Avicenna on the Nature of Mathematical Objects

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ABSTRACT: Some authors have proposed that Avicenna considers mathematical objects, i.e., geometric shapes and numbers, to be mental existents completely separated from matter. In this paper, I will show that this description, though not completely wrong, is misleading. Avicenna endorses, I will argue, some sort of literalism, potentialism, and finitism.

RÉSUMÉ : Certains auteurs ont proposé qu'Avicenne considérait les objets mathématiques, à savoir les formes géométriques et les nombres, comme étant mentaux et complètement séparés de la matière. Dans cet article, je vais montrer que cette description, qui n'est pas complètement fausse, est cependant trompeuse. Avicenne approuve, je soutiendrai, une sorte de littéralisme, de potentialisme et de finitisme.

Keywords: Avicenna, mathematical objects, estimation, immateriality, separability, dependency on materiality

1. Introduction

Although many impressive studies have dealt with Avicenna's views on the philosophy of logic, his views on the philosophy of mathematics[1] have been

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[1] It is well known that Avicenna, following the Aristotelian tradition, considers some sciences, like music and astronomy, to be branches of mathematical sciences. But, having the modern conception of *mathematics* in mind, I will focus only on geometry and arithmetic. More precisely, I will focus on what Avicenna (2005, I.3, 17, l. 10)

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largely neglected in the scholarship on Islamic-Arabic philosophy. Avicenna's approach to philosophical questions about mathematics has been discussed in only a few works.[2] There are at least two reasons why the existing literature is unable to provide a plausible overall understanding of Avicenna's philosophy of mathematics. First, although some features of Avicenna's philosophical views on mathematics have been touched upon in the literature, many crucial and significant questions remain unanswered. For example, to the best of my knowledge, no one has discussed the details of Avicenna's views on the nature of mathematical truths and the ontological grounds of their necessity. Second, there are some views on the philosophy of mathematics attributed to Avicenna in the literature that seem to be universally accepted as his own; however, there is strong textual evidence suggesting that these attributions are imprecise or even false. I will discuss one such view in this paper. I think these facts demonstrate a clear need for further studies on Avicenna's philosophy of mathematics. In this paper, I will confine myself to the ontology of mathematics, and will try to shed new light on Avicenna's views concerning the nature of mathematical objects (e.g., numbers and geometric shapes).

What are mathematical objects? Do they exist at all? If yes, where? What is their nature? These are some of the questions to which I will try to identify Avicenna's answer. Some authors, without addressing the details of his views on the philosophy of mathematics, have hastily concluded that Avicenna's position regarding the nature of mathematical objects is simply Aristotelian.[3] These authors have overlooked two facts: on the one hand, in the absence of a careful

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calls 'pure mathematics.' So, when I speak of Avicenna's philosophy of mathematics, I speak of his philosophical views concerning three dimensional Euclidean geometry and the arithmetic of natural numbers. However, there is a difference between Avicenna's understanding of the notion of natural numbers and ours. He, following Aristotle (*Metaphysics* XIV, 1088a6-8), believes that *one* is not a number, and that numbers begin with *two*. See Avicenna (2005, III.3-4). For the sake of simplicity, I do not tackle his views concerning the arithmetic of other rational or irrational numbers. These problems are briefly discussed by Rashed (1984) and (2008, Sec. 2).

2 McGinnis (2007, 185, n. 41) confirms the lack of sufficient studies on Avicenna's philosophy of mathematics. To the best of my knowledge, Ardeshir's (2008) paper is the only work devoted exclusively to Avicenna's philosophy of mathematics. Papers by Al-Daffa and Stroyls (1984) and Rashed (1984) have focused mostly on Avicenna's technical innovations in mathematics, rather than on his philosophical views. In his recent (2016) book, Tahiri argues that Avicenna's philosophical understanding of mathematics plays an influential role in the development of his general theory of knowledge. Tahiri tries to establish this claim by discussing some aspects of Avicenna's philosophy of arithmetic.

3 See Al-Daffa and Stroyls (1984, 90) and McGinnis (2006a, 68).

inspection of Avicenna's writings on the ontology of mathematics, it is perilous to ascribe a full-blown Aristotelian position to him. There are many topics on which Avicenna's views differed, either in part or in full, from those of Aristotle. Therefore, only a detailed textual analysis can reveal whether the ontology of mathematics is one of those topics. On the other hand, there is a wide range of different, even mutually inconsistent, positions ascribed to Aristotle concerning the ontology of mathematics.[4] These positions vary from a *fictionalist* one (according to which mathematical objects do not exist in any sense) to a *literalist* position (according to which such objects do literally exist in the material world).[5] Merely stating that Avicenna is Aristotelian does not help us to situate Avicenna's philosophy of mathematics in relation to this diverse set of views on the ontology of mathematical objects. More substantial clarification is in order.

There is a growing tendency in the scholarship on Avicenna to defend an interpretation according to which he believes that mathematical objects are mental existents. John McGinnis, Mohammad Ardeshir, Allan Bäck, and Hassan Tahiri uphold this interpretation.[6] They believe that "Avicenna's ontology implies that mathematical objects are mental objects"[7] and that he sees these "objects as mental constructs abstracted from concrete physical objects."[8] Given this understanding of Avicenna, mathematical objects are mental entities purely abstracted and separated from matter. Although they are not abstract

4 For a classic work on Aristotle's philosophy of mathematics, see Apostle (1952). For a recent work on this topic, see Bostock (2012). Franklin (2014) defends a modern reconstruction of an Aristotelian philosophy of mathematics.

5 Mueller (1970, 1990) and Lear (1982) attribute variations of literalism to Aristotle. Hussey (1991) defends a fictionalist interpretation of Aristotle. The strengths and weaknesses of these interpretations have been discussed by Corkum (2012). White (1993) discusses a spectrum of miscellaneous interpretations of the nature and location of mathematical objects in the framework of Aristotle's philosophy.

6 See, respectively, McGinnis (2006a), Ardeshir (2008), Bäck (2013), and Tahiri (2016). While McGinnis believes that Avicenna is fully Aristotelian concerning the nature of mathematical objects, Bäck and Tahiri distinguish Avicenna's view from Aristotle's. It seems that Bäck, like Hussey (1991), considers interpreting Aristotle in a fictionalist framework to be tendentious (Bäck 2013, 100), but Tahiri attributes a *potentialist* position to Aristotle (Tahiri 2016, Sec. 3.3) according to which mathematical objects (at least numbers) only potentially exist. Fictionalism and potentiallism are two distinct, though not necessarily incompatible, positions.

7 Ardeshir (2008, 43).

8 McGinnis (2006a, 68).

Platonist[9] entities with extramental independent (or autonomous) existence, they are mental constructions and intentional objects[10] entirely separated from matter. Some of the proponents of this position have no hesitation in interpreting Avicenna's philosophy of mathematics as a *constructivist* or *intuitionist* philosophy, in the modern senses of these notions.[11]

There is an underrepresented view, on the other hand, which says that mathematical objects "are always [i.e., even in our minds] mixed with matter, but not, however, with a specific kind of matter […]. As objects of mathematical knowledge, they undergo a degree of abstraction whereby the mathematician will consider their properties dissociated from any specific kind of material, but not, however, from any matter whatsoever."[12] Both of these views (i.e., the view that mathematical objects are mental objects completely separated from matter and the view that mathematical objects are separable from any specific matter but not from any matter whatsoever) are *to some extent* true. But I will show that, as interpretations of Avicenna, they are imprecise.

In the following section, I will draw a general sketch of Avicenna's views on the nature of mathematical objects. I will show that in his philosophical system geometric shapes and numbers are accidents of material substances existing in the physical world. They are associated with specific kinds of matter in the

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9 Avicenna criticizes mathematical Platonism in Chs. 2 and 3 of Bk. VII of *The Metaphysics of The Healing*. For a commentary on these chapters, see Marmura (2006). A full examination of the tenability of his objections to different versions of Platonism would merit an independent study.

10 I borrow the phrase 'intentional objects' from Tahiri's (2016) preferred terminology. According to his understanding of Avicenna, mathematical objects, and particularly numbers, "are intentional objects, the product of a specific intentional act that makes it possible to generate objects beyond the sensible experience such as infinite numbers" (2016, 41). Tahiri believes that *intentionality* is the most substantial notion in Avicenna's metaphysics: "If there is one word that can sum up Ibn Sīnā's al-Ilāhiyāt, it is without doubt *intentionality*" (2016, 69). Tahiri's understanding of intentionality seems very similar to Crane's (2001, 2013) view, according to which all mental phenomena are intentional. I seriously doubt the reliability of such an interpretation of Avicenna. In particular, I think that Tahiri overestimates the significance of the notion of intentionality (in the sense mentioned) in interpreting Avicenna. However, I avoid further discussion on this issue in the present paper. See Banchetti-Robino's (2004) and Black (2010) for Avicenna's treatment of intention and intentionality.

11 See McGinnis (2006a, 64) and Tahiri (2016, Sec. 5.2.1). While they emphasize the affinities between Avicenna's *ontology* of mathematical objects and the modern constructivist/intuitionist ontology of mathematics, Ardeshir (2008, 57-58) highlights similarities between Avicenna's *epistemology* of mathematics and the modern intuitionist epistemology of mathematics.

12 Marmura (2005, xix). Marmura (1980) defends the same position.

extramental world but, in our minds, they can be separated from matter to different degrees. In Sections Three and Four, I will clarify that geometric shapes and numbers differ with respect to the mode and degree of their separability from matter.[13] Although both are separable from specific kinds of matter in our minds, geometric shapes, contrary to numbers, are inseparable from materiality itself. Geometric shapes have some sort of *ontological admixture* with material forms that is retained, even in the mind. Numbers, on the other hand, can be separated from materiality and all material forms in the mind. But, inasmuch as they are the subject of mathematical studies, they should still be *considered* as receptive of the accidents they (i.e., numbers) may have only when they are in numbered material things. Numbers, therefore, have some sort of *epistemological admixture* with materiality. In Section Five, I will show that Avicenna endorses the existence of *perfect* mathematical objects in the external world. I will argue that there is no serious obstacle preventing us from attributing a full-blown literalism to him. Independently of the accuracy of such an attribution, the number of mathematical objects that do *actually* exist, in either the extramental or the mental realm, is finite, or so I will argue. There are an infinite number of mathematical objects that only *potentially* exist. So, the attribution of some sort of *finitism* and *potentialism* to Avicenna is unavoidable. In the last section, I conclude by discussing the main points on which I diverge from the mainstream understandings of Avicenna's philosophy of mathematics.

2. Mathematical Objects: A General Picture

Do mathematical objects exist? One may consider this question to be a paraphrased form of a more specific question: *Are mathematical objects mind-independent substances?* Nonetheless, from the perspective of Avicenna's philosophy, we should distinguish these two questions. His answer to the former question, but not the latter, is *trivially* positive. In Avicenna's philosophy, *existence* (*wujūd*) and *thingness*/*objecthood* (*shay'iyya*) are distinct but coextensive concepts.[14]

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[13] None of the aforementioned studies on Avicenna's philosophy of mathematics has investigated distinctions between the nature of geometric objects (i.e., geometric shapes) and the nature of arithmetical objects (i.e., numbers). Although Ardeshir (2008) discusses some general points about the subject matter of geometry (Sec. 2.1), his main discussion on the ontology of mathematical objects is focused on the nature of numbers (Sec. 2.2). Tahiri (2016) confines himself even more to the nature of numbers. Nonetheless, in a few footnotes, he briefly discusses the views of Farabi (20, n. 20), Avicenna (33, n. 17), and Averroes (54, n. 6) concerning the distinctions between numbers and geometric shapes. I will return to his note on Avicenna later in this paper.

[14] See, for example, Avicenna (2005, I.5). For a comprehensive study on Avicenna's treatment of the notion of *shay'iyya*, see Wisnovsky (2000).

This justifies the interchangeable use of 'existent' and 'object' in the context of Avicenna's philosophy.[15] It also entails that not only mathematics but all sciences are pre-scientifically committed to the existence of their subject matters, i.e., their objects. Every science, inasmuch as it is a science, studies some things or objects. Moreover, thingness/objecthood and existence are coextensive, such that science studies some existents and, consequently, carries ontological commitments to its subject matter. Subject matters of all sciences exist, but that does not entail that they exist in the same way. Existence can be qualified in many different ways, and everything exists in a certain way. Avicenna believes that mathematical objects do exist, but not as mind-independent substances. Hence, his answer to the second question is negative.

Mathematical objects or subject matters of mathematical studies are *quantities* (*kammiyyāt*). They are either (a) *continuous* (*muttaṣil*) quantities or magnitudes (*maqādīr*), which are geometric objects (or shapes), or (b) *discrete* (*munfaṣil*) quantities or numbers (*aʿdād*), which are arithmetical objects. Both of these two groups of mathematical objects are *accidents* of material substances,[16] which have *mind-independent* existence, but as accidents dependent on material substances, rather than as autonomous substances. Therefore, mathematical objects are not primarily mental constructions. However, we can separate them, in our minds, from the particular material substances to which they are attached in the extramental realm. Nonetheless, even in our minds, they have some sort of dependency on matter and materiality. A careful analysis of Avicenna's writings on the classification of the sciences[17] reveals that only subjects of metaphysical studies can be completely released from all sort of dependencies on matter and materiality.

According to Avicenna's categorization of the sciences, two sciences are distinct either because they study objects with different natures or because they study objects with the same nature but from different aspects

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15 However, this view raises some controversial problems. For example, quiddity (*māhiyya*) as quiddity is something, so it should have some sort of existence. But this result seems in tension with one of Avicenna's famous doctrines, according to which quiddity is neutral relative to existence. The solution lies in the fact that existence can be qualified in different modes. See Marmura (1979, 1992), Black (1999), and Bertolacci (2012) for more discussions on this issue.

16 In Ch. 3 of Bk. III of *The Metaphysics of the Healing*, Avicenna argues that numbers are accidents. In the next chapter of the same book, he argues that magnitudes are accidents too.

17 Avicenna discusses this issue in several places. See, among others, Ch. 2 of Bk. I of *Isagoge* (1952), Chs. 1-3 of Bk. I of *The Metaphysics of the Healing* (2005), and Chs. 1-2 of the Metaphysics part of *Dānishnāmah* (2004). In his monumental paper (1980), Marmura discusses the detail of Avicenna's classification of sciences in the *Isagoge*. See also Gutas (2003).

(*ḥaythiyyāt*).[18] He believes that theoretical sciences are divided into natural sciences, mathematics, and metaphysics. Every object of a natural science is mixed with a specific kind of matter in both the external world and the mind. It *may* be possible to abstract this object, in the mind, from the specific kind of matter with which it is mixed. However, if we do so, the abstracted object cannot be the subject of theoretical studies in a natural science,[19] but should instead be studied by mathematics or metaphysics. Every object of a natural science, inasmuch as it is the subject of theoretical studies in a natural science, is associated with a specific kind of matter. The objects of mathematics are similarly mixed with specific kinds of matter in the external world. Nevertheless, we can separate these objects, in our minds, from all particular kinds of matter. Nonetheless, this does not mean that mathematical objects are completely separated from materiality itself and that they have no dependency on matter. An object free from any kind of dependency on materiality cannot be the object of mathematical study; it should be studied in metaphysics. Given this classification, mathematical objects are separable from any specific kind of matter, but they still have some sort of dependency on materiality itself. The following passage supports this understanding:

> **TEXT # 1:** The various kinds of the sciences therefore either [(a)] treat the consideration of existents inasmuch as they are in motion, both in cognitive apprehension (*taṣawwuran*) and in subsistence, and are related to materials of particular species; [(b)] treat the consideration of existents inasmuch as they separate from materials of a particular species in cognitive apprehension, but not in subsistence; or [(c)] treat the consideration of existents inasmuch as they are separated from motion and matter in subsistence and cognitive apprehension.
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> The first part of the sciences is natural science. The second is pure mathematical science, to which belongs the well-known science of number, although knowing the nature of number inasmuch as it is number does not belong to this science. The third part is divine science [i.e., metaphysics]. Since the existents are naturally divided into these three divisions, the theoretical philosophical sciences are these.[20]

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18 Marmura (1980, 240) believes that Avicenna appeals to an *ontological* basis for his categorization of the sciences. Admittedly, there are some phrases in Avicenna's writings that seemingly support this claim. But a detailed investigation of his writings shows that his classification is grounded on an intertwined group of *ontological* and *epistemological* criteria. Sometimes he distinguishes two sciences because of the different objects they study; this is an ontological ground. But he also, as we will see, accepts that two distinct sciences may study the same object from different aspects; this can be considered to be an epistemological ground.

19 For a recent work on Aristotle's treatment of the notion of abstraction, see Bäck (2014); for studies on different aspects of Avicenna's theory of abstraction, see Hasse (2001) and McGinnis (2006b).

20 Avicenna (1952, I.2, 14, l. 3-10). English translations of all passages from *Isagoge*, I.2 are Marmura's in his (1980) paper, unless otherwise specified.

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According to this passage, the objects of natural science are mixed with specific kinds of matter in both the extramental world and the mind. Mathematical objects are similarly associated with specific kinds of matter in extramental reality, but they can be separated from all specific kinds of matter in the mind. This passage does not explicitly say whether mathematical objects still have some sort of materiality or dependency on materiality in the mind. However, there is a hint that this is the case. It seems that if we purify number of all characteristics of materiality, then the result is number inasmuch as it is number which, as Avicenna says in the above text, is the subject of metaphysical, not mathematical, studies. Admittedly, we need more persuasive evidence to support the dependency of mathematical objects on materiality in the mind. The nature of this dependency (if there is such) is itself unclear. So, it is better to address the subtleties of Avicenna's view about geometric shapes and numbers. In the next section, I will discuss his views on geometric shapes.

3. Geometric Objects

Avicenna believes that geometric shapes, even in our minds, have some sort of *necessary* association with materiality. They are separable from all specific materials in our minds, but not from materiality itself. I will try to establish and expand this rendition of Avicenna by gleaning textual evidence for it from his various works. I start by analyzing a passage from the *Isagoge*:

> **TEXT # 2:** The things existing in external reality whose existence is not by our choice and action are first divided into two divisions: [(I)] one consists of things that are mixed with motion; [(II)] the second of things that do not mix with motion, for example, mind and God. The things that mix with motion are of two modes. They are either [(Ia)] such that they have no existence unless they undergo admixture with motion, as for example, humanness, squareness and the like; or [(Ib)] they have existence without this condition. The existents that have no existence unless undergoing admixture with motion are of two divisions. They are either [(Ia-1)] such that, neither in subsistence nor in the estimation (*al-wahm*) would it be true for them to be separated (*tujarrada*) from some specific matter (*māddatan mu'ayyanah*) as for example, the form of humanness and horseness; or else, [(Ia-2)] this would be true for them in the estimation but not in subsistence, as for example, squareness. For, in the case of the latter, its acquisition as a form (*taṣawwuruhu*) does not require that it should be given a specific kind of matter (*naw' māddah*) or that one should pay attention to some state of motion.[21]

If we consider what Avicenna says in this text about the quiddity (*mahiyya*) of squareness as his general view about quiddities of geometric shapes, then we should conclude that for him these quiddities have no existence unless

[21] Avicenna (1952, I.2, from 12, l. 11 to 13, l. 4).

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undergoing admixture with motion and matter.[22] In other words, it is impossible for them to be fully detached from materiality. Geometric shapes, *inasmuch as they are geometric shapes*, are necessarily mixed with materiality (because they lie under one of the subdivisions of the group (Ia) mentioned in the text). Nonetheless, according to this text, our *estimation* (*wahm*) has the ability to separate geometric shapes from all specific kinds of matter with which they may be mixed in the external world (because they lie under the group (Ia-2) of objects mentioned in the text). Geometric shapes, inasmuch as they are geometric shapes, are not necessarily associated with a specific kind of matter, though they are mixed with materiality.[23] A square, inasmuch as it is square, is not necessarily mixed with gold, wood, or any other specific kind of matter, but it is necessarily associated with materiality. Therefore, contrary to the concepts of *non-wooden triangle* or *non-golden triangle*, which are easily intelligible, the concept of *immaterial triangle* is a self-contradictory and unintelligible concept,[24] just as impossible as *round square*. Materiality is integrated with the quiddities of geometric shapes. Avicenna says that the core of the truth about geometric shapes, which Platonists and Pythagoreans have not ascertained, is that:

> **TEXT # 3**: [T]he *definitions* of geometric [shapes] among mathematical [objects] do not *utterly* dispense with matter, even though they can do without any given species of matter.[25]

Here, Avicenna explicitly embraces the notion that association with materiality is a characteristic of not only the extramental existence of geometric shapes,

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22 I have supposed that the admixture with *motion* is equivalent to the admixture with *matter*. Many authors have endorsed this equivalency in Avicenna's writings. For example, Hasse (2013, 115, n. 28) writes: "In the *Introduction* to *al-Shifā*, Avicenna differentiates beings mixed with motion (matter) from those unmixed, for which he gives 'the intellect and God' as examples." McGinnis (2010, 37) offers the same treatment of these two notions. TEXT # 5 confirms that Avicenna uses these two notions equivalently. But, according to some commentaries, movability is not equivalent to materiality for Aristotle. See Porro (2011, 278-279).

23 In contrast with geometric shapes, the quiddity of humanness, inasmuch as it is quiddity of humanness, is mixed with a specific kind of matter; i.e., flesh and blood. So, it cannot be abstracted from either materiality or even this specific kind of matter.

24 Humanness is inseparable from not only materiality, but also the particular kind of matter from which human beings are constituted; i.e., flesh and blood. Therefore, the concept of *immaterial humanness* and the concept of *humanness separated from flesh and blood* are both self-contradictory.

25 Avicenna (2005, VII.2, 249, ll. 2-4). I have modified Marmura's translation by putting 'shapes' instead of 'figures,' 'do not utterly' instead of 'absolutely do not,' and 'species' instead of 'kind.' The italics are mine.

but also of their definitions. This text explicitly shows that Ardeshir's reading of Avicenna, according to which mathematical objects are "not combined with matter in definition but with matter in existence,"[26] is misleading if not wrong. Geometric shapes, even in our minds, are connected to matter. However, it remains obscure how it is possible for a mental existent to be mixed with matter but not with a specific kind of matter. Obviously, when we consider a geometric shape in our minds as an object of our cognition, it is fully separated from the materiality that exists in the physical world. So, the materiality from which we cannot separate geometric shapes in our estimation is not the former kind of materiality existing in the extramental world.[27] Geometric shapes are associated with some sort of *estimative* or, in Aristotelian terms, *intelligible matter* which may be considered as the cognitive counterpart to the perceptible materiality in the physical world.[28] We can say, at least metaphorically, that geometric shapes are mixed with some sort of estimative or intelligible matter, which is neither any specific kind of matter we have in the external world, nor separable from geometric shapes. A significant consequence of this inseparability from intelligible matter is that geometric shapes are necessarily associated with material forms (*ṣuwar māddiyya*). Avicenna says:

> **TEXT # 4:** [The subject matter of geometry, i.e., magnitude (*miqdār*)] does not separate from matter except in the act of estimation and *does not separate [even in the estimation] from the form that belongs to matter*.[29]

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26 Ardeshir (2008, 45).

27 Having a mental concept of something in the mind does not necessarily guarantee that that thing is separable from matter. Consider Eiffel Tower and its mental counterpart, i.e., the concept EIFFEL TOWER. These two things, according to Avicenna, have the same quiddity; the quiddity of Eiffel Tower, which can accept two distinct modes of extramental and mental existence. The concept of EIFFEL TOWER, inasmuch as it is a concept, is mental and therefore, in a trivial sense, separated from matter. In this sense, *anything* of which we have a concept is trivially separated from matter in the mind. However, this is definitely not what Avicenna means by separability from matter in the mind. It seems, rather, that, according to Avicenna, X is separable from Y in the mind if and only if it is possible to conceive X without Y. We can conceive squareness without woodenness. For we can conceive a non-wooden, say golden, square. Therefore, squareness is separable from woodenness. However, according to Avicenna, we cannot conceive squareness without materiality (as we will see, it means: without the intelligible matter or material form). Consequently, squareness is inseparable from materiality in the mind. For a recent study on Avicenna's understanding of the notions of immateriality and separability, see Porro (2011).

28 Porro (2011, 294) upholds this interpretation.

29 Avicenna (2005, III.4, 84, ll. 31-32). I have slightly modified Marmura's translation. Particularly, I prefer to translate '*miqdār*' into 'magnitude,' not 'measure.' The italics are mine.

Geometric objects, inasmuch as they are what they are, are necessarily attached to intelligible matters. They are always in the forms of material objects.[30] So, they have some sort of *ontological admixture* and *association* with (or dependency on) materiality, or, more precisely, on material forms.[31] In our estimation, we can separate them from all the particular matters mixed with which they may exist in the physical world; nevertheless, they remain attached to their material forms.[32] It is impossible to conceive of geometric objects as being separated from their material forms. I shall now turn to an investigation of the nature of numbers.

4. Numbers

Avicenna's position on the status of numbers differs slightly from his views on the nature of geometric objects. Numbers have no necessary association with material forms, but, inasmuch as they are the subject of arithmetical studies, they still have some sort of dependency on materiality. This long passage sets out the main characteristics of numbers:

> **TEXT # 5:** Regarding those things that can mix with motion, but have an existence other than this, these [include] such things as individual identity (*al-huwiyyah*), unity, plurality and causality […]. These are either: [(a)] regarded inasmuch as they are [the things] they are (*min ḥaythu hiya hiya*), in which case viewing them in this way does not differ from looking at them inasmuch as they are abstracted—for they would then be among [the things examined through] the kind of examination that pertains to things not inasmuch as they are in matter, since these, inasmuch as they are themselves (*min ḥaythu hiya hiya*) are not in matter; or, [(b)] regarded inasmuch as an accidental thing that has no existence except in matter has occurred to them. This latter is of two divisions, It is either the case [(b1)] that that accident cannot be apprehended by the estimative faculty as existing except in conjunction with being related to specific matter and motion—for example considering one inasmuch as it is fire or air, plurality inasmuch as it is the [four] elements, causality inasmuch as it is warmth or coldness, and intellectual substance inasmuch as it is soul, that is, a principle of motion even though it in itself (*bi-ḏātihi*) is separable—or [(b2)] that that accident, even though it cannot occur except in relation to matter and mixed with motion, is such

30 By 'the *form* (*ṣūra*) that belongs to matter,' Avicenna means nothing more than the *shape* of material objects, or so it seems. Geometric objects are inseparable from intelligible matter. Therefore, they cannot be conceived without material shape. It is worth remembering that Avicenna had no understanding of geometry in dimensions higher than three. See Avicenna (2005, III.4, from 89, l. 25 to 90, l. 7). If he had, his view on the necessary association of geometric objects with material form might have changed.

31 See also Avicenna (2005, III.4, 85, ll. 14-16 and from 86, l. 34 to 87, l. 2).

32 See also Avicenna (2009a, I.8, 59, Sec. 6).

> that its state can be apprehended by the estimation and discerned without looking at the specific matter and motion in the aforementioned way of looking. The example of this would be addition and subtraction, multiplication and division, determining the square root and cubing, and the rest of the things that append (*talhaqu*) to number. For all this attaches to number either in men's faculty of estimation, or in the existents that are subject of motion, division, subtraction and addition. Apprehending this as a form (*taṣawwuru ḏālik*), however, involves a degree of abstraction that does not require the specifying of matters of certain species.[33]

This text reveals that numbers, inasmuch as they are numbers, are mixed with neither any specific kind of matter, nor even materiality itself. Therefore, they can be found and considered in three different forms, considered inasmuch as: (1) they are what they are, fully separated from materiality (i.e., category (a) of the text), (2) they are accidents of material things, associated with a specific kind of matter (i.e., category (b1) of the text), (3) they are accidents of material things, but dissociated from any specific kind of matter (i.e., category (b2) of the text). Subject matters of arithmetic are numbers only when they are considered in the latter form.

Numbers mixed with some specific kinds of matter should be studied in natural science. For example, the number four, inasmuch as it is accidentally true of the four elements, should be studied in natural science. On the other hand, numbers, inasmuch as they are what they are, fully separated from materiality, should be studied in metaphysics. To be the subject of arithmetical studies, numbers should be considered as accidents of material things. They are associated with materiality, but not necessarily with a specific kind of matter. The concept of *immaterial number*, contrary to the concept of *immaterial triangle*, is plausibly intelligible.[34] When Avicenna discusses numbers in his metaphysics, he does indeed have a fully immaterial conception of numbers in mind.[35]

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33 Avicenna (1952, I.2, from 13, l. 4 to 14, l. 2). I have slightly modified Marmura's translation of the Arabic phrase '*mawjūdāt mutiḥarrika munqasima mutifarriqa wa mujtami'a*.' His translation is 'existents that move, divide, separate and combine,' while my translation is 'existents that are subject of motion, division, subtraction and addition.' I think that my translation is more faithful the context of this passage, which is about mathematics and mathematical operations.

34 The concept of immaterial triangle is self-contradictory, at least if by 'materiality' we mean 'association of material form.'

35 Therefore, numbers are similar neither to certain objects of metaphysics, e.g., God and mind, which are necessarily immaterial, nor to certain objects of natural science or mathematics, e.g., humanness and squareness, which are necessarily material. Numbers are *contingently* mixed with matters.

This discussion shows that there is a dissimilarity between geometric shapes and numbers. Geometric shapes, inasmuch as they are what they are, are necessarily associated with materiality or material forms. As we saw in TEXT # 1, TEXT # 2, and TEXT # 3, Avicenna believes that inseparability from materiality is included in the definition of geometric shapes. Therefore, the dependency of geometric objects on materiality is an *ontological* dependency, in contrast to the dependency of numbers on materiality, which is only *epistemological*. Numbers, inasmuch as they are what they are, have no necessary accompaniment with materiality. But, inasmuch as they are the subject of arithmetical studies, we should *consider* or *regard* them as things dependent on materiality (i.e., accidents of material things). Therefore, numbers are not *ontologically* intertwined with materiality. It is only the consideration (*naẓar*) of the arithmetician that preserves numbers in association with materiality (not the ontological status of numbers inasmuch as they are numbers). Hence, numbers, inasmuch as they are the subject of arithmetical studies, have some sort of *epistemological* dependency on materiality. I will now show why we need to consider this dependency for numbers, and why numbers, inasmuch as they are numbers, cannot be the subject of mathematical studies:

> **TEXT # 6:** [N]umber can be found in separable things and in natural things […]. Number whose existence is in things separate [from matter] cannot become subject to any relation of increase or decrease that may occur but will only remain as it is. Rather, it is only necessary to posit it in such a way that it becomes receptive to any increase that happens to be, and to any relation that happens to be when it exists in the matter of bodies (which is, potentially, all modes of numbered things) or when [number] is in the estimative faculty. In both these states, it is not separable from nature.
>
> Hence the science of arithmetic, inasmuch as it considers number (*yanẓuru fi al-ʿadad*), considers it only after [number] has acquired that aspect possessed by it when it exists in nature. And it seems that the first consideration [or theoretical study] of [number that the science of arithmetic undertakes] is when it is in the estimative faculty having the description [mentioned] above; for this is an estimation [of number] taken from natural states subject to addition and subtraction and unification and division.
>
> Arithmetic is thus neither a consideration [or study] of the essence of number nor a consideration [or study] of the accidents of number inasmuch as it is absolute number, but [it is a study] of its accidental occurrences with respect to its attaining a state receptive of what has been indicated [above]. It is either material, then, or [it] pertains to human estimation dependent *(yastanidu)* on matter.[36]

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[36] Avicenna (2005, I.3, from 18, l. 18 to 19, l. 8). I have slightly modified Marmura's translation. More precisely, I prefer to translate '*naẓar*' and '*yanẓuru*' respectively to 'consideration' and 'considers,' rather than 'theoretical studies' and 'studies.' I have also translated '*an tajtami' va taftariq*' into 'subject to addition and subtraction,' rather than 'subject to combination and separation.'

This text contains at least three important points: First, it corroborates the view that number can be found in both inseparable and separable things.[37] This means that number, inasmuch as it is number, is neutral with respect to materiality. As we saw, geometric shapes are not neutral in this respect. This alone, however, is not enough for us to conclude that numbers, inasmuch as they are the subject of arithmetical studies, are neutral with respect to materiality. Here we find the second point I want to make. Number, inasmuch as it is the subject matter of arithmetic, should be capable of participating in relations of decrease, increase, addition, subtraction, etc. And it is so only when it exists as accidents of material things.[38] In our estimative faculty, we can separate numbers from all particular materials while preserving those aspects that they have acquired only after admixture with materiality. We separate numbers from materiality in our estimation, but *consider* them *as if* they had some material aspects and capabilities.

In another phrase, very similar to the view Avicenna expresses in the first sentence of TEXT # 6, he says that number "would apply to [both] sensible and non-sensible things. Thus, inasmuch as it is number, it is not attached to sensible things."[39] Ardeshir concludes from this that "discussion about number and its relations should be understood as abstracted from sensible objects, not when it may belong to sensible objects. So, discussion about numbers is not about sensible objects."[40] But this interpretation is misleading. As we saw, discussion of numbers, *inasmuch as they are numbers*, should be understood as abstracted from sensible objects, but discussion of numbers and their *mathematical* relations, *inasmuch as they are the subject of arithmetical studies*, are not completely independent of sensible or, more precisely, material objects.

The third interesting point in this text is that Avicenna does not say that numbers in our estimation are attached to (or associated with) materiality or nature; he says only that they have some sort of dependency (*istinād*) on nature, which seems weaker than an ontological association with materiality. That is what I call 'epistemological' dependency on matter.

What Avicenna says in TEXT # 6 is intended to *refute* the claim of a person who might say: "The purely mathematical things examined in arithmetic and geometry are also 'prior to nature'—particularly number, for there is no dependency at all for its existence on nature because it cannot

37 See also Avicenna (2009a, I.8, 57), where he says that the identity (*huwiyyah*) of number "does not require any dependence relation upon either natural or non-natural things."

38 See also Avicenna (2009a, I.8, 57).

39 Avicenna (2005, I.2, 8, ll. 15-17).

40 Ardeshir (2008, 46).

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be found in nature."[41] Tahiri confusingly considers this passage to be something that Avicenna *believed*. He writes:

> This specificity of arithmetic is stressed by many 19th century mathematicians like Gauss who strikingly expressed a similar view in his letter to Olbers (1817) following the discovery of non-Euclidean geometries: "geometry must not stand with arithmetic which is purely a priori" (Gauss 1900, vol. VIII, p. 177). Ibn Sīnā would wholly agree with Gauss, since for him the concept of number is so pure that even time is not essential to its construction.[42]

This interpretation is clearly false, and not just because it is anachronistic. According to Avicenna, as I have shown, arithmetic has an epistemological dependency upon nature. To summarize:

(I) Geometric shapes, inasmuch as they are geometric shapes, are mixed with estimative matter (or, consequently, with material forms). They have an ontological dependency on materiality.
(II) Numbers, inasmuch as they are numbers, are not mixed with materiality; but, inasmuch as they are the subject of arithmetical studies, they should be considered as mixed with materiality. They have an epistemological dependency on materiality.

Numbers, are, in a sense, more abstractable or more separable than geometric shapes. The subject matter of arithmetic is somehow closer to the subject matter of metaphysics. Hence, it is plausible to expect that the difference in the ontological status of numbers and geometric shapes would lead to further epistemological differences between geometry and arithmetic. However, I will refrain from engagement in this debate, which merits independent study.

In his discussion of the division of sciences, McGinnis says that, for Avicenna, "[t]hose existents that can be conceptualized without matter, even though they are necessarily mixed with motion and never subsist without matter, are the subject of mathematical sciences."[43] My analysis shows that this picture of the nature of mathematical objects suffers from imprecision. If by 'matter' McGinnis means particular kinds of matter existing in the physical world, then he is right. Mathematical objects can be conceptualized without any particular matter. However, if he intends to convey materiality *qua* materiality by 'matter,' then his interpretation is false. Mathematical objects, inasmuch as they are the subject of mathematical studies, cannot be abstracted from the materiality itself.

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41 Avicenna (2005, I.3, 17, ll. 10-13).

42 Tahrir, (2016, 33, n. 17).

43 McGinnis (2010, 36).

Unfortunately, we cannot arrive at a reliable understanding of Avicenna's view on the subject matter of the theoretical sciences by consistently fixing one of these two meanings of 'matter' in McGinnis's book. He writes:

> [T]here are three major branches of theoretical sciences: the natural sciences, the mathematical sciences, and the science of metaphysics. These divisions correspond respectively with whether the objects investigated by the science must necessarily subsist as well as be conceptualized together with motion and matter; necessarily subsist together with matter and motion but need not to be conceptualized as such; or need neither subsist nor be conceptualized together with matter and motion.[44]

As we saw, if by 'matter' he means materiality itself, then mathematical objects cannot be conceptualized without matter. On the other hand, if by 'matter' he means particular matters existing in the physical world, then subsisting and being conceptualized without matter (in this new sense) is not sufficient for being the subject of metaphysical investigation. Objects of metaphysics are separated from all material forms and from materiality itself, not merely from special kinds of matter.[45]

5. Actual and Potential Perfect Objects

Mathematical objects, as we saw, are primarily accidents of material things in the external world. Mathematical enquiry is, therefore, primarily about accidents of material objects, not about independent immaterial entities. We can obtain a universalized conception of mathematical objects by abstracting them, via our faculty of estimation, from all particular materials with which they may be mixed in the extramental world. This purification procedure can, in principle, end in the production of some intelligible forms of *exact* and *perfect* mathematical objects that are not easily perceptible in tangible objects.[46] But does this necessarily mean that there are no *perfect* mathematical objects in the physical world? Does it necessarily mean that perfect mathematical objects are

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44 McGinnis (2010, 37).

45 Admittedly, the source of this ambiguity has its source in Avicenna's own writings, where he uses the term 'matter' equivocally. See, for example, Avicenna (2004, 4-5).

46 What we perceive in our ordinary perceptual experiences only approximates the ideal shape of celebrity geometric objects, e.g., perfectly straight lines, circles, parabolas, etc. So, at first glance, it might seem that there should exist nothing in nature with exactly these shapes. If so, this fact provides a strong motivation for the view that perfect mathematical objects (i.e., objects which exactly satisfy the mathematical definition of those ideal shapes) are merely mental constructions that do not really exist in the extramental world. Nonetheless, Avicenna believes that these perfect objects can—and some of them really do—exist in the extramental world, or so I will argue.

merely mental constructions that have no counterpart in the external world? I will argue, in this section, that Avicenna endorses the existence of at least some perfect mathematical objects in the extramental world.

The most important evidence for Avicenna's agreement with some sort of literalism is that, wherever he discusses the nature of mathematical objects in his writings, he affirms that mathematical objects exist in the external world in association with determinate kinds of matter (as accidents of specific material particulars). However, he simultaneously insists that mathematical forms are not sensible natural forms.[47] Some might object that, although this provides strong evidence for the existence of mathematical objects in the extramental world (albeit not as independent substances), this cannot count as evidence that those objects are perfect. I think that this objection is untenable. If it stood, then Avicenna would need to distinguish between two sorts of perfect and imperfect mathematical objects, such that the latter could exist in the external world mixed with matter, but the former could exist only in the mind. He would need to say that, for example, *quasi* circular objects (which approximate the ideal shape of a circle but do not really satisfy the mathematical definition of a circle) could exist in the extramental world but *quiet* circular objects (which perfectly satisfy the mathematical definition of a circle) could exist only in the mind. This he does not do.[48] As we will see, he denies that what we *see* in the external world are perfect mathematical objects, but this is so just because mathematical forms are not sensible (visible) forms, not because we see a mathematical form that is imperfect. Paying attention to the epistemological formalities that Avicenna proposes for grasping mathematical forms and producing mathematical concepts will show how mathematical literalism can, by and large, be compatible with Avicenna's philosophy.

Interestingly, the mental faculty involved in apprehending mathematical concepts is *estimation*. Discussing the details of the role estimation plays in forming mathematical concepts and attaining mathematical knowledge is outside the scope of this paper. But consideration of some other objects of the estimative faculty may help us to reach a better understanding of the existential mode of mathematical objects in the extramental world. According to Avicenna's epistemology, estimation is a bodily faculty with a distinct and unique cognitive power that lies between imagination and intellect in the hierarchy of cognitive faculties. Some of its activities are common to both humans and animals, while others are exclusively human. Looking at one of its activities will give

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47 See, for example, Avicenna (2005, III.4, 85, ll. 10-16) and (2009a, I.8, 57-58). Mathematical forms can exist in sensible things but they are not themselves sensible forms.

48 In his attack on Platonism, he defends the view that the geometric shapes that exist in the external world and the intelligible geometric forms that we have in our minds have the same quiddities. See Avicenna (2005, VII.3, especially 249-250).

us a better understanding of the role of estimation in perceiving mathematical objects in the external world; this activity is likened to *incidental perception*.[49] When somebody perceives the sweetness of a yellow cake just by seeing it, she has an incidental perception. She has perceived a sensible form without employing the right perceptive faculty that we usually use to apprehend similar sensible forms. She has perceived the sweetness of the cake without tasting it. Such an apprehension, according to Avicenna, is feasible only because of our estimative faculty. It enables us to 'see' the sweetness of a yellow cake. Supposing that her apprehension is reliable and that the cake really is sweet, she has apprehended, by the aid of her estimation, something which *really exists* in the external world but is imperceptible by the sense she uses.[50] It can be argued, analogously, that when we see a triangular wooden shape or a group of three balls we apprehend perfect mathematical objects (a perfect triangle or the number three[51]) that really exist in the extramental world as accidents of material objects; but they are not visible, or available, to our sensory faculties.[52] Estimation, among its other roles, enables us to apprehend those things that actually exist in the extramental world but are invisible.[53]

49 For more on Avicenna's treatment of estimation and the other roles that he attributes to this faculty, see Black's (1993) seminal paper.

50 For more on the details of the mechanism of incidental perception, see Black (1993, 25-27).

51 Of course, it is more precise to say that what exist in the physical world are *instantiations* of triangularity and threeness.

52 Admittedly, there are some dissimilarities between the roles estimative faculty plays in incidental perception and apprehension of mathematical objects. In incidental perception, our estimation enables us to perceive something that is the proper object of the sense-perceptual faculty X using data we receive instead from the faculty Y. Therefore, estimation performs what the faculty X can normally perform. In apprehension of mathematical objects, however, estimation performs what no sense-perceptual faculty can perform, because mathematical objects are not sensible at all. So, it might seem better to analogize mathematical perception to the apprehension of some non-sensible intentions, such as pleasantness, goodness, friendship, and hostility. Avicenna believes that, although these intentions are not themselves sensible, they can be perceived through the perception of some sensible forms, albeit by the aid of estimation. For example, we can apprehend the goodness of a friend through what we perceive by our senses from her. Nonetheless, some dissimilarities rise again. Some of these intentions, contrary to mathematical objects, are not necessarily associated with materiality. They can be properties of some immaterial objects (e.g., God is good). The moral is that each analogy has limitations.

53 Tahiri (2016) has, strangely, overlooked the significant role Avicenna accords to the estimative faculty in attaining knowledge of mathematical objects. His translation of 'uhām al-nās' to 'people's beliefs' (31, n. 13) is just one of the signs of his negligence.

The existence of numbers as perfect mathematical objects in the external world seems more defensible than the existence of perfect geometric objects. The twoness of two tomatoes is as perfect as the twoness of two books or the twoness of one imaginary Santa Claus and one (hopefully real!) Christmas gift. They are different instantiations of the same universal concept, i.e., twoness. Twoness is not directly sensible, like whiteness or warmth, but we apprehend it thanks to the estimative faculty, and it is as real as the existence of any ordinary accidents that material objects may have. Hence, there is no serious hindrance to the attribution of arithmetic literalism to Avicenna.

The existence of perfect geometric objects, on the other hand, might seem more improbable. At first glance, it seems obvious that there is no perfect geometric object in the physical world. Since everything there has width, length, and depth, there is no perfect line with no width; hence, there is no perfect triangle. In fact, there are some passages from which someone may conclude that Avicenna endorses this view, i.e., anti-literalism. For example, he accepts that, when we want to prove a geometric theorem based on a composition of geometric shapes that we have drawn on a piece of paper, what we have drawn are *not* perfect geometric objects, and what we are trying to prove is *not* about those *visible* figures. He endorses a view that he attributes to Aristotle:

> **TEXT # 7:** The drawn line and the drawn triangle are not drawn because the demonstration needs them. The demonstration [of a geometric theorem] is [demonstrated] on a line which is really [i.e., perfectly] straight and width-less; and [it is demonstrated] on a triangle which has really [i.e., perfectly] straight sides with the same length. This triangle and that line [drawn on the paper] are rather for preparation of the mind to imagine. Demonstration is [demonstrated] on the intelligible, not sensible or imaginable (*mutakhayyal*) [forms].[54]

There is no doubt that perfect geometric objects cannot be drawn. Moreover, they are completely invisible. But this does not necessarily entail that there is no perfect geometric object in the external world.[55] There is some evidence that supports the claim that Avicenna accepts the existence of perfect geometric objects in the external world, though not as sensible things. It can be argued, compatibly with Avicenna's philosophy, that the role of estimation is not merely to

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54 Avicena (1956, II.10, 186). The translation is mine.

55 Marmura, in a note on his translation of *The Metaphysics of The Healing*, writes: "Geometer's circle is a partial abstraction by the estimative faculty of circles that exist in sensible matter. This need not exclude the existence of 'perfect' circles in material things, a notion rejected by atomists" (Avicenna 2005, 397, n. 5 of III.9). It seems that in (at least some of) his arguments against atomism, Avicenna presupposes the possibility of the existence of perfect geometric objects in the external world. See Avicenna (2009b, III.4, 284-285), in particular McGinnis's notes.

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construct perfect mental objects that have no counterpart in the extramental world. Estimation, at least in some cases, helps us to apprehend some sort of *non-sensible perfection* that really exists in the extramental world. There are some passages that support this construal. For example, Avicenna says that the ascertained doctrine is that:

> **TEXT # 8:** Point exists only in line, which is in surface, which is in body, which is in matter.[56]

From one perspective, this text is a criticism of Platonism. Avicenna believes that perfect geometric objects have no immaterial independent existence. From another perspective, it is a confirmation of the actual existence of perfect mathematical objects (e.g., point and line) in the extramental world. It is worth noting that the actual existence of point, line, and surface in the external world do not entail their separability from each other outside the mind. Avicenna emphasizes that we can separate point from line, line from surface, and surface from body (*jism*) *only* in our estimation.[57] They exist in the external world, but they are not distinctly perceptible and cannot be separately predicated upon material particulars. Analogously, we can say that perfect triangles exist in the external world (e.g., in triangular bodies), but they are not distinctly perceptible and cannot be separately predicated upon material particulars. Given these considerations, the actual existence of *at least some* perfect geometric objects in the physical world seems, by and large, compatible with the tenor of Avicenna's writings on the nature of mathematical objects. So, it is not incautious to say that he endorses some sort of geometric literalism.

Undoubtedly, a great deal of work is needed to establish literalism as a plausible view about the nature of mathematical and especially geometric objects. But my concern here is the compatibility and consonance of literalism (in the sense described) with Avicenna's philosophy, rather than the plausibility of the view itself. My arguments show that he, by and large, endorses some sort of literalism.[58]

Even if we accept the actual existence of some perfect mathematical objects in the physical world, it is undeniable that most mathematical objects do not exist in the physical world. If the number of material objects is finite,[59] then there are some large numbers (larger than the number of all objects that we

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56 Avicenna (2005, VII.3, 254, ll. 25-27). I have slightly modified Marmura's translation.

57 See Avicenna (2005, III.4, 86-87).

58 Nonetheless, if literalism is the actual existence of mathematical objects as physical *substances*, then Avicenna expressly rejects the doctrine. For Avicenna, mathematical objects, inasmuch as they are *accidents* of material things, exist in the extramental realm.

59 There is a consensus, according to which Avicenna rejects the idea of actual infinity.

have in the extramental world) that are not accidents of any group of numbered material objects; so, those numbers do not literally exist. On the other hand, as Avicenna admits, there are many geometric objects that do not exist in the physical world.[60] These objects, by the aid of the imaginative and estimative faculty,[61] can, in principle, be constructed in the mind. But, obviously, there are infinitely many of these objects that have never been constructed. Consider a very large number (larger than the number of physical objects and larger than the largest number we have ever thought of), or a very strange geometric shape that nobody has never thought about. These objects do not exist, either as accidents of material objects in the physical world or as objects constructed by imagination and estimation in a human mind. Of course, if someone decides to construct such an object in her mind, she *may* succeed. But before that, these objects do not actually exist. Nevertheless, we can still attribute some sort of *potential* existence to these objects. We have the *potentiality* to create them in our minds, so they *potentially* exist. Therefore, Avicenna is somehow a *potentialist*: at least some mathematical objects only potentially exist. More precisely, some mathematical species exist only in a potential sense of existence.[62]

6. Conclusion

According to Avicenna, mathematical objects are, in the first instance, accidents of material objects, so they exist in the extramental realm mixed with particular materials. Nonetheless, they are not themselves material or natural forms.

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60 See Avicenna (1957, Namat. III, Ch. 7, 336-337). Interestingly, he does not say that no geometric shape exists in the physical world; he says that most do not exist in the physical world. This means that he accepts the actual existence of at least some of geometric objects in the external world. However, it does not automatically entail that those objects are perfect.

61 For a study of Avicenna's treatment of the roles of these cognitive faculties, see Black (1993, 2000).

62 I do not claim that Avicenna commits to a vast ontology of non-existent objects. The quantifier 'there are' in the sentence that 'there are some mathematical objects that only potentially exist' should not be read as having ontological claim. My aim in this section is simply to emphasize that, although Avicenna believes in the existence of mathematical objects (i.e., numbers and geometric shapes) in the external world, he does not believe that all numbers and all geometric shapes one can, in principle, think about really do exist in the external world. Contrary to a mathematical Platonist who endorses the actual existence of an infinite number of mathematical objects (though not as concrete objects), Avicenna's philosophy allows only a finite number of mathematical species, either in the external world or even in the mind, to exist. He nonetheless accepts the possibility of creating *new* mathematical objects. This is what I mean by attributing potentialism to Avicenna.

In our minds, we can separate them, by our estimation, from all the determinate kinds of matter with which they may be associated. However, the degrees of separability of geometric objects differ from that of numbers. Geometric objects, inasmuch as they are what they are, are inseparable from materiality *qua* materiality. We can separate them from all specific kinds of matter, but not from materiality itself; such objects are necessarily attached to material forms, even in our estimation. So, geometric objects have some sort of *ontological dependency* on materiality. Numbers, on the other hand, are completely separable from matter. Inasmuch as they are what they are, they have no dependency on materiality. They can be found in association with, or separate from, materiality. But numbers as the subject of arithmetical studies should be receptive to decrease and increase, and should have the capability of being subject to addition, subtraction, multiplication, and addition. Numbers are receptive to such accidents only when they are applied to material things. So, if we want to have a conception of number, inasmuch as it is the subject of arithmetical studies, we should *consider* it as something associated with matter. Number in its nature has no ontological dependency on materiality but, as a subject of mathematical studies, should be considered as mixed with matter. So, numbers, inasmuch as they are the subject matter of arithmetic (but not inasmuch as they are numbers) should be considered in accompaniment with materiality. Therefore, they have some sort of *epistemological dependency* on materiality.

Avicenna endorses the existence of perfect mathematical objects in the external world. He believes that mathematical objects can literally exist in the extramental world as accidents of material things, though not as independent substances. They are not sensible forms but they can be perceived by the aid of the estimative faculty.

In any case, the number of mathematical species that actually exist in either the extramental or mental world is finite. Most mathematical objects only potentially exist. They have no actual existence, whether extramental or mental. They can, in principle, be constructed either in the extramental world by creating new objects and increasing the number of material objects in the world, or in the mind with the aid of imagination and estimation. So, Avicenna is somehow a *literalist*, a *finitist* and a *potentialist*. He does not think that mathematical objects can be released from all the ontological or epistemological dependencies they may have on materiality; this is what distinguishes my view from that of McGinnis, Ardeshir and Tahiri. However, geometric objects and numbers have different degrees or different kinds of dependencies on materiality, and this is what distinguishes my view from that of Michael Marmura.

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