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REVIEW

The Association for Symbolic Logic publishes analytical reviews of selected books and articles in the field of symbolic logic. The reviews were published in ***The Journal of Symbolic Logic*** from the founding of the JOURNAL in 1936 until the end of 1999. The Association moved the reviews to this BULLETIN, beginning in 2000.

The Reviews Section is edited by Ernest Schimmerling (Managing Editor), Thomas Colcombet, Mark Colyvan, Samuel Coskey, Bradd Hart, Steffen Lempp, Bernard Linsky, Rahim Moosa, Henry Towsner, Benno van den Berg, and Kai Wehmeier. Authors and publishers are requested to send, for review, copies of books to *ASL, Box 742, Vassar College, 124 Raymond Avenue, Poughkeepsie, NY 12604, USA*.

M. GROHE, ***Descriptive Complexity, Canonisation, and Definable Graph Structure Theory***, Cambridge University Press, Cambridge, 2017, x + 544 pp.

Descriptive complexity characterises complexity classes by the type of logic needed to express the problems in the class. An important goal in descriptive complexity is to find a logic expressing exactly the properties computable in polynomial time. The main logical framework of this book is an extension of first-order logic using inflationary fixpoint formulas and counting features, denoted IFP+C. All the properties definable in IFP+C are computable in polynomial time, but the converse is not true as Cai, Fürer, and Immerman showed in 1992.

A class of graphs is said to have canonisation if each graph of the class has a canonical representation. An isomorphism test between two graphs of the class is then just an equality test between their canonical representation. Hence polynomial time computable canonisations induce polynomial time tests for the isomorphism problem. Furthermore, a canonisation definable in IFP+C implies an isomorphism test using the Weisfeiler–Lehman (WL) algorithm with a suitable dimension. IFP+C definable canonisations have another important consequence as it implies that IFP+C can define all polynomial time computable properties over this class.

Graph structure theory is the branch of graph theory decomposing a graph into simpler components using a tree structure. An usual application is the design of fast algorithms taking advantage of the tree structure. Another application is to compute canonisations by induction on the tree structure of the decomposition. The decompositions used in this book are definable in IFP+C. This is necessary for deriving definable canonisations. This requires the decompositions to be invariant under isomorphisms. Hence this requires to tune-up the usual definitions and this is the core of the book.

Part I of the book describes the material mentioned above. It introduces the logic IFP+C, definable canonisation, WL isomorphism test and the links between IFP+C definable canonisation, the WL algorithm and PTime. It also introduces definable structure theory and its consequences in terms of canonisation. Part I culminates with the fact that trees, graphs of bounded tree-width and planar graphs have IFP+C definable canonisation. This part is self content and very nicely presented. I think it is the first time where all these materials are gathered in a single textbook. It could be the material for an advanced course in logic, algorithm, and complexity theory.

Part II is more involved. It revisits known decomposition algorithms and shows how each of them can be turned into IFP+C definable decompositions. Part II contains the main result of the book: if a class of graphs excludes a nontrivial minor then it has an IFP+C definable canonisation. In particular it implies that IFP+C captures PTime over this class and that the isomorphism test can be done using the WL algorithm with some suitable dimension. This is

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an outstanding result and this is the only place where its full proof is given. The isomorphism test for these classes can now be done using a simple combinatorial algorithm avoiding the previous complex group theoretic techniques. This part of the book is clearly more involved and hard to digest for someone not familiar with structural graph theory.

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