. We assume that the cost of collecting labor income taxes from formal workers is small and constant, whereas tax enforcement spending is proportional to the expected informal tax revenue. We also assume that benefits paid by the government are proportional to hours worked in the formal sector, which affects agents' labor supply decisions in the formal and informal sectors by reducing the marginal effective tax rate.

The model is built on Chari et al. (1996) with three main differences: (i) the tax system is complete but imperfect, (ii) auditing is costly, and (iii) the government can create incentives, in the form of benefits available only to formal workers, for agents to remain in the formal sector. Chari and Kehoe (1999) define an economy's tax system as complete if the number of tax rates a social planner can select is equal to the number of commodities, and incomplete if the number of tax instruments is smaller than the number of commodities. Our model has a complete but imperfect set of tax instruments: The government has enough instruments to tax formal labor income and punish tax evaders, but enforcement is imperfect. Agents try to evade taxes by underreporting labor income or working in the informal sector. The government audits a certain fraction of the population and imposes an evasion penalty proportional to the tax evaded. We focus on the economically interesting case where penalties are bounded (i.e., we rule out perfect and zero detection probabilities and infinite penalty schemes). As a consequence, only a portion of the economy's informal labor income is taxed, rendering the tax system complete but imperfect.

We find that when tax enforcement is imperfect, deviations from the Friedman rule are optimal; the optimal nominal interest rate is not zero. A tax system characterized by evasion and costly tax enforcement constrains the government's ability to raise revenue. Inflation then becomes a "second best" tax because of the unavailability of alternative taxes. The size of the deviation from the Friedman rule depends on the tax enforcement policies and the cost of detecting informal activities. Our focus is on analyzing quantitatively the effects of these three tools to fight informal activities by keeping workers in the formal sector, allowing the government to reduce distortions.[5] To analyze the effects of enforcement on monetary policy quantitatively, we calibrate the model for the United States, using U.S. estimates for labor tax rates, benefits, tax evasion penalties, and the detection probability.

We show quantitatively that policies that create incentives to work in the formal sector or improve tax enforcement can significantly decrease both the optimal interest rate and inflation. We find that more generous benefits have a greater impact on the optimal interest rate than a high probability of detection. In other words, policies that reward work in the formal sector are more effective than policies that punish work in the informal sector, a result driven by the data and our calibration strategy. Benefits create incentives for agents to increase their formal labor supply, i.e., to allocate more time to the productive sector of the economy, increasing output and government tax revenue, and thus reducing the need to rely on an inflation tax. The government could potentially discourage informal activities if it had access to either higher detection probabilities or very harsh penalties. However, detection probabilities are typically low because of social norms that limit "cruel and unusual punishments."

We also investigate the sensitivity of the optimal inflation and interest rate to a variety of fiscal environments. For example, Eurozone members have very different tax and enforcement policies and represent a natural source of information. We perform numerical exercises based on these differences that show how different tax enforcement structures induce different optimal inflation and interest rates. Our results suggest that monetary policies that keep nominal interest rates and inflation low are constrained optimal only when distortions associated with informality are reduced. The different fiscal environments observed in Europe, which we take as given, motivate counterfactual experiments. Our results indicate that when governments pursue different levels of spending and differ in the efficiency of their public administration and willingness to fight tax evasion, their optimal monetary policies will differ.

The paper is organized as follows: Section 2 presents the model. We state the Ramsey problem and consider the optimality of the Friedman rule in the presence of tax evasion and costly tax collection. In Section 3, we calibrate the model and solve it numerically. We verify the implications of the model for the United States. The Friedman rule fails, as expected, but we show that optimal deviations are small. In Section 4 we show that different tax enforcement structures lead to different optimal inflation and interest rates. Section 5 concludes.

2. THE MODEL

2.1. The Economy

The economy is populated by a large number of identical, infinitely lived consumers in discrete time. There are no information problems and markets are complete. Agents are endowed with one unit of time, which can be spent on formal work l_t^F, informal work l_t^I or leisure h_t. There is no aggregate uncertainty. Agents value consumption and leisure, with period utility function $U(c_{1t}, c_{2t}, h_t)$, where c_{1t} and c_{2t} are cash and credit goods, respectively. Let $\beta \in (0, 1)$ be the discount factor and $U(\cdot)$ be a strictly concave, twice continuously differentiable function, separable in consumption and leisure, that satisfies the INADA conditions.

Cash and credit goods are distinguished solely by the means of payment, which is determined by a cash-in-advance constraint. This simple cash–credit model is chosen to make the results comparable with the existing literature. The cash–credit goods specification creates a distinction between an inflation tax and a consumption tax. A consumption tax distorts the labor–leisure decision, whereas the inflation tax distorts this margin and, in addition, distorts the cash–credit goods decision. Previously accumulated currency is not needed to purchase credit goods, which can be purchased with contemporaneously earned income. The distinction is also relevant because informal payments usually occur in cash.[6]

There is a complete set of tax instruments: The government can perfectly observe and tax formal labor income at rate τ_t^F. Government-sponsored benefits are available only to formal workers, which creates an incentive for agents to increase their formal labor supply. These benefits b_t are proportional to formal, reported, and taxed work. There is also a tax on informal income, but it is an imperfect instrument. Agents attempt to evade formal taxes by working in the informal sector, and there is a probability of being caught. Given detection probability $\pi_t \in (0, 1)$,[7] the government punishes tax evaders by imposing a penalty λ_t.[8] We assume that λ_t is proportional to the amount of tax evaded and is bounded. That is, $\lambda_t \in (0, \lambda_M)$, where λ_M is the exogenous maximal penalty level that the government can impose. The upper bound on the penalty is motivated by the fact that countries usually adopt a range for this type of penalty. We abstract from other types of penalties such as imprisonment.

By imposing a positive penalty, the government ensures that an audited individual will pay at least as much as he would if he paid the tax voluntarily. If the penalty is high enough, the agent will choose to pay the tax voluntarily rather than risk being caught working in the informal sector and paying a heavy penalty. The nature of the informal sector is such that agents can reduce their exposure to audit risk by reducing the time they allocate to informal activities. The expected or effective penalty rate on wage income earned in the informal sector is thus given by $\pi_t\lambda_t$. We rule out the cases where no penalty is imposed ($\pi_t\lambda_t = 0$).[9]

Given a probability of detection $\pi_t \in (0, 1)$, the consumer's problem is to maximize expected discounted lifetime utility

$$\max_{\{c_{1t}, c_{2t}, h_t, M_{t+1}, B_{t+1}\}_{t=0}^{\infty}} E\left[\sum_{t=0}^{\infty} \beta^t U(c_{1t}, c_{2t}, h_t)\right] \quad (1)$$

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subject to

$$M_{t+1} + B_{t+1} = R_t B_t + M_t - p_t c_{1t} - p_t c_{2t} + p_t(1 - \tau_t^F + b_t) w_t^F l_t^F + p_t(1 - \pi_t \lambda_t) w_t^I l_t^I, \quad (2)$$

$$p_t c_{1t} \leq M_t, \quad (3)$$

$$l_t^F + l_t^I + h_t = 1, \quad (4)$$

$$-B \leq \frac{B_t}{p_t} \leq B, \quad (5)$$

$$0 \leq M_t, c_{1t}, c_{2t}, h_t, l_t^I, l_t^F. \quad (6)$$

M_t is money holdings, B_t is nominal bonds, R_t is the interest rate paid on bonds, p_t is the price level, and w_t^F and w_t^I are wage rates for formal and informal labor, respectively.

The agent receives a return on bonds acquired previously, faces a cash-in-advance constraint, consumes, and then acquires new bonds and new cash for the next period with the remaining income. Constraint (2) is the representative agent's budget constraint. The left-hand side is the nominal value of the assets held at the beginning of the next period. The first term on the right-hand side is the value of nominal debt bought in the current period. The next two terms are the agent's unspent cash. The fourth term is the payment for credit goods. The last two terms are the net formal and informal labor income, respectively. Earnings in the formal sector are a linear function of hours of work, with net earnings in this sector given by $(1 - \tau_t^F + b_t) w_t^F l_t^F$. The use of an expected budget constraint in the household problem implies that the earnings in the informal sector do not fluctuate with the audit status [Lemieux et al. (1994); Fugazza and Jacques (2003); Turnovsky and Basher (2009).[10]

Regarding the agent's expected informal labor income, the agent gets $(w_t^I l_t^I - \lambda_t w_t^I l_t^I)$ with probability π_t and $(w_t^I l_t^I - 0)$ with probability $(1 - \pi_t)$. The agent's expected income if caught is $\pi_t(1 - \lambda_t) w_t^I l_t^I$, and $(1 - \pi_t) w_t^I l_t^I$ otherwise. Hence, expected informal income is $(1 - \pi_t \lambda_t) w_t^I l_t^I$, where punishment $\lambda_t w_t^I l_t^I$ is linear in the amount of tax the agent tries to evade.

Money is introduced and withdrawn through open market operations. Purchases of cash goods must satisfy a cash-in-advance (CIA) constraint (3). Inflation causes people to substitute away from activities that require cash, such as consumption, for activities that do not require cash, such as leisure. If agents in this economy wish to reduce cash holdings in response to higher inflation, they can only do so by reducing consumption of cash goods. The total time spent on formal work, informal work, and leisure is 1 in equation (4). Ponzi schemes are ruled out by constraint (5). The first-order conditions with respect to $c_{1t}, c_{2t}, l_t^F, l_t^I$, and B_{t+1}, M_{t+1} and the equilibrium conditions are in Appendix A.1.

There is a representative firm with technology $F(l_t^F, l_t^I)$ that exhibits constant returns to scale. Labor services are the only factor of production. The final consumption good is a composite produced by both the formal and informal sectors. In every period the firm takes prices as given and competitive pricing ensures that workers are paid their marginal products. Profit maximization implies $w_t^F = F_{l^F}(l_t^F, l_t^I)$ and $w_t^I = F_{l^I}(l_t^F, l_t^I)$, where $F_{l^F}(t)$ and $F_{l^I}(t)$ denote the marginal products of formal and informal labor, respectively.[11]

The government finances an exogenously given expenditure (g_t) through printing new money, issuing new bonds, collecting labor income taxes from formal workers, enforcing the tax code, and imposing penalties on informal workers. It also incurs costs to collect formal and informal taxes. Government spending to collect formal taxes (s_t^F) is assumed to be small and constant.[12] On the other hand, we assume that tax enforcement spending to collect informal taxes (s_t^I) is proportional to the expected informal tax revenue ($\pi_t \lambda_t w_t^I l_t^I$), given by

$$s_t^I = \phi \pi_t \lambda_t w_t^I l_t^I. \tag{7}$$

Each additional unit of revenue from the informal sector costs $\phi > 0$ to collect and the government devotes a total amount s_t^I to this purpose (Turnovsky and Basher, 2009).

Given a probability of detection π_t, the government's period budget constraint is

$$p_t g_t + p_t \left(s_t^F + s_t^I\right) + R_t B_t = B_{t+1} + M_{t+1} - M_t + p_t\left[(\tau_t^F - b_t) w_t^F l_t^F + \pi_t \lambda_t w_t^I l_t^I\right]. \tag{8}$$

The left-hand side of equation (8) contains government expenditures (g_t), spending to collect taxes (s_t^F, s_t^I), and current period debt service. The terms on the right-hand side are government revenues generated by asset sales, formal tax revenue, and informal tax revenue, respectively.

The resource constraint in this economy is

$$c_{1t} + c_{2t} + g_t + s_t^F + s_t^I = F(l_t^F, l_t^I). \tag{9}$$

DEFINITION 1. *A competitive equilibrium is a sequence of allocations* $x = \{x_t\}_{t=0}^{\infty}$, *where* $x_t = (c_{1t}, c_{2t}, l_t^F, l_t^I, M_t, B_t)$, *a sequence of prices* $p = \{p_t, w_t^I, w_t^F\}_{t=0}^{\infty}$, *a sequence of government policies* $\tau = \{g_t, \tau_t^F, b_t, \lambda_t\}_{t=0}^{\infty}$, *a sequence of government bond prices* $R = \{R_t\}_{t=0}^{\infty}$, *and a given probability of detection* $\pi_t \in (0, 1)$ *such that: (i) given* (p, τ, R), *the allocation sequence* $x = \{x_t\}_{t=0}^{\infty}$ *solves the representative agent's utility maximization problem, and (ii) given* $\{g_t, \tau_t^F, b_t, \lambda_t\}_{t=0}^{\infty}$, *the resource constraint is satisfied each period.*

2.2. The Ramsey Problem

The social planner's goal is to maximize a representative agent's utility subject to raising an exogenously determined amount of revenue for the government, taking into account the equilibrium reactions of consumers and firms to the distortionary tax system. That is, the Ramsey planner maximizes the discounted present value of an agent's utility subject to a resource constraint (9), a government budget constraint (8), and the constraints imposed by household and firm optimization. Thus, a Ramsey problem characterizes the set of allocations that can be implemented as a competitive equilibrium with distorting taxes subject to resource and implementability constraints.[13] The implementability constraint is derived in Appendix A.2, and is given by

$$\sum_{t=0}^{\infty} \beta^t \left[c_{1t} U_1(t) + c_{2t} U_2(t) - (1 - h_t) U_3(t)\right] = 0. \tag{10}$$

This constraint is the consumer budget constraint with both taxes and prices substituted out by the first-order conditions from the agent's utility maximization problem. It restricts the set of allocations that can be implemented as a competitive equilibrium with distorting taxes.

PROPOSITION 1 (Ramsey Allocation). *For a given probability of detection* $\pi_t \in (0, 1)$*, the consumption and labor allocations in the Ramsey equilibrium solve the Ramsey problem*

$$\max_{\{c_{1t}, c_{2t}, h_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t U(c_{1t}, c_{2t}, h_t) \tag{11}$$

subject to (9)*,* (10)*, and*

$$U_1(t) \geq U_2(t). \tag{12}$$

The solution of the Ramsey problem is an allocation that maximizes social welfare, subject to the restriction that it can be decentralized as a competitive equilibrium with taxes. The proof of Proposition 1 follows directly from Chari et al. (1996) and is omitted. It has two parts: (i) allocations in a competitive equilibrium must satisfy implementability constraint (10) and resource constraint (9) and, conversely, (ii) any allocation satisfying (9), (12), and (10) can be decentralized as a competitive equilibrium. When the tax system is complete but imperfect, implementability constraint (10) is unchanged; i.e., it holds regardless of the nature of the tax system. In this environment, (10) and (9) fully describe the set of competitive allocations that can be attained through feasible government policies, and an allocation from the centralized problem can be implemented as a competitive equilibrium. Money earns a gross nominal return of 1 and, from the consumer problem, at equilibrium $R \geq 1$. Thus, in any equilibrium constraint (12) must hold.

2.3. The Optimality of the Friedman Rule

In this section, we study the optimal monetary policy in the presence of informal activities and costly enforcement. The expected exogenously given informal penalty $\pi_t \lambda_t$ is an imperfect instrument for reducing the wedge between the marginal rate of substitution of consumption and informal labor and the marginal rate of transformation. In this economy, the Friedman rule is not necessarily the optimal monetary policy. The government's consumption expenditures, g_t, are assumed to be constant, and we study this economy in a steady state.

As in Chari et al. (1996), we consider utility functions separable between consumption and leisure of the form

$$U(c_1, c_2, h) = V(w(c_1, c_2), h), \tag{13}$$

where w is homothetic. A utility function of this particular form satisfies the following conditions:

$$\frac{U_{11}c_1 + U_{21}c_2}{U_1} = \frac{U_{12}c_1 + U_{22}c_2}{U_2}, \tag{14}$$

$$\frac{U_{31}}{U_1} = \frac{U_{32}}{U_1} = \frac{V_{12}}{V_1}. \tag{15}$$

PROPOSITION 2. *Suppose the detection probability* $\pi \in (0, 1)$ *is exogenously given. If the utility function is given by* (13) *and tax enforcement spending is positive* ($s^I > 0$)*, then the Friedman rule is not necessarily the optimal monetary policy.*

Proof. See Appendix A.3. ■

When informal sector activities cannot be perfectly observed and such activities are costly to monitor, the optimal policy is to set a positive inflation tax, in addition to two other distortionary tax instruments, namely a positive formal income tax and a positive evasion penalty. A positive income tax increases the distortion in agents' consumption–leisure choice. A positive inflation tax affects the agents' decisions to hold real balances. On the other hand, a positive evasion penalty affects the worker's informal labor supply. The government spends resources to collect informal labor income taxes, and this spending is a deadweight loss. However, higher evasion penalties deter agents from working in the informal sector, reducing the distortion on the second margin.

In the absence of tax evasion and tax collection costs and for the same preference specifications, the Friedman rule is the optimal monetary policy. If taxes are not costly to collect, i.e., $s^I = 0$, and informal labor is not an input of the production function, then this model boils down to the Chari et al. (1996) model. We illustrate this result quantitatively as well.

3. QUANTITATIVE ANALYSIS

3.1. Measurement and Calibration Strategy

We use the United States as our baseline economy. Following Chari et al. (1991, 1996) we assume the following CES utility function:

$$U(c_{1t}, c_{2t}, h_t) = (1-\eta)\frac{1}{v}\log\left[(1-\sigma)(c_{1t})^v + \sigma(c_{2t})^v\right] + \eta \log h_t. \tag{16}$$

Preference parameters η, σ, and v represent the work–leisure time allocation, the cash–credit goods weight, and the coefficient of risk aversion, respectively.

Aggregate output is given by a constant–returns to scale production function of the form

$$F(l_t^F, l_t^I) = \left[\alpha(l_t^I)^\rho + (1-\alpha)(l_t^F)^\rho\right]^{\frac{1}{\rho}}. \qquad \textbf{(17)}$$

Output is a function of the two types of labor, formal (l_t^F) and informal (l_t^I), with elasticity of substitution $1/(1-\rho)$. The technology parameter α denotes the percentage of informal labor in production.[14]

Assuming these particular functional forms for utility, equation (16), and production function, equation (17), the steady state Ramsey problem is as follows:

$$\max_{\{c_1,c_2,h\}} \left\{(1-\eta)\frac{1}{v}\log\left[(1-\sigma)(c_1)^v + \sigma(c_2)^v\right] + \eta\log(1-l^F-l^I)\right\} \qquad \textbf{(18)}$$

subject to

$$c_1+c_2+g+s^F+\phi\left[F_{l^I} - \frac{\eta}{(1-\eta)}\frac{(1-\sigma)(c_1)^v+\sigma(c_2)^v}{(1-l^F-l^I)\sigma(c_2)^{v-1}}\right]l^I = F(l^F,l^I), \quad \textbf{(19)}$$

$$(1-\eta)+\eta\frac{l^F+l^I}{1-l^F-l^I} = 0, \qquad \textbf{(20)}$$

$$(1-\sigma)(c_1)^{v-1} \geq \sigma(c_2)^{v-1}, \quad \textbf{(21)}$$

$$c_1, c_2, l^F, l^I \geq 0. \qquad \textbf{(22)}$$

Equation (19) is the resource constraint and corresponds to (9), (20) is the implementability constraint corresponding to (10), and (21) is an analogue of condition (12). The expression for the optimal interest rate for this economy is obtained by the procedure described in the proof of Proposition 2. For the particular functional forms (16) and (17), the optimal steady state interest rate R^* is the solution of the expression

$$\left(1+(v-1)\Gamma\phi l^I\right)R^* + \left[(v-1)\Gamma\phi l^I\left(\frac{\sigma}{1-\sigma}\right)^{\frac{1}{v-1}}\right]R^{*\frac{2v-1}{v-1}} = 1, \qquad \textbf{(23)}$$

where $\Gamma = [\eta/(1-l^F-l^I)(1-\eta)]$. Notice that (23) is not an explicit expression for the optimal interest rate because it depends on endogenous variables, which are functions of preference and production parameters, as well as tax enforcement policies and the tax collection cost parameter ϕ.

We calibrate the model for the United States, the baseline economy. Suppose that government consumption expenditures (g_t) are constant and assume that the solution to the Ramsey problem converges to a steady state. We solve for the optimal steady state interest rate in terms of preference and production parameters. The parameters of the model are η, v, σ, β, α, and ρ. The baseline values are summarized in Table 1.

TABLE 1. Baseline values

			Value	Source
Preferences	β	Discount factor	0.96	U.S. data
	η	Work–leisure time allocation	0.75	Time data
	σ	Cash–credit goods weight	0.57	Chari et al. (1991)
	v	Risk–aversion term	0.83	Chari et al. (1991)
Technology	ρ	Elasticity of substitution formal–informal labor	0.71	Lemieux et al. (1994)
	α	Percentage of informal labor in production	0.2731	Calibrated
Government	τ^F	Tax rate on formal income	0.29	OECD (2004b)
	b	Social Security contribution	0.053	SourceOECD
	π	Probability of detection	0.093	IRS
	λ	Administrative penalty for tax evasion	0.35	OECD (2004a, 2004c)
	s^F	Formal tax collection spending	0.0052	IRS, OECD (2004c)
	ϕ	Tax enforcement parameter	0.39	Calibrated

A period is a year and we assume that the discount factor β is 0.96. One-fourth of an agent's time is allocated to market activities (40 hours per week). In the steady state, the implementability constraint implies that $h = \eta$; thus η equals 0.75.[15] For the values of σ and v, we use 0.57 and 0.83, respectively, estimated by Chari et al. (1991) for the United States. The elasticity of substitution between formal and informal labor, ρ, is assumed to be 0.71, based on the estimate for Canada in Lemieux et al. (1994). Because no similar data are available for the United States, and given the similarities between the two economies, we use this parameter as a proxy for ρ in the United States.

The formal tax rate τ_t^F is assumed to be the "all-in" marginal tax rate for employees. This tax rate is calculated as the combined central and subcentral government income tax plus employee social security contributions, as a percentage of gross wage earnings. We consider workers earning 100% of the average wage level or average production wage (APW), which is (in national currency) the average annual gross wage earnings of adult, full-time manual workers in the manufacturing sector [OECD (2004b)]. The marginal tax rate may influence how many hours are worked, as it gives the amount of extra wage income an individual worker keeps after taxes.[16] For the United States, the "all-in" marginal tax rate is 29% and we set $\tau^F = 0.29$.

Formal employees receive a variety of nonwage compensation in addition to wages or salaries. These benefits include group insurance (health, dental), disability income protection, retirement benefits, employer-provided day care, sick leave, vacation, and social security. In our quantitative exercise, we use public pension spending as a proxy for benefits that induce workers to remain in the formal sector. These benefits are related to formal labor income and measures are comparable across countries.[17] We do not consider benefits such as education, roads, and security services, for instance, because the government cannot exclude informal workers. In the United States, public expenditure on pensions was equivalent to 5.3% of GDP in 2005, and we set $b = 0.053$.

To construct the effective penalty rate on wage income earned in the informal sector, we must identify values for the probability of detecting an informal worker (π) and the penalty for tax evasion (λ). The probability of detecting an agent working underground or evading taxes is related to auditing procedures and the monitoring technology. In the tax enforcement literature and in tax administration studies, several measures are used as proxies for the detection probability. Detection probabilities based on the rate of tax returns audited (frequency of audit) are very low. In the United States, according to Andreoni et al. (1998), the audit rate for individual tax returns was 1.70% in 1995. Data from the Internal Revenue Service (IRS) indicate that this rate dropped to 0.93% in the period 1996–2002.[18]

Statutory penalties for understatement of tax liability are generally imposed as a percentage of the additional tax payable and vary according to the seriousness of the offence. In the United States, evasion penalties are applied at rates of 20 and 75% of the portion of underpaid tax or fraud, respectively [Andreoni et al. (1998)]. We assume the lower rate of 20% as our baseline value. We set $\lambda = 0.35$ and interpret it as follows. In the United States, an agent detected working in the informal sector is expected to pay the tax rate he was trying to evade ($\tau^F = 0.29$), plus an additional penalty of 20% of the evaded amount, which implies an informal tax rate of 35% ($0.29 \times (1 + 0.20) = 0.35$).

Schneider and Enste (2000) estimate that the size of the informal sector (% of GDP) is 8.7% for the United States in 2000. The production parameter α is chosen so that the size of the informal sector corresponds to 8.7% of the total output and satisfies the following expression derived from the consumer and firm first-order conditions:

$$\frac{l^I}{l^F} = \left(\frac{\alpha}{1-\alpha} \frac{1-\pi\lambda}{1-\tau^F} \right)^{\frac{1}{1-\rho}}. \qquad \textbf{(24)}$$

We choose the government expenditure parameter g_t so that the size of the government corresponds to 21% of formal output at the resulting Ramsey equilibrium. National revenue authorities compute and publish a "cost of collection" for formal sector taxes. The ratio is computed by comparing the annual costs of administration incurred by a revenue authority with the revenue collected over the course of a fiscal year [OECD (2004c)]. This can be expressed as the amount the government spends to collect 100 units of revenue. According to the IRS, the United States spends \$0.52 to collect \$100 of formal taxes, and we set $s^F = 0.0052$.

Spending to collect formal taxes can be interpreted as the cost of running the IRS and carrying out its main tasks. However, enforcement spending requires additional effort and resources. Data on parameter ϕ is not readily available for the United States or other countries and we calibrate it using equilibrium condition (23), derived from the solution of the Ramsey problem. Enforcement cost parameter

TABLE 2. Model implications: selected variables (United States)

	Interest rate (%)	Inflation rate (%)	Informal sector (%)	Cash good	Credit good	C/Y ratio
Model	6.45	5.24	6.89	0.061	0.669	75.95
Data	6.45	2.74	8.70	0.057	0.625	70.86

ϕ is chosen so that the interest rate corresponds to the average long-term interest rate for the period 1991–1999 and satisfies equation (23). For the United States, this interest rate (denoted as $\bar{R}$) is 6.45%. Manipulating equation (23), we obtain an expression for ϕ:

$$\phi = \left(1 - \bar{R}\right)\left((v-1)\Gamma l^{I}\left\{\bar{R} + [\sigma/(1-\sigma)]^{\frac{1}{v-1}}\,\bar{R}^{\frac{2v-1}{v-1}}\right\}\right)^{-1}. \qquad \mathbf{(25)}$$

Values for parameters σ, η, and v were described previous. From the implementability constraint, in the steady state, $l^F + l^I = 0.25$, which together with the fact that the size of the informal sector in the United States is 8.7% of GDP implies that $l^I = 0.02$. For our baseline economy, the calibrated informal tax collection spending parameter ϕ is 0.39, indicating that the government would spend U.S.$39 to collect U.S.$100 of informal taxes.[19]

3.2. Implications of the Model

Assuming the baseline values in Table 1, we now evaluate the implications of the model for inflation, the size of the informal sector, cash goods consumption, credit goods consumption, and the total consumption–output ratio. Following Chari et al. (1991), c_1 is real money balances and c_2 is aggregate consumption minus real money balances. We measure real money balances by the monetary base and consumption by consumption expenditures. The size of the informal sector is measured as the proportion of informal income with respect to total output. The model overpredicts the inflation rate, cash and credit goods, and the consumption–output ratio. The size of the informal sector predicted by the model is lower than the value observed in the data. By construction, the estimated optimal interest rate is equal to the average long-term interest rate for the period 1991–1999, repeated in Table 2 for completeness.

These results can be explained in part by our calibration strategy and the choice of values for the preference, technology, and enforcement parameters. To further explore the implications of the model for the optimal interest rate and inflation, we conduct sensitivity analysis in which we vary key parameters of the model. The results for the optimal monetary policy for the United States are reported in Table 3. The base value case corresponds to $v = 0.83$ and $\rho = 0.71$. The exercise is repeated with different values of v and ρ.

TABLE 3. Optimal monetary policy and welfare cost

		$\rho = 0.60$	$\rho = 0.71$	$\rho = 0.80$
	α	0.223	0.273	0.374
$v = 0.75$	R^*	18.48	9.71	3.32
	$\Pi*$	16.74	8.32	2.19
	Welfare cost (%)	5.38	4.09	2.83
$v = 0.83$	R^*	12.15	6.50	2.25
	Π^*	10.67	5.24	1.16
	Welfare cost (%)	3.95	2.66	1.41
$v = 0.90$	R^*	6.97	3.78	1.31
	Π^*	5.69	2.63	0.27
	Welfare cost (%)	2.70	1.41	0.15

Observe that the share of informal labor in production α increases with the elasticity of substitution parameter ρ, which reflects the substitutability between formal and informal labor in the production function. For a fixed risk-aversion parameter v, the optimal interest rate R^* (and the optimal inflation rate) decreases as ρ increases. For instance, for $v = 0.83$, when ρ increases from 0.60 to 0.80, the optimal interest rate decreases from 12.15 to 2.25%. This result differs from previous studies that suggested that the higher ρ is, the higher the interest rate and inflation will be [Cavalcanti and Villamil (2003); Yesin (2004)]. If the government has no instruments to detect and tax informal activities, it must rely on an inflation tax to raise revenue and finance its expenditures. In our model, the government is able to collect tax revenue imperfectly from the informal sector, relying less on seigniorage revenue.

Table 3 also reports the welfare costs of deviating from the Friedman rule. First, we calculate welfare U^*, defined as agents' utility in equation (16), for an economy without an informal sector or tax collection costs. This is the same economy studied by Chari et al. (1991), where the Friedman rule is optimal. Then we calculate the agent's utility for our economy, under different values for v and ρ, and compute the percentage deviation from the welfare results under the Friedman rule. We denote this as the welfare cost of deviating from an optimal monetary policy characterized by a nominal interest rate equal to zero. The results suggest that deviations from the Friedman rule are more costly when the elasticity of substitution is relatively low and both inputs of production are "necessary." When informal activities are only imperfectly taxed and auditing is costly, the government raises revenue through a second-best inflationary tax, which reduces welfare.

Consumption and welfare U^* are higher when formal and informal labor inputs are more substitutable. The results indicate that economies with a higher elasticity of substitution between formal and informal labor would enjoy higher consumption and welfare. When $\rho = 1$, the elasticity of substitution between formal and informal labor is infinity and these inputs are perfect substitutes in

TABLE 4. Optimal interest rate (%) for the United States as benefits (b) and detection probability (π) vary

	$b = 0.053$	$b = 0.10$	$b = 0.20$
$\pi = 0.0093$	6.50	5.34	3.63
$\pi = 0.40$	5.69	4.67	3.16
$\pi = 0.80$	4.93	4.04	2.74

production. Productivity in the formal sector is higher than in the informal sector, and the optimal policy would imply that the net (after-tax) wage rate is higher in the formal than in the informal sector. Workers would allocate more time to the productive sector of the economy, i.e., the formal sector. This increases output, and consequently consumption and welfare are higher.

Finally, we investigate how policies relevant to agents' decision to work in the informal sector affect the optimal monetary policy quantitatively. Notice that in our setup, an increase in government-sponsored benefits is equivalent to a decrease in labor income taxes, and this implies higher net formal earnings. Similarly, the effective penalty rate on wage income earned in the informal sector ($\pi\lambda$) can be changed either through the probability of detection or through the evasion penalty. Table 4 reports the optimal interest rate for the United States when benefits and the evasion probability vary and the base values case corresponds to the upper left cell. All other parameters are kept at the values in Table 1.

Policies that create incentives to work in the formal sector and improve enforcement of tax legislation decrease both the optimal interest rate and inflation. Either a higher probability of detection or more generous benefits affect optimal monetary policy by reducing distortions in the economy. Benefits, however, have a greater impact on the optimal interest rate. In other words, the optimal interest rate is more elastic to changes in benefits than to changes in the detection probability. For instance, if we double the probability of detection from 40 to 80%, the predicted interest rate drops from 5.69 to 4.93% for the United States. On the other hand, for an increase in benefits from 10 to 20%, the reduction in the optimal interest rate is greater, from 5.34 to 3.63%. In the presence of informal activities and tax evasion, the results suggest that policies that reward work in the formal sector are more effective. Benefits create incentives for agents to increase their formal labor supply, i.e., to allocate more time to the productive sector of the economy. This increases output and consequently government tax revenue, reducing the deviations from the Friedman rule and the need to rely on an inflation tax.

4. TAX ENFORCEMENT AND OPTIMAL MONETARY POLICY

We now investigate how different enforcement policies affect the optimal monetary policy. The goal of this exercise is to show the sensitivity of the optimal inflation

TABLE 5. Tax enforcement policies parameters and optimal monetary policy

	τ^F	λ	b	size	Inflation rate (%)	Interest rate (%)
United States	0.29	0.35	0.053	8.7	5.24	6.50
Austria	0.43	0.43	0.122	10.2	5.36	4.14
Belgium	0.55	0.64	0.070	23.2	11.21	9.76
Denmark	0.49	0.98	0.054	18.2	9.07	7.71
Finland	0.45	0.47	0.075	18.3	9.11	7.75
France	0.33	0.36	0.106	15.3	7.77	6.46
Germany	0.58	0.67	0.110	16.3	8.22	6.89
Greece	0.29	0.29	0.107	28.6	13.38	11.85
Ireland	0.26	0.26	0.026	15.8	8.00	6.68
Italy	0.44	0.44	0.115	27.0	12.75	11.24
Netherlands	0.45	0.53	0.047	13.0	6.70	5.43
Portugal	0.25	0.29	0.086	22.6	10.96	9.52
Spain	0.30	0.30	0.070	22.6	10.96	9.52
Sweden	0.36	0.50	0.064	19.1	9.46	8.09
United Kingdom	0.33	0.33	0.055	12.6	6.51	5.25

and interest rate to a variety of fiscal environments. The numerical exercises are motivated by the very different tax and enforcement policies observed in Eurozone countries. However, our exercises are purely counterfactuals within the baseline U.S. economy. We do not investigate why policies differ across economies; rather we take policies as given and study the implications of these counterfactual policies for optimal monetary policy in the United States.

Table 5 reports data for 14 European countries' tax, enforcement, and informal sector size parameters that we will use in the counterfactual exercises. Notice that marginal labor tax rates, benefits, and the evasion penalty are higher in Austria, France, Germany, Italy, and the Scandinavian countries than in the United States. Greece and Portugal have higher benefits but lower taxes and penalties for tax evasion. In Ireland, all policy instruments are lower than in the United States, whereas in Spain and in the United Kingdom labor taxes and benefits are higher and penalties are lower. The size of the informal sector is higher in all 14 countries than in the United States.[20]

The last two columns of Table 5 indicate what the optimal inflation and interest rate would be, respectively, if the United States were to adopt each counterfactual policy. The exercise fixes the enforcement spending parameter (ϕ), the probability of detection (π), the technology parameter (α), the elasticity of substitution parameter (ρ), and the preference parameters (σ, υ) at the baseline U.S. values, and then varies the policy parameters governing the tax evasion penalty (λ), the formal labor income tax (τ^F), government benefits (b), and the size of the informal sector (size).[21] Observe that if the marginal labor tax, benefits, evasion penalty, and informal sector in the United States were increased to the levels observed in Denmark, the optimal inflation and interest rates would be higher as well. In general, this is the result for the majority of fiscal environments and tax structures in Table 5. Higher taxes on labor, not compensated for by more government-sponsored benefits, and a low expected penalty (recall that the probability of detection is kept at the United States level of 0.93%) induce agents to work more in the expanded informal sector. As a consequence, the government collects less formal taxes and must raise revenue through inflation. The combination of generous government benefits and a large informal sector leads to higher optimal inflation and interest rates, which is the case for environments similar to Greece, Italy, Portugal, and Spain.

Overall this exercise illustrates that when tax enforcement is imperfect, deviations from the Friedman rule are optimal (the optimal nominal interest rate is not zero). If the government can only tax informal workers and tax evaders with a given probability, it is optimal to increase the tax on money via positive inflation. This is a second-best way to reduce (given) distortions from the informal sector. Previous studies have attempted to explain deviations from the Friedman Rule in the presence of tax evasion and informal activities. In Nicolini (1998), the quantitative effect of tax evasion on optimal monetary policy is small, even in economies with large underground sectors. Cavalcanti and Villamil (2003) show that the optimal inflation tax can be positive, ranging from 0 to 22%, for alternative calibrations. Yesin (2004) explores the relevance of tax collection costs (the cost of collecting formal taxes). Her model performs well only for a small group of countries, and the optimal interest rate ranges from 5 to 43%. In Koreshkova (2006), the optimal policy takes into account the inefficiency of the informal sector. However, the optimal inflation rate in her model is very high, with an optimal inflation rate of 80% per annum. Our model predicts optimal inflation and interest rates in the range of 4 to 22% for a variety of fiscal environments.

Finally, the counterfactual exercise clearly indicates that different tax enforcement structures induce different optimal inflation and interest rates. Lower interest rates and inflation are consistent with low tax evasion and efficient enforcement systems. The different environments in Europe represent a challenge to the stability of the European Monetary Union. Since the introduction of the Euro in 1999, the mismatch between the EU's advanced economic and monetary union and an incomplete fiscal union has become problematic, particularly after the recent global financial crisis led to a sharp deterioration of many European Union countries' public finances. Of particular concern are the PIIGS, an acronym for Portugal, Ireland, Italy, Greece, and Spain. Although the specific problems are different in each country, high government spending, structural rigidities, and tax evasion and/or corruption have led to budget problems. Countries are now pursuing measures to reduce excessive government spending, improve inefficient public administration, and fight tax evasion. Our study shows the potential difficulties of trying to maintain a monetary union when countries have highly diverse fiscal environments.

5. CONCLUSIONS

This paper shows the effect of tax evasion and informal activities on optimal monetary policy. Positive inflation and interest rates are optimal when enforcement spending is positive and tax enforcement is imperfect. Tax enforcement policies can justify modest deviations from the Friedman Rule. We show quantitatively that policies that create incentives to work in the formal sector and improve the enforcement of tax legislation decrease both the optimal interest rate and inflation. The quantitative exercises show that different tax enforcement structures induce different optimal inflation and interest rates. Lower interest rates and inflation are consistent with low tax evasion and efficient enforcement systems. Eurozone countries have coordinated their monetary policies, but still lack fiscal policy coordination. Our study indicates that fiscal and structural conditions are highly relevant to the design of optimal monetary policy.

NOTES

1. Informal activities are defined as all income-generating activities that do not comply with tax obligations, i.e., mainly tax evasion and noncompliance with economic legislation. We use the terms tax evasion and informal activities interchangeably.

2. Nicolini (1998) shows that the Friedman rule fails when there is an underground sector and money is introduced by means of a cash-in-advance constraint. Cavalcanti and Villamil (2003) show that the Friedman rule is not optimal and welfare costs are asymmetric when there are an informal sector and a shopping-time constraint. Yesin (2004) studies an economy with an informal sector and shows that the costs of collecting formal income taxes can partly explain the observed deviations from the Friedman rule across countries. Koreshkova (2006) investigates quantitatively the importance of the public-finance motive for inflation with a tax-evading sector and finds a positive relationship between the size of the underground economy and the inflation rate. For discussions of general conditions for the optimality of the Friedman rule and reasons for deviations from it see, for instance, Aizenman (1983), Vegh (1989) or De Fiore (2000).

3. For instance, among Eurozone countries, the size of the informal sector (as a proportion of the GDP) varies from a low of 10% in Austria to a high of 29% in Greece. Figures for Scandinavian countries are around 18% of GDP [Schneider and Enste (2000)].

4. Researchers have identified two main reasons that workers become informal: (i) to evade taxation and labor market regulations [Johnson et al. (1998); Friedman et al. (2000); Fugazza and Jacques (2003)] and (ii) in response to bureaucracy and corruption among other institutional and enforcement conditions [Busato and Chiarini (2004); Choi and Thum (2005)]. For a discussion about sectoral shift and wealth distribution in the presence of an (urban) informal sector, see Yuki (2008).

5. This paper is also related to the literature on tax evasion and enforcement, for instance, Becker (1968), Allingham and Sandmo (1972), Slemrod and Yitzhaki (2002), and Polinsky and Shavell (2005).

6. Chugh (2009) demonstrates that the precise timing of financial markets and goods markets in a simple cash good/credit good model does not matter for the main results in the Ramsey literature on optimal fiscal and monetary policy.

7. This probability is exogenous and independent of government enforcement expenditures.

8. The magnitude of penalties is the object of debate in the enforcement literature. If the government is free to choose the penalties, Becker (1968), Chander and Wilde (1998), Krasa and Villamil (2000), and Polinsky and Shavell (2005), among others, have shown that (extremely) severe penalties are optimal. However, less-than-maximum fines can be optimal when enforcement is uncertain [see Polinsky and Shavell (2005) for a survey] or social norms impose economic restrictions on the penalty function [see Marhuenda and Ortuno-Ortin (1997)].

9. Previous studies have analyzed the extreme cases. Either the probability of detecting a tax evader is equal to one and the agent must pay tax on "informal" income [Jones et al. (1997)], or the government cannot detect a tax evader (informal worker), which implies a detection probability of zero.

10. An alternative to this approach would be to allow the consumer's realized income to depend on whether the consumer was actually caught. This would result in an intraperiod distinction between audited and nonaudited agents. See Arbex and Turdaliev (2010).

11. See Faia (2008) for a discussion of optimal monetary policy and capital accumulation.

12. De Fiore (2000) and Yesin (2004) incorporate this feature into their models and show that it is not sufficient to account for deviations from the Friedman rule. Also, OECD (2004c) data support this assumption.

13. Implicit in this approach is that the central bank sets its policy according to the solution to the Ramsey problem. The central bank's problem is then to choose values of the monetary instruments such that the representative agent's utility is maximized.

14. These functional forms for the utility and production functions are standard in quantitative studies of the Friedman rule. See for instance Cavalcanti and Villamil (2003) and Yesin (2004). See also Easterly (1993).

15. We also consider the case $\eta = 0.67$. The results are not sensitive to this choice.

16. Average tax rates may influence the decision to enter (or exit) the labor market, as they affect how much total net income after tax changes if one decides to join (or exit) the labor market. The average "all-in" tax rate for the United States is 24.1%. The results in the next section are not sensitive to the choice between marginal and average tax rates.

17. Pension systems vary across countries and no single model fits all. Generally, there is a mix of public and private provision. Public pensions are statutory, most often financed on a pay-as-you-go basis — where current contributions pay for current benefits—and managed by public institutions.

18. Similarly, Busato and Chiarini (2004) estimate a 3% probability of detection in Italy and, according to the Canada Revenue Agency, these audit rates fell from 1.01% in 2002 to 0.75% in 2006 in Canada.

19. According to IRS Commissioner Mark Everson [Kenney (2005)], enforcement of the tax code is expected to pay for itself and the benefit/cost ratio is more than four to one. This means that an additional dollar spent on enforcement returns at least four dollars to the government, or $25 to collect $100 of informal taxes.

20. We treat the size of the informal sector as an enforcement policy variable. Although evasion penalty λ affects the size of the informal sector in theory, in practice it does not have a large role because when it is multiplied by 0.0093 (the probability of detection), the "expected" tax on informal labor is very small for all countries. Cavalcanti and Villamil (2003) also fix the size of the informal sector. See Antunes and Cavalcanti (2007) for a model where this size is determined endogenously.

21. Despite the availability of data on enforcement measures, obtaining a comparable measure of the probability of detection across countries is quite challenging.

REFERENCES

Aizenman, J. (1983) Government size, optimal inflation tax and tax collection costs. *Eastern Economic Journal* 9(2), 103–105.

Allingham, M. and A. Sandmo (1972) Income tax evasion: A theoretical analysis. *Journal of Public Economics* 1, 323–338.

Andreoni, J., B. Erard, and J. Feinstein (1998) Tax compliance. *Journal of Economic Literature* 36, 818–860.

Antunes, A. and T. Cavalcanti (2007) Start up costs, limited enforcement, and the hidden economy. *European Economic Review* 51(1), 203–224.

Arbex, M. and N. Turdaliev (2011) Optimal monetary and audit policy with imperfect taxation. *Journal of Macroeconomics* 33(2), 327–340.

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Arseneau, D. (2007) The inflation tax in an open economy with imperfect competition. *Review of Economic Dynamics* 10, 126–147.

Becker, G. (1968) Crime and punishment: An economic approach. *Journal of Political Economy* 76(2), 169–217.

Busato, F. and B. Chiarini (2004) Market and underground activities in a two-sector dynamic equilibrium model. *Economic Theory* 23, 831–861.

Cavalcanti, T. and A. Villamil (2003) Optimal inflation tax and structural reform. *Macroeconomics Dynamics* 7, 333–362.

Chander, P. and L. Wilde (1998) A general characterization of optimal income tax. *Review of Economics Studies* 65(1), 165–183.

Chari, V., L. Christiano, and P. Kehoe (1991) Optimal fiscal and monetary policy: Some recent results. *Journal of Money, Credit and Banking* 23(3), 519–539.

Chari, V., L. Christiano, and P. Kehoe (1996) Optimality of the Friedman rule in economies with distorting taxes. *Journal of Monetary Economics* 37, 203–223.

Chari, V.V. and P. Kehoe (1999) Optimal fiscal and monetary policy. In: J. B. Taylor and M. Woodford (eds.), *Handbook of Macroeconomics*, vol. 1C, pp. 1671–1745. Amsterdam: North-Holland.

Choi, J. and M. Thum (2005) Corruption and the shadow economy. *International Economic Review* 46(3), 817–836.

Chugh, S. K. (2009) Does the timing of the cash-in-advance constraint matter for optimal fiscal and monetary policy? *Macroeconomic Dynamics* 13, 133–150.

De Fiore, F. (2000) The Optimal Inflation Tax When Taxes Are Costly to Collect. Working paper 38, European Central Bank.

Easterly, W. (1993) How much do distortions affect growth? *Journal of Monetary Economics* 32, 187–212.

Faia, E. (2008) Ramsey monetary policy with capital accumulation and nominal rigidities. *Macroeconomic Dynamics* 12, 90–99.

Friedman, E., S. Jonhson, D. Kaufmann, and P. Zoido-Lobaton (2000) Dodging the grabbing hand: the determinants of unofficial activity in 69 countries. *Journal of Public Economics* 76, 459–493.

Friedman, M. (1969) The optimum quantity of money. In M. Friedman (ed.), *The Optimum Quantity of Money and Other Essays*, pp. 1–50. Chicago: Aldine.

Fugazza, M. and J.F. Jacques (2003) Labor market institutions, taxation and the underground economy. *Journal of Public Economics* 88, 395–418.

Johnson, Simon, Daniel Kaufmann, and Pablo Zoido-Lobaton (1998) Regulatory discretion and the unofficial economy. *American Economic Review* 88, 387–392.

Jones, L., R. Manuelli and P. Rossi (1997) On the optimal taxation of capital income. *Journal of Economic Theory* 73, 93–117.

Kenney, A. (2005) Year in review: Everson evaluates state of IRS, pledges strong agenda for 2005. *Insurance Tax Review* (Feb.), 288.

Koreshkova, T. (2006) A quantitative analysis of inflation as a tax on the underground economy. *Journal of Monetary Economics* 53, 773–796.

Krasa, S. and A. Villamil (2000) Monitoring the Monitor: An Incentive Structure for a Financial Intermediary. Manuscript, University of Illinois at Urbana–Champaign.

Lemieux, T., B. Fortin, and P. Frechette (1994) The effect of taxes on labor supply in the underground economy. *American Economic Review* 84(1), 231–254.

Marhuenda, F. and I. Ortuno-Ortin (1997) Tax enforcement problems. *Scandinavian Journal of Economics* 9(91), 61–72.

Nicolini, J. (1998) Tax evasion and the optimal inflation tax. *Journal of Development Economics* 55, 215–232.

OECD (2004a) Benefits and Wages: OECD Indicators. SourceOECD.

OECD (2004b) Recent Tax Policy Trends and Reforms in OECD Countries. SourceOECD, No. 9.

OECD (2004c) Tax Administration in OECD Countries: Comparative Information Series. SourceOECD.

Polinsky, M. and S. Shavell (2005) The Theory of Public Enforcement of Law. Working Paper 11780, NBER.

Schneider, F. and D. Enste (2000) Shadow economies: Size, cause and consequences. *Journal of Economics Literature* 38, 77–114.

Slemrod, J. and S. Yitzhaki (2002) Tax avoidance, evasion and administration. In J. Auerbach and M. Feldstein (eds.), *Handbook of Public Economics*, vol. 3. Amsterdam: Elsevier Science.

Turnovsky, S. and M. A. Basher (2009) Fiscal policy and the structure of production in a two-sector developing economy. *Journal of Development Economics* 88, 205–216.

Vegh, C. (1989) Government Spending and Inflationary Finance: A Public Finance Approach. IMF Staff Paper 36.

Yesin, A. (2004) Tax Collection Cost, Tax Evasion and Optimal Interest Rates. Working paper 04.02, Study Center Gerzensee.

Yuki, K. (2008) Sectoral shift, wealth distribution and development. *Macroeconomic Dynamics* 12, 527–559.

APPENDIX

A.1. THE HOUSEHOLD'S PROBLEM

Let $\beta^t \eta_t$ and $\beta^t \mu_t$ be the Lagrange multipliers for the consumer budget constraint (2) and the CIA constraint (3), respectively. Let $U_i(t) = U_i(c_{1t}, c_{2t}, h_t)$ denote the marginal utility of element i, and likewise for $U_{ij}(t)$, where $i, j = 1, 2, 3$. The first-order conditions with respect to $c_{1t}, c_{2t}, l_t^F, l_t^I, B_{t+1}, M_{t+1}$ are

$$U_1(t) = p_t(\eta_t + \mu_t), \quad \textbf{(A.1)}$$

$$U_2(t) = p_t\eta_t, \quad \textbf{(A.2)}$$

$$U_3(t) = p_t\eta_t \left(1 - \tau_t^F + b_t\right) w_t^F, \quad \textbf{(A.3)}$$

$$U_3(t) = p_t\eta_t \left(1 - \pi_t\lambda_t\right) w_t^I, \quad \textbf{(A.4)}$$

$$\eta_t = \beta\eta_{t+1}R_{t+1}, \quad \textbf{(A.5)}$$

$$\eta_t = \beta(\eta_{t+1} + \mu_{t+1}),$$

and the equilibrium conditions can be represented as

$$\tau_t^F - b_t = 1 - \frac{U_3(t)}{U_2(t)}\frac{1}{w_t^F}, \quad \textbf{(A.6)}$$

$$\pi_t\lambda_t = 1 - \frac{U_3(t)}{U_2(t)}\frac{1}{w_t^I}, \quad \textbf{(A.7)}$$

$$R_{t+1} = \frac{U_2(t)}{\beta U_{c_2}(t+1)}(1 + \Pi_{t+1}), \quad \textbf{(A.8)}$$

$$\Pi_{t+1} = \beta\frac{U_1(t+1)}{U_2(t)} - 1, \quad \textbf{(A.9)}$$

where $\Pi_{t+1} = (p_{t+1} - p_t)/p_t$ is the inflation rate. Given that the gross nominal return on money is equal to one, in any equilibrium $R_{t+1} \geq 1$, because otherwise the consumer could make infinite profits by buying money and selling bonds. Agent equilibrium conditions (A.8) and (A.9) imply that $U_1(t) \geq U_2(t)$ must hold in any equilibrium.

A.2. THE RAMSEY PROBLEM

To derive the implementability constraint, we multiply the period-t budget constraint (2) and the period-t cash-in-advance constraint (3) by their Lagrange multipliers $\beta^t \eta_t$ and $\beta^t \mu_t$, respectively, for each period t, and sum over t. All the money terms except M_0 and all the bond terms except B_0 will cancel out because of the use of the first-order conditions of the agent's optimization problem. This yields the following:

$$\sum_{t=0}^{\infty} \beta^t \left[c_{1t} U_1(t) + c_{2t} U_2(t) - (1 - h_t) U_3(t)\right] = \eta_0 R_0 (M_0 + B_0).$$

We assume that the government is constrained in its policy choices in the first period and sets the initial stock of nominal assets, $M_0 + B_0$, equals to zero. If the initial stock is either negative or positive, welfare is maximized by setting the initial price level to either negative infinity or infinity, respectively. The assumption that $M_0 + B_0 = 0$ does not affect the results but simplifies the calculations. Hence we obtain the following expression:

$$\sum_{t=0}^{\infty} \beta^t \left[c_{1t} U_1(t) + c_{2t} U_2(t) - (1 - h_t) U_3(t)\right] = 0.$$

A.3. PROOF OF PROPOSITION 2

Consider the Ramsey problem described in Proposition 1 and assume that the solution to this problem converges to a steady state. Let ξ and γ be the Lagrange multipliers on the implementability constraint (10) and the resource constraint (9), respectively. This problem can be written in the Lagrangian form,

$$\mathscr{L} = U(c_1, c_2, h) + \xi \left[c_1 U_1 + c_2 U_2 - (1 - h) U_3\right] \quad \textbf{(A.10)}$$

$$+ \gamma \left[F(l^F, l^I) - c_1 - c_2 - g - s^F - \phi \left(1 - \frac{U_3}{U_2} \frac{1}{F_{l^I}}\right) F_{l^I} l^I\right], \quad \textbf{(A.11)}$$

where (7) and (A.7) are used to express s^I in the resource constraint. The first-order conditions with respect to c_1 and c_2 are, respectively,

$$(1 + \xi) U_1 + \xi [U_{11} c_1 + U_{21} c_2 - (1 - h) U_{31}] = \gamma \left[1 - \phi \left(\frac{U_{31} U_2 - U_3 U_{21}}{U_2^2}\right) l^I\right], \quad \textbf{(A.12)}$$

$$(1 + \xi) U_2 + \xi [U_{12} c_1 + U_{22} c_2 - (1 - h) U_{32}] = \gamma \left[1 - \phi \left(\frac{U_{32} U_2 - U_3 U_{22}}{U_2^2}\right) l^I\right]. \quad \textbf{(A.13)}$$

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Dividing (A.12) and (A.13) by U_1 and U_2, respectively, and noting that $U_{3i}/U_i = V_{12}/V_1$ for $i = 1, 2$, we have

$$(1+\xi)+\xi\left[\frac{U_{11}c_1+U_{21}c_2}{U_1}-\frac{V_{12}}{V_1}(1-h)\right]=\frac{\gamma}{U_1}\left[1-\phi\left(\frac{U_{31}U_2-U_3U_{21}}{U_2^2}\right)l^I\right], \quad \textbf{(A.14)}$$

$$(1+\xi)+\xi\left[\frac{U_{12}c_1+U_{22}c_2}{U_2}-\frac{V_{12}}{V_1}(1-h)\right]=\frac{\gamma}{U_2}\left[1-\phi\left(\frac{U_{32}U_2-U_3U_{22}}{U_2^2}\right)l^I\right]. \quad \textbf{(A.15)}$$

Using (14), we have that the left-hand sides of (A.14) and (A.15) have the same value. This implies that

$$\frac{\gamma}{U_1}\left[1-\phi\left(\frac{U_{31}U_2-U_3U_{21}}{U_2^2}\right)l^I\right]=\frac{\gamma}{U_2}\left[1-\phi\left(\frac{U_{32}U_2-U_3U_{22}}{U_2^2}\right)l^I\right]. \quad \textbf{(A.16)}$$

After this equation is rearranged, it follows that

$$\frac{U_1}{U_2}=\frac{1-\phi\left(\frac{U_{31}U_2-U_3U_{21}}{U_2^2}\right)l^I}{1-\phi\left(\frac{U_{32}U_2-U_3U_{22}}{U_2^2}\right)l^I}. \quad \textbf{(A.17)}$$

Equation (A.17) is an expression for the optimal steady state interest rate R^*. The right-hand side of this equation can be less than, equal to, or greater than one. When it is greater than 1, the Friedman rule is not optimal (see the consumer's first-order condition $U_1/U_2 = R$ in steady state). When it is less than or equal to 1, constraint (12) will bind and the Friedman rule is optimal. In the presence of a costly tax system and imperfect tax enforcement, the cost to collect revenue from the informal sector (s^I), more precisely ϕ, plays an important role in the determination of the optimal monetary policy. ■