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Reply to Sider

Timothy Williamson

Wykeham Professor of Logic, University of Oxford, Oxford, UK

In 'On Williamson and simplicity in modal logic' (Sider 2016), Ted Sider makes a case against my application of an abductive methodology to modal logic, and in particular against my use of simplicity and strength as joint criteria for preferring some theories over others in that field.

1. Fundamental and non-fundamental abductions

My disagreement with Sider is best understood in the context of our agreement on much else concerning methodology. We agree that a broadly abductive methodology is appropriate for theory choice in many fields, including physics, chemistry, archaeology, criminology, fundamental metaphysics and fundamental logic. We also share a full-bloodedly realist conception of those fields as primarily engaged in the pursuit of truth: the abductive methodology is independent of any pragmatist or otherwise anti-realist view of what is at stake in theory choice. Sider is willing to apply a modestly realist conception to quantified modal logic too: either necessitism is true and contingentism false or *vice versa*, and we may hope to find out which. He even seems ready to grant that *if* we apply the abductive criteria of simplicity and strength to theory choice in modal logic, they will favour necessitism over contingentism.

However, in Section 4 of his paper, Sider argues that we should *not* apply the abductive criteria of simplicity and strength to theory choice in modal logic. For present purposes, □ is to be read as expressing metaphysical necessity, and he has argued elsewhere that metaphysical necessity does not cut reality at a fundamental joint, but rather makes a messy, arbitrary, disjunctive distinction (Sider 2011, 266–291). In the present paper, he argues that metaphysically modal inquiry lacks the sort of successful track record needed to back up the claim that it asks fundamental questions. For Sider, modal logic, metaphysically interpreted, does not investigate the sort of area where we can expect exceptionless strong and simple laws.

CONTACT Timothy Williamson timothy.williamson@philosophy.ox.ac.uk

Sider concedes that the legitimate application of an abductive methodology is not restricted to fundamental science, since it may legitimately be applied in many non-fundamental sciences too, such as chemistry, archaeology and criminology, and even to everyday practical problems. In all those areas, we need to use some form of inference to the best explanation, in order to dismiss infinitely many gruesomely complicated alternative explanations that ingenuity could devise but good sense would not take seriously. However, Sider argues, the applicability of abduction in those areas depends on their relation to fundamental physics, because violations of simple non-fundamental principles would require violations of simple fundamental principles too. By contrast, he assumes, modality lacks that connection with fundamental physics. Thus, on Sider's view, modal logic is methodologically worse off than criminology.

One may question Sider's justification of abduction for non-fundamental science in terms of abduction for fundamental physics. He suggests that 'a sudden, huge, isolated exception to otherwise simple physical generalizations […] would also constitute a sudden, huge, and isolated exception to otherwise simple chemical generalizations'. Perhaps so. But his next step is to claim: 'More generally, *any* sudden, huge, and isolated exception to otherwise simple chemical generalizations would require some such physical complexity or other; thus we assume a priori that these [chemical] exceptions are unlikely' (because the physical exceptions are unlikely). That generalization goes from chemical anomalies to physical anomalies, whereas the premise it is supposed to generalize went from physical anomalies to chemical anomalies; it is unclear just what connection between them Sider intends. In any case, how do such considerations help a detective rank an elaborate conspiracy theory involving international Freemasons below more mundane hypotheses as explanations of what appears to be a local gang killing? The conspiracy theory entails no violation of accepted physical laws.

The gap between fundamental physics and many legitimate applications of abduction to non-fundamental matters is much wider than Sider suggests. In particular, given that there are exceptionless strong and simple generalizations in fundamental terms, why should we also expect there to be exceptionless strong and simple generalizations in non-fundamental terms? The non-fundamental nature of the latter terms might unsuit them for capturing such generalizations. As Sider notes, it is not enough for the non-fundamental terms to be based somehow or other on fundamental terms, since if 'green' and 'blue' can be reduced to fundamental physical vocabulary, so too can 'grue' and 'bleen', yet the latter are far less suited to abduction than the former. Sider suggests that the disjunctive nature of 'grue' and 'bleen' is the problem. However, if 'gang killing' is reducible to the language of fundamental physics, it will presumably be as something like an infinite disjunction of infinite conjunctions whose disjuncts are not unified by any distinctive similarity at the fundamental physical level. Analogous issues arise even concerning the relation of macroscopic physics to microscopic physics. A proper account of abduction for non-fundamental branches of science may need to pay more attention to their quite significant degree of methodological autonomy.

2. Modal physics

Sider concedes the legitimacy of an abductive methodology in both fundamental and non-fundamental physics. He takes for granted that doing so implies nothing good about the application of such a methodology to modal inquiry, perhaps because he takes the language of physics to be purely non-modal. Since so much of physics is expressed in mathematical notation, without modal operators, one could easily get that impression. But mathematics becomes physics only when the variables are assigned physical meanings. Sometimes those physical meanings are themselves implicitly or explicitly modal.

For instance, a *phase space* consists of the set of all *possible* states of a physical system at a time, mathematically structured in some way (geometrically, topologically, or the like), typically including a dynamics to describe how the system evolves through a series of states over time. With pardonable exaggeration, a recent history of the idea of a phase space begins: 'Listen to a gathering of scientists in a hallway or a coffee house, and you are certain to hear someone mention phase space' (Nolte 2010, 33). As examples, it cites stock prices in economics, the dust motes in Saturn's rings, and high-energy particles in an accelerator. Phase spaces are used in even the most fundamental physics, such as quantum mechanics. When scientists reason about phase space, they often talk simply of 'states' without making the qualification 'possible' explicit, but it is tacitly understood, because no trajectory traced out by the system according to the dynamics over a complete history goes through every state in the space (typically, each trajectory is of lower dimension than the space itself). The key mathematical structure is defined over the space as a whole, not trajectory by trajectory. Each state may be represented by the values of the physically relevant parameters. For instance, if the system comprises *n* particles, each of which varies on six abstract dimensions, then a given state may be represented by a 6*n*-tuple, and the whole system is 6*n*-dimensional. In reasoning about the system, a scientist uses ordinary non-modal mathematics, but once the mathematics is applied the intended interpretation is modal. For example, let X be the set of states in which the system meets a condition C, and Y the set of states in which it meets a condition D. Then the scientist may write '$X \subseteq Y$', which looks purely non-modal, but the intended reading is 'The system meets D in every possible state in which it meets C', in other words, 'Necessarily if the system meets C then it meets D'.

Reasoning about a phase space is like reasoning about the set of worlds in a standard model of modal logic, and even more like reasoning about the set of ordered pairs of possible worlds and times in a standard model of modal temporal logic, which has temporal as well as modal operators. Since the standard model theory of modal logic and modal temporal logic is done in a metalanguage without modal or temporal operators, it too uses ordinary non-modal mathematics (set theory), but once the mathematics is applied to an intended model over a set of genuine worlds or ordered pairs of genuine worlds and times, the intended interpretation is modal.

States, like worlds or world-time pairs, are understood as jointly exhaustive and mutually exclusive. They are maximally specific in all relevant respects, for although they are typically states of a local physical system, not the whole universe, that system is treated in isolation. Just as we can equate propositions in a model of modal logic with sets of worlds, and propositions in a model of modal temporal logic with sets of world-time pairs, we can equate propositions in a phase space with sets of states. The set X of states corresponds to the proposition that the physical system is in a state belonging to X.

Even though the system does not traverse all the states in the phase space in any given history, they are all treated as possible states of the system absolutely, not relative to the standpoint of one state or another. A state space has no proper analogue of the accessibility relation between worlds in a model of modal logic, by which only worlds in a restricted subset are treated as possible from the standpoint of a given world. Of course, some states and not others are future from the standpoint of a given state, but that is a temporal rather than modal matter. In effect, for a phase space the only necessary proposition is the set of all states, and any nonempty set of states is possible. It follows that the propositional modal logic of phase spaces is S5, on which possibility and necessity are not themselves contingent matters. Thus mathematical reasoning about a phase space is not modally neutral; there is an implicit commitment as to which modal logic is appropriate.

The analogy between states in phase space and world-time pairs is not quite identity, because the dynamics may cycle: in some cases, if all the relevant parameters take given values at one time, they will take all those same values again a fixed length of time later (and so on ad infinitum, if the dynamics is deterministic). Thus the system may be in the same state at different times. Nevertheless, the analogy is strong enough to bring out the modal aspect of phase spaces. It is more than a purely formal analogy, because it is rooted in the physicists' own conception of the states as the possible states of a physical system.

The analogy can be carried into higher-order logic. Just as in modal temporal logic one can interpret quantification into sentence position, given a model, as quantification over propositions (sets of world-time pairs), so in the corresponding modal quasi-temporal logic for phase spaces one can interpret quantification into sentence position, given a phase space, as quantification over propositions (sets of states), and thereby formalize quantification over sets of states in the language. The logic of such quantification will be (higher-order) necessitist rather than contingentist, because which sets of states one

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is quantifying over does not depend on which state is current. Thus analogues of the Barcan and converse Barcan formulas will hold. Elsewhere, I develop the modal quasi-temporal logic corresponding to phase spaces in dynamical systems theory in much more detail (Williamson 2016). More generally, physicists' free use of mathematical reasoning over phase spaces no more indicates neutrality on modal matters than does metaphysicians' free use of mathematical reasoning over intended possible world models of quantified S5.

For present purposes, there is no need to insist on the point that physicists' reasoning about phase spaces embodies implicitly necessitist assumptions. What matters most is that when they investigate the phase space of a physical system, they are asking and answering implicitly modal questions. They are exploring the dynamical structure of possible states of the system. Physics is not a modality-free zone.

A potential source of confusion is that when scientists investigate the phase space of a physical system, they often do so in the spirit of *model-building*, following the widespread scientific practice of making simplified mathematical models of the target system in order to facilitate understanding of its key features. Thus they may not believe that the dynamics is perfectly accurate; they may not even believe that every state in the model is really physically possible. For instance, some standard models of predator–prey interaction specify the evolution of population numbers for the two species by differential equations, which treat the population numbers as continuous variables, even though the answers to 'how many?' questions should be discrete. A more general kind of simplification is that interactions between the target system and the rest of the universe are ignored. However, this kind of indirect model-building strategy is widespread throughout most sciences. It is applied to both modal and non-modal questions, and is quite consistent with a full-bloodedly realist attitude towards the target physical system. It does not detract from the concern of physics with modal matters.

3. Physical and metaphysical modalities

At this point, Sider might concede a modal dimension to physics, but deny that the modality is of the right kind for my purposes. Indeed, when physicists speak of 'all possible states' of a system, they may well not intend all *metaphysically* possible states. For instance, physicists may build into the dynamics various physical assumptions about the working of the system without assuming them to be metaphysically necessary. They may not even intend them to be physically necessary, in the strong sense of following from the laws of physics; they may instead embody more specific assumptions about, say, the dust motes in Saturn's rings that can be held fixed for their purposes. The issue is not whether they comprehend all metaphysically possible states of the system but whether they are relevantly related to its metaphysically possible states.

If the states in phase space were only intended to be *epistemically* possible, in other words, consistent with what we know about the system, they would not be relevant in the right way for my purposes. For suppose that some scientists conjecturally propose a dynamics for the phase space of a given system (perhaps in the form of some differential equations governing the specified parameters). The dynamics implies, for instance, that *necessarily* if the system is in state s_0 then in one second it will be in state s_1. The scientists cannot simply replace the strict conditional 'necessarily if' by a material one (perhaps governed by 'always'), for that would make the conjecture about s_0 vacuously true if the system actually never is in s_0: but if they happen to know that for some accidental reason the system actually never is in s_0, that does not verify the conjectured dynamical relation between s_0 and s_1. But it is also not to the point for them to conjecture that it is *epistemically* necessary that if the system is in s_0 then in one second it will be in s_1, which is in effect to conjecture that they *know* that if the system is in s_0 then in one second it will be in s_1. They know that they do not know so much (by hypothesis the dynamics is conjectural), and anyway their aim is to investigate the physical system itself, not their knowledge about it. They are, after all, physicists, not epistemologists. It is clear that the relevant modality for their conjectures to be about is not epistemic necessity but some sort of *physical* necessity, perhaps weaker than nomic necessity.

It is widely agreed that what is physically possible is also metaphysically possible. Someone might doubt the entailment on the grounds that logical consistency with the laws of nature suffices for nomic possibility but not for metaphysical possibility. However, such a view of nomic possibility makes it unduly sensitive to syntactic form, for instance by implausibly making it nomically possible for Hesperus to be distinct from Phosphorus. I argue in more detail elsewhere against such a conception (Williamson 2016). Anyway, whether 'Physical system S is in state *s*' is logically consistent with the laws of nature depends on delicate issues about the semantics of the terms 'S' and '*s*', of a kind with which physicists are hardly bothering when they consider the states in the phase space as the possible states of S. It is reasonable to assume that physical possibility in the physicists' sense entails metaphysical possibility. I will make that assumption henceforth, but the arguments below could also be adapted to the weaker assumption that physical possibility of the relevant sort is just *good evidence* for metaphysical possibility.

By Sider's own lights, when we read $\Box$ and $\Diamond$ as expressing *physical* necessity and *physical* possibility, in a sense appropriate to phase spaces as studied in physics, we can legitimately apply abductive criteria such as simplicity and strength to our choice of theory in quantified modal logic, because such theories are formulated purely in terms drawn from core non-modal logic and from physics, which he treats as unexceptionable for such purposes. Thus we can argue abductively in the manner of *Modal Logic as Metaphysics* for a restricted version of necessitism: it is *physically* necessary that everything is

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physically necessarily something. Call that principle 'physical necessitism', and its negation 'physical contingentism'; call the original version of necessitism in the book 'metaphysical necessitism', and its negation 'metaphysical contingentism'. By what was assumed above, metaphysical necessitism entails physical necessitism, but there is no obvious argument for the converse. Why should not some truths be physically necessary but metaphysically contingent? Ontology might be constant across the inner realm of physical possibilities, yet start varying in the outer realm of physically impossible metaphysical possibilities. But let us consider what physical necessitism already gives us.

The first point to notice is that the usual alleged common sense counter-examples to (metaphysical) necessitism all involve cases that are *physically* possible, not just metaphysically possible—indeed, if they were physically impossible, that would severely weaken their claim to common sense status. For instance, it was physically possible for Wittgenstein to have had a child, not just metaphysically possible. Similarly, it was physically possible, not just metaphysically possible, for any of us never to have been conceived or born. Thus, if the usual common sense cases are counterexamples to metaphysical necessitism, they are equally counterexamples to *physical* necessitism. Conversely, if physical necessitism is true, then most metaphysical contingentists are wrong about what is going on in the cases they regard as the most glaring counterexamples to metaphysical necessitism, and so far their necessitist opponents are right. That would already be a disaster for metaphysical as well as physical contingentism, since it would deprive both views of their strongest and most natural motivation.

A further corollary of physical necessitism is that metaphysical contingentists would still be stuck with much of the ontological inflation to which they object in metaphysical necessitism. For there would still be all the non-concrete entities that in some physical possibility are children of Wittgenstein, and so on. That is so whether or not one accepts Sider's claim that 'there is surely some sort of presumption to minimize ontology'. My own view is that one should minimize ontology only insofar as doing so contributes towards more general abductive values, such as simplicity and strength. For example, in the foundations of set theory, the tendency is towards new axioms that *maximize* rather than minimize set-theoretic ontology, because they make for a simpler, stronger, and more unified conception of the set-theoretic universe. But there is no need to press issues about ontological parsimony here.

In brief, Sider's methodological reservations leave unscathed an abductive case like that in *Modal Logic as Metaphysics* for physical necessitism in place of metaphysical necessitism, and once one accepts physical necessitism, one has already paid most of the price of metaphysical necessitism in pre-theoretically surprising consequences. In those circumstances, the point of holding out against metaphysical necessitism is obscure.

4. Modal logic's track record

There are also reasons independent of physical modality to reject Sider's methodological strictures on modal logic. He depicts it as having a poor track record, one 'mostly within philosophy'. That is clearly false of modal logic as a technical discipline, which is a flourishing branch of mathematical logic now being advanced more by mathematicians and computer scientists than by philosophers. Under dynamic, temporal and epistemic interpretations it has been extensively applied in both computer science and theoretical economics. The phrase 'mostly within philosophy' is apt for modal logic only when the modal operators are read metaphysically. It is hardly surprising that philosophers are those most interested in the metaphysical reading of the modal operators, just as they are those most interested in the unrestricted reading of the quantifiers. Sider accepts the latter reading as fundamental.

Of course, *Modal Logic as Metaphysics* adopts the metaphysical readings of the modal operators. What is the track record of modal logic with those readings? A formula is *metaphysically universal* if and only if its universal generalization is true on the metaphysical reading of the modal operators and the unrestricted reading of the quantifiers (Williamson 2013, 93–95). At the level of ordinary propositional modal logic, it is clear beyond reasonable doubt that all theorems of the modal system T are metaphysically universal formulas and that all metaphysically universal formulas are theorems of the stronger modal system S5. For T imposes only the compelling requirements that what obtains *can* obtain and that necessity is closed under logical consequence (in a language without tricky modal operators such as 'actually'), while anything stronger than S5 puts a specific finite upper bound to the number of mutually exclusive possibilities (Williamson 2013, 111). I will not argue here against sceptics who doubt that there are non-actual possibilities, just as I will not argue against sceptics who doubt that there is a physical world. When philosophers discuss metaphysical necessity and possibility, much of the time they use only formulas of modal depth 1, that is, formulas in which modal operators occur, but not in the scope of further modal operators. But every theorem of S5 of modal depth 1 is already a theorem of T, so all the candidate logics agree on such formulas. There is no serious dispute as to which formulas of modal depth 1 are metaphysically universal. Although some formulas of greater modal depth are more contentious, there is considerable convergence amongst modal metaphysicians on S5 as the correct propositional modal logic for metaphysical modality—for good reason, as I have argued elsewhere (Williamson 2016).

For quantified modal logic, contingentists and necessitists disagree even over formulas of modal depth 1 (such as $\forall x \Box \exists y\, x = y$). But those theoretical disagreements should not obscure the extensive agreement on the basics. For instance, necessitists and contingentists may well agree that any first-order modal formula true in all variable domain Kripke models is metaphysically universal (since constant domain models are a special case of variable domain models). Although counterpart theorists may reject a few of those theorems, their revisionism does not go far, on pain of discrediting counterpart theory. Of course, agreement is not truth, but there is no serious reason for suspecting that modal logicians have got the modal logic of metaphysical modality radically wrong. Even on its metaphysical reading, modal logic has a far more successful track record than do most branches of metaphysics.

5. Metaphysical modality as the maximal objective modality

I have not discussed Sider's disjunctive conception of metaphysical necessity, as a ragbag of disparate kinds of truth. If such a conception were correct, metaphysical modality might well be an unfit subject for systematic theorizing, abductive or otherwise. My own understanding of it is quite different. In ordinary thought we can get at it through counterfactuals, as what would be the case no matter what was the case (Williamson 2007). At a more theoretical level, we can approach the same metaphysical modality as the limiting case of a kind of modality we may call *objective*, by contrast with other categories such as *epistemic* modality. Physical possibility is an example of an objective modality; so are various grades of practical possibility. At least to a first approximation, objective modalities correspond to what linguists classify as *circumstantial* readings of natural language modal auxiliaries such as 'can' and 'must'. The category is similar to that of objective interpretations of probability, by contrast with epistemic, subjective and purely logical interpretations. Metaphysical possibility is simply the maximal type of objective possibility, in the sense that something is metaphysically possible if and only if it has at least one objective type of possibility. Physical possibility as investigated by physics is an objective type of possibility and so automatically entails metaphysical possibility, in vindication of a claim made earlier.

The objective modalities are closed under various operations. For instance, if $\Diamond_1$ and $\Diamond_2$ are types of objective possibility, $\Diamond_1\Diamond_2$ is also a type of objective possibility. In particular, therefore, if $\Diamond$ is metaphysical possibility, which is an objective type of possibility, then $\Diamond\Diamond$ is also a type of objective possibility, and so entails metaphysical possibility because the latter is the maximal type of objective possibility. That vindicates the S4 axiom $\Diamond\Diamond p \rightarrow \Diamond p$, perhaps the most controversial theorem of S5, which goes some way towards vindicating the convergence noted above on S5 as the propositional logic of metaphysical modality.

I defend the conception of metaphysical modality as the maximal objective modality in more detail elsewhere (Williamson 2016). It is arguably closer to what many metaphysicians have understood by the term than is Sider's disjunctive conception.

If metaphysical modality is just the limiting case of a category of modality many types of which are explored by the natural and social sciences, the presumption should be that we may legitimately apply the same sort of abductive methodology in theorizing about metaphysical modality that is applied in theorizing all over the natural and social sciences, just as *Modal Logic as Metaphysics* did. In particular, that gives us two converging lines of argument for *metaphysical* necessitism. We can give a direct abductive argument for it, along the lines of the book. But we can also give a more indirect argument, by first arguing abductively for various types of *physical* necessitism, and then arguing by a further abductive step for metaphysical necessitism as the strong and simple principle that unifies all the necessitist principles for more specific types of objective modality.

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