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INCOME INEQUALITY AND ECONOMIC GROWTH WITH ALTRUISTIC BEQUESTS AND HUMAN CAPITAL INVESTMENT

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In this paper we explore how income inequality affects growth in a dynastic family model with bequests (physical capital) and investment in human capital for children. For tractability, we abstract from factor markets and focus on household production, which is prevalent in developing countries. We explore a joint distribution of bequests and human capital and track the evolution of income distribution across generations. We show that initial inequality has a positive indirect effect on average output growth by lowering the ratio of physical to human capital, besides its standard negative direct effect. If education is mainly privately (publicly) provided, then income inequality retards (promotes) growth outside the balanced growth path. On the balanced growth path, inequality always hinders growth.

Keywords: Income Inequality, Human Capital, Bequests, Growth

1. INTRODUCTION

In this paper we develop a theory that captures how income inequality affects growth in a dynastic family model with human capital investment in children and with bequests to children for physical capital accumulation. In our model, children's human capital depends on three factors: their innate ability, parental investment in education, and positive spillovers from average parental human capital (through channels such as state financing of education). For tractability, we abstract from factor markets and focus on household production, which is prevalent in developing countries. We explore a joint distribution of bequests and human capital and track the evolution of income distribution across generations.

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Most related studies in this line ignore the role of bequests and use human capital investment with credit constraints as the sole channel for the evolution of wealth across generations [e.g., Becker and Tomes (1986); Bénabou (1996); Eckstein and Zilcha (1994); Glomm and Ravikumar (1992); Tamura (1991); Zhang (1996)]. In these existing models, a mean-preserving spread in a single type of capital that has diminishing returns in production or in education reduces average output and slows average capital accumulation and growth. It is interesting to explore whether this existing result may change if there are two types of capital that are complementary in production and subject to diminishing returns and credit constraints.

So far, very few studies on this theme have included two types of capital, and all of them differ from our model in some key assumptions. For example, although Galor and Moav (2004) and Zhang (2005) included two complementary types of capital, they maintained the assumption of perfect competition in the final good sector with complete markets for labor and capital. This assumption, together with a Cobb–Douglas production function, allows them to focus on aggregate production. However, given a focus on aggregate production, a mean-preserving spread in physical capital (bequests or savings) will not exert any direct effect on production or capital accumulation. In this case, inequality impedes growth only through human capital, with diminishing returns in the education sector, which is essentially similar to the models with just one type of capital. Zilcha (2003) also considers bequests and life-cycle savings for physical capital accumulation and education investment for human capital accumulation. However, his paper has a different focus, looking into how things may differ between two economies with different tastes for utility components derived from bequest giving and education investment.

All of these models ignore labor market frictions, which may have intriguing implications for the relationship between growth and inequality, especially in early development. To capture these implications, as well as to achieve model tractability, we assume away factor markets and focus on production organized by each household. (We will argue later that this assumption can nevertheless capture some realistic features emerging from market frictions.) The importance of household production for poor countries in the absence of markets is well described by Locay (1990). In this framework we show that if the role of parental investment is large (small) enough relative to that of average human capital in educating children, then income inequality hinders (promotes) growth outside the balanced growth path. On the balanced growth path, in the long run, income inequality always retards growth. Our results therefore differ significantly from those in the literature.

The reasons for this difference in results are as follows. When production takes place in each family with diminishing marginal products, the variances of both types of capital that make up income inequality will affect average capital accumulation. Specifically, a mean-preserving spread in physical capital reduces average output, decelerates average physical capital accumulation, and lowers the ratio of physical to human capital. Similarly, a mean-preserving spread in human capital reduces average output, decelerates average human capital accumulation, and raises the ratio of physical to human capital. Because average output growth depends inversely on the ratio of physical to human capital, higher initial income inequality may be conducive to average output growth if it can lower the ratio of physical to human capital by raising the variance of physical capital. There are thus two opposing forces of income inequality acting on average output growth. The net growth effect of income inequality will then depend on the relative strengths of these two opposing forces.

A key factor determining the relative strengths of the direct and indirect effects of income inequality is the degree of the externality of average human capital in education. If the externality is weak in countries with little public spending on education, then parental influence on children through education is strong, and the negative force of income inequality on growth via human capital investment tends to dominate. Conversely, if the externality is strong in countries with heavy public spending on education, then the negative force of income inequality on growth via human capital investment is weak and the indirect force via the ratio of physical to human capital may dominate on the transitional path. In the long run the economy will eventually converge to a unique balanced growth path on which sustainable growth is driven by human capital accumulation, as in growth models with both physical and human capital [e.g., Lucas (1988)].

However, unlike initial inequality, long-run income inequality merely reflects innate ability inequality. In our model, a mean-preserving spread of innate ability is found to raise the variance of human capital (through education) more than that of physical capital, thereby always being harmful to long-run growth driven by human capital accumulation. Given that public schooling is commonly available at both primary and secondary levels in developed countries, our mechanism suggests a possible positive growth effect of income inequality; however, this is rather limited, because without factor markets, our model is not well suited to developed economies. Conversely, in poor countries, where there is typically very limited access to formal schooling, we expect a negative growth effect of inequality. The rest of the paper proceeds as follows. Section 2 lays out the model. Sections 3 and 4 provide analytical and numerical results, respectively. Section 5 concludes.

2. THE MODEL

In this model there is an infinite sequence of overlapping generations of agents who live for two periods. Each old agent has one child, and each generation has unit mass. Young agents learn through education and play no role in decision making. Each old agent has one unit of labor time and devotes it inelastically to working. The old agents allocate resources to their own consumption, their children's education, and bequests to their children.

The preferences of an old agent are assumed to be

$$V_t = \ln c_t + \rho E_t V_{t+1}, \quad 0 < \rho < 1, \tag{1}$$

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where c_t stands for parental consumption, V_{t+1} is the child's welfare, E_t is the parent's expectation at t conditional on the initial state of human capital and received bequests, and ρ is the subjective discount factor. The log specification of utility is necessary for a global characterization of the model, which will be particularly useful in our analysis when we start outside the balanced growth path.[1]

We assume borrowing constraints and the absence of factor markets, so that every family organizes its own production and finances child education subject to available resources at the beginning of its planning horizon.[2] Empirical evidence using cross-country data indicates higher returns to schooling in poor countries with lower rates of school enrollment than in rich countries [see, e.g., Heckman et al. (2006); Psacharopoulos and Patrinos (2004)]. This evidence suggests that credit markets for human capital investment are incomplete in these poor countries.

Under these assumptions, an old agent produces output y_t using a received bequest k_t and embodied human capital h_t according to the technology

$$y_t = Dk_t^{\alpha} h_t^{1-\alpha}, \quad D > 0, \quad 0 < \alpha < 1, \tag{2}$$

where D is total factor productivity and α the share parameter of physical capital. The justification of this strong assumption is twofold. First, the model with a joint distribution of human and physical capital would lose tractability if we instead assumed perfect factor markets, as in the standard models, because it would be difficult to work out the distribution of the sum of capital income from bequests and wage income from human capital. One gain of this assumption is to allow altruistic parents to leave bequests in this dynastic family model. The relevance of taking bequests into account builds on Laitner (1992, 2002) and Laitner and Ohlsson (2001), who have argued that altruistic bequests are important in accounting for inequality in future generations. In contrast, the related studies that adopt a dynastic family model similar to (1) have not taken bequests into account (e.g., Bénabou (1996); Tamura (1991); Zhang (1996, 2005)]. Clearly, assuming away bequests in this type of model can lead to suboptimal solutions, because parental altruism, once strong enough, motivates not only human capital investment in children but also bequests to children.

Second, this assumption can capture some realistic factors that may have intriguing implications for how inequality affects growth. By saying so, we mean complementarity between human and nonhuman capital at an individual household level; that is, raising one production factor also raises the marginal product of the other factor. This sort of complementarity at an individual household level appears relevant in the real world. It is relevant in developing countries in which factor markets are underdeveloped for large rural populations because of a lack of transportation and communication, among other reasons; see Zhang (1999, 2002). Even in developed countries, it is particularly relevant for those who run self-employed small businesses, as well as in other cases to some extent. For example, nonhuman assets can help workers achieve greater labor market mobility for higher wages, given various labor market frictions such as costs of job search and relocation. On the other hand, human capital may help households find higher rates of return on their nonhuman assets, because in the real world, household investment of such assets may involve substantial time inputs and the returns to nonhuman assets depend positively on their knowledge and skills.

These relevant situations are not reflected in the standard models with perfect markets, because every individual faces the same market rate of return on human or nonhuman capital. Therefore, in these standard models, the complementarity between human and nonhuman capital applies only to firms in the production sector, represented equivalently by an aggregate production function, not to individuals in the household sector. Because bequests are converted into physical capital one for one, it is the mean, not the variance, of bequests or physical capital in the population that matters in production in these standard models. The most realistic scenario should lie between complete and missing markets, which is outside the scope of this paper.[3]

An old agent allocates his output to his own consumption, investment in the education of his child q_t, and a bequest k_{t+1} to his child:

$$c_t = y_t - q_t - k_{t+1}. \tag{3}$$

The distribution of k_t, $\eta_t(k_t)$, is lognormal, with $\eta_t(\cdot)$ following a marginal distribution $\ln k_t \sim \mathcal{N}(\mu_{kt}, \Delta^2_{kt})$, and a bivariate lognormal distribution with human capital (to be specified later). The length of one period in this overlapping-generations model is around 30 years, corresponding approximately to the average lifetime of various forms of physical capital, such as equipment and structures, in the real world. It is thus reasonable to assume that physical capital depreciates fully in one period; otherwise the model loses tractability.[4]

The human capital of a child depends on his own innate ability in learning ξ,[5] the parental investment q_t, and the average human capital of the parent generation in the economy H_t:[6]

$$h_{t+1} = \xi q_t^{\epsilon} H_t^{1-\epsilon}, \quad 0 < \epsilon < 1. \tag{4}$$

This form of human capital accumulation is used by Tamura in a series of papers (1991, 1996, 2002, 2006). The inclusion of average human capital here reflects outside-family factors that affect education, such as community characteristics and state financing for schools. Thus, the size of ϵ may mainly depend on the education system in a particular country. For example, in developed countries with free and compulsory public schooling, the size of ϵ can be very small. On the other hand, in developing countries with little state financing of education, the size of ϵ can be very large.

The distributions of ξ and h_t, $\phi(\xi)$ and $\psi_t(h_t)$ respectively, are lognormal, with $\phi(\cdot)$ following a distribution $\ln \xi \sim \mathcal{N}(\mu_\xi, \Delta^2_\xi)$ and with $\psi_t(\cdot)$ following a marginal distribution $\ln h_t \sim \mathcal{N}(\mu_{ht}, \Delta^2_{ht})$. The distribution of innate ability is independent of the distributions of human capital and bequests. The mean of innate ability for each generation, $E(\xi) = \exp(\mu_\xi + \Delta^2_\xi/2)$, is a constant, whereas the mean of human capital and that of bequests, $H_t \equiv E(h_t) = \exp(\mu_{ht} + \Delta^2_{ht}/2)$ and

$K_t \equiv E(k_t) = \exp(\mu_{kt} + \Delta_{kt}^2/2)$, can vary over time. Also, parental investment in the education of children has to be made before realizing their ability in learning. The bivariate normal distribution of human and physical capital is $(\ln h_t, \ln k_t) \sim \mathcal{N}(\mu_{ht}, \mu_{kt}, \Delta_{ht}^2, \Delta_{kt}^2, \mathrm{cov}(\ln h_t, \ln k_t))$ where μ, Δ^2, and $\mathrm{cov}(\cdot)$ refer to the means, the variances, and the covariance, respectively. To the best of our knowledge, this joint lognormal distribution of human and physical capital has not been analyzed in the literature. Finally, human capital embodied in each worker fully depreciates in one period in this overlapping-generations model.

3. EQUILIBRIUM AND RESULTS

Starting with a received bequest k_t and human capital h_t, a parent decides on the optimal intertemporal allocation by solving the concave programming problem

$$V_t(k_t, h_t) = \max_{k_{t+1}, q_t} \left\{\ln\left(Dk_t^{\alpha} h_t^{1-\alpha} - q_t - k_{t+1}\right) + \rho E_t V_{t+1}(k_{t+1}, h_{t+1})\right\}, \quad \mathbf{(5)}$$

subject to the education technology (4) and given the sequence (H_t, K_t), where (3) is used to substitute out c_t. The first-order conditions for (5) are given by

$$k_{t+1}: \quad \frac{1}{c_t} = \rho E_t \frac{\partial V_{t+1}}{\partial k_{t+1}}, \quad \mathbf{(6)}$$

$$q_t: \quad \frac{1}{c_t} = \rho E_t \frac{\partial V_{t+1}}{\partial h_{t+1}} \left(\frac{\epsilon h_{t+1}}{q_t}\right). \quad \mathbf{(7)}$$

The envelope conditions are

$$k_t: \quad \frac{\partial V_t}{\partial k_t} = \frac{\alpha y_t}{k_t c_t}, \quad \mathbf{(8)}$$

$$h_t: \quad \frac{\partial V_t}{\partial h_t} = \frac{(1-\alpha) y_t}{h_t c_t}. \quad \mathbf{(9)}$$

An equilibrium in this economy is characterized by the first-order conditions, the envelope conditions, the budget constraints, and the technologies of production and education.

The fractions of output spent on consumption, investment in child education, and bequests are denoted by $\Gamma_c \equiv c_t/y_t$, $\Gamma_q \equiv q_t/y_t$, and $\Gamma_k \equiv k_{t+1}/y_t$, respectively. Given the log utility, Cobb–Douglas technologies, and the full depreciation of physical and human capital per period, we expect the optimal proportional allocation rule $(\Gamma_c, \Gamma_k, \Gamma_q)$ to be constant over time on the entire equilibrium path. From these equilibrium conditions, the equilibrium solution for the proportional allocation rule is given by

$$\Gamma_c = 1 - \rho[\alpha + \epsilon(1-\alpha)]; \quad \Gamma_q = \epsilon\rho(1-\alpha); \quad \Gamma_k = \alpha\rho, \quad \mathbf{(10)}$$

which is indeed constant over time. The solution in (10) satisfies all the equilibrium conditions and is therefore the solution over the entire equilibrium path of the economy. With this time-invariant proportional allocation rule, we can then track down the equilibrium transitional dynamics of the economy.

The evolutions of physical and human capital are determined by the following equations:

$$\ln k_{t+1} = \ln \Gamma_k D + \alpha \ln k_t + (1-\alpha) \ln h_t, \tag{11}$$

$$\ln h_{t+1} = \ln \xi + \epsilon \ln \Gamma_q D + \alpha\epsilon \ln k_t + \epsilon(1-\alpha) \ln h_t + (1-\epsilon) \ln H_t. \tag{12}$$

Equations (11) and (12) can now be used to characterize how the means, the variances, and the covariance of the distributions of (the logs of) of physical and human capital, $\ln k_t$ and $\ln h_t$ respectively, evolve over time:

$$\mu_{kt+1} = \ln \Gamma_k D + \alpha \mu_{kt} + (1-\alpha)\mu_{ht}, \tag{13}$$

$$\Delta^2_{kt+1} = \alpha^2 \Delta^2_{kt} + (1-\alpha)^2 \Delta^2_{ht} + 2\alpha(1-\alpha)\mathrm{cov}(\ln h_t, \ln k_t), \tag{14}$$

$$\mu_{ht+1} = \mu_\xi + \epsilon \ln \Gamma_q D + \alpha\epsilon \mu_{kt} + (1-\alpha\epsilon)\mu_{ht} + (1-\epsilon)\Delta^2_{ht}/2, \tag{15}$$

$$\Delta^2_{ht+1} = \Delta^2_\xi + \alpha^2\epsilon^2 \Delta^2_{kt} + \epsilon^2(1-\alpha)^2 \Delta^2_{ht} + 2\epsilon^2\alpha(1-\alpha)\mathrm{cov}(\ln h_t, \ln k_t), \tag{16}$$

$$\mathrm{cov}(\ln h_{t+1}, \ln k_{t+1}) = \epsilon\alpha^2 \Delta^2_{kt} + \epsilon(1-\alpha)^2 \Delta^2_{ht} + 2\epsilon\alpha(1-\alpha)\mathrm{cov}(\ln h_t, \ln k_t). \tag{17}$$

It is clear from these equations that the next-period values of the variances and the covariance of $\ln k$ and $\ln h$ depend positively on their current values.

Taking logs on both sides of the production function in (2), the mean and the variance of the distribution of output $\ln y_t$ are given by

$$\mu_{yt} = \ln D + \alpha \mu_{kt} + (1-\alpha)\mu_{ht}, \tag{18}$$

$$\Delta^2_{yt} = \alpha^2 \Delta^2_{kt} + (1-\alpha)^2 \Delta^2_{ht} + 2\alpha(1-\alpha)\mathrm{cov}(\ln h_t, \ln k_t). \tag{19}$$

Clearly, the variance of (the log of) output Δ^2_{yt} is increasing with the variances and the covariance of physical and human capital. Note that in models with only one type of capital as the sole channel to transmit parental influences to children [e.g., Bénabou (1996); Glomm and Ravikumar (1992); Zhang (1996)], the covariance is trivially equal to zero by construction. Our model is therefore more general. As in the related literature, income inequality in our model is measured by Δ^2_{yt}.

From (14) to (19), we can determine how income inequality in the parents' generation influences the distributions of their children's capital:

$$\Delta^2_{kt+1} = \Delta^2_{yt}; \quad \Delta^2_{ht+1} = \Delta^2_\xi + \epsilon^2 \Delta^2_{yt}; \quad \mathrm{cov}(\ln h_{t+1}, \ln k_{t+1}) = \epsilon \Delta^2_{yt}. \tag{20}$$

That is, the variance of bequests received by the next generation is equal to the variance of output produced by the current generation, whereas the variance of human capital of the next generation is increasing with both the variance of innate ability and the variance of output produced by the current generation. Here, income inequality in the parents' generation affects the variance of physical capital, or bequests, of their children's generation one for one, whereas it affects the variance of human capital of their children's generation less than one for one. Also, the covariance of human and physical capital in the children's generation depends positively on income inequality in the parents' generation less than one for one. On the other hand, updating (19) by one period, the variances and the covariance of physical and human capital in the children's generation will fully determine income inequality in the children's generation. In short, income inequality evolves from one generation to another through positive interactions with the variances and the covariance of physical and human capital.

From (19) and (20), the evolution of income inequality is given by the following equation:

$$\Delta^2_{yt+1} = (1-\alpha)^2\Delta^2_{\xi} + [\alpha + \epsilon(1-\alpha)]^2\Delta^2_{yt}. \tag{21}$$

According to this equation, income inequality Δ^2_{yt} converges globally toward its steady-state level $\Delta^2_{y\infty} = (1-\alpha)^2\Delta^2_{\xi}/\{1-[\alpha+\epsilon(1-\alpha)]^2\}$ at a rate $1 - [\alpha + \epsilon(1-\alpha)]^2$. By this expression, the steady-state variance of the income distribution is proportional to the variance of innate ability; this proportional factor is increasing with ϵ (the role of parental investment relative to average human capital in education). When ϵ approaches unity (i.e., no role for average human capital in education), the level of long-run income inequality will approach infinity. This special case is inconsistent with the fact that the values of income inequality in the real world are always bounded, and therefore is ignored in the rest of our analysis. These results can be summarized by the following proposition.

PROPOSITION 1. *Income inequality Δ^2_{yt} breaks down into the variances and the covariance of physical and human capital* (Δ^2_{kt}, Δ^2_{ht}, $\mathrm{cov}(\ln k_t, \ln h_t)$) *according to weights α^2, $(1-\alpha)^2$, and $2\alpha(1-\alpha)$, respectively, and converges globally to $\Delta^2_{y\infty} = (1-\alpha)^2\Delta^2_{\xi}/\{1-[\alpha+\epsilon(1-\alpha)]^2\}$.*

We now derive the rates of growth in average physical and human capital:

$$\begin{aligned}
\ln(1+g_{Kt}) &\equiv \ln(K_{t+1}/K_t) = \ln\Gamma_k D - (1-\alpha)\ln(K_t/H_t) - \alpha\Delta^2_{kt}/2 \\
&\quad - (1-\alpha)\Delta^2_{ht}/2 + \Delta^2_{yt}/2 \\
&= \ln\Gamma_k D - (1-\alpha)\ln(K_t/H_t) - \alpha(1-\alpha)(\Delta^2_{kt} + \Delta^2_{ht})/2 \\
&\quad + \alpha(1-\alpha)\mathrm{cov}(\ln h_t, \ln k_t),
\end{aligned} \tag{22}$$

$$\begin{aligned}
\ln(1+g_{Ht}) &\equiv \ln H_{t+1}/H_t = \ln E(\xi) + \epsilon\ln\Gamma_q D + \alpha\epsilon\ln(K_t/H_t) - \alpha\epsilon\Delta^2_{kt}/2 \\
&\quad - \epsilon(1-\alpha)\Delta^2_{ht}/2 + \epsilon^2\Delta^2_{yt}/2 \\
&= \ln E(\xi) + \epsilon\ln\Gamma_q D + \alpha\epsilon\ln(K_t/H_t) - \alpha\epsilon(1-\alpha\epsilon)\Delta^2_{kt}/2 \\
&\quad - \epsilon(1-\alpha)[1-\epsilon(1-\alpha)]\Delta^2_{ht}/2 + \epsilon^2\alpha(1-\alpha)\mathrm{cov}(\ln h_t, \ln k_t).
\end{aligned} \tag{23}$$

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In both of these equations, the growth rates of average physical and human capital, g_{Kt} and g_{Ht}, are decreasing with the variances, Δ^2_{kt} and Δ^2_{ht}, and increasing with the covariance, $\mathrm{cov}(\ln h_t, \ln k_t)$.

The growth rate of average output can now be derived as follows:

$$\begin{aligned}
\ln(1+g_{Yt}) &\equiv \ln Y_{t+1}/Y_t = \alpha \ln \Gamma_k D + (1-\alpha)\ln E(\xi) - \alpha(1-\alpha)\Delta^2_\xi/2 \\
&\quad + \epsilon(1-\alpha)\ln \Gamma_q D - \alpha(1-\alpha)(1-\epsilon)\ln(K_t/H_t) + \alpha(1-\alpha)(1-\epsilon)\Delta^2_{kt}/2 \\
&\quad + (1-\alpha)^2(1-\epsilon)\Delta^2_{ht}/2 - \{1-[\alpha+\epsilon(1-\alpha)]^2\}\Delta^2_{yt}/2 \\
&= \alpha \ln \Gamma_k D + (1-\alpha)\ln E(\xi) - \alpha(1-\alpha)\Delta^2_\xi/2 + \epsilon(1-\alpha)\ln \Gamma_q D \\
&\quad - \alpha(1-\alpha)(1-\epsilon)\ln(K_t/H_t) + \alpha\{(1-\alpha)(1-\epsilon) - \alpha + \alpha[\alpha+\epsilon(1-\alpha)]^2\} \\
&\quad \times \Delta^2_{kt}/2 + (1-\alpha)^2\{[\alpha+\epsilon(1-\alpha)]^2 - \epsilon\}\Delta^2_{ht}/2 \\
&\quad - \alpha(1-\alpha)\{1-[\alpha+\epsilon(1-\alpha)]^2\}\mathrm{cov}(\ln h_t, \ln k_t). \qquad (\mathbf{24})
\end{aligned}$$

Note that the growth rate of average output is decreasing with the covariance between human and physical capital. Also, note that how average output growth responds to the variances of the two types of capital depends on parameterizations, particularly on the size of ϵ. For small enough ϵ (i.e., a weak enough role of parental investment relative to average human capital in education), it is clear that the coefficients on Δ^2_{kt} and Δ^2_{ht} are positive.

In addition, because the growth rate of average output is decreasing with the ratio of physical to human capital in (24), income inequality may have a positive effect on output growth if it has a negative effect on the ratio of physical to human capital. Hence, income inequality may affect output growth indirectly through changing the ratio of physical to human capital. This can be seen from the evolution of the ratio of physical to human capital derived next:

$$\begin{aligned}
\ln K_{t+1}/H_{t+1} &= \ln \Gamma_k D - \ln E(\xi) - \epsilon \ln \Gamma_q D + \alpha(1-\epsilon)\ln K_t/H_t - \alpha(1-\epsilon) \\
&\quad \times [1-\alpha(1+\epsilon)]\Delta^2_{kt}/2 - (1-\epsilon)(1-\alpha)[\alpha - \epsilon(1-\alpha)]\Delta^2_{ht}/2 \\
&\quad + \alpha(1-\alpha)(1-\epsilon^2)\mathrm{cov}(\ln h_t, \ln k_t). \qquad (\mathbf{25})
\end{aligned}$$

If we assume that the share parameter for physical capital is set as $\alpha < 1/2$, as widely assumed in the literature, then the variance of physical capital always has a negative effect on the ratio of physical to human capital, according to (25). Under the same assumption, the variance of human capital may have a positive effect on the ratio of physical to human capital when ϵ is large enough (i.e., there is a weak enough human capital externality in education).

What does this mean for growth? Because the growth rate of average output is decreasing with the ratio of physical to human capital in (24), a greater variance of physical capital is conducive to output growth by reducing the ratio of physical to human capital. Likewise, a greater variance of human capital is harmful to growth if the role of parental investment relative to average human capital in education is strong enough. This means that parental influences become a comparatively important factor in determining a child's education within a family. The conventional view that income inequality is harmful to income growth should therefore be examined with more scrutiny. This conventional view may refer to how initial income inequality affects growth on the transitional path, or how income inequality affects growth on the balanced growth path.

To answer these questions, one has to convert the variances and the covariance of the two types of capital into the conventional measure of income inequality, Δ_{yt}^2. From (21), the solution for the dynamic path of income inequality, Δ_{yt}^2 for $t > 0$ can be derived in terms of the initial Δ_{y0}^2 and the steady-state $\Delta_{y\infty}^2$ as

$$\Delta_{yt}^2 = (1 - \Gamma^t)\Delta_{y\infty}^2 + \Gamma^t \Delta_{y0}^2, \quad \Gamma \equiv [\alpha + \epsilon(1-\alpha)]^2 \in (0, 1), \tag{26}$$

which is globally convergent. The solution for the variances of physical and human capital, Δ_{kt}^2 and Δ_{ht}^2 respectively, for $t > 0$ can now be expressed in terms of income inequality Δ_{yt}^2 as

$$\Delta_{kt}^2 = (1 - \Gamma^{t-1})\Delta_{y\infty}^2 + \Gamma^{t-1}\Delta_{y0}^2, \tag{27}$$

$$\Delta_{ht}^2 = \Delta_\xi^2 + \epsilon^2(1 - \Gamma^{t-1})\Delta_{y\infty}^2 + \epsilon^2\Gamma^{t-1}\Delta_{y0}^2. \tag{28}$$

A key observation that emerges from these two equations is that the effect of income inequality on the variance of human capital varies in magnitude with ϵ. Specifically, the smaller the role of parental investment in education relative to average human capital (smaller ϵ), the weaker the effect of income inequality on the variance of human capital (relative to that on the variance of physical capital).

Another interesting observation in (27) and (28) is that there are different sensitivities of the variances of physical and human capital to initial income inequality and ability inequality. On one hand, the variance of physical capital is clearly more responsive to initial income inequality than is the variance of human capital. On the other hand, the variance of physical capital is less responsive to ability inequality than is the variance of human capital in the long run, which is shown later. Linking $\Delta_{y\infty}^2$ to Δ_ξ^2 in (27) and (28) with $t \to \infty$, we obtain the steady-state variances of $\ln k_t$ and $\ln h_t$, respectively:

$$\Delta_{k\infty}^2 = \Delta_{y\infty}^2 = \left\{\frac{(1-\alpha)^2}{1 - [\alpha + \epsilon(1-\alpha)]^2}\right\}\Delta_\xi^2, \tag{29}$$

$$\Delta_{h\infty}^2 = \left\{1 + \frac{\epsilon^2(1-\alpha)^2}{1 - [\alpha + \epsilon(1-\alpha)]^2}\right\}\Delta_\xi^2. \tag{30}$$

Equations (27)–(30) lead to the following proposition:

PROPOSITION 2. *The steady-state variances* $(\Delta_{h\infty}^2, \Delta_{k\infty}^2, \Delta_{y\infty}^2)$ *and their sensitivities to a change in the variance of innate ability are ranked as* $\Delta_{h\infty}^2 > \Delta_{k\infty}^2 = \Delta_{y\infty}^2$ *and* $d\Delta_{h\infty}^2/d\Delta_\xi^2 > d\Delta_{k\infty}^2/d\Delta_\xi^2 = d\Delta_{y\infty}^2/d\Delta_\xi^2\ (> 0)$. *For* $t \in (0, \infty)$, *the variance of physical capital is more sensitive to initial income inequality than is the variance of human capital:* $d\Delta_{kt}^2/d\Delta_{y0}^2 > d\Delta_{ht}^2/d\Delta_{y0}^2$ (> 0).

Proof. To compare the coefficients on Δ_ξ^2 on the right-hand sides of (29) and (30), we construct the following function:

$$B(\alpha) \equiv (1-\alpha)^2 - 1 + [\alpha + \epsilon(1-\alpha)]^2 - \epsilon^2(1-\alpha)^2.$$

Clearly, the proof at hand is just to show that $B(\alpha) < 0$ under $\alpha \in (0,1)$ and $\epsilon \in (0,1)$. Note that $B(1) = B(0) = 0$. The first derivative of B is

$$B'(\alpha) = -2(1-\alpha) + 2[\alpha + \epsilon(1-\alpha)](1-\epsilon) + 2\epsilon^2(1-\alpha)$$

with $B'(0) = -2(1-\epsilon) < 0$ and $B'(1) = 2(1-\epsilon) > 0$. The second derivative of B is

$$\begin{aligned} B''(\alpha) &= 2 + 2(1-\epsilon)^2 - 2\epsilon^2 \\ &= 4(1-\epsilon) > 0. \end{aligned}$$

Thus, as α rises from 0 to 1, B falls initially from 0 and then rises back to 0 with only one turning point. With $\alpha \in (0,1)$ and $\epsilon \in (0,1)$, $B < 0$ must hold.

To establish the last part of this proposition, note that according to (27) and (28), we have $d\Delta_{kt}^2/d\Delta_{y0}^2 = \Gamma^{t-1} > \epsilon^2\Gamma^{t-1} = d\Delta_{ht}^2/d\Delta_{y0}^2$ for $t \in (0,\infty)$, with $\Gamma \in (0,1)$ and $\epsilon \in (0,1)$. ∎

According to Proposition 2, the steady-state variances of physical capital and income are equal and smaller than the steady-state variance of human capital. Also, the latter is more sensitive to changes in ability inequality than are the former two on the balanced growth path, because innate ability in this model impacts directly on education. On the transitional path, the variance of physical capital is more responsive to initial income inequality than is the variance of human capital, as we noted earlier. The results in Proposition 2 are important in helping us understand why the effect of income inequality on growth in the short run may differ from its effect in the long run.

The entire equilibrium path of the economy can now be fully described by tracking down the two-dimensional evolutions of the ratio of physical to human capital and the variance of output. Starting with an initial income inequality $\Delta_{y0}^2 \in (0,\infty)$ and an initial capital ratio $K_0/H_0 \in (0,\infty)$, and knowing the steady-state inequality $\Delta_{y\infty}^2 \in (0,\infty)$, the equilibrium solution for the evolution of the capital ratio K_t/H_t is

$$\begin{aligned} \ln\frac{K_t}{H_t} &= \alpha^t(1-\epsilon)^t \ln\frac{K_0}{H_0} + \left(\Theta - \Lambda_1\frac{\Delta_{y\infty}^2}{2}\right)\left[\frac{1-\alpha^t(1-\epsilon)^t}{1-\alpha(1-\epsilon)}\right] \\ &\quad + \Lambda_2\left(\frac{\Delta_{y0}^2}{2} - \frac{\Delta_{y\infty}^2}{2}\right)\left[\frac{\Gamma^t - \alpha^t(1-\epsilon^t)}{\Gamma - \alpha(1-\epsilon)}\right], \end{aligned} \tag{31}$$

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where $\Theta \equiv \ln \Gamma_k D - \ln E(\xi) - \epsilon \ln \Gamma_q D$, $\Lambda_1 \equiv (1-\epsilon)[2\alpha(1-\epsilon) - \epsilon]$, and

$$\Lambda_2 \equiv (1-\epsilon)\left\{1+\epsilon - \frac{\alpha + (1-\alpha)\epsilon^2}{[\alpha + (1-\alpha)\epsilon]^2}\right\}.$$

Note that, given a standard value of the share parameter of physical capital in production α, the signs of Λ_1 and Λ_2 will depend on the value of ϵ. In particular, if ϵ is small enough (strong enough externalities from average human capital), then $\Lambda_1 > 0$ and $\Lambda_2 < 0$. Therefore, for a small enough ϵ, initial income inequality clearly has a negative effect on the ratio of physical to human capital in the short run (i.e., $t < \infty$). Also, on the balanced growth path with $t = \infty$, the transitory terms with exponents containing t and with bases ranging in (0, 1) in (31) will eventually vanish, leading to the following steady-state ratio of physical to human capital:

$$\lim_{t\to\infty} \ln \frac{K_t}{H_t} = \Theta - \Lambda_1 \frac{\Delta^2_{y\infty}}{2}.$$

According to this steady-state capital ratio and the definition of Λ_1 in (31), income inequality clearly has a negative effect on the ratio of physical to human capital on the balanced growth path whenever ϵ is small enough so that $\Lambda_1 > 0$.

Through these negative effects on the ratio of physical to human capital, income inequality tends to increase the growth rate of average output per worker both on the transitional path and on the balanced path. This indirect effect exists in this model because of the complementarity between human and physical capital at the individual household level in the absence of factor markets. Specifically, in this model, the average or aggregate physical capital K_{t+1} is a sum of individual bequests $k_{t+1} = \Gamma_k y_t(h_t, k_t)$, where $y_t(\cdot, \cdot)$, defined in (2), is increasing and concave in individual states (h_t, k_t). According to this concave conversion, a mean-preserving spread in the individual bequest k_t will reduce the average or aggregate level of physical capital in the next period, giving rise to a possible decline in the ratio of average physical to average human capital.[7]

Besides this indirect effect, there is of course a negative direct effect of income inequality on the growth rate of average output per worker along the entire equilibrium path of the economy, as in the existing studies with one type of capital. The essence of this direct effect, when production takes place at a family level rather than at the aggregate level, is the loss in average output from a mean-preserving spread in each of the two types of capital, with diminishing marginal products in production. As investments in physical and human capital are proportional to output in each family, the loss in average output in turn decelerates average capital accumulation. Thus, we need to examine the net effect of income inequality on the growth rate of average output per worker in both the short run and the long run.

Another interesting observation is that models with only human capital and without physical capital correspond to a special case of our model with $\alpha = 0$ and zero bequests. In this special case, we would always have $\Lambda_1 = -\epsilon(1-\epsilon) < 0$

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and $\Lambda_2 = \epsilon(1-\epsilon) > 0$, so that the capital ratio in (31) becomes

$$\lim_{\alpha \to 0} \ln \frac{K_t}{H_t} = \Theta + \epsilon(1-\epsilon)^2 \frac{\Delta^2_{y\infty}}{2} + \epsilon(1-\epsilon)\frac{\Delta^2_{y0}}{2}.$$

Given the positive coefficients in this equation on both $\Delta^2_{y\infty}$ and Δ^2_{y0}, it would be impossible for the ratio of physical to human capital to channel a positive effect of income inequality toward growth in output per worker on either the transitional or the balanced path. This observation suggests that it should be essential to incorporate both physical and human capital in order to generate a positive effect of income inequality on growth in output per worker.

As mentioned earlier, when ϵ is smaller, a rise in income inequality has a stronger effect on the variance of physical capital relative to the variance of human capital in (26)–(28), which transforms into a stronger negative effect on the ratio of physical to human capital in (25). Because output growth depends inversely on the ratio of physical to human capital, this stronger negative effect on the ratio of physical to human capital is transformed into a stronger positive effect on output growth in our model with both bequests and human capital.

Combining the evolutions of the variances of physical and human capital and the ratio of physical to human capital with (24), we can obtain the reduced-form solution for the growth rate of output per worker as follows:

$$\begin{aligned}\ln(1+g_{yt}) &= \alpha \ln \Gamma_k D + (1-\alpha)\ln E(\xi) + \epsilon(1-\alpha)\ln \Gamma_q D \\ &\quad - \alpha(1-\alpha)(1-\epsilon)\Theta \\ &\quad \times \left[\frac{1-\alpha^t(1-\epsilon)^t}{1-\alpha(1-\epsilon)}\right] - (1-\alpha)\alpha^{t+1}(1-\epsilon)^{t+1}\ln\frac{K_0}{H_0} + F\frac{\Delta^2_{y0}}{2} + G\frac{\Delta^2_{y\infty}}{2}, \qquad (\mathbf{32})\end{aligned}$$

where

$$\begin{aligned}F(\epsilon) &\equiv [\alpha(1-\alpha) + \epsilon^2(1-\alpha)^2](1-\epsilon)\Gamma^{t-1} - (1-\Gamma)\Gamma^t \\ &\quad - \alpha(1-\alpha)(1-\epsilon)\Lambda_2\left[\frac{\Gamma^t - \alpha^t(1-\epsilon)^t}{\Gamma - \alpha(1-\epsilon)}\right]\end{aligned}$$

and

$$\begin{aligned}G(\epsilon) &\equiv [\alpha + (1-\alpha)\epsilon^2](1-\alpha)(1-\epsilon)(1-\Gamma^{t-1}) - (1-\Gamma)(1-\Gamma^t) \\ &\quad + \alpha(1-\alpha)(1-\epsilon)\left\{\Lambda_1\left[\frac{1-\alpha^t(1-\epsilon)^t}{1-\alpha(1-\epsilon)}\right] + \Lambda_2\left[\frac{\Gamma^t-\alpha^t(1-\epsilon)^t}{\Gamma-\alpha(1-\epsilon)}\right]\right\} \\ &\quad - \left\{\frac{[\alpha-(1-\alpha)(1-\epsilon)](1-\Gamma)}{1-\alpha}\right\}.\end{aligned}$$

In this reduced-form solution, the growth rate of output per worker is decreasing with the initial ratio of physical to human capital as in the literature, and, as expected, is increasing with the mean of the ability distribution. The coefficients

F and G on initial and steady-state income inequality, respectively, in this reduced-form solution comprise all the direct and indirect effects of income inequality that we have mentioned so far. These coefficients therefore indicate the net effects on the growth rate of output per worker of initial and steady-state income inequality on transitional and balanced growth paths, respectively. Also, note that the growth rate of average output is globally convergent to its steady-state level.

Now, we show in the following proposition that there is an inverse relationship between output growth and income inequality on the balanced growth path, which supports the conventional view that income inequality is harmful to growth.

PROPOSITION 3. *The growth rate of average output converges globally to a unique steady-state level. On the balanced growth path, steady-state income inequality* $\Delta^2_{y\infty}$ *has a negative effect on the steady-state growth rate. At the limit, as* $\epsilon \to 0$*, this negative effect vanishes.*

Proof. The global convergence of the growth rate to a unique steady-state level is obvious. To derive the long-run relationship between inequality and growth, we set $t = \infty$ in (32) and obtain

$$G_\infty = \frac{1}{(1-\alpha)[1-\alpha(1-\epsilon)]} X(\epsilon),$$

where

$$\begin{aligned} X(\epsilon) &= -[1-\alpha(1-\epsilon)](1-\Gamma) + (1-\alpha)(1-\epsilon)[1-\alpha(1-\epsilon)](1-\Gamma) \\ &\quad + 2\alpha^2(1-\alpha)^2(1-\epsilon)^3 - \epsilon\alpha(1-\alpha)^2(1-\epsilon)^2 \\ &\quad + \alpha(1-\alpha)^2(1-\epsilon)[1-\alpha(1-\epsilon)] + \epsilon^2(1-\alpha)^3(1-\epsilon)[1-\alpha(1-\epsilon)]. \end{aligned}$$

Because sign $G_\infty =$ sign X, our proof focuses on X. Note that if $\epsilon = 0$, then $\Gamma = \alpha^2$, and that if $\epsilon = 1$, then $\Gamma = 1$. Then, it is easy to verify that $\lim_{\epsilon\to 1} X = \lim_{\epsilon \to 0} X = 0$ (immediately implying the last part of the proposition).

We now need to see how X responds to a change in ϵ:

$$\begin{aligned} X'(\epsilon) &= \alpha[\Gamma - 1 + (1-\alpha)(1-\epsilon)(1-\Gamma) + \alpha(1-\alpha)^2(1-\epsilon) \\ &\quad + (1-\alpha)^3(1-\epsilon)\epsilon^2] \\ &\quad + (1-\alpha)[1-\alpha(1-\epsilon)]\{2[\alpha+\epsilon(1-\alpha)] - (1-\Gamma) - 2(1-\alpha)(1-\epsilon) \\ &\quad \times [\alpha + \epsilon(1-\alpha)] - \alpha(1-\alpha) - (1-\alpha)^2\epsilon^2 + 2(1-\alpha)^2(1-\epsilon)\epsilon\} \\ &\quad - 6\alpha^2(1-\alpha)^2(1-\epsilon)^2 - \alpha(1-\alpha)^2(1-\epsilon)^2 + 2\epsilon\alpha(1-\alpha)^2(1-\epsilon). \end{aligned}$$

Note that $\lim_{\epsilon\to 0} X' < 0$ and $\lim_{\epsilon \to 1} X' > 0$. Combining the features of X and X', we have $X < 0$ for ϵ being near either 0 or 1 (not equal to). The remaining step for $X < 0$ for all $\epsilon \in (0, 1)$ is to show that $X'' > 0$ for $\epsilon \in (0, 1)$. That is, X falls from zero at $\epsilon = 0$ and then rises to zero again at $\epsilon = 1$, with only one turning point in X at a unique value of $\epsilon \in (0, 1)$.

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Taking the second derivative of X, we have

$$X''(\epsilon) = 2\alpha(1-\alpha)\{2[\alpha+\epsilon(1-\alpha)] - (1-\Gamma)$$
$$- 2(1-\alpha)(1-\epsilon)[\alpha+\epsilon(1-\alpha)]$$
$$-\alpha(1-\alpha) - (1-\alpha)^2\epsilon^2 + 2(1-\alpha)^2(1-\epsilon)\epsilon\}$$
$$+ [1-\alpha(1-\epsilon)](1-\alpha)^2\{2 + 4[\alpha+\epsilon(1-\alpha)] - 4(1-\alpha)\epsilon\}$$
$$+ 12\alpha^2(1-\alpha)^2(1-\epsilon) + 4\alpha(1-\alpha)^2(1-\epsilon) - 2\epsilon\alpha(1-\alpha)^2.$$

It can be verified that $\lim_{\epsilon\to 0} X'' > 0$ and that $X''' = 0$ for $\epsilon \in [0, 1]$. Thus, $X'' > 0$ for $\epsilon \in (0, 1)$ as needed for $X < 0$ for all $\epsilon \in (0, 1)$. ■

Because the level of long-run income inequality $\Delta^2_{y\infty}$ is proportional to the variance of the ability distribution Δ^2_{ξ}, the meaning of Proposition 3 is that a mean-preserving spread in ability reduces the steady-state rate of growth in average output. The intuition for this result is as follows. According to Proposition 2, we have seen that ability inequality has a greater influence on human capital inequality than on physical capital inequality along the balanced growth path. Because sustainable growth in output per worker in this type of model is driven by human capital accumulation,[8] a rise in ability inequality leads to a negative net effect on the steady-state rate of growth in the long run.

At a deeper level, this net effect reflects opposing forces of long-run income inequality on growth. On one hand, as mentioned earlier, for a small enough ϵ (strong enough externalities from average human capital), there is an indirect positive effect of income inequality on growth in output per worker on the balanced growth path through the ratio of physical to human capital. On the other hand, besides this indirect effect, there is also a direct effect of income inequality on average output growth that occurs through decelerating capital accumulation. The negative net effect indicates that the direct negative effect always dominates in the long run for the reasons given previously. However, in the limiting case when the role of parental investment in human capital vanishes ($\epsilon \to 0$), the direct and indirect forces cancel out exactly, and therefore the net effect of long-run inequality on long-run growth vanishes too.[9]

The result contained in Proposition 3 agrees with the conventional view that income inequality is harmful to growth. This result also agrees with the weight of empirical evidence [e.g., Alesina and Rodrik (1994); Easterly (2001); Perotti (1996); Persson and Tabellini (1994)], which indicates a negative growth effect of income inequality. However, recent evidence in Barro (2000) suggests that income inequality retards growth in poor countries but promotes growth in richer ones. Also, empirical evidence in Forbes (2000) suggests a positive growth effect of inequality in the short and medium term.

Note that empirical researchers can only observe how initial income inequality affects output growth in a finite number of years; e.g., samples within 1960–2000 are popularly used in the literature because of data availability. In what follows, we thus focus on how initial income inequality affects average output growth on the transitional path in order to reconcile the theory of how growth responds to income inequality with the recent empirical evidence. Combining (24) with (26)–(31), we obtain the following key result in this paper:

PROPOSITION 4. *For $\alpha < 1/2$ and $0 < t < \infty$, if ϵ is large enough, then initial income inequality Δ_{y0}^2 has a negative effect on the growth rate of output per worker. Conversely, if ϵ is small enough, then initial income inequality Δ_{y0}^2 has a positive effect on the growth rate of output per worker.*

Proof. The coefficient on Δ_{y0}^2 in the growth equation (32) is given by $F(\epsilon)$. Clearly, $\lim_{\epsilon\to 1} F(\epsilon) = 0$ and $\lim_{\epsilon\to 1} F'(\epsilon) = 1-\alpha > 0$. Consequently, $F(\epsilon) < 0$ must hold if ϵ is large enough.

On the other hand, if $\epsilon \to 0$, then $\Gamma \to \alpha^2$ and $\Lambda_2 \to 1 - 1/\alpha < 0$. Thus, we have

$$\lim_{\epsilon\to 0} F(\epsilon) = (1-\alpha)\alpha^{t-1}[1-(1+\alpha)\alpha^{t+1}] > 0 \quad \text{for } \alpha < 1/2 \text{ and } 0 < t < \infty.$$

Consequently, $F(\epsilon) > 0$ must hold if ϵ is small enough. ∎

The results contained in Proposition 4 differ substantially from the conventional view that high income inequality retards growth in models with just one type of capital or in models with two types of capital, perfect competition, and a complete labor market. In our model, the net growth effect of income inequality depends on the relative strength of the two opposing forces. If the role of parental investment relative to the human capital externality in education is large, then the negative effect of income inequality on growth via human capital investment is strong and tends to dominate the other effect from the ratio of physical to human capital. As a result, income inequality retards growth. Conversely, if the role of the human capital externality is large, then the negative effect of income inequality on growth via human capital investment is weak and tends to be dominated by the other effect.

Also, the strength of these opposing effects (hence the net effect of inequality on growth) depends inversely on the length of time away from the initial period. As can be seen clearly in (27) and (28) with $\Gamma \in (0, 1)$, the larger the value of t, the smaller the effects of initial income inequality on the variances of physical and human capital. Recall that in Proposition 2 the variance of physical capital is more sensitive to initial income inequality than the variance of human capital during the transition. Combining this with the previous discussion on the role of ϵ, it is intuitive that, when ϵ is small, the positive growth effect of initial income inequality via the ratio of physical to human capital can be strong initially. As time approaches infinity, all the terms of the coefficient on initial income inequality F approach zero. That is, all the transitory opposing effects of initial inequality disappear on the balanced growth path. Thus, in the long run, the relationship

TABLE 1. Effect of income inequality on per worker income growth: $\alpha = 0.33$

	Coefficient on income inequality in the growth equation					
ϵ	$t = 1$	$t = 2$	$t = 3$	$t = 4$	$t = 5$	$t = 6$
0.05	0.3878	0.1426	0.0475	0.0153	0.0048	0.0015
0.10	0.2559	0.0954	0.0314	0.0098	0.0030	0.0009
0.15	0.1593	0.0614	0.0203	0.0063	0.0019	0.0005
0.20	0.0870	0.0358	0.0122	0.0038	0.0011	0.0003
0.25	0.0320	0.0157	0.0058	0.0019	0.0006	0.0002
0.30	−0.0103	−0.0008	0.0003	0.0002	0.0001	0.0000
0.35	−0.0430	−0.0148	−0.0050	−0.0016	−0.0005	−0.0002
0.40	−0.0681	−0.0271	−0.0103	−0.0038	−0.0014	−0.0005
0.45	−0.0870	−0.0381	−0.0158	−0.0064	−0.0026	−0.0010
0.50	−0.1007	−0.0480	−0.0218	−0.0097	−0.0043	−0.0019
0.55	−0.1098	−0.0566	−0.0281	−0.0138	−0.0067	−0.0033
0.60	−0.1145	−0.0639	−0.0345	−0.0186	−0.0099	−0.0053
0.65	−0.1152	−0.0694	−0.0409	−0.0240	−0.0141	−0.0082
0.70	−0.1118	−0.0727	−0.0466	−0.0297	−0.0190	−0.0121
0.75	−0.1044	−0.0732	−0.0508	−0.0352	−0.0244	−0.0169
0.80	−0.0928	−0.0701	−0.0526	−0.0394	−0.0296	−0.0222
0.85	−0.0768	−0.0624	−0.0505	−0.0408	−0.0330	−0.0267
0.90	−0.0563	−0.0490	−0.0427	−0.0372	−0.0323	−0.0282
0.95	−0.0308	−0.0287	−0.0268	−0.0251	−0.0234	−0.0219

between income inequality and growth goes back to the negative one stated in Proposition 3.

Note that the restriction $\alpha < 1/2$ that emerges from Proposition 4 is widely accepted in the literature; i.e., the share parameter of physical capital is smaller than the share parameter of effective labor in the Cobb–Douglas production function. Given the standard value of $\alpha = 1/3$, the two factors that determine the sign of the growth effect of initial income inequality are the role of parental investment relative to the human capital externality (ϵ) and the length of time from the initial period ($t > 0$). Theoretically as well as practically, it is interesting to find the critical value of the key parameter ϵ through simulations that alter the sign of the growth effect of initial income inequality, which is our next task.

4. NUMERICAL RESULTS

The simulation procedure is simple: holding $\alpha = 1/3$, we change the value of ϵ from very small to very large gradually and see what happens to the coefficient $F(\epsilon)$ on initial income inequality in the growth equation. In doing so, we can also change time t for some finite number of periods and see how persistent the growth effect of initial income inequality will be. The results are reported in Table 1.

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TABLE 2. School enrollment and public education spending across countries

Type of countries by income	Number of countries	Average GDP per capita 1960–1985 (1980 U.S.$)	Primary school enrollment 1960–1985 (%)	Secondary school enrollment 1960–1985 (%)	Public education spending/GDP 1960–1985 (%)
Poor	38	580	62.3	13.2	3.8
Middle-income	35	1793	93.3	32.3	4.2
Rich	34	5925	100	62.8	7.8

Note: Poor countries have average per capita GDP (average for 1960–1985 per year) below \$1,000 (1980 U.S.\$); middle-income countries have average per capita GDP between \$1,000 and \$3,000; rich countries have average per capita GDP above \$3,000. Data are based on the data set in Barro (1991). School enrollment rates and the ratios of public education spending to GDP are the averages for 1960–1985.

It is quantitatively important to note in Table 1 that the sign switch occurs in the plausible range $0.35 \geq \epsilon \geq 0.25$. When $\epsilon < 0.25$, the sign of the growth effect of initial income inequality is always positive in Table 1, whereas the opposite is true when $\epsilon > 0.35$. Another observation is that this effect diminishes over time (close to zero when $t = 6$); that is, the effect of initial income inequality on growth fades away after six generations.

In the real world, the value of ϵ may depend on how education is organized within a particular country. In Table 2, we report remarkable differences in per capita GDP, primary/secondary school enrollment rates, and the ratio of public education spending to GDP across poor, middle-income, and rich countries, using the data set in Barro (1991).[10] According to Table 2, nearly all children have primary education and many (most) children at the due age have secondary education in middle-income (rich) countries. In fact, in many middle-income countries and almost all rich countries, there is access to compulsory public schooling for 9–12 years. In these countries, where many school-age children attend public schools,[11] we expect a dominating role of outside-family factors in the determination of education for a child such that $\epsilon < 0.3$ becomes possible.

By contrast, according to Table 2, poor countries' public education spending as a fraction of GDP is less than half of rich countries' on average. Also, per capita GDP in poor countries is less than 10% of that in rich countries on average in Table 2. Consequently, the level of public education spending per child in poor countries is less than 5% of the corresponding level in rich countries on average. As a result, many children in poor countries did not finish primary education and the vast majority of children in these countries did not have secondary education in Table 2. With a lack of formal schooling in these poor countries, we expect a key role of inside-family factors in the determination of education for a child, such that $\epsilon > 0.3$. Given that this division between poor and richer countries by the value of ϵ appears to be plausible in the real world, we may interpret the results in Table 1 as follows: whereas income inequality reduces the growth rate in poor countries, it raises the growth rate in richer countries.

The apparent positive associations of school enrollment rates with income and with public education spending in Table 2 are consistent with some of the well-known observed facts of structural transformation in Kuznets (1973). According to Kuznets (1973), during development, there are shifts from agriculture to industry to services, from home work to employee status, and from informal to formal education, among others; and there are increasing roles for governments. The significant involvement of governments in education spending in all groups of countries at all income levels in Table 2 also suggests the possibility of imperfect credit markets, among other factors, as far as human capital investment is concerned for many families.

Our result differs substantially from those in the literature. In the theoretical literature there have been two mechanisms generating a negative effect of income inequality on economic growth. The first mechanism involves income inequality engendering political pressure for redistribution or political instability [e.g., Alesina and Perotti (1996); Alesina and Rodrik (1994); Benhabib and Rustichini (1998); Persson and Tabellini (1994)]. The second mechanism involves individuals facing credit constraints and their capital or skills evolving over time according to a concave function exhibiting diminishing returns [e.g., Bénabou (1996); Eckstein and Zilcha (1994); Galor and Zeira (1993); Glomm and Ravikumar (1992); Tamura (1991, 1992); Zhang (1996, 2005)]. In this latter mechanism a mean-preserving spread in initial capital causes an efficiency loss under diminishing returns in production or education, thereby hindering average capital accumulation and growth.

On the other hand, the classical approach postulates that the rich have a higher propensity to save than the poor and hence inequality promotes capital accumulation and growth [e.g., Bourguignon (1981); Kaldor (1957); Keynes (1920); Lewis (1954); Murphy et al. (1989b); Smith (1776)]. Only Galor and Moav (2004), who combine this classical approach with a credit constraint on investment in human capital, have identified a nonmonotonic relationship between growth and inequality in a two-stage development process. The early development is driven by physical capital accumulation, whereas the later development is driven mainly by human capital accumulation. They show that in the early stage income inequality promotes growth by promoting savings, but in the later stage it hinders growth by worsening credit constraints.

There have also been numerous empirical studies estimating how inequality affects growth from cross-country data. Most of them have shown evidence of a negative growth effect of inequality [e.g., Alesina and Rodrik (1994); Easterly (2001); Perotti (1996); Persson and Tabellini (1994)]. By contrast, Forbes (2000) has found evidence of a positive growth effect of inequality in the short and medium term, when many poor countries are excluded because of the problem of data quality. Also, Barro (2000) has found a negative (positive) effect of inequality on growth among poor (richer) countries and a nonsignificant effect in the whole sample, as opposed to the theory of Galor and Moav. However, Banerjee and Duflo (2003) cast doubts on the validity of this type of estimation.

As empirical researchers can only observe variables in finite periods in the short and medium term, our interpretation is consistent with the empirical evidence in Barro (2000). Our result with a small enough ϵ is also consistent with the evidence of a positive growth effect of inequality in Forbes (2000). In particular, because many poor countries were excluded from the sample because of poor data quality in Forbes's work, those included in the sample may be dominated by countries with strong public support for education. However, we should interpret the relationship of the results in our model with the evidence in rich countries with caution, because of our abstraction from factor markets.

Our results are complementary to a set of papers in addition to those we mentioned earlier. Tamura (1996) presents a model with conditional human capital spillovers. In his model, there can be rising inequality in the world before there is integration and convergence. Also, real output per worker would be falling as inequality was rising, and then it would sharply increase once all agents or countries chose to grow. This is also consistent with the work of Galor and Weil (2000) and Galor and Moav (2002). In those papers there would be accelerating growth as the number of families that chose growth increased. When eventually all agents choose growth, inequality will begin to decline and decline rapidly as growth accelerates. Also, Tamura (2001) shows how a model of public education can produce convergence even when revenues are not shared among the parents. In doing so, he shows how reduced-form diminishing returns to schooling expenditures can arise from a model with class size and relative teacher quality. Overall, our mechanism for the relationship between inequality and growth and convergence is just one of many possible mechanisms for dealing with this complex issue.

5. CONCLUDING REMARKS

In this paper, we have analyzed how income inequality affects growth by allowing both bequests and human capital investment to transmit income inequality from generation to generation, in the absence of factor markets. As a consequence, the mechanism through which income inequality affects growth in average output in this model differs from those in related studies. On one hand, higher income inequality in this model has a negative direct effect on growth in average output by decelerating capital accumulation. On the other hand, it also has a positive indirect effect on growth by lowering the ratio of physical to human capital.

This positive indirect effect occurs partly because production takes place at a family level in the absence of factor markets, and partly because income inequality has a stronger effect on the variance of physical capital than on the variance of human capital over a short time horizon. This indirect effect cannot exist in models with one type of capital, and cannot be positive in models with two types of capital and perfect competition in the production sector. The net effect of income inequality on growth depends on the relative strengths of these two

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opposing forces, which are determined in turn by the degree of the human capital externality.

Our Propositions 3 and 4 show that if the role of parental investment is large (small) enough relative to average human capital in educating children, then income inequality has a negative (positive) effect on growth outside the balanced growth path. Given a standard value of physical capital's share parameter, we have found numerically that the critical level of the externality in education to divide the signs of the impact of income inequality on growth outside the balanced growth path is approximately 70%. On the balanced growth path, we have shown that income inequality merely echoes ability inequality and is always harmful to growth.

Given the fact that education is more publicly provided in the rich countries than in the poor countries, our results suggest a likely negative (positive) net effect of income inequality on growth in poor (rich) countries. These results offer a possible explanation for some recent evidence that suggests a positive growth effect of income inequality. As we mentioned earlier, both assumptions of missing or complete markets are strong when we consider the real world situations. Thus, we should interpret the different results from these different assumptions with caution. In particular, because our model is better suited for developing countries than for developed countries, the suggested positive effect of inequality on growth is only of limited relevance. The most realistic situation in developed countries is somewhere between the two polar cases. Investigation into this more realistic scenario is difficult and merits future research.

Finally, our analysis of a joint lognormal distribution of human and physical capital appears to be novel in its own right, and leads to some intriguing results. Explicitly, we have shown how income inequality breaks down into the variances and the covariance of human and physical capital, and how income inequality evolves over time through influencing the variances and the covariance. These results may help model specifications in empirical studies on earnings at a micro level and on growth at a macro level, both of which usually impose a lognormal distribution and control for proxies of human and physical capital.

NOTES

1. The log or Cobb–Douglas specification for preferences has been popularly used in the related literature on income inequality [e.g., Bénabou (1996); Galor and Moav (2004); Glomm and Ravikumar (1992); Zilcha (2003)] for the sake of tractability.

2. In the related literature, borrowing constraints are widely assumed for human capital investment in order to allow human capital to channel the influence of inequality on growth. In some models such as Loury (1981) and Mookherjee and Ray (2003), capital markets are entirely absent, an assumption we have adopted in this model. Murphy et al. (1989a) also consider imperfect competition as a possible contributing factor to industrialization.

3. What is essential for our results is strong enough complementarity between the human and nonhuman factors at an individual household level, which will become clear later. In other words, the essence of results in this paper may still hold when the assumption of missing markets is replaced by a more realistic one that can capture such complementarity.

4. Like the log preference and the Cobb–Douglas production function, full depreciation of capital in one period is another popular simplifying assumption in the related studies.

5. In Murphy et al. (1991), the allocation of talent to rent-seeking areas or entrepreneurship has different implications for growth. In this paper, we treat innate ability as a contributing factor in individual learning, as in Bénabou (1996) and Zhang (2005).

6. The results will be similar if parental time inputs are allowed in the education of their children, so that $h_{t+1} = \xi[q_t^{\nu}(e_t h_t)^{1-\nu}]^{\epsilon} H_t^{1-\epsilon}$, where $e_t h_t$ stands for the effective time input by a parent. We thus abstract from parental time inputs for simplicity.

7. If there were perfect factor markets, as in the standard model, an individual's income would be $r_t k_t + w_t h_t$, where the interest rate r and the wage rate w are determined by the average or aggregate levels of physical and human capital. Thus, the average or aggregate level of physical capital K_{t+1} would be a sum of individual bequests $k_{t+1} = \Gamma_k(r_t k_t + w_t h_t)$, which is a linear function of individual states (h_t, k_t). A mean-preserving spread in k_t in this case would not affect average or aggregate physical capital K_{t+1}.

8. In the absence of human capital, this model would reduce to a neoclassical growth model without long-run growth. In the absence of physical capital, however, this model would still yield sustainable growth in the long run.

9. The other limiting case with $\epsilon \to 1$ is not relevant, because then inequality would be ever-rising without any upper bound.

10. We choose \$1,000 and \$3,000 (1980 U.S. dollars), respectively, as the critical levels to divide 107 countries with available data into the three groups. In this way, we get similar numbers of countries for the three groups.

11. As cited by Glomm and Ravikumar (1992), the vast majority of primary and secondary students in the United States have attended public schools since the late 19th century.

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