COROTATION SOLUTIONS IN THE ELLIPTIC ASTEROIDAL PROBLEM WITH STOKES DRAG

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Abstract. We have searched for stable stationary solutions of the resonant elliptic restricted problem of three bodies with Stokes drag. Results for the 1/2 external Jovian resonance are shown as example, including a comparison with numerical results of an N-body simulation.

1. INTRODUCTION

Recently, a number of studies (e.g., Weidenschilling & Davies, 1984; Patterson, 1987) have attempted to explain the present nearly resonant structure of the solar system by means of capture into resonance in a gas-rich scenario for planetary formation. These studies have shown that, under certain conditions, the orbit-orbit resonance of a mass-less particle with a planet can counteract the orbital decay due to gas drag and trap the body, not allowing further decay.

Weidenschilling and Davies used a simple analytical model for the motion in the neighborhood of a resonance to show that such capture can only occur in external resonances; Patterson obtained cases in which the orbits of the planetesimals can evolve into an apparent corotation. However, in both cases, the analytical models were simple and stationary points were studied only in the momenta, without regard to the rest of the variables.

In this paper we present preliminary results in the study of solutions of the asteroidal planar elliptic problem of three bodies with Stokes drag, averaged over the synodic period near a $(p+q)/p$ resonance. The purpose of this study is to use a rather complete model to search for stable stationary solutions of the system.

2. EQUATIONS OF MOTION

In rectangular heliocentric coordinates, the equations of motion for a mass-less particle can be written as:

$$\mathbf{x}' = -\frac{\partial F}{\partial \mathbf{y}}, \qquad \mathbf{y}' = \frac{\partial F}{\partial \mathbf{x}} + \mathbf{Y}(\mathbf{x},\mathbf{y}); \tag{1}$$

with $\mathbf{x}$ the coordinates and $\mathbf{y}$ the conjugate momenta. $F(\mathbf{x},\mathbf{y})$ is the Hamiltonian of the conservative system and $\mathbf{Y}(\mathbf{x},\mathbf{y})$ is the acceleration due to Stokes drag:

$$F(\mathbf{x},\mathbf{y}) = \frac{1}{2}|\mathbf{y}|^2 + U(\mathbf{x}), \qquad \mathbf{Y}(\mathbf{x},\mathbf{y}) = C(\mathbf{y} - \alpha\mathbf{\Omega}\times\mathbf{x})$$

where C is a coefficient and $\alpha\mathbf{\Omega}\times\mathbf{x}$ is the velocity of the gas. We suppose a negative radial pressure gradient which causes the orbital velocity of the gas to be slightly less than Keplerian velocity ($\alpha = 0.995$). For this reason, the particle may suffer an orbital decay even when in a circular orbit.

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The next steps are the transformation in Keplerian variables and the averaging of the right-hand sides of the equations over the synodic period, in a resonant scenario (see Ferraz-Mello, 1992). The non-dissipative terms are transformed as usual (see Klafke et al., these proceedings). The averaging of the dissipative terms involves cumbersome elliptic functions and are done using the tables of Gradshteyn and Ryzhik. The tables of undefined integrals were used to allow us to check the results. Al last, the averaged system was tranformed to the resonant variables (k, h, L, σ_1),

$$k = e\cos(\phi/q - \varpi), \qquad L = \sqrt{\mu a}$$

$$h = e\sin(\phi/q - \varpi), \qquad \sigma_1 = \phi/q - \varpi_1$$

where $\phi = (p+q)\lambda_1 - p\lambda$. The variables with subscript 1 correspond to the perturbing body. The resulting system can be written as:

$$\frac{dk}{dt} = \frac{\beta}{L}\frac{\partial R}{\partial h} - h\frac{d\sigma_1}{dt} - \frac{k\beta}{L(1+\beta)}\frac{dL}{dt} - k\beta P_1 \tag{2}$$

$$\frac{dh}{dt} = -\frac{\beta}{L}\frac{\partial R}{\partial h} + k\frac{d\sigma_1}{dt} - \frac{h\beta}{L(1+\beta)}\frac{dL}{dt} - h\beta P_1 \tag{3}$$

$$\frac{dL}{dt} = r\left(h\frac{\partial R}{\partial k} - k\frac{\partial R}{\partial h} - \frac{\partial R}{\partial \sigma_1}\right) + P_2 \tag{4}$$

$$\frac{d\sigma_1}{dt} = W + (2rL/\mu)\frac{\partial R}{\partial a} - \frac{\beta r}{L(1+\beta)}\left(h\frac{\partial R}{\partial h} + k\frac{\partial R}{\partial k}\right) \tag{5}$$

where:

$$r = p/q, \qquad P_1 = \frac{1}{2}C(1+\alpha)$$

$$W = (1+r)n_1 + rn, \qquad P_2 = -CL(1 - \alpha + 5\alpha e^2/16)$$

Since we are only interested in orbits with low excentricity, we used the Laplacian expansion for the disturbing function. The terms P_1 and P_2 are the contributions of the gas drag to the system. It is noted that there is no integral of motion.

3. COROTATION SOLUTIONS

In this presentation, we have searched for the mono-parametric families of corotation solutions (as a function of the drag coefficient C) in all first and second order commensurabilities (internal an external). As the disturbing body we have considered Jupiter, in a fixed ellipse of eccentricity $e_1 = 0.05$.

In the particular case in which corotation centers are considered as possible points of captures into resonance, only stable solutions are of interest. Even though singular points were found in all resonances, only external commensurabilies showed the existence of linearly stable solutions. Figure 1 shows corotation centers (as function of C),for one of these resonances (1/2).

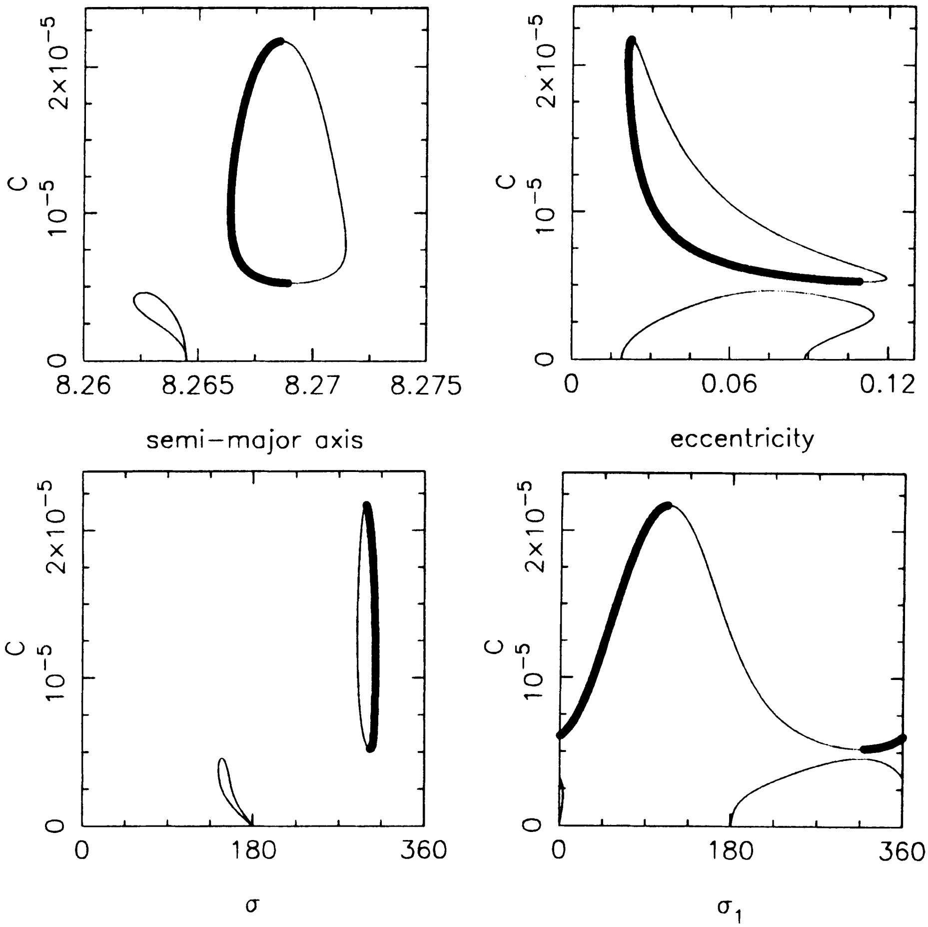

Fig. 1. Corotation centers, as a function of the drag parameter C, for the external 1/2 Jovian resonance. Broad lines indicate linearly stable solutions.

We can see the existence of two different families of solutions, one which is the analytical continuation of the corotation points in the Hamiltonian system without drag (see Ferraz-Mello et. al., these proceedings), and one with only exists in the dissipative case. Broad lines indicate linearly stable solutions.

4. CAPTURE IN COROTATION

In order to study whether a particle under the effect of Stokes drag can be captured in a corotation, we have preformed an N-body numerical simulation of a mass-less third body, started originally with semi-major axis equal to 9 A.U. and eccentricity $e = 0.01$. The value adopted for the drag coefficient was $C = 8.0 \times 10^{-6} days^{-1}$, chosen as to be inside the range of capture of the 1/2 resonance. Figure 2 shows the result of the simulation. The particle's orbit spirals until it reaches the resonance,

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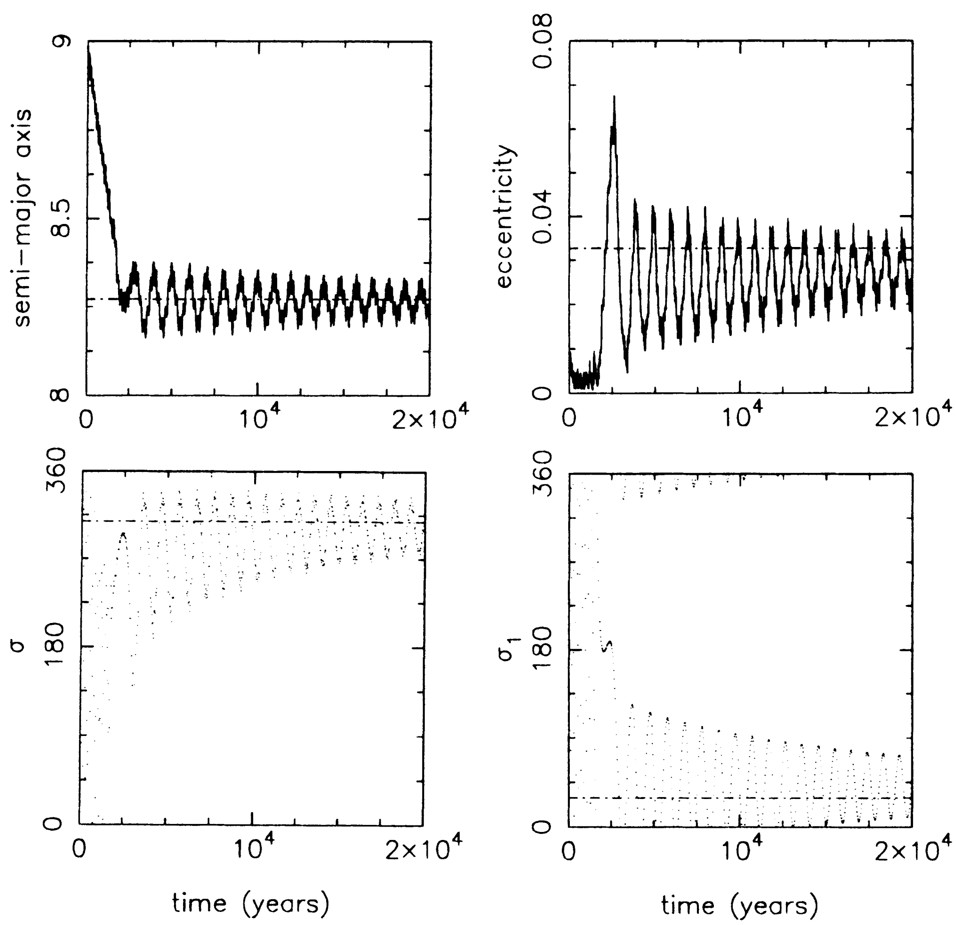

Fig. 2. N-body simulation of a capture into corotation. Doted line indicates predicted value.

it is then captured and the semi-major axis shows a damped oscillation around the resonant value. Similar behaviour is shown by the rest of the variables. In all cases, the observed equilibrium values show good agreement with those predicted by the model.

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