J. Fluid Mech. (2017), *vol.* 817, *pp.* 171–182. © Cambridge University Press 2017
doi:10.1017/jfm.2017.130

The discharge plume parameter Γ_d and its implications for an emptying–filling box

O. Vauquelin[1,†], E. M. Koutaiba[1,2], E. Blanchard[2] and P. Fromy[2]

[1]Aix-Marseille Université, Laboratoire IUSTI, UMR CNRS 7343, 5 rue Enrico Fermi, 13 453 Marseille CEDEX 13, France

[2]Centre Scientifique et Technique du Bâtiment, 84 avenue Jean Jaurès, 77 447 Marne-la-Vallée, France

(Received 20 April 2016; revised 22 February 2017; accepted 22 February 2017; first published online 16 March 2017)

The natural ventilation flow driven by an internal buoyant plume in a box involving an upper opening (vent) located at the ceiling (for the outflow) and a large lower opening at the floor (for the inflow) is examined theoretically in a general non-Boussinesq case. Analytical solutions of this emptying–filling box problem allow the characteristics of the flow at the vent to be determined. From these characteristics, a non-dimensional parameter Γ_d (called the discharge plume parameter) is expressed. This parameter characterizes the initial balance of volume, buoyancy and momentum fluxes in the plume-like flow that forms above the vent. We then note that the value of Γ_d allows the buoyant fluid layer depth in the box to be estimated, which is a new and interesting result for natural ventilation problems. Following previous experimental results, the decrease of the vent discharge coefficient C_d when Γ_d increases is discussed and a theoretical model based on plume necking is proposed. The emptying–filling box model is then extended for a variable C_d (depending on Γ_d). Even though the discharge coefficient may be markedly reduced at high values of Γ_d, our results show that this only affects transients and the steady state of an emptying–filling box for relatively thin buoyant fluid layers.

Key words: convection, mixing, plumes/thermals

1. Introduction

The fluid mechanics of displacement ventilation in a room with vents at floor and ceiling levels was examined theoretically by Linden, Lane-Serff & Smeed (1990). Their model (known as the 'emptying–filling box' model) considered a turbulent plume driven by a point source of buoyancy and a stable stratification with an upper layer of buoyant fluid underneath the ceiling. This buoyant layer is fed by the entraining plume and drives a flow through the upper vent due to the hydrostatic pressure difference. In both transients and at steady state, the emptying–filling box model allows the thickness and the mean temperature (or density) of the buoyant layer to be estimated, as well as the flow rate of buoyant fluid through the upper vent. In this model, the filling mechanism is similar to that which had been considered by

† Email address for correspondence: olivier.vauquelin@univ-amu.fr

https://doi.org/10.1017/jfm.2017.130 Published online by Cambridge University Press

Baines & Turner (1969) in their 'filling box' model, and the draining flow (emptying process) is estimated using Bernoulli's theorem.

In the Boussinesq limit, one of the major results obtained by Linden *et al.* (1990) is that, at steady state, the buoyant layer thickness is a function of a non-dimensional geometrical parameter only, which is defined from the vent areas, the height of the room and the discharge coefficient C_d. In most engineering applications, this discharge coefficient is taken as a constant approximately 0.6–0.7. However, in two conference papers, Hunt & Holford (2000) and Holford & Hunt (2001) quantified experimentally the dependence of C_d on density contrast for a horizontal vent involved in a process of emptying or simultaneous filling and emptying of a box. Their experimental results (discussed further in § 4.1) showed that C_d can rapidly decrease with increasing density contrast between the buoyant and ambient fluids. Such a finding suggests that assuming a constant value of C_d could lead to an overestimate of flow rates in natural ventilation problems for which density (or temperature) differences may be significant.

Contrary to what the titles of their papers may suggest, Holford and Hunt's corrections for C_d were not based explicitly on the density contrast but on the value of a dimensionless discharge (plume) parameter Γ_d which quantifies the ratio between buoyancy and momentum in the plume-like flow that develops outside the box from the vent. We can therefore imagine that the behaviour of the external plume (above the vent) is not simply an indicator since it can control the state within the box by modifying the vent discharge coefficient value. The main objective of this paper is to address this issue.

The discharge plume parameter Γ_d depends not only on the density contrast, but also on the outflow velocity and the size of the vent. Thus, for an emptying–filling box problem, the value of Γ_d could in theory be estimated from the output data of the model proposed by Linden *et al.* (1990). Since this model was originally developed using the Boussinesq approximation, we first reformulate in § 2 the general non-Boussinesq case. The expression of the discharge plume parameter Γ_d is then obtained explicitly in § 3. In § 4, the dependence of the discharge coefficient C_d on Γ_d is discussed on the basis of Holford and Hunt's experiments and a simple theoretical model is proposed. Implications of a modified C_d (via Γ_d) for both steady state and transients of an emptying–filling box are then examined and discussed in § 5.

2. The non-Boussinesq 'emptying–filling box' model

2.1. *Configuration and hypothesis*

As shown in figure 1, we consider the simultaneous filling and emptying of a cuboid box of height H and (constant) horizontal surface area S. The box has an upper opening of area A located at the ceiling and a large lower opening at the floor. Note that this lower opening is large enough that the pressure at floor level is the same as that outside the box. For this reason, only the upper opening is referred to as a vent in what follows. For simplicity, we assume that this vent is a circular opening of diameter D. A (non-Boussinesq) point source of buoyancy flux B is set at the floor ($z = 0$). An internal turbulent plume then rises, impinges with the ceiling and flows along it to form a thin layer in the upper part of the box at the initial time $t = 0$. Hydrostatic pressure difference then allows the light (or hot) fluid to escape through the vent. The interface between the buoyant fluid layer and ambient air descends over time until steady state is reached. Both during the transients and at steady state, a situation of ideal displacement flow is assumed which supposes: a negligible

https://doi.org/10.1017/jfm.2017.130 Published online by Cambridge University Press

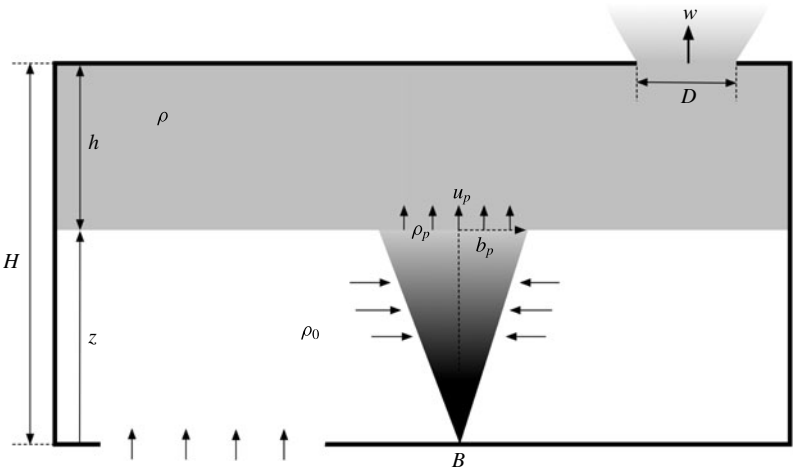

FIGURE 1. Schematic of the studied configuration.

'overturning' of the initial buoyant layer after colliding with the lateral walls (Kaye & Hunt 2007) and a lower opening (for the incoming ambient air) large enough to overcome bidirectional flows at the vent or an inlet jet strongly mixing the interface (Coffey & Hunt 2010; Hunt & Coffey 2010). For the sake of simplicity, heat transfers (Faure & Le Roux 2012; Lane-Serff & Sandbach 2012) and external wind effects (Hunt & Linden 2001; Li & Delsante 2001; Coomaraswamy & Caulfield 2011) are ignored.

2.2. *The internal plume model*

Following the turbulent plume theory proposed by Morton, Taylor & Turner (1956), we model the internal buoyant release using the self-similar solutions of a non-Boussinesq axisymmetric plume in which the classical Morton's entrainment coefficient is multiplied by the square root of the plume local density to account for the non-Boussinesq effects (Ricou & Spalding 1961; Rooney & Linden 1996; Woods 1997). Note that these solutions are similar to those used for fire plumes (Heskestad 1984). Using a 'top-hat' formalism, the plume characteristics (mean vertical velocity u_p, radius b_p and mean density deficit η_p) are then expressed as functions of the vertical coordinate z, thus

$$u_p(z) = a^{1/3} B^{1/3} z^{-1/3}, \tag{2.1}$$

$$b_p(z) = \frac{6\alpha}{5} z \left[\frac{\rho_0}{\rho_p(z)}\right]^{1/2} = \frac{6\alpha}{5} z \sqrt{1+\eta_p(z)}, \tag{2.2}$$

$$\eta_p(z) = \frac{\rho_0 - \rho_p(z)}{\rho_p(z)} = \frac{4}{3g} a^{2/3} B^{2/3} z^{-5/3}, \tag{2.3}$$

where ρ_p is the plume mean density (at location z), ρ_0 is the ambient density, g is the gravitational acceleration, α is the entrainment coefficient (generally taken to be approximately 0.1) and a is a constant equal to $25/48\pi\alpha^2$. The buoyancy flux B is an invariant which can be expressed from the plume characteristics as:

$$B = \frac{\eta_p g \pi u_p b_p^2}{1+\eta_p}. \tag{2.4}$$

https://doi.org/10.1017/jfm.2017.130 Published online by Cambridge University Press

For a buoyant fluid released from a finite area source, these solutions can be vertically shifted using a virtual origin correction at location z_v, z may be replaced by $z - z_v$ in relations (2.1)–(2.3), and z may therefore be considered as the height above this virtual origin. The determination of z_v has been extensively studied in the past. Convenient analytical solutions may be found in Candelier & Vauquelin (2012) for a wide range of non-Boussinesq round plumes.

2.3. *Emptying–filling box conservation equations*

Conservation equations are written for the buoyant layer of thickness h and density ρ shown in figure 1. This layer is fed by the internal plume (at height $z = H - h$) and the buoyant fluid escapes at the mean velocity w through the vent of area A located on the ceiling of the box. These equations are, for the mass flux and the buoyancy flux, respectively:

$$\frac{\mathrm{d}\rho Sh}{\mathrm{d}t} = \rho_p \pi b_p^2 u_p\big|_{z=H-h} - \rho w A, \tag{2.5}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\left[g\frac{\Delta\rho}{\rho_0}Sh\right] = B - g\frac{\Delta\rho}{\rho_0}wA, \tag{2.6}$$

with $\Delta\rho = \rho_0 - \rho$. Note that by combining these two relations, we naturally obtain the volume flux conservation equation for the buoyant layer.

The outflow velocity w can be estimated by applying Bernoulli's theorem between a point located at the interface and a point located at the vent. Denoting C_d the vent discharge coefficient, this may be expressed as

$$\frac{1}{2}\rho\left(\frac{w}{C_d}\right)^2 = \Delta\rho g h. \tag{2.7}$$

We first replace b_p, u_p and η_p from (2.1), (2.2) and (2.3) as functions of z in equation (2.5) and we then introduce the dimensionless variables $\zeta = z/H$, $\eta = \Delta\rho/\rho$ and $\omega = w/\sqrt{gH}$. Following some algebraic manipulations, equations (2.5), (2.6) and (2.7) can then be rewritten as

$$\frac{\mathrm{d}\zeta}{\mathrm{d}\tau} = \Lambda\omega - \kappa\Theta^{1/2}(\zeta^{5/3} + \Theta), \tag{2.8}$$

$$\frac{\mathrm{d}\eta}{\mathrm{d}\tau} = \frac{1+\eta}{1-\zeta}\kappa\Theta^{1/2}(\Theta - \eta\zeta^{5/3}), \tag{2.9}$$

$$\omega = C_d\sqrt{2\eta(1-\zeta)}, \tag{2.10}$$

with $\Lambda = A/H^2$ a geometrical parameter, $\tau = g^{1/2}H^{3/2}t/S$ the dimensionless time, κ a constant equal to $18\sqrt{3}\alpha^2\pi/25$ and $\Theta = 4a^{2/3}B^{2/3}/3gH^{5/3}$. Note that the dimensionless parameter Θ corresponds to the value of η_p for $z = H$ (or $\zeta = 1$), namely the value of the density deficit in the initial layer that is formed at the ceiling at $t = 0$. This parameter also characterizes the strength of the buoyant source relative to the box height.

At steady state, the time derivative terms vanish. Combining (2.8), (2.9) and (2.10) yields:

$$\frac{\zeta_{ss}^{5/3}(\zeta_{ss}^{5/3} + \Theta)^2}{1 - \zeta_{ss}} = 2\frac{\Lambda^2 C_d^2}{\kappa^2}, \tag{2.11}$$

https://doi.org/10.1017/jfm.2017.130 Published online by Cambridge University Press

with ζ_{ss} the dimensionless interface height at steady state. Note that under the Boussinesq approximation ($\Theta \approx 0$) it may be found that ζ_{ss} only depends on the geometrical parameter Λ, as highlighted by Linden *et al.* (1990). Subsequently, the density difference η_{ss} and the dimensionless outflow velocity ω_{ss} at steady state can be expressed as a function of ζ_{ss}:

$$\eta_{ss} = \frac{\Theta}{\zeta_{ss}^{5/3}} \quad \text{and} \quad \omega_{ss} = \frac{\kappa \Theta^{1/2}}{\Lambda}(\zeta_{ss}^{5/3} + \Theta). \tag{2.12a,b}$$

3. The discharge plume parameter Γ_d

A vertical turbulent plume which results from the upward discharge of a light fluid in a heavier ambient fluid of density ρ_0 may be characterized by a non-dimensional function $\Gamma(z)$ called the 'plume function'. The vertical coordinate, z, is measured positively upwards with the source of the plume at $z=0$. By recasting the conservation equations of Morton *et al.* (1956), this function was introduced by Hunt & Kaye (2005) from a combination of the fluxes (volume, buoyancy and momentum) and later extended in the general non-Boussinesq case by Michaux & Vauquelin (2008) from the local plume variables (density ρ_p, radius b_p and velocity u_p). The plume function quantifies locally the ratio between buoyancy and momentum and can be seen as a normalized form of a Richardson number. Using 'top-hat' profiles, it is expressed as

$$\Gamma(z) = \frac{5}{8\alpha}\frac{\eta_p g b_p}{u_p^2\sqrt{1+\eta_p}}. \tag{3.1}$$

For a pure (or ideal) plume, $\Gamma(0)=1$ by definition, and the plume function turns out to be invariant with z. For a non-pure plume, $\Gamma(z)$ monotonically evolves from its value at the source $\Gamma(0)$ to unity at large heights. The value of the plume function at $z=0$, which is called the 'plume parameter', was identified and discussed a long time ago by Morton (1959). In Morton & Middleton (1973) a classification was proposed to distinguish the forced plumes ($\Gamma(0) < 1$) initially dominated by momentum from the lazy plumes ($\Gamma(0) > 1$) initially dominated by buoyancy.

In an emptying–filling box problem, the flow at the upper vent is characterized by its vertical velocity w, its density deficit η and its equivalent radius which can be expressed as $\sqrt{A/\pi}$ with A the vent area. From relation (3.1), the plume parameter Γ_d of the plume-like flow that arises from the vent then reads

$$\Gamma_d = \frac{5}{8\alpha}\frac{\eta g\sqrt{A/\pi}}{w^2\sqrt{1+\eta}} = \frac{5}{16\alpha}\frac{1}{C_d^2}\frac{\sqrt{\Lambda/\pi}}{(1-\zeta)\sqrt{1+\eta}}, \tag{3.2}$$

and is now called the 'discharge plume parameter'. Note that this plume-like flow arising from the vent will be simply referred as 'the plume' in the following. It corresponds to the external plume of the problem and should not be confused with the internal plume within the box.

Since ζ decreases (from 1 to smaller values) during transients of an emptying–filling box, relation (3.2) indicates that Γ_d must be infinite at the beginning. Since η increases (from Θ to higher values), Γ_d always decreases over time ($\partial\Gamma_d/\partial\zeta > 0$ and $\partial\Gamma_d/\partial\eta < 0$) except if the variables ζ and η overshoot their final location (see Kaye & Hunt 2004 and Vauquelin 2015). In such a case the time derivative of Γ_d,

$$\frac{\mathrm{d}\Gamma_d}{\mathrm{d}\tau} = \Gamma_d\left[\frac{1}{1-\zeta}\frac{\mathrm{d}\zeta}{\mathrm{d}\tau} - \frac{1}{2}\frac{1}{1+\eta}\frac{\mathrm{d}\eta}{\mathrm{d}\tau}\right], \tag{3.3}$$

https://doi.org/10.1017/jfm.2017.130 Published online by Cambridge University Press

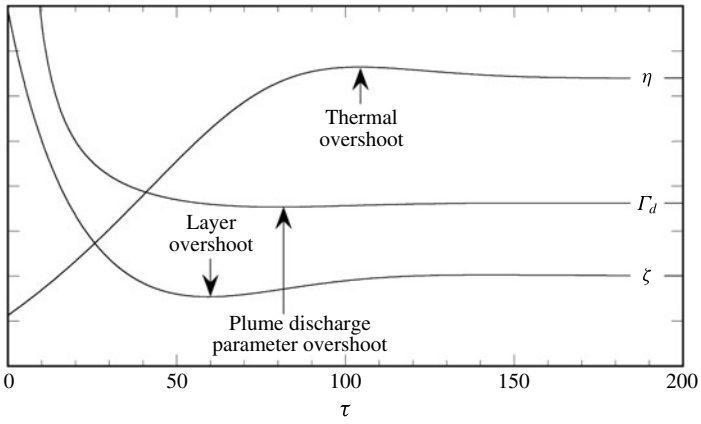

FIGURE 2. Evolution and overshoot occurrence of ζ, η and Γ_d as a function of the dimensionless time τ. Calculations were made for $\Lambda = 0.02$ and $\Theta = 0.5$.

indicates that Γ_d can overshoot and reach a minimal value if $\mathrm{d}\zeta/\mathrm{d}\tau$ and $\mathrm{d}\eta/\mathrm{d}\tau$ are both positive, that is to say between the layer overshoot ($\mathrm{d}\zeta/\mathrm{d}\tau = 0$) and the so-called thermal overshoot ($\mathrm{d}\eta/\mathrm{d}\tau = 0$). This behaviour is illustrated in figure 2 from a numerical integration of conservation equations (2.8), (2.9) and (2.10) for a particular case where $\Lambda = 0.02$ and $\Theta = 0.5$.

At steady state, by using relations (2.11) and (2.12) in (3.2), we find that:

$$\Gamma_d = \left(\frac{3}{8}\right)^{5/4} C_d^{-5/2} \left(\frac{\zeta_{ss}}{1-\zeta_{ss}}\right)^{5/4} \quad \Longrightarrow \quad \zeta_{ss} = \frac{1}{1 + \dfrac{3}{8C_d^2 \Gamma_d^{4/5}}}. \tag{3.4}$$

This result is of considerable practical value since it demonstrates that the thickness of the buoyant layer (or the smoke free height for fire safety issues) is a function of the plume discharge parameter alone and is independent of the source buoyancy flux and of the geometry. Hence, it is possible to estimate the interface location at steady state from the 'laziness' of the plume at the vent alone. In particular, if we consider the classical value $C_d \approx 0.6$ we find that the plume at the vent will be lazy ($\Gamma_d > 1$) as long as $z_{ss}/H = \zeta_{ss} \gtrsim 0.49$, and then forced ($\Gamma_d < 1$) at lower locations of the interface. Furthermore, for lazy plumes, a contraction (also called a neck) can occur as soon as the plume parameter exceeds 1.5 or 2.5, depending on the density contrast (Fanneløp & Webber 2003; Michaux & Vauquelin 2008), that is to say in the range $0.53 \lesssim \zeta_{ss} \lesssim 0.67$ according to relation (3.4). These results are summarized in figure 3 in which a classification of the plume rising through the vent is given as a function of the interface location at steady state. We should note, of course, that the numerical values indicated in figure 3 are related to the assumptions used: 'top-hat' profiles and constant entrainment coefficient α.

4. How C_d depends on Γ_d

4.1. *Holford and Hunt's experimental results*

Holford and Hunt's experiments investigated a displacement mode of ventilation. A layer of salt solution drained out through an opening in the base of a container

https://doi.org/10.1017/jfm.2017.130 Published online by Cambridge University Press

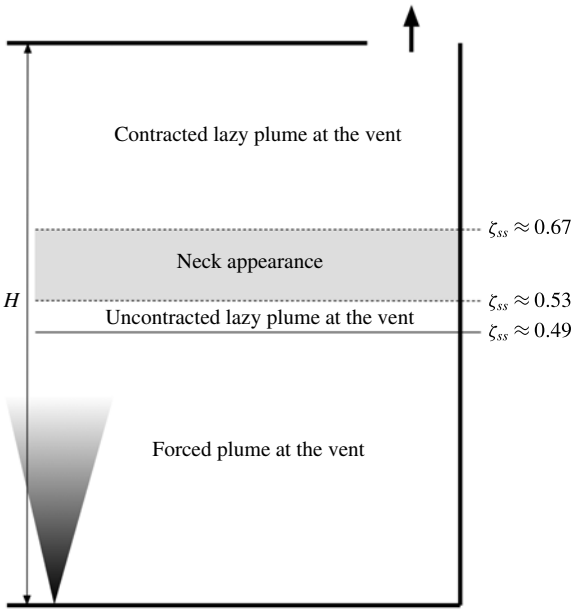

FIGURE 3. Classification of the plume at the vent as a function of the interface location at steady state (for $C_d = 0.6$).

immersed in a larger water-filled tank (note that there is a switch in directions between salt and heat). Their first paper (Hunt & Holford 2000) reported only a few experiments, carried out for a restricted range of Γ_d lying between 0 and 40 in steady state experiments that involved simultaneous filling and emptying of the immersed container. A quadratic fit:

$$C_d = 0.71 - 1.910^{-4}\Gamma_d^2 \quad \text{for } 0 \leqslant \Gamma_d \leqslant 40, \tag{4.1}$$

was proposed. The authors justified this fit with the observation that it is consistent with $\mathrm{d}C_d/\mathrm{d}\Gamma_d = 0$ at $\Gamma_d = 0$. Although the data were somewhat dispersed, these early experiments clearly indicate a rapid decrease of the discharge coefficient C_d when the plume discharge parameter Γ_d rises above 10. Their second paper (Holford & Hunt 2001) reported additional data obtained from transient draining experiments. The discharge coefficient C_d was deduced from a comparison between the experimental volume flow rate and that given by an ideal model. A best fit was derived in two regions, for a wider range of Γ_d:

$$C_d = \begin{cases} 0.63 & \text{if } 0 < \Gamma_d < 4.9, \\ 1.11\Gamma_d^{-0.356} & \text{if } 4.9 < \Gamma_d < 500. \end{cases} \tag{4.2}$$

It can be seen that a correction on the discharge coefficient is then required for a discharge plume parameter that exceeds the critical value $\Gamma_d^{crit} = 4.9$, and thus for a lazy plume exhibiting a neck (i.e. a contraction relatively close to the source). In their analyses, the authors considered that this buoyancy-induced contraction reduces the fraction of the opening area occupied by the discharge, giving rise to reduced values of C_d.

https://doi.org/10.1017/jfm.2017.130 Published online by Cambridge University Press

4.2. *A proposal for a simple model*

Following the idea of an opening area reduction due to a plume contraction introduced by Holford & Hunt (2001), we can consider that if an unmodified discharge coefficient $C_d^\star$ (~ 0.6) can be applied to the reduced area A_{neck}, then the discharge coefficient has to be modified considering the effective area A of the vent. This would mean that $C_d^\star A_{neck} = C_d A$, i.e. $C_d/C_d^\star = (b_{neck}/b)^2$ where b_{neck} is the plume radius at the neck and b is the radius (or the equivalent radius) of the vent.

For non-Boussinesq lazy plumes, Michaux & Vauquelin (2008) showed that the plume radius $b_p(z)$ and the plume density deficit $\eta_p(z)$ can be expressed from the plume function $\Gamma(z)$ and the known initial values at $z=0$ (indicated with the subscript i) as follows:

$$\frac{b_p(z)}{b_i} = \left[\frac{1+\eta(z)}{1+\eta_i}\right]^{1/2} \left[\frac{\Gamma(z)}{\Gamma_i}\right]^{1/2} \left[\frac{\Gamma_i - 1}{\Gamma(z) - 1}\right]^{3/10}, \tag{4.3}$$

$$\frac{\eta_p(z)}{\eta_i} = \left[\frac{\Gamma_i}{\Gamma(z)}\right]^{1/2} \left[\frac{\Gamma(z) - 1}{\Gamma_i - 1}\right]^{1/2}. \tag{4.4}$$

Thus, for a contracting lazy plume, by considering z at the neck location and the (initial) values of the plume variables at the vent (radius b, density deficit η and plume parameter Γ_d), relations (4.3) and (4.4) lead to

$$\left(\frac{b_{neck}}{b}\right)^2 = \frac{1+\eta\left(\dfrac{\Gamma_d}{\Gamma_{neck}}\right)^{1/2}\left(\dfrac{\Gamma_{neck}-1}{\Gamma_d - 1}\right)^{1/2}}{1+\eta} \frac{\Gamma_{neck}}{\Gamma_d}\left(\frac{\Gamma_d - 1}{\Gamma_{neck} - 1}\right)^{3/5}. \tag{4.5}$$

For non-Boussinesq plumes, the value of Γ_{neck} is the real root of a third-order polynomial form whose coefficients depend on the value of η (see Michaux & Vauquelin 2008). This introduces an additional parameter, so we will simply consider the two asymptotic cases corresponding to low and to high density contrasts in the following.

For low density contrasts ($\eta \ll 1$, i.e. in the Boussinesq approximation) the value of the plume function at the neck location is exactly 5/2 (Fanneløp & Webber 2003) and the previous relation then becomes:

$$\frac{C_d}{C_d^\star} = \frac{5}{2^{2/5}3^{3/5}} \frac{(\Gamma_d - 1)^{3/5}}{\Gamma_d} \simeq 1.96 \frac{(\Gamma_d - 1)^{3/5}}{\Gamma_d} \quad \text{for } \Gamma_d > \Gamma_d^{crit} = 5/2. \tag{4.6}$$

For large density contrasts ($\eta \to \infty$) the value of the plume function decreases towards 5/4 (Michaux & Vauquelin 2008) yielding:

$$\frac{C_d}{C_d^\star} = \frac{\sqrt{5}}{2^{4/5}} \frac{(\Gamma_d - 1)^{1/10}}{\Gamma_d^{1/2}} \simeq 1.28 \frac{(\Gamma_d - 1)^{1/10}}{\Gamma_d^{1/2}} \quad \text{for } \Gamma_d > \Gamma_d^{crit} = 5/4. \tag{4.7}$$

In both cases, when $\Gamma_d \gg 1$, we find that $C_d \propto \Gamma_d^{-2/5}$ is qualitatively consistent with the correlation (4.2) derived by Holford & Hunt (2001) from their draining experiments.

The fits (4.1) and (4.2) proposed in Hunt & Holford (2000) and Holford & Hunt (2001) are plotted with the theoretical wrap-around relations (4.6) and (4.7) illustrated in figure 4. The first fit given by the relation (4.1) does not seem to fully reflect physical reality probably because it was derived from a very limited number of data

https://doi.org/10.1017/jfm.2017.130 Published online by Cambridge University Press

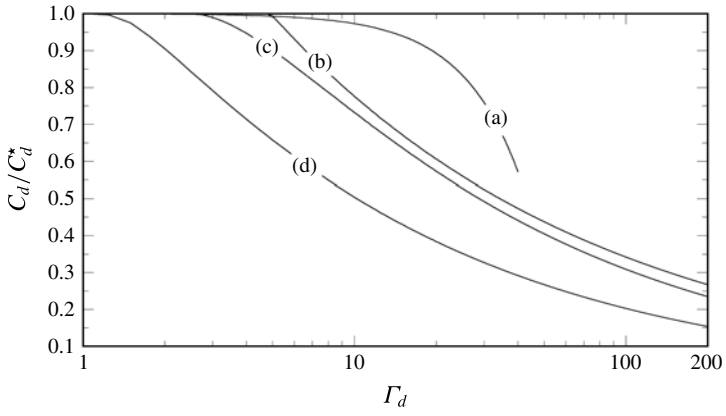

FIGURE 4. Decrease of the discharge coefficient (divided by the constant unmodified discharge coefficient $C_d^\star$) as a function of the discharge plume parameter Γ_d. Here, (a) is the best fit from Hunt & Holford (2000), (b) is the best fit from Holford & Hunt (2001), (c) is the relation (4.6) for low density contrasts and (d) is the relation (4.7) for high density contrasts.

points. The second fit given by relation (4.2) and the theoretical relation (4.6) for low density contrasts are more likely to agree. However, whatever the hypothesis with respect to the density contrast, the main difference between the theoretical relations in (4.6) and (4.7) and Holford and Hunt's second correlation is due to our choice of Γ_d^{crit} (theoretically 5/2 for low density contrasts and 5/4 for large density contrast instead of the experimental value 4.9). This difference is not really surprising since our theoretical model based on the necking effect only is a somewhat simplified view of the physics of such a discharge flow. Furthermore, the present plume theory considers 'top-hat' profiles and a constant entrainment coefficient, which is probably, in the near field of a lazy plume, a source of additional uncertainties, as recently discussed by van Reeuwijk & Craske (2015) and Carlotti & Hunt (2017).

Nevertheless, in spite of the simplicity of our approach, by rescaling the theoretical relations (4.6) and (4.7) using the suitable (experimental) critical value $\Gamma_d^{crit} = 4.9$, the coefficients in these relations become 2.16 and 1.89 (instead of 1.96 and 1.28), respectively, in good agreement with (4.2) as shown in figure 5.

5. The modified emptying–filling box model

In this section, the emptying–filling box equations are solved for a variable discharge coefficient $C_d = f(\Gamma_d)$. In particular, comparisons are made with results obtained for an unmodified discharge coefficient $C_d^\star = 0.6$. For these comparisons we have chosen to use the theoretical relation (4.6), modified to be in accordance with the critical value $\Gamma_d^{crit} = 4.9$ found experimentally by Holford & Hunt (2001):

$$C_d = \begin{cases} 0.6 & \text{if } \Gamma_d < 4.9, \\ 1.3\dfrac{(\Gamma_d - 1)^{3/5}}{\Gamma_d} & \text{if } \Gamma_d > 4.9. \end{cases} \tag{5.1}$$

Note that according to relation (3.4), the critical value $\Gamma_d^{crit} = 4.9$ corresponds to a dimensionless layer height $\zeta_{ss}^{crit} \approx 0.77$ which indicates that consideration of a variable

https://doi.org/10.1017/jfm.2017.130 Published online by Cambridge University Press

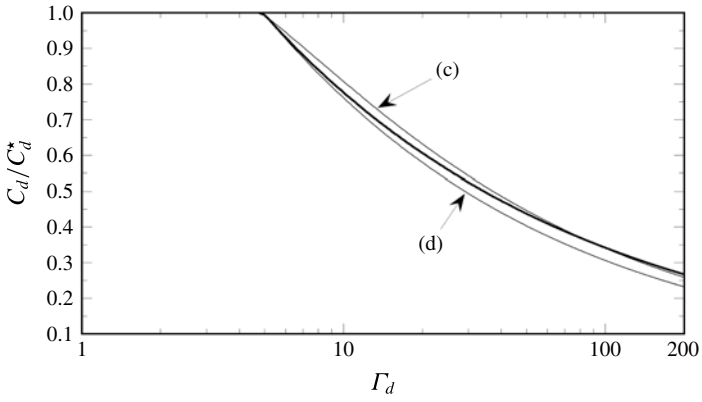

FIGURE 5. Decrease of the discharge coefficient (divided by the constant unmodified discharge coefficient $C_d^\star$) as a function of the discharge plume parameter Γ_d. Here, the bold line is the best fit from Holford & Hunt (2001), (c) is the relation (4.6) and (d) is the relation (4.7) both corrected to agree with the experimental critical value $\Gamma_d^{crit} = 4.9$.

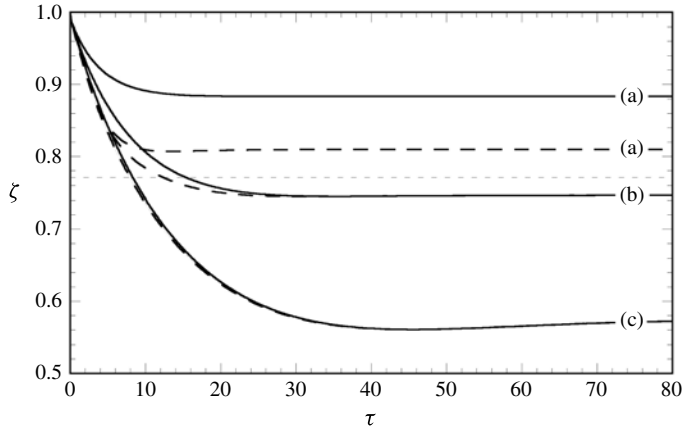

FIGURE 6. Time evolution of the dimensionless interface height ζ for $\Lambda = 0.1$ and three different values of Θ: 0.5 (a); 1 (b); 1.5 (c). The solid line represents a constant discharge coefficient $C_d = 0.6$, and the dashed line the modified discharge coefficient given in relation (5.1).

C_d will have an impact on steady state predictions only for relatively thin upper layers (lower than $\backsim 23\,\%$ of the box height).

Figure 6 presents the results of a numerical integration of conservation equations (2.8), (2.9) and (2.10) with and without correction of the discharge coefficient, for a geometrical parameter $\Lambda = 0.1$ and three values of the dimensionless parameter Θ (0.5, 1 and 1.5). Curves (a) for $\Theta = 0.5$ reveal faster filling, a slight overshoot and a deeper layer of buoyant fluid at steady state when considering the modified discharge coefficient. By doubling the value of Θ (that is, by multiplying the buoyancy flux by $2^{3/2}$), it can be seen from curves (b) that ζ drops below the critical value ($\zeta^{crit} = 0.77$) during transients and the steady state is then unaffected by the correction of the discharge coefficient. Filling is still faster with the modified model. Curves (c) for

https://doi.org/10.1017/jfm.2017.130 Published online by Cambridge University Press

$\Theta = 1.5$ no longer allow us to distinguish between the two models since ζ rapidly drops below ζ^{crit} during transients.

6. Conclusion

In the simultaneous filling (by an internal plume issuing from a point source of buoyancy) and emptying of a box, the characteristics of the plume-like flow that develops outside the box from an upper vent allows a 'plume parameter' to be constructed: the 'discharge plume parameter' which we denote Γ_d. This non-dimensional parameter quantifies the ratio between buoyancy and momentum in the flow at the vent and it can be expressed analytically from the output data of the emptying–filling box model assuming a displacement ventilation mode. In the transient phase of an emptying–filling box, the discharge plume parameter decreases from an infinite value (at $t = 0$) to a finite value at steady state which allows the height of the interface (between air and buoyant fluid) to be estimated whatever the source buoyancy flux. In particular, the plume above the vent is buoyancy dominated at the beginning and can become momentum dominated as soon as the box is (approximately) half-filled. Previous experiments have shown that the discharge coefficient C_d of a draining process may fall dramatically below its classical value ($C_d \sim 0.6$–0.7) for large values of the discharge plume parameter. By including the relation that links C_d to Γ_d in the emptying–filling box model, we show that transients and steady state are broadly unchanged provided the buoyant layer depth is greater than approximately one-quarter of the box height. For thinner buoyant layers however, estimations made with an unmodified C_d significantly underestimate the buoyant layer depth in both transients and steady state of an emptying–filling box.

REFERENCES

BAINES, W. D. & TURNER, J. S. 1969 Turbulent buoyant convection from a source in a confined region. *J. Fluid Mech.* **37**, 51–80.

CANDELIER, F. & VAUQUELIN, O. 2012 Matched asymptotic solutions for turbulent plumes. *J. Fluid Mech.* **699**, 489–499.

CARLOTTI, P. & HUNT, G. R. 2017 An entrainment model for lazy turbulent plumes. *J. Fluid Mech.* **811**, 682–700.

COFFEY, C. J. & HUNT, G. R. 2010 The unidirectional emptying box. *J. Fluid Mech.* **660**, 456–474.

COOMARASWAMY, I. A. & CAULFIELD, C. P. 2011 Time-dependent ventilation flows driven by opposing wind and buoyancy. *J. Fluid Mech.* **672**, 33–59.

FANNELØP, T. K. & WEBBER, D. M. 2003 On buoyant plumes rising from area sources in a calm environment. *J. Fluid Mech.* **497**, 319–334.

FAURE, X. & LE ROUX, N. 2012 Time dependent flows in displacement ventilation considering the volume envelope heat transfers. *Build. Environ.* **50**, 221–230.

HESKESTAD, G. 1984 Engineering relations for fire plumes. *Fire Safety J.* **7**, 25–32.

HOLFORD, J. M. & HUNT, G. R. 2001 The dependence of the discharge coefficients on density contrast – experimental measurements. In *Proceedings 14th Australasian Fluid Mechanics Conference* (ed. B. B. Dally), pp. 123–126. University of Adelaide.

HUNT, G. R. & COFFEY, C. J. 2010 Emptying boxes – classifying transient natural ventilation flows. *J. Fluid Mech.* **646**, 137–168.

HUNT, G. R. & HOLFORD, J. M. 2000 The discharge coefficient – experimental measurement of a dependence on density contrast. In *Proceedings of 21st International AIVC Conference*, pp. 12–24. Air Infiltration and Ventilation Centre, Document AIC-PROC-21-2000.

HUNT, G. R. & KAYE, N. B. 2005 Lazy plumes. *J. Fluid Mech.* **533**, 329–338.

https://doi.org/10.1017/jfm.2017.130 Published online by Cambridge University Press

HUNT, G. R. & LINDEN, P. F. 2001 Steady-state flows in an enclosure ventilated by buoyant forces assisted by wind. *J. Fluid Mech.* **426**, 355–386.

KAYE, N. B. & HUNT, G. R. 2004 Time-dependent flows in an emptying filling box. *J. Fluid Mech.* **520**, 135–156.

KAYE, N. B. & HUNT, G. R. 2007 Overturning in a filling box. *J. Fluid Mech.* **576**, 297–323.

LANE-SERFF, G. F. & SANDBACH, S. D. 2012 Emptying non-adiabatic filling box: the effects of heat transfers on the fluid dynamics of natural ventilation. *J. Fluid Mech.* **701**, 386–406.

LI, Y. & DELSANTE, A. 2001 Natural ventilation by combined wind and buoyancy forces. *Build. Environ.* **36**, 59–71.

LINDEN, P. F., LANE-SERFF, G. F. & SMEED, D. A. 1990 Emptying filling boxes, the fluid mechanics of natural ventilation. *J. Fluid Mech.* **212**, 309–335.

MICHAUX, G. & VAUQUELIN, O. 2008 Solutions for turbulent buoyant plumes rising from circular sources. *Phys. Fluids* **20**, 066601.

MORTON, B. R. 1959 Forced plumes. *J. Fluid Mech.* **5**, 151–163.

MORTON, B. R. & MIDDLETON, J. 1973 Scale diagrams for forced plumes. *J. Fluid Mech.* **58**, 165–176.

MORTON, B. R., TAYLOR, G. I. & TURNER, J. S. 1956 Turbulent gravitational convection from maintained and instantaneous sources. *Proc. R. Soc. Lond.* A **234**, 1–23.

VAN REEUWIJK, M. & CRASKE, J. 2015 Energy-consistent entrainment relations for jets and plumes. *J. Fluid Mech.* **782**, 333–355.

RICOU, F. P. & SPALDING, D. B. 1961 Measurements of entrainment by axisymmetrical turbulent jets. *J. Fluid Mech.* **11**, 21–32.

ROONEY, G. G. & LINDEN, P. F. 1996 Similarity considerations for non-Boussinesq plumes in an unstratified environment. *J. Fluid Mech.* **318**, 237–250.

VAUQUELIN, O. 2015 Oscillatory behaviour in an emptying-filling box. *J. Fluid Mech.* **781**, 712–726.

WOODS, A. W. 1997 A note on non-Boussinesq plumes in an incompressible stratified environment. *J. Fluid Mech.* **345**, 347–356.

https://doi.org/10.1017/jfm.2017.130 Published online by Cambridge University Press