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Relativistic Gaussian laser beam self-focusing in collisional quantum plasmas

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Abstract

Propagation of Gaussian X-ray laser beam is presented in collisional quantum plasma and the beam width oscillation is studied along the propagation direction. It is noticed that due to energy absorption in collisional plasma, the laser energy drops to an amount less than the critical value of the self-focusing effect and consequently, the laser beam defocuses. It is found that the oscillation amplitude of the laser spot size enhances while passing through collisional plasma. For the greater values of collision frequency, the beam width oscillates with higher amplitude and defocuses in a shallower plasma depth. Also, it is realized that in a dense plasma environment, the laser self-focusing occurs earlier with the higher oscillation amplitude, smaller laser spot size and more oscillations.

Keywords: Beam width parameter; Dielectric constant; Quantum plasma; Self-focusing; X-ray laser

1. INTRODUCTION

Recent continuous advances in ultra-intense short-pulse lasers and their various applications motivated the research activities in this field such as mono-energetic electron generation, ion block acceleration, and inertial confinement fusion with the fast igniter scheme (Sari *et al.*, 2005; Koyama *et al.* 2006; Badziak *et al.*, 2006). When an intense laser beam propagates in the plasma, due to the induced quivering motion of electrons, the plasma refractive index changes (Hora, 1975; Sadighi-Bonabi *et al.*, 2009). In this condition, the plasma behaves initially similar to a positive lens that decreases the laser spot size and continues its focusing and defocusing through plasma (Hora, 1985; Faure *et al.*, 2002; Pukhov, 2003). In order to achieve a better interaction of laser with plasma, deeper penetration of high-intensity beams in plasma is required. The self-focusing effect plays an important role in the recent advances of the laser–plasma interaction and particularly in fast ignition systems. It is noticed that this effect enables the laser beam to propagate over several Rayleigh lengths in plasma (Schlenvoigt *et al.*, 2007; Boyd *et al.*, 2008).

The laser self-focusing has been studied in the interaction of the laser beam with both homogeneous and inhomogeneous plasmas (Upadhyay *et al.*, 2002; Varshney *et al.*, 2006; Kaur & Sharma, 2009; Sharma & Kourakis, 2010). Prakash studied the propagation of a Gaussian laser beam in a radial inhomogeneous medium with multi-photon absorption (Prakash, 2005). Furthermore, in the classical regime, the propagation of intense Gaussian laser beam in collisional and collisionless plasmas has been studied by many researchers (Upadhyay *et al.*, 2002; Sharma *et al.*, 2003; Sharma & Kourakis, 2010; Prakash, 2005; Sodha & Sharma, 2006; Varshney *et al.*, 2006; Kaur & Sharma, 2009; Etehadi Abari & Shokri, 2012; Gupta *et al.*, 2013; Jafari Milani *et al.*, 2014). In principle, classical plasma is introduced by high temperature and low density, while quantum plasma is characterized by high density and low temperature (Shukla, 2009; Chandra *et al.*, 2012). Distinction between the classical and the quantum models for plasma is determined by the parameter $\chi = T_F/T$, where T_F and T represent the Fermi temperature and the plasma temperature, respectively. If the plasma temperature is equal to or less than the electron Fermi temperature ($\chi \geq 1$), then the quantum effects are dominant and the relevant statistical distribution changes from Maxwell–Boltzmann to Fermi–Dirac. The Fermi temperature could be defined as follows (Landau & Lifshitz, 1980):

$$k_B T_F = E_F = \frac{\hbar^2}{2m_e}(3\pi^2 n_e)^{2/3} \tag{1a}$$

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$$\chi = \frac{T_F}{T} = \frac{1}{2}(3\pi^2)^{2/3}(n_e \lambda_B^3)^{2/3} \tag{1b}$$

n_e represents the electron plasma density. On the other hand, the quantum effects could be measured by the thermal de Broglie wavelength, $\lambda_B = \hbar/(m_e k_B T)^{1/2}$ ($\hbar$, m_e, and k_B are the rationalized Planck's constant, the electron mass, and the Boltzmann constant, respectively). λ_B explains roughly the spatial extension of the particle wave function by considering the quantum uncertainty. Therefore, the quantum effects are important when the de Broglie wavelength of the electron is equal to or greater than the average inter-electron distance, $n_e^{-1/3}$, that is, $n_e \lambda_B^3 \geq 1$ (Manfredi, 2005; Shukla & Eliasson, 2010). In the classical regime, the de Broglie wavelength is small enough to ignore overlapping of the wave functions and quantum interferences and consider the particles as points. It is realized that the quantum effects get more effective with the increase in the plasma density or the decrease in the plasma temperature (Patil & Takale, 2013). Another important parameter in quantum plasma, characterized as the ratio of the interaction energy, E_{int}, to the Fermi energy, E_F, is the quantum coupling parameter, g_Q,

$$g_Q = \frac{E_{int}}{E_F} = \frac{2}{(3\pi^2)^{2/3}} \frac{e^2 m_e}{\hbar^2 \varepsilon_0 n_e^{1/3}} \tag{2}$$

For $g_Q \geq 1$, the quantum plasma is collisional and for $g_Q < 1$, it is collisionless and the mean-field effects are dominant (Manfredi, 2005). Hence, for large plasma densities, the quantum plasma is collisionless. By considering the Pauli's exclusion principle, one could find, with increasing of plasma density, the average kinetic energy of the plasma also increases and causes to decrease the quantum coupling parameter (Manfredi, 2005).

Quantum plasmas is strongly sound in many environments, that is, in astrophysical systems (Opher *et al.*, 2001), biophotonics (Barnes *et al.*, 2003), neutron stars (Chabrier *et al.*, 2002), ultra-cold plasmas (Killian, 2006), ultra-small electronic devices (Markowich *et al.* 1990), laser-produced plasmas (Kremp *et al.*, 1999; Andreev, 2000; Kremp *et al.*, 2005; Marklund, 2005; Becker *et al.*, 2006), fast ignition (Hu & Keitel, 1999; Andreev, 2000; Azechi, 2006; Marklund & Shukla, 2006; Shukla & Stenflo, 2006; Glenzer & Redmer, 2009), micro plasmas (Becker *et al.*, 2005), quantum well, and quantum diodes (Ang *et al.*, 2006; Ang & Zhang, 2007). The distribution of electrons in quantum plasma is explained by the Wigner function (Wigner, 1932; Hillery *et al.*, 1984; Kozlov & Smolyanov, 2007). In recent years, instabilities in plasma, propagation of magneto-acoustic soliton and ion-acoustic solitary Fermi temperature have been studied in quantum plasma physics (Hussain & Mahmood, 2011; Ghosh *et al.*, 2012; Chandra & Ghosh, 2012). Eliasson & Shukla (2008) has presented the fluid equations of quantum plasma and the dielectric function of an unmagnetized collisionless quantum plasma has been also introduced (Ali & Shukla, 2006). In 1970, Mermin derived the dielectric permittivity for collisional quantum plasma (Mermin, 1970). Moreover, Latyshev derived the dielectric permittivity using a kinetic equation in the momentum space in the relaxation approximation (Latyshev & Yushkanov, 2014).

The nonlinear effects are present more effectively in quantum plasma than in the classical case (Shukla *et al.*, 2006; Shukla & Eliasson, 2010). In the quantum regime, the laser spot size oscillates with greater frequency and less amplitude while propagating deeper in the medium. Therefore, these effects will result in stronger self-focusing compared with the classical regime (Shukla *et al.*, 2006; Shukla & Eliasson, 2010; Marklund & Brodin, 2007; Bulanov *et al.*, 2009; Patil *et al.*, 2013). Despite the fact that the laser self-focusing in collisionless quantum plasma has been studied over the last decades (Manfredi, 2005; Ali & Shukla, 2006; Shukla *et al.*, 2006, Shukla and Eliasson (2010); Na & Jung, 2009; Habibi & Ghamari, 2014), the interaction of relativistic laser intensities with collisional quantum plasma has never been presented.

The current study is devoted to investigate the self-focusing of the relativistic Gaussian X-ray laser beam in collisional quantum plasma. Using the ansatz for the electric field in the wave equation, together with the Wentzel–Kramers–Brillouin (WKB) and the paraxial approximations, a mathematical formulation for the beam-width parameter in collisional quantum plasma is obtained. By considering the dielectric permittivity derived by Latyshev (Latyshev & Yushkanov, 2014), the evolution of the beam-width parameter is introduced along the propagation direction. It is noticed that in collisional plasma, the laser beam width initially oscillates along the propagation direction (focusing) and then defocuses due to divergence and energy absorption. Greater collision frequencies result in the higher energy absorption rate, and as a consequence, the laser spot size oscillates with higher amplitude and defocuses earlier. Furthermore, the effect of the collision frequency and the plasma density on the self-focusing conditions is thoroughly explained. It is noticed that in denser plasmas, the laser self-focusing occurs earlier with higher oscillation amplitude, smaller spot size, and more oscillations.

2. THEORY

The cylindrical coordinate system is used to study the propagation of a Gaussian laser beam along the z-axis. In this coordinate, the scalar wave equation is,

$$\frac{\partial^2 E}{\partial z^2} + \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial E}{\partial r}\right) + \frac{\omega^2}{c^2}\varepsilon(r,z)E = 0 \tag{3}$$

Akhmanov *et al.* (1968) and Sodha *et al.* (1974) suggest the solution of Eq. (3), as the following ansatz:

$$E(r,z) = A(r,z)\exp\left(i\omega t - i\int_0^z k(z)\,dz\right), \tag{4}$$

where $k = \sqrt{\varepsilon_{0r}}\omega/c$, and ϵ_{0r} is the real part of the linear dielectric constant. By considering the WKB approximation

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for slowly converging and diverging beams, and neglecting $\partial^2 A/\partial z^2$, the following equation is obtained by substituting Eq. (4) into Eq. (3):

$$2ik(z)\frac{\partial A(r,z)}{\partial z} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial A(r,z)}{\partial r}\right) + \frac{\omega^2}{c^2}(\varepsilon(r,z) - \varepsilon_{0r})\,A(r,z) \quad (5)$$

The complex amplitude, $A(r,z)$, is expressed below:

$$A(r,z) = A_0(r,z)\,\exp(-ik(z)S(r,z)), \quad (6)$$

$S(r,z)$ is the eikonal function which is complex for collisional plasmas (Wang *et al.*, 2011),

$$S(r,z) = S_r(r,z) + i\,S_i(r,z), \quad (7)$$

In this relation, S_r and S_i are real functions, where S_i represents the decay of the laser intensity during propagation in an absorbing plasma. In the paraxial approximation, the dielectric constant of absorbing plasma can be written similar to the case of non-absorbing plasma (Wang *et al.*, 2011),

$$\varepsilon(r,z) = \varepsilon_0(z) + \varepsilon_2(r,z) \quad (8)$$

For collisional plasma, the dielectric constant is a complex function. Therefore,

$$\varepsilon_0(z) = \varepsilon_{0r}(z) + i\varepsilon_{0i}(z) \quad (9a)$$

$$\varepsilon_2(r,z) = \varepsilon_{2r}(r,z) + i\varepsilon_{2i}(r,z) \quad (9b)$$

Substituting $A(r,z)$ and $S(r,z)$ from Eqs. (6) and (7) into Eq. (5), and separately equating the real and imaginary parts to zero, Eqs. (10a) and (10b) are resulted,

$$2\frac{\partial S_r}{\partial z} + \left(\frac{\partial S_r}{\partial r}\right)^2 - \left(\frac{\partial S_i}{\partial r}\right)^2 = \frac{1}{k^2 A_0}\left(\frac{\partial^2 A_0}{\partial r^2} + \frac{1}{r}\frac{\partial A_0}{\partial r}\right) + \frac{2}{kA_0}\frac{\partial A_0}{\partial r}\frac{\partial S_i}{\partial r} + \frac{1}{k}\left(\frac{\partial^2 S_i}{\partial r^2} + \frac{1}{r}\frac{\partial S_i}{\partial r}\right) + \frac{\varepsilon_{2r}}{\varepsilon_{0r}} \quad (10a)$$

$$\frac{\partial A_0^2}{\partial z} + \frac{\partial A_0^2}{\partial r}\frac{\partial S_r}{\partial r} + A_0^2\left(\frac{\partial^2 S_r}{\partial r^2} + \frac{1}{r}\frac{\partial S_r}{\partial r}\right) + 2A_0^2 k\left(\frac{\partial S_i}{\partial z} + \frac{\partial S_i}{\partial r}\frac{\partial S_r}{\partial r}\right) = A_0^2 k\frac{\varepsilon_{0i} + \varepsilon_{2i}}{\varepsilon_{0r}} \quad (10b)$$

In the next step, S_r and S_i are defined (Sodha *et al.*, 1974; Prakash, 2005),

$$S_r(r,z) = \varphi_r(z) + \frac{r^2}{2f}\frac{df}{dz} \quad (11a)$$

$$S_i(r,z) = \varphi_i(z) + \frac{r^2}{2}\beta_i(z) \quad (11b)$$

where ϕ_r and ϕ_i represent the axial phase and attenuation functions, independently; r and f are the radial coordinate of the cylindrical system and the dimensionless beam-width parameter, respectively. Considering Eq. (8) and the paraxial approximation, ϵ_{2r} and ϵ_{2i} of Eq. (9b) are represented,

$$\varepsilon_{2r}(r,z) = r^2\theta_r(z) \quad (12a)$$

$$\varepsilon_{2i}(r,z) = r^2\theta_i(z) \quad (12b)$$

Besides, for the laser beam with the Gaussian intensity distribution, the real amplitude function, A_0, is given as the following:

$$A_0(r,z) = \frac{A_{00}}{f(z)}\exp\left(\frac{-r^2}{2r_0^2 f^2(z)}\right) \quad (13)$$

A_{00} and r_0 are the initial electric field amplitude and the initial laser beam width, respectively. Expanding A_0 in a power series of r^2 and substituting Eqs. (11a), (12a) and (13) into Eq. (10a), and by separately equating the r independent and r^2 terms to zero, the two following relations are obtained:

$$\frac{d\phi_r(z)}{dz} = \frac{\beta_i(z)}{k(z)} - \frac{1}{k^2(z)\,r_0^2 f^2(z)} \quad (14a)$$

$$\frac{1}{f(z)}\frac{d^2 f(z)}{d^2 z} = \left(\frac{1}{r_0^2 k(z) f^2(z)} - \beta_i(z)\right)^2 + \frac{\theta_r(z)}{\varepsilon_{0r}(z)} \quad (14b)$$

Similarly Eqs. (11b), (12b), and (13) are substituted into Eq. (10b), and these relations are resulted,

$$2\frac{d\phi_i(z)}{dz} = \frac{\varepsilon_{0i}(z)}{\varepsilon_{0r}(z)}, \quad (15a)$$

$$\frac{d\beta_i(z)}{dz} + \frac{2}{f(z)}\frac{df(z)}{dz}\beta_i(z) = \frac{\theta_i(z)}{\varepsilon_{0r}(z)} \quad (15b)$$

More to the point, in quantum plasma physics, by using the quantum kinetic Wigner–Vlasov–Boltzmann (WVB) equation and the Bhatnagar, Gross, and Krook (BGK) collision integral in the coordinate space, the plasma dielectric function can be derived (Latyshev & Yushkanov, 2014),

$$\varepsilon = 1 + \frac{\hbar\, e^2}{\pi^2\gamma m_e \omega q^2}\int R(k,q,\omega,v)kq\,dk \quad (16)$$

$$R(k,q,\omega,v) = \frac{f_{k+q/2} - f_{k-q/2}}{E_{k+q/2} - E_{k-q/2}}\left(1 - \frac{\hbar(\omega+iv)(1-\alpha(q,\omega,v))}{E_{k+q/2} - E_{k-q/2} + \hbar(\omega+iv)}\right) \quad (17)$$

$$1 - \alpha(q,\omega,v) = \frac{\omega(B(q,0) - B(q,\omega+iv))}{\omega B(q,0) + ivB(q,\omega+iv)} \quad (18a)$$

$$B(q,\omega+iv) = \frac{1}{4\pi^3}\int\frac{f_{k+q/2} - f_{k-q/2}}{E_{k+q/2} - E_{k-q/2} + \hbar(\omega+iv)}dk \quad (18b)$$

$$E_k = \frac{\hbar^2 k^2}{2m}, \quad f_k = (\exp(E_k/k_B T) + 1)^{-1} \quad (18c)$$

where f_k and q represent the electron distribution and the wave vector, respectively. Also, ω and v are the laser frequency and the collision frequency of the plasma electrons, seperately. By considering Eqs. (4), (6), (7), and (13), the relativistic factor, $\gamma = \sqrt{1 + e^2 EE^*/(m_e^2 \omega^2 c^2)}$, is expressed as a function of the radial coordinate, r, and the beam-width parameter, f,

$$\gamma = \left[1 + \frac{e^2}{m_0^2 \omega^2 c^2} \frac{A_{00}^2}{f^2} \exp\left(\frac{-r^2}{r_0^2 f^2}\right)\right]^{1/2} \tag{19}$$

This equation can be expanded in a power series of r^2. It should be realized that by using Eqs. (16)–(18) and (19), the dielectric constant is calculated and divided into real and imaginary parts. Then all parts of the dielectric constant in Eqs. (8) and (9), that is, ϵ_{0r}, ϵ_{2r}, ϵ_{0i}, and ϵ_{2i} are obtained and according to Eq. (12), the θ_r and θ_i functions are found. It is better to express Eqs. (14) and (15) in terms of the succeeding dimensionless variables,

$$\xi = z/(r_0^2 k), \quad \beta_i = \beta/(r_0^2 k), \quad \varphi = k\varphi_i \tag{20}$$

ξ represents the dimensionless distance that is concerned to the Rayleigh length, and c is the velocity of light in vacuum. By utilizing these dimensionless variables, Eqs. (14) and (15) take these forms,

$$f'' = f\left(\beta - \frac{1}{f^2}\right)^2 + \left(\frac{r_0 \omega}{c}\right)^2 \frac{p_0}{2f^3} \theta_r \left(1 + \frac{p_0}{f^2}\right)^{-3/2} \tag{21a}$$

$$\beta' = \frac{-2f'}{f}\beta + \left(\frac{r_0 \omega}{c}\right)^2 \frac{p_0}{2f^4} \theta_i \left(1 + \frac{p_0}{f^2}\right)^{-3/2} \tag{21b}$$

$$\varphi' = \frac{\varepsilon_{0i}}{2}\left(\frac{r_0 \omega}{c}\right)^2 \left(1 + \frac{p_0}{f^2}\right)^{-1/2} \tag{21c}$$

where $e^2 A_{00}^2/(m_0^2 \omega^2 c^2) = p_0$ is the dimensionless quantity proportional to the laser beam power. Assuming a Gaussian intensity distribution and the initial plane wave front, the boundary conditions are,

$$f = 1, \quad df/d\xi = 0, \quad \beta = 0, \quad \varphi = 0 \text{ at } \xi = 0 \tag{22}$$

Additionally, from Eqs. (4), (6), (7), (13) and (20), the irradiation intensity, $I(r, \xi)$ is

$$\begin{aligned} I(r, \xi) = EE^* &= \frac{A_{00}^2}{f^2} \exp\left[-\frac{r^2}{r_0^2}\left(\frac{1}{f^2} - \beta\right)\right] \exp(2\varphi) \\ &= \frac{A_{00}^2}{f^2} \exp\left(-\frac{r^2}{r_0^2 F^2}\right) \exp(2\varphi) \end{aligned} \tag{23}$$

and subsequently it is deduced that the modified beam-width parameter should have the following form:

$$\frac{1}{F^2} = \frac{1}{f^2} - \beta \tag{24}$$

In Eq. (23), exp(2φ) is related to the energy attenuation. It should be realized when the nonlinear absorption in the plasma is considered, f and F are different parameters. The axial intensity is determined by $f\,(\propto 1/f^2)$ but the radial intensity depends on both f and the modified beam-width parameter, F. Therefore, in this manner F (not f) corresponds to the beam width, that is, $r_0 F$. From now on, for simplicity, "modified beam-width parameter" is meant by "beam-width parameter" through the text.

3. RESULTS

By means of the fourth-order Runge–Kutta method, Eqs. (21) are numerically solved under the boundary conditions given by Eq. (22). Equation (23) is plotted with plasma and laser parameters as follows: $\omega = 1.778 \times 10^{20}\ \mathrm{s}^{-1}$, $T = 1000\ °\mathrm{K}$, $r_0 = 20\ \mu\mathrm{m}$, $n_e = 4 \times 10^{22}\ \mathrm{cm}^{-3}$, and $p_0 = 1$. The plasma temperature and density are chosen in a way that the conditions of collisional quantum plasma, that is, $g_Q \geq 1$ and $n_e \lambda_B^3 \geq 1$, are satisfied. As the laser beam in collisional plasma is focused, the spatial diffraction becomes stronger and it grows until becoming predominant. The laser spot size increases after a minimum value and converges again, showing an oscillatory behavior.

Figure 1 makes a comparison for the laser self-focusing effect between two distinct mediums, a collisional plasma with $v = 0.5\omega_p$ (dashed curve) and a collisionless plasma (solid curve). It is noticed that initially the beam-width parameter shows similar oscillations to ξ for both cases. Due to energy absorption in the collisional plasma, the laser beam power reduces and when it becomes lower than the critical value of the self-focusing, the diffraction effects overcome the focusing. Accordingly, F initially oscillates with ξ (self-focusing) and then defocuses due to energy absorption. For the collisional plasma, however, the oscillation amplitude of the laser beam width enhances by passing through the plasma, and the laser beam defocuses at a few Rayleigh lengths.

Figure 2 shows the dependency of the beam-width parameter on the propagation distance for various collision frequencies, $v = 0.2\omega_p$ (solid), $0.5\omega_p$ (dash), and $0.9\omega_p$ (dash-dot). For greater collision frequencies, the laser absorption rate increases. Therefore, the oscillation amplitude of the beam width enhances and the laser beam defocusing occurs sooner.

Figure 3 compares the beam-width parameter for different plasma densities, $n_e = 4 \times 10^{22}\ \mathrm{cm}^{-3}$ (solid), $n_e = 4 \times 10^{21}\ \mathrm{cm}^{-3}$ (dash-dot), and $n_e = 10^{21}\ \mathrm{cm}^{-3}$ (dash). In denser plasmas, the self-focusing length is reduced, and the laser spot size acquires a smaller minimum and higher oscillation amplitude. These results are valid for both collisional and collisionless quantum plasmas (Patil & Takale, 2013). In other words, higher plasma densities induce earlier laser self-focusing. Also, by increasing the plasma density, energy absorption increases subsequently and the laser beam defocuses quicker.

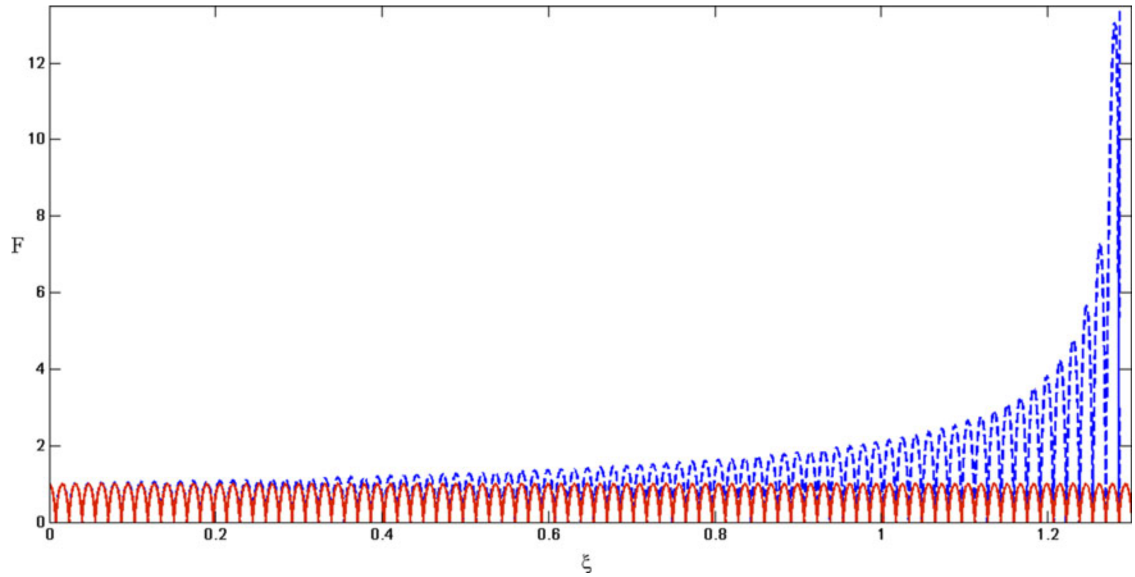

Fig. 1. Comparing the beam-width parameter for the collisionless plasma (solid) and the collisional plasma with $\nu = 0.5\omega_p$ (dash) at $T = 1000$ K, $n_e = 4 \times 10^{22}$ cm^{-3}.

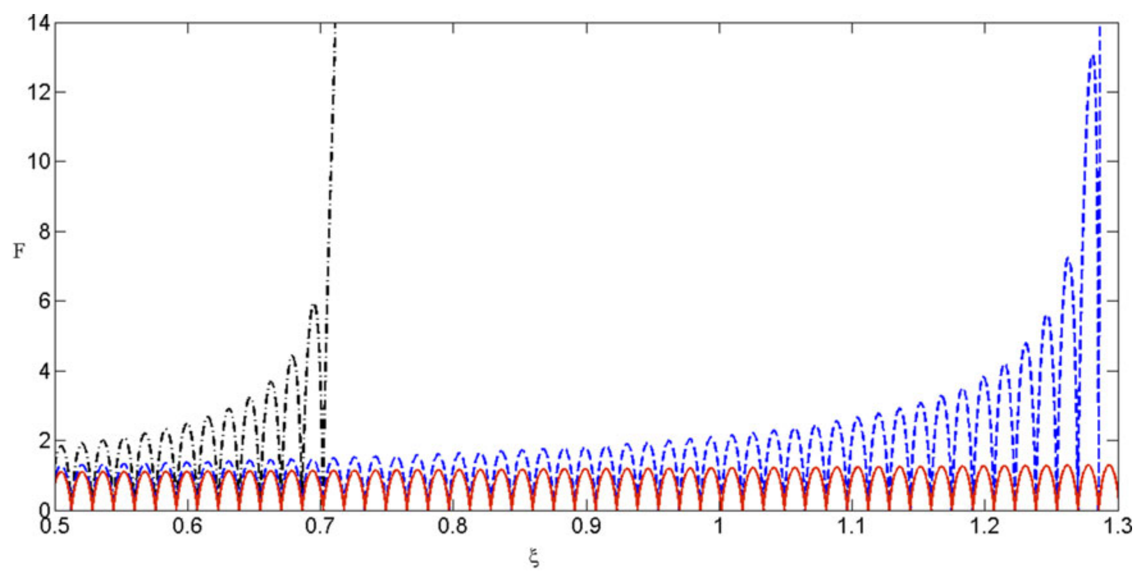

Fig. 2. Variation of the beam-width parameter with ξ for different collision frequencies, $\nu = 0.2\omega_p$ (solid), $\nu = 0.5\omega_p$ (dash), and $\nu = 0.9\omega_p$ (dash-dot) with the temperature and density similar to the case of Fig. 1

4. CONCLUSION

The propagation of relativistic Gaussian X-ray laser beam is studied in collisional quantum plasma. In this scheme based on the obtained equations for the beam-width parameter, the laser power becomes lower than the critical value of the self-focusing effect. Therefore, the divergence effects overcome the self-focusing effects and the laser beam defocuses after a few oscillations in the plasma. On account of energy absorption in the plasma, it is found that in the propagation of the laser beam through collisional plasma, the oscillation amplitude of the laser beam width increases. In addition, it is noticed that for the greater collision frequencies, the laser energy absorption rate enhances and the laser spot size oscillates with higher amplitude and defocuses sooner. Furthermore, for higher densities in collisional plasmas, early and soon self-focusing with smaller spot size could be achieved.

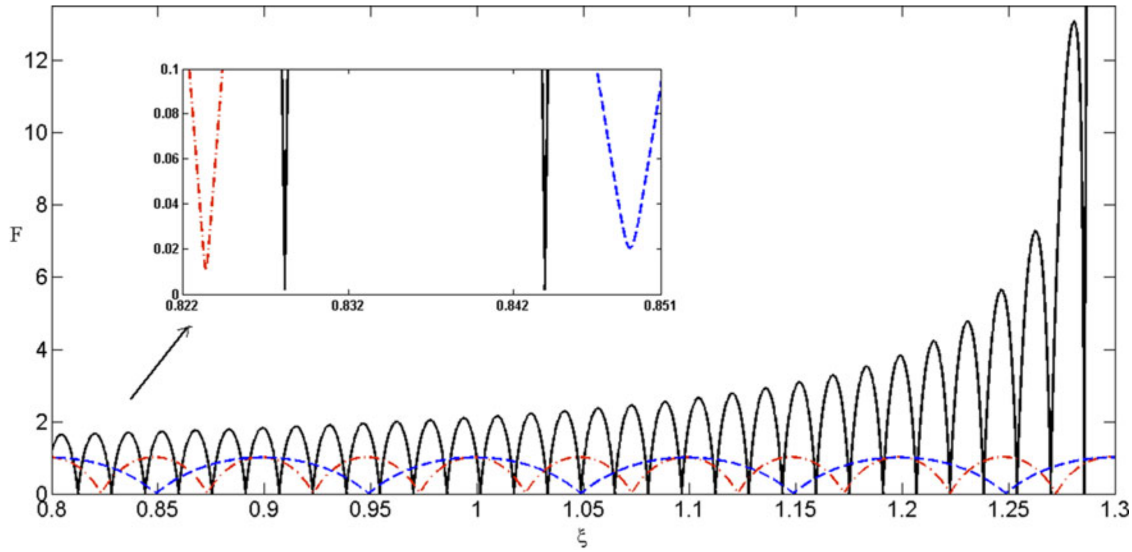

Fig. 3. Comparing the beam-width parameter for various plasma densities, $n_e = 4 \times 10^{22}\ \mathrm{cm}^{-3}$ (solid), $n_e = 4 \times 10^{21}\ \mathrm{cm}^{-3}$ (dash-dot), and $n_e = 10^{21}\ \mathrm{cm}^{-3}$ (dash) at $T = 1000$ K, $\nu = 0.5\omega_p$.

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