

Session V
CENTRAL STARS

THE ATMOSPHERES AND SPECTRA OF CENTRAL STARS

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ABSTRACT

After a brief discussion of the determination of the effective temperature, the surface gravity and the chemical composition of the central stars, we describe the typical problems arising in the computation of static and dynamical models of the very hot atmospheres of these objects. Special attention is paid (1) to the strong non-greyness of the atmosphere, (2) the electron-scattering contribution to the source function, (3) the contribution of the higher ions of C, N, O and Ne to the absorption coefficient, (4) the importance of radiation-pressure effects. The application of the computed energy distribution to the determination of the ionization stratification of the nebula and to the calculation of the bolometric correction is discussed.

A formation of emission edges due to the Schuster effect does not seem to occur in any of the static *non-grey* model atmospheres which have been computed so far. The reason is that the steep temperature gradient in the uppermost layers of these models strongly favors the formation of absorption edges. Using very simple dynamical models by Schmid-Burgk (1967) we discuss the possibility of a considerable flattening of this temperature gradient by a hydrodynamic outflow driven by radiative acceleration. We argue that the Schuster effect may be much more important, if these atmospheres are not in hydrostatic equilibrium.

The position of central stars of different spectral type in the $T_{\text{eff}}-g$ plane is discussed.

1. Introduction

A study of the atmospheres of central stars is of great interest for the following reasons:

(1) It enables us to calculate the energy distribution of the true surface fluxes of these stars, thus permitting:

- (a) a theoretical determination of the ionization stratification of the nebulae,
- (b) the definition of an improved scale of Zanstra temperatures,
- (c) the computation of improved bolometric corrections.

(2) It must finally offer a possibility to understand the rather unusual line spectra (e.g. of type WR, Of and NGC 246, see below) of many of these objects, thus permitting a direct determination of chemical abundances.

(3) We have an opportunity to study the physics of very hot stellar atmospheres in the range $4 \times 10^4 \text{°K} \lesssim T_{\text{eff}} \lesssim 2 \times 10^5 \text{°K}$.

Let us assume that the structure of the atmosphere of a central star is determined by the effective temperature T_{eff} , the surface gravity g and the chemical composition

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X_i of the atmosphere. This excludes (tentatively) the possibility of magnetic fields or rotation playing an important role in determining the structure of these atmospheres, but it does not exclude the possible instability of the atmosphere due to a strong 'radiative acceleration' (see below).

It is a very fortunate circumstance that in the case of central stars T_{eff} and g can be determined without understanding the rather complex line spectra of these objects. In this respect the study of central star atmospheres is even simpler than the investigation of other stellar atmospheres. Consequently we shall discuss in this paper:

- (1) the determination of T_{eff} and g (without using line spectra),
- (2) the computation of static and dynamic model atmospheres for these stars, the calculation of the frequency distribution of the emergent fluxes and application to the determination of the ionization structure of the nebula, and only finally
- (3) we shall briefly consider the spectra of central stars (which are not yet understood in detail).

2. The Determination of T_{eff} , g and the Chemical Composition

The knowledge of the Zanstra temperatures and the luminosity L (determined either by the $R(M)$ - or the $R(N_e)$ -method, see below) together with a fairly crude estimate of the masses of the central stars may be used to determine T_{eff} and g in the following way. Tentatively, T_{eff} may be identified with the Zanstra temperature*, g may be determined using the well-known relations:

$$L = 4\pi R_s^2 \sigma T_{\text{eff}}^4 \quad (1)$$

$$g = \frac{GM_s}{R_s^2} \quad (2)$$

where L and the stellar mass M_s are assumed to be known. R_s is the stellar radius, G the gravitational constant and σ the Stefan-Boltzmann constant.

Consequently the necessary empirical data are the Zanstra temperature, the luminosity and the mass of the central star. Let us discuss briefly the determination of these data:

(A) It is well known that in Zanstra's method one counts the number of photons beyond the Lyman limit (of H I, He I or He II) by measuring the number of photons in one or several recombination lines of these ions and compares this to the number of photons in the visual region. It is very important that the Zanstra temperature be identified with the effective temperature only for a nebula which is optically thick in the Lyman continuum – otherwise the Zanstra temperature gives only a lower limit

* Strictly speaking a new scale of Zanstra temperatures based on the wavelength distribution of the emergent flux for model atmospheres should be used. Such a Zanstra-temperature scale has been determined by Capriotti and Kovach (1968), using the model atmospheres by Böhm and Deinzer (1966).

for T_{eff} . Since we are only interested in definite values for T_{eff} , some criteria for the optical thickness of the nebula have to be applied. Such criteria have been worked out by Harman and Seaton (1966). They are based on comparison of the recombination lines HeII 4686 and H β (optical thickness in HeII), HeI 4471 and H β (optical thickness in HeI, provided HeII is absent) and on the presence of OI lines (optical thickness in HI). The importance of using the HeII lines for a study of the optical thickness of a nebula has been emphasized earlier, especially by Wurm and Singer (1952). Harman and Seaton (1966) have given a list of 42 central stars, for which the nebula is optically thick in at least one Lyman continuum and for which, consequently, fairly reliable effective temperatures can be determined.

(B) The determination of the luminosity of central stars requires a knowledge of the distance of the planetary, the apparent brightness of the central star and the bolometric correction for such a high-temperature object. The distance is equal to the ratio (R/ϕ) of the linear and the angular radius. Two methods have been used for the determination of R :

(1) Often one uses the so-called Shklovsky method (see Minkowski and Aller, 1954; Shklovsky, 1956), in which it is assumed that all planetary nebulae have approximately the same mass. The method makes use of the relation between the surface brightness $S_{\text{H}\beta}$ in H β and the electron density N_e

$$S_{\text{H}\beta} = \frac{\epsilon}{3} RN_e^2 h\nu_{\text{H}\beta} \alpha_{\text{H}\beta}. \tag{3}$$

ϵ is the filling factor, R the radius of the nebula, $\alpha_{\text{H}\beta}$ the recombination coefficient for hydrogen leading to the emission of H β . In addition one applies the obvious relation between the ionized mass of the nebula and N_e :

$$M = \frac{4\pi}{3} R^3 \epsilon N_e m_{\text{H}}. \tag{4}$$

If M is assumed to be known and $S_{\text{H}\beta}$ has been measured, R can be determined from (3) and (4). For obvious reasons this procedure has been called the $R(M)$ -method by Seaton (1966). Osterbrock (1960), O'Dell (1962), Khromov (1962) and Seaton (1966) have tried to check the assumptions of the $R(M)$ -method. The assumption of constant mass for all nebulae seems to be justified as a crude approximation. Obviously the mass given by Equation (4) is only identical with the total nebular mass for an optically thin nebula, so that this method can be applied only in this case.

Seaton (1966) has developed an approximate theory of $R(M)$ in the optically thick case. He finds the following relation between $R(M)$ and the actual radius R of an optically thick nebula

$$\frac{R(M)}{R_0} = \left(\frac{R}{R_0} \right)^{2/5} \left(\frac{L_0}{L} \right)^{1/5}, \tag{5}$$

showing that $R(M)$ increases monotonically with increasing R , but decreases with increasing L of the central star. Since in the early stages of a star following the Harman-Seaton sequence the luminosity of the central star is rather low, the $L^{-1/5}$ -dependence overcompensates the $R^{2/5}$ -dependence of $R(M)$. This leads to an explanation of the fact that there exists a minimum of the observed $R(M)$ -values as found by O'Dell (1963). This possibility gives us additional confidence in the applicability of the $R(M)$ -method.

(2) The second method is the so-called $R(N_e)$ -method (Harman and Seaton, 1964; Seaton, 1966). In this procedure one determines N_e from the ratio of forbidden-line intensities (e.g. from the ratio $[\text{OII}] 3726/[\text{OII}] 3729$), and then uses Equation (3). The disadvantage of this method is its strong dependence on an exact knowledge of the filling factor and the details of the density distribution within the nebula. This method has been applied by Seaton (1966) to those nebulae which are optically thick in hydrogen and to which consequently the $R(M)$ -method cannot be applied.

(C) The mass can be estimated only in a rather crude way. We may follow the argument by O'Dell (1963) and Osterbrock (1966) that the distribution of the planetary nebulae vertical to the galactic plane indicates $M \approx 1.2 M_\odot$ for the original star (according to the distributions for stars of different mass given by Schmidt, 1963). This original mass of $1.2 M_\odot$ is compatible with our present ideas about the evolution of these objects. Furthermore, if the nebula contains $0.6 M_\odot$ (as suggested by Seaton, 1966), we get a mass of $0.6 M_\odot$ for the central star in satisfactory agreement with the masses of normal D A white dwarfs (Weidemann, 1963).

Using the methods described under (A) and (B), T_{eff} and L can be determined and the HR-diagram can be drawn. This can be most easily done using nebulae which are optically thick in HeII (thus permitting the derivation of a reliable HeII Zanstra temperature) and optically thin in HI (so that the $R(M)$ -method can be applied). The main part of the Harman-Seaton sequence (see especially Seaton, 1966) is defined by these objects. Only the L of the very early (low-luminosity) part of the Harman-Seaton sequence has to be determined by the $R(N_e)$ -method. O'Dell (1968) has pointed out that the results for the Magellanic Clouds (by Miss Webster) indicate that the luminosities for the early phases of the Harman-Seaton sequence might be larger than the values derived by Seaton (1966). From our point of view it is also important to note that the central stars of the nebulae which are optically thick in HeII and HI seem to have considerably higher T_{eff} (up to $2 \cdot 10^5 \text{ }^\circ\text{K}$) than the maximum value reached by the Harman-Seaton sequence ($T_{\text{eff}} \approx 1 \cdot 10^5 \text{ }^\circ\text{K}$).

Having available the HR-diagram of the central stars and the mass estimates described in paragraph (C), we can calculate the position of the central stars in the $T_{\text{eff}}-g$ diagram, which is an extremely useful starting-point for model-atmosphere calculations. Such a diagram for the Harman-Seaton sequence and for the original sequence of O'Dell (1963) is shown in Figure 1. We have also drawn the curve (a straight line in the doubly logarithmic diagram) which separates the region permitting hydrostatic

solutions from the region in which such solutions are impossible (because the radiative acceleration g_{rad} is larger than the surface gravity g). This curve is defined by the relation

$$g = \frac{\sigma T_{\text{eff}}^4}{c} \sigma_{\text{el}}, \tag{6}$$

where σ_{el} is the cross-section for Thomson scattering. The simple relation (6) is only valid if Thomson scattering is the dominant absorption mechanism close to the instability limit. This seems to be the case (Böhm and Deinzer, 1966).

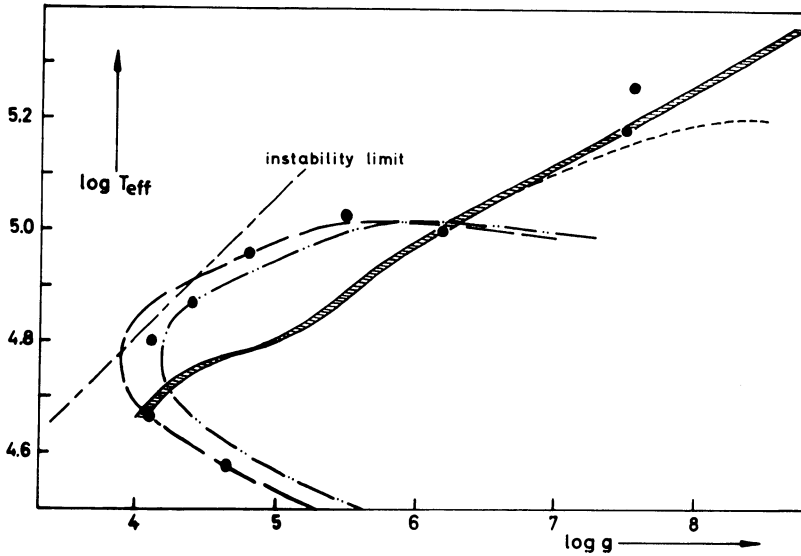


FIG. 1. The schematic Harman-Seaton sequence for $M = 1 M_{\odot}$ (-.-.) and $M = 0.5 M_{\odot}$ (---) and the (old) O'Dell sequence (widened line) in the $\log g$ - $\log T_{\text{eff}}$ -plane. The dots correspond to the position of computed non-grey model atmospheres. The instability limit is defined by Equation (6).

Finally we have to consider the determination of chemical abundances. Since there is little hope of gaining direct information without first understanding the structure of these atmospheres, the best possible working hypothesis is to assume that the central star atmospheres have the same chemical composition as the nebulae. We have used the nebular chemical composition given by Aller (1965).

3. Model Atmospheres and Energy Distribution of Surface Fluxes

Having T_{eff} , g and the chemical composition available, we can now proceed to the computation of model atmospheres.

It is useful to first give a brief list of the typical difficulties encountered in the calculation of very hot stellar atmospheres:

(1) Because of the strong absorption edges of He II and the higher ions of C, N, O and Ne these atmospheres are strongly *non-grey*.

(2) Especially in the intermediate part of the Harman-Seaton sequence, the electron-scattering contribution to the source function becomes rather large. This fact leads to a considerable increase in the complexity of radiative equilibrium computations.

(3) The 'radiative acceleration' has to be taken into account in the calculation of the hydrostatic equilibrium, i.e. the surface gravity g has to be replaced by

$$g - \frac{\pi}{c} \int_0^{\infty} (\kappa_{\nu} + \sigma_{el}) F_{\nu} d\nu, \quad (7)$$

where F_{ν} is the monochromatic radiative flux*. Since the middle part of the Harman-Seaton sequence lies either very close to or actually crosses the instability line (see Figure 1), we have to keep in mind the possibility that (7) may be negative in large parts of the atmosphere and that therefore no hydrostatic solution is possible.

(4) The radiative equilibrium is probably influenced considerably by strong lines in the far ultraviolet.

(5) Even the 'classical' model atmospheres in this $T_{\text{eff}}-g$ -range are rather extended. We find e.g. for a non-grey atmosphere of $T_{\text{eff}} = 7.4 \times 10^4 \text{ K}$ and $g = 2.51 \times 10^4 \text{ cm sec}^{-2}$ a thickness of $1.2 \times 10^{10} \text{ cm}$. This has to be compared to the radius of $7 \times 10^{10} \text{ cm}$ for such a star. It is obvious that treating such an atmosphere as plane-parallel is only a crude approximation.

Model atmospheres of central stars have been computed by Gebbie and Seaton (1963), Gebbie (1967), Böhm and Deinzer (1965, 1966) and Böhm (1965, 1967). Gebbie and Seaton have paid special attention to point 2 (influence of electron scattering on S_{ν}) but have assumed a grey temperature stratification and have neglected the contribution of the high ions of C, N, O, and Ne to the absorption coefficient. Böhm and Deinzer, on the other hand, have constructed non-grey models with a high degree of flux constancy. They have included the higher ions of C, N, O and Ne in the computation of the absorption coefficient, but they have used an iteration method for the determination of the electron-scattering part of the source function. This procedure converges only sufficiently fast if at least in the deeper parts of the atmosphere electron scattering is not too important. An ideal method would be a combination of both procedures (Gebbie-Seaton and Böhm-Deinzer). At the moment, however, this would require a very large amount of computing time, since the Gebbie-Seaton method would have to be applied for every frequency in every iteration step of the flux-iteration method.

Both groups have taken into account point 3 (radiative acceleration) and both have neglected points 4 and 5.

* In the following we have neglected the surface effects. These are due to the very rapid increase of F very close to the surface in wavelength regions in which κ_{ν} is very large compared to $\bar{\kappa}$.

These authors have restricted themselves to the consideration of hydrostatic models. Presently Schmid-Burgk (1967) is carrying out an investigation of hydrodynamic models for central stars in which (7) is negative in a large part of the atmosphere (see below).

Let us first consider the construction of model atmospheres in hydrostatic and radiative equilibrium. In the usual form of the hydrostatic equation, g has to be replaced by Equation (7). The condition of radiative equilibrium can be expressed as

$$F = \Sigma F_v = \Sigma \phi_v(S_v) = \text{const.} \tag{8}$$

(independent of depth), where the integral operator ϕ_v is defined as

$$\phi_v(S_v) = 2 \left\{ \int_{\tau_v}^{\infty} S_v(t_v) E_2(t_v - \tau_v) dt_v - \int_0^{\tau_v} S_v(t_v) E_2(\tau_v - t_v) dt_v \right\}. \tag{9}$$

The source function S_v is

$$S_v = \frac{\kappa_v}{\kappa_v + \sigma_{e1}} B_v + \frac{\sigma_{e1}}{\kappa_v + \sigma_{e1}} J_v. \tag{10}$$

J_v is the monochromatic mean intensity and has to be determined for every frequency separately by solving the Schwarzschild-Milne integral equation

$$J_v = A_v \left(\frac{\kappa_v}{\kappa_v + \sigma_{e1}} B_v \right) + A_v \left(\frac{\sigma_{e1}}{\kappa_v + \sigma_{e1}} J_v \right), \tag{11}$$

where

$$A_v(S_v) = \frac{1}{2} \int_0^{\infty} S_v(t_v) E_1(|t_v - \tau_v|) dt_v. \tag{12}$$

The most effective way to solve (8) is the use of a ‘temperature correction procedure’ (cf. Unsöld, 1951; Böhm, 1954; Avrett and Krook, 1963; Lucy, 1964; Henyey, 1967) in which one calculates ΣF_v for a given approximation of the temperature stratification and then converts the difference ΔF between the computed total flux and the required constant flux ($\sigma/\pi T_{\text{eff}}^4$) into a correction ΔB of the approximate stratification of the Planck function. Böhm and Deinzer have used Lucy’s (1964) method which is a modification of the Unsöld (1951) flux-iteration procedure. The following relation between ΔB and ΔF has been used :

$$\Delta B - \frac{\kappa_J}{\kappa_P} J = B - \frac{\kappa_J}{\kappa_P} \left\{ \frac{\Delta F(0)}{2} + \frac{3}{4} \int_0^{\tau} \frac{\kappa_F}{\kappa_P} \Delta F d\tau \right\}. \tag{13}$$

Here the three mean values κ_J , κ_P and κ_F of the absorption coefficient are defined by:

$$\kappa_J = \frac{1}{J} \int_0^\infty \kappa_\nu J_\nu d\nu; \quad \kappa_P = \frac{1}{B} \int_0^\infty \kappa_\nu B_\nu d\nu; \quad \kappa_F = \frac{1}{F} \int_0^\infty (\kappa_\nu + \sigma_{cl}) F_\nu d\nu. \quad (14)$$

The method of Böhm and Deinzer is an iteration scheme, in which one calculates $\Sigma\phi_\nu(S_\nu)$ (see Equations (8) and (9)) and then applies (13). Between two successive 'flux-iteration' steps one iterates Equation (11) twice for every frequency using the B_ν found by the flux iteration.

Gebbie and Seaton assume that the stratification of B_ν is known in advance (grey atmosphere). Consequently we only have to solve Equation (11). This is done by rewriting (11) as an integral equation for:

$$\psi_\nu = \frac{\sigma_{cl}}{\kappa_\nu + \sigma_{cl}} \{J_\nu - B_\nu\} \quad (15)$$

subtracting out the singularity of the integrand and replacing the integral by a gaussian sum. This leads to a system of linear inhomogeneous equations which can be solved.

Böhm and Deinzer (1966), and Böhm (1967) have calculated 9 models in the temperature range $3.8 \times 10^4 \text{ }^\circ\text{K} \leq T_{\text{eff}} \leq 1.8 \times 10^5 \text{ }^\circ\text{K}$. These models lie partly on the Harman-Seaton and partly on the O'Dell sequence (see Figure 1). Gebbie has computed 9 model atmospheres for the range $1.0 \times 10^5 \text{ }^\circ\text{K} \leq T_{\text{eff}} \leq 2.5 \times 10^5 \text{ }^\circ\text{K}$; $4.8 \leq \log g \leq 8.0$ using the Gebbie-Seaton method.

It is important to note that the non-grey models show (like non-grey models of cooler stars) a very steep temperature gradient close to the surface (Böhm and Deinzer, 1965). This is a reaction of the atmosphere to the rapid increase of F_ν in the ν -region with large κ_ν when one approaches the surface.

With one exception ($T_{\text{eff}} = 4.625 \times 10^4$; $g = 1.26 \times 10^4$) it has been possible to construct model atmospheres with a flux constancy of better than 1%.

What kind of emergent flux distributions do we get for these models? Here we have to distinguish between the results of Gebbie and Seaton (1963) and Gebbie (1967) on the one hand, and Böhm and Deinzer (1966) and Böhm (1967) on the other. For the reason discussed above the results of Böhm and Deinzer show effects of the steep ('non-grey') temperature decrease at the surface, whereas such effects do not appear in the results of Gebbie and Seaton. Furthermore Gebbie and Seaton neglect absorption by the higher ions of C, N, O and Ne. This has the consequence that for wavelengths which are sufficiently small compared to 228 \AA (Lyman edge of He II), electron scattering becomes the only absorption mechanism. Therefore they get a rather high F_ν (larger than the black-body radiation for $T = T_{\text{eff}}$) in this frequency range. On the other hand the absorption by high ions of C, N, O and Ne may have been somewhat

overestimated by Böhm and Deinzer, who used the hydrogen-like approximation for these ions.*

A possibly very important effect in these high-temperature atmospheres is the Schuster-effect as pointed out by Gebbie and Seaton (1963). It is well known (cf. Kourganoff, 1963) that J_ν is smaller than B_ν close to the surface in those ν -regions in which B_ν increases only slowly with τ_ν . This is the case e.g. in the 'red' part of the spectrum. ('Red' means: at a wavelength considerably longer than the λ corresponding to the maximum of the Planck function.) In this case S_ν increases abruptly as we pass through the absorption edge going from longer to shorter wavelengths: As shown by Equation (10), the jump in κ_ν has the effect that S_ν jumps from a lower value (close to J_ν) to a higher value (close to B_ν): it shows an emission edge. If S_ν behaves in this way in a large part of the atmosphere, one might expect to find an *emission edge* also in the emergent flux $F_\nu(0)$. On the other hand, one has to keep in mind that there is also present the well-known effect which usually leads to the formation of an *absorption edge*: One looks deeper into the atmosphere on the long wavelength side of the edge

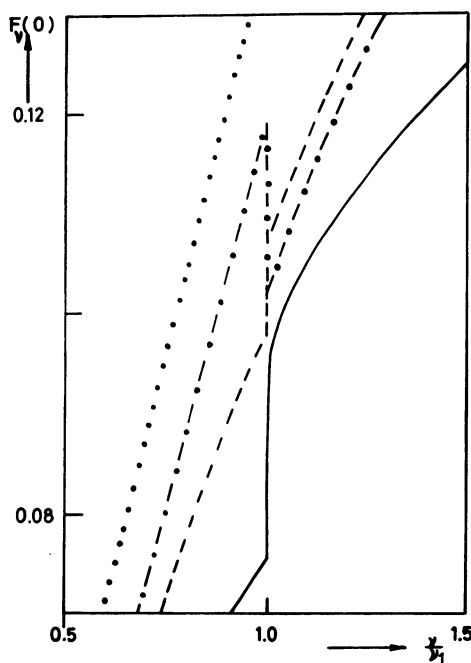


FIG. 2. The frequency distribution of $F_\nu(0)$ at the H γ -Lyman edge for grey model atmospheres of $T_{\text{eff}} = 1.0 \times 10^5$ and $\log g = 4.8$ (—); $\log g = 5.0$ (---) and $\log g = 5.2$ (-·-·-). The dotted curve shows the black-body distribution. This figure has been taken from Gebbie (1967).

* A computation using absorption coefficients determined by the quantum-defect method (Flower, 1968) has just been started.

than on the short wavelength side. Consequently one might expect that there will be a certain amount of cancellation between both effects and that a fairly accurate knowledge of the atmospheric structure will be required in order to decide whether an emission or absorption edge will be present.

As pointed out by Mrs. Gebbie (1967), the formation of an emission edge will be favored in atmospheres of relatively low g (see Figure 2). This is due to the great importance of electron scattering in atmospheres of low surface gravity.

It is also important to note that, according to the statement made above, the formation of an emission edge will be favored by a small temperature gradient and will be suppressed by a steep T -gradient. Consequently in a non-grey atmosphere (having a rather steep T -gradient close to the surface; see above) there is a strong tendency towards the suppression of emission edges. So it is understandable that Gebbie and Seaton (using grey models) often find emission edges for the H I Lyman continuum, whereas the same edge appears in absorption for all non-grey models (Böhm and Deinzer, 1966; Böhm, 1967) which have been computed. Typical diagrams of $F_\nu(0)$ for two atmospheric models of different T_{eff} and g are presented in Figure 3. Besides a rather strong absorption edge of He II and a fainter H I edge, one sees also rather pronounced absorption edges of ions like Ne IV, O V, Ne V etc. One important aspect is the strong depression of the $F_\nu(0)$ (as compared to the Planck function) shortward of

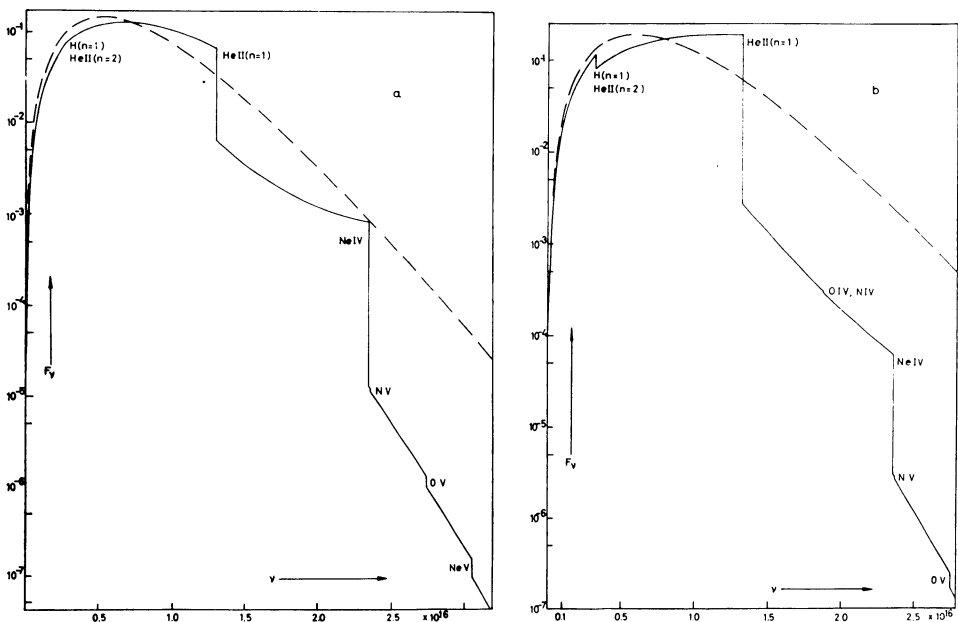


FIG. 3. $F_\nu(0)$ for model atmospheres with $T_{\text{eff}} = 9.12 \times 10^4 \text{ K}$, $g = 6.3 \times 10^4 \text{ cm sec}^{-2}$ (a), and $T_{\text{eff}} = 1.0 \times 10^5 \text{ K}$, $g = 1.5 \times 10^6 \text{ cm sec}^{-2}$ (b). The broken curve shows the black-body distribution for the same temperature.

$\lambda 228$ in general and especially shortward of the NeIV edge. It is obvious that this depression must have a strong influence on the ionization stratification of the nebula. Since we have used only a crude approximation for the computation of the absorption coefficients of the C-, N-, O- and Ne- ions, it is important to ask: How strongly do these results depend on an accurate knowledge of the absorption coefficient for these ions? We have made one test calculation for a star of $T_{\text{eff}} = 7.4 \times 10^4 \text{ }^\circ\text{K}$; $g = 2.51 \times 10^4 \text{ cm sec}^{-2}$, in which we have arbitrarily reduced the absorption coefficient for all C-, N-, O- and Ne-ions by a factor of 6.67 as compared to our earlier calculations. Surprisingly the general qualitative picture is changed rather little by these large changes in the absorption coefficient. The main reason is that the absorption coefficient immediately shortward of the two most important absorption edges (OIV – NIV and NV) is so high that one essentially sees only the ‘surface’ of the atmosphere in these wavelength regions. Even after an artificial reduction of κ_ν in this ν -range by a factor 6.67 one still essentially sees only the surface. These ideas can be applied only to $T_{\text{eff}} \approx 7 \cdot 10^4 \text{ }^\circ\text{K}$; in other T_{eff} ranges the influence of an error in κ_ν may be much larger.

The models can be used to determine a better scale of Zanstra temperatures and improved bolometric corrections for central stars. Improved Zanstra temperatures have been determined by Capriotti and Kovach (1968). They find (for a fixed ratio of the measured $H\beta$ intensity to the continuum intensity) higher Zanstra temperatures for non-grey model atmospheres than for black-body radiation. Using the models by Böhm and Deinzer (1966) they find e.g. a Zanstra temperature of $1.18 \times 10^5 \text{ }^\circ\text{K}$ for a star which would have a Zanstra temperature of 1.0×10^5 on the black-body scale. Bolometric corrections for model atmospheres have been computed by Böhm (1967*b*) and have been compared to those calculated for black-body radiation. The results are given in Table 1.

Table 1
Bolometric corrections for central stars of planetary nebulae.
(non-grey model atmospheres)^a

Model No.	$T_{\text{eff}} (\text{ }^\circ\text{K})$	$g (\text{cm sec}^{-2})$	References ^b	B.C.N.G.R. ^c	B.C.B. ^d
1	3.800×10^4	4.5×10^4	II	-3.65	-3.36
2	4.625×10^4	1.26×10^4	I	-4.27	-3.93
3	6.310×10^4	1.26×10^4	I	-5.26	-4.85
4	7.400×10^4	2.51×10^4	II	-5.73	-5.34
5	9.100×10^4	6.3×10^4	I	-6.36	-5.97
6	1.000×10^5	1.5×10^6	I	-6.81	-6.27
7	1.060×10^5	3.16×10^5	I	-6.89	-6.44
8	1.500×10^5	3.06×10^7	I	-7.93	-7.54
9	1.800×10^5	8.6×10^7	II	-8.25	-8.11

^a All bolometric corrections given here refer to the colour V of the U-B-V system.
^b Paper from which the $F_\nu(0)$ has been taken: (I) = Böhm and Deinzer (1966), (II) = Böhm (1967).
^c B.C.N.G.R. = bolometric correction for the non-grey model atmosphere.
^d B.C.B. = bolometric correction for black-body radiation with $T = T_{\text{eff}}$.

The relatively large deviations of $F_\nu(0)$ from black-body radiation in some ν -regions has important consequences for the ionization stratification of the nebula. These have been studied by Goodson (1967). Since all central stars which have been investigated have a higher $F_\nu(0)$ than the black-body curve between the He I and He II Lyman edges and a lower $F_\nu(0)$ shortward of $\lambda 228$, one finds e.g. much more O III and less O II in a nebula excited by a 'star' with a non-grey model atmosphere than in a nebula excited by black-body radiation of the same T_{eff} . The reason is that the O II ionization edge lies between the He I and He II edges, in the ν -region where the radiation field is rather strong. On the other hand the O III ionization edge occurs in a ν -region in which the radiation field is strongly depressed. Consequently there will be relatively few O II and O IV ions, but relatively many O III ions. A similar effect occurs in the ionization of N. This is clearly shown in Figures 4 and 5 in which we compare the ionization stratification which one gets for the computed $F_\nu(0)$ of a non-grey model atmosphere of $T_{\text{eff}} = 46250^\circ\text{K}$ and the stratification following from the black-body energy distribution for the same T_{eff} . It is important to note that the black-body model leads to a pronounced Ne v zone, whereas one gets much less Ne v from the $F_\nu(0)$ of a non-grey atmosphere. As pointed out by Williams (1967), this leads to difficulties in the interpretation of the observed spectra of planetaries. There is still some hope that these can be avoided when a calibration of the Zanstra-temperature scale is used which is based on the computed $F_\nu(0)$ for non-grey atmospheres (Capriotti and Kovach, 1968). If this should not be the case we have to ask: How can the strong ultraviolet (for $\lambda 228 \text{ \AA}$) deficiency be avoided? From what has been stated above it is

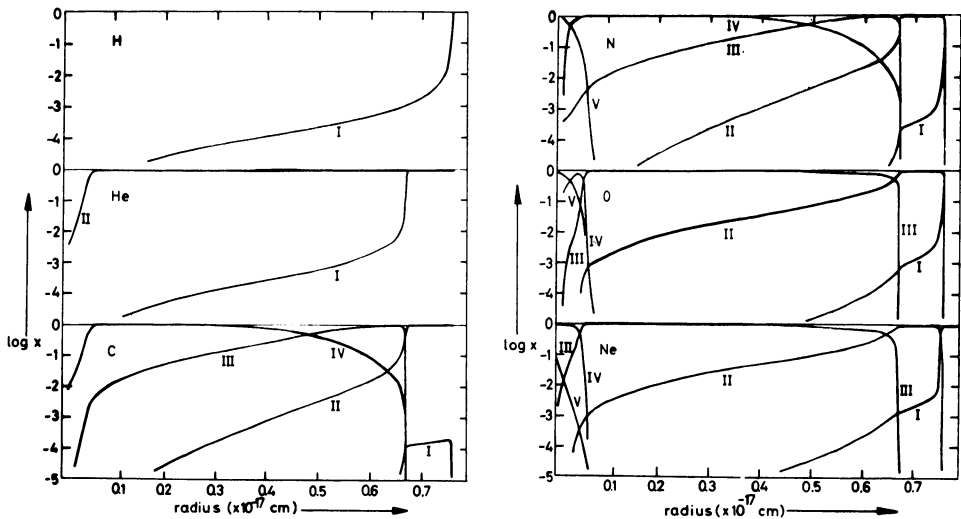


FIG. 4. The logarithm of the abundance x of the different ions (designated by Roman numbers) of H, He, C, N, O and Ne as a function of the distance from the central star in a planetary nebula of constant density excited by stellar black-body radiation of $T = 46250^\circ\text{K}$. After Goodson (1967).

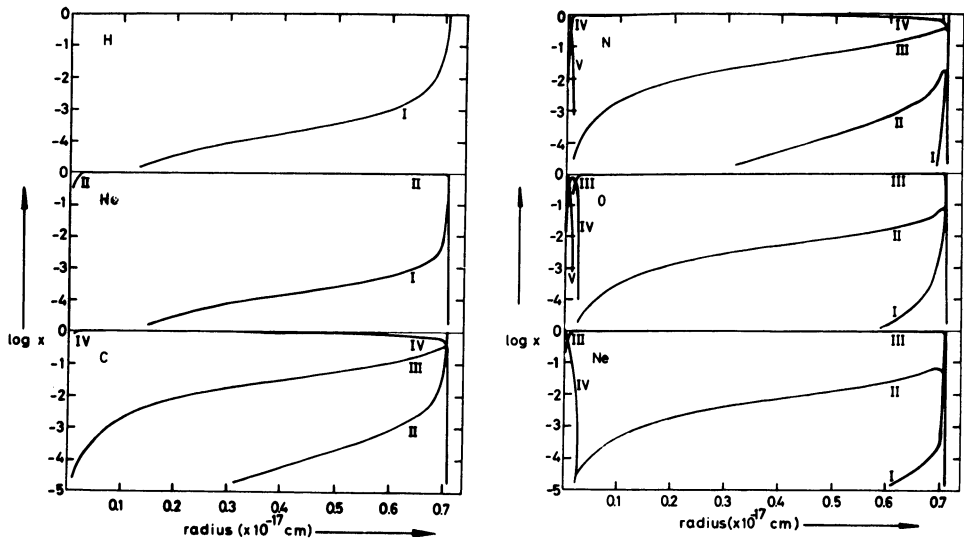


FIG. 5. The same as Figure 7, but with excitation by the radiative flux $F_{\nu}(0)$ of a non-grey model atmosphere of the same temperature and $g = 1.24 \times 10^4 \text{ cm sec}^{-2}$. After Goodson (1967).

obvious that we get a smaller deficiency if either the surface g is low or the temperature gradient is sufficiently flat. Whether a lower surface g alone would be sufficient to convert the ultraviolet deficiency into an ultraviolet excess as required by Williams is not known at present. How could we get a flatter temperature gradient in the high layers of the atmosphere?* One important possibility is that a considerable fraction of all central star atmospheres are not in complete hydrostatic and radiative equilibrium, but are continuously losing mass, e.g. due to the instability discussed in connection with Equation (7). The possible evidence for such a process is partly empirical (central stars having Wolf-Rayet spectra, large emission line width in central stars of the NGC 246 type; see Greenstein and Minkowski, 1964) and partly theoretical (a hydrodynamical explanation of planetary nebulae probably requires a stellar wind from the central star; Mathews, 1966). Schmid-Burgk (1967) has started a study of the hydrodynamics of a hot atmosphere in which Equation (7) is negative in a large part of the atmosphere. At the moment his calculations are not yet finished and he has available only certain similarity solutions (Sedov, 1959) of the problem. In order to find such similarity solutions the original depth-dependence of the temperature and the ρ - and T -dependence of κ has to be simplified in such a drastic manner that they are no longer realistic from a physical point of view. The problem is considered as a time-dependent one in the sense that one starts out from a stable situation and then

* It would not make sense to argue that the observations show that a grey stratification is 'better' than the non-grey one, unless physical arguments could be given for this peculiar result.

lets the total radiative flux F grow with time while keeping g fixed. Schmid-Burgk's solutions show that the $T(\tau)$ becomes much flatter as time goes on (and consequently the outflow of mass becomes more important). We may suspect that this will be also true for models which are more realistic from a physical point of view.* Consequently one expects that the especially steep temperature gradient in the upper layers of the non-grey atmospheres will be flattened and that therefore the $F_{\nu}(0)$ will more easily be influenced by the Schuster-effect. Moreover, the depression in the far ultra-violet will be reduced considerably. In Figure 6 the $T(\tau)$ -relation is shown as a function

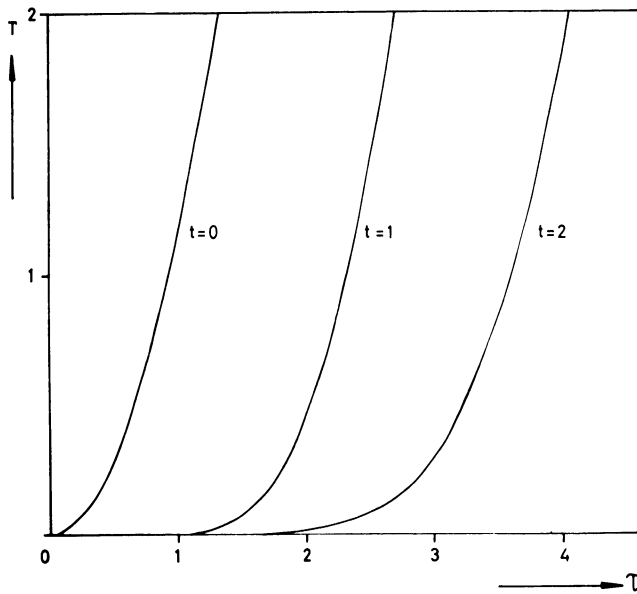


FIG. 6. The $T(\tau)$ relation as a function of time for Schmid-Burgk's similarity solutions.

of time for one of Schmid-Burgk's similarity solutions. The initial stratification is a polytropic atmosphere with $\Gamma = 11$, which means that

$$\rho \propto z^{1/10} \quad (16a)$$

$$T \propto z, \quad (16b)$$

where z is the geometrical depth. The opacity is assumed to be proportional to $\rho T^{-3/5}$. The time-dependence of F is assumed to be:

$$F \propto t. \quad (17)$$

* Schmid-Burgk is now doing such calculations, in which the system of partial differential equations is solved numerically. The main difficulty is of course a correct treatment of the radiative terms.

The diagram clearly shows the increasing flattening of the $T(\tau)$ -relation with increasing time.

4. Spectra of Central Stars

We are still very far from understanding the spectra of central stars in any detail. Since all classical methods of quantitative spectral analysis do not seem to work for most of these rather peculiar objects, the only promising approach seems to be the following:

- (1) Use the available information on T_{eff} and g to construct model atmospheres (as described in the preceding sections);
- (2) Predict the line spectra for these models;
- (3) Compare the predicted and the observed spectra and improve the models by a trial and error method.

Steps 2 and 3 have not yet been carried out. (But, as we have seen in the preceding section, even the use of the empirical information on the far UV continuous spectra already leads to better model atmospheres.) As has been emphasized by Aller and Liller (1967), a direct spectroscopic analysis should be possible for the absorption-line O stars among the central stars.* But even when considering these objects one has to keep in mind that they probably have a considerably higher g than normal O stars.

Very useful surveys of the empirical data on the spectra of central stars have been given by Vorontsov-Velyaminov (1953), Aller (1956), Aller and Liller (1967), and Perek and Kohoutek (1967).

Six different types of spectra seem to occur, namely:

- (1) Wolf-Rayet spectra, mainly of type WC or intermediate between WC and WN. (Only recently Bertola (1964) has found a central star of spectral type WN.)
- (2) Of-spectra (e.g. NGC 2392; see Wilson, 1948) often showing some deviations from the spectra of population-I Of-stars (cf. Oke, 1954).
- (3) Spectra intermediate between Of- and Wolf-Rayet type.
- (4) Absorption-line O spectra.
- (5) Purely continuous spectra (within the presently obtainable spectral resolution).
- (6) Spectra of the NGC 246 type (which are high-excitation spectra with strong O VI emission lines). Very high temperature objects of this type have been studied by Greenstein and Minkowski (1964).

Among the brightest planetary nuclei there is a fairly large number of Wolf-Rayet stars. There is some hope that some fundamental questions about the nature of the Wolf-Rayet phenomenon may be answered using empirical information about planetary nuclei. One may ask: Can the instability of the atmospheres of Wolf-Rayet stars be due to the action of the 'radiative acceleration' discussed above? If that should be the case all Wolf-Rayet stars should lie beyond (or at least close to) the instability line drawn in Figure 1. From this point of view it is interesting to find out whether stars of

* A determination of ionization temperatures in these objects has been carried out by Aller (1948) following the work of Petrie (1947) for normal O stars.

the six different spectral groups cover different regions of the g - T_{eff} -plane. It is obvious that such a study can be especially useful from our point of view, if we restrict ourselves to objects for which reasonably accurate values of g and T_{eff} can be given. Consequently we shall limit our study to the objects studied by Harman and Seaton (1966) and Seaton (1966), since these authors have paid special attention to the determination of the optical thickness of the nebulae which are used for the determination of T_{eff} and L . Even so, in view of the uncertainties in the determination of T_{eff} and g (see above), we have to expect a large scatter of the points. Spectral classification is available (see Perek and Kohoutek, 1967; Aller, 1956; Aller and Liller, 1967) for 28 of the 42 objects with known values of T_{eff} and g . These objects have been listed in Table 2.

Table 2
Relation between spectral type, effective temperature and surface gravity for 28 central stars

Object	Adopted Spectral Type for Central Star ^a	T_{eff} (K)	g (cm sec ⁻²)
NGC 40	WC 8	3.39×10^4	1.66×10^5
IC 351	W	9.12×10^4	5.75×10^4
NGC 1501	WC 6	7.25×10^4	7.25×10^4
NGC 1535	cont.	7.41×10^4	1.86×10^4
IC 418	WC 7	4.27×10^4	2.88×10^4
NGC 2022	cont.	9.12×10^4	5.13×10^4
IC 2149	O 7	4.90×10^4	8.13×10^2
NGC 2371-2	cont.	1.00×10^5	1.91×10^5
NGC 2392	Of	6.76×10^4	9.33×10^3
NGC 3242	cont.	9.33×10^4	7.41×10^4
NGC 6058	cont.	7.25×10^4	2.51×10^4
NGC 6210	Of	5.01×10^4	3.16×10^6
NGC 6309	cont.	9.55×10^4	3.98×10^4
NGC 6543	W	6.61×10^4	1.48×10^4
NGC 6572	W	6.17×10^4	8.71×10^3
NGC 6751	WC 6	7.59×10^4	1.70×10^4
NGC 6804	cont.	7.25×10^4	6.61×10^3
BD + 30°3639	WC 8	4.57×10^4	9.12×10^4
NGC 6826	Of + W	6.92×10^4	1.32×10^4
NGC 6853	cont. + He I 5876	1.32×10^5	6.76×10^6
NGC 6891	O 7	5.50×10^4	4.37×10^3
NGC 6905	WC 6	1.02×10^5	1.48×10^5
NGC 7008	cont.	9.77×10^4	3.16×10^5
NGC 7009	cont.	8.13×10^4	5.75×10^4
NGC 7026	WC 6	9.77×10^4	2.63×10^4
IC 5217	W	7.41×10^4	2.82×10^4
NGC 7662	cont.	1.00×10^5	3.72×10^4
HD 138403	Of	4.27×10^4	2.69×10^3

^a From Perek and Kohoutek (1967), Aller (1956), Aller and Liller (1967).

Of these 28 central stars 11 are WR-stars, 3 Of-stars; 2 absorption-line O stars; 1 object is intermediate between Of and WR, 11 show a continuous spectrum (without detectable absorption or emission lines). Unfortunately there is only one star of the NGC 246 type in Seaton's list, namely the central star of NGC 246 itself. But NGC 246 is probably optically thin for H α and HeII. Consequently the Zanstra temperature defines only a lower limit for T_{eff} and the central star cannot be located in the g - T_{eff} -plane. The position of central stars of different spectral type in the g - T_{eff} -plane is shown in Figure 7.

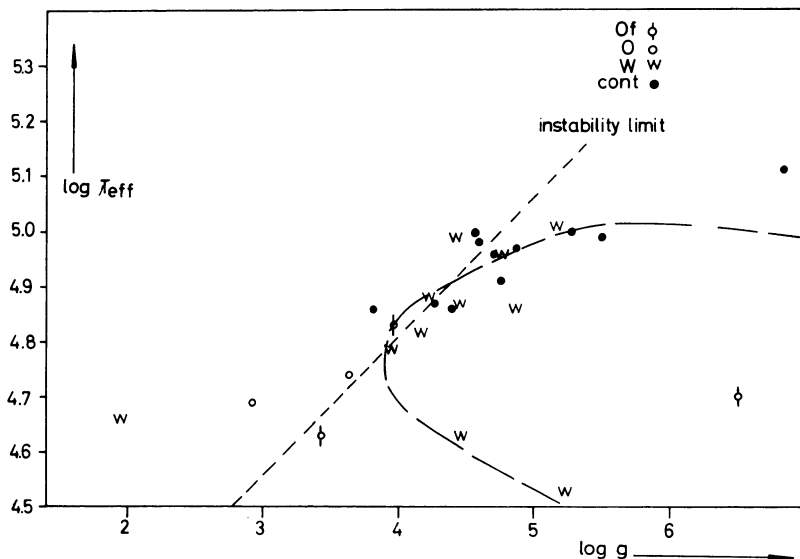


FIG. 7. The position of central stars of different spectral types in the g - T_{eff} -plane.

Though the scatter is considerable, certain trends can easily be recognized. The stars with 'purely continuous' spectra occur only in the high-temperature part of the Harman-Seaton sequence (starting at about $T_{\text{eff}} \approx 7.5 \times 10^4 \text{ K}$). Among the stars available in our list all objects in advanced evolutionary stages (up to $\log g \approx 7$, $T_{\text{eff}} \approx 10^5 \text{ K}$) seem to belong to this spectral class. (It would be interesting to know, how the very hot objects of NGC 246 type fit into this picture.) The Wolf-Rayet stars occur over a wide range of temperatures ($3.3 \times 10^4 \text{ K} \lesssim T_{\text{eff}} \lesssim 1 \times 10^5 \text{ K}$), but they are all rather close to the instability limit, except for the rather low-temperature objects for which the scatter is enormous. (These are the objects for which the $R(N_c)$ -method has been used.) It is difficult to decide, whether the value of g could be wrong by more than 2 orders of magnitude in an object like the central star of NGC 40. If that should be the case this Wolf-Rayet star could be also shifted into the neighbourhood of the instability limit and it would be very convincing to assume that the Wolf-

Rayet phenomenon has to do with the radiative acceleration as discussed above in connection with Equations (6) and (7). One difficulty with this hypothesis is that the Of-stars and even the absorption-line O stars seem to lie as close to the instability limit as the WR stars. With the very limited accuracy of giving a star's position in $T_{\text{eff}}-g$ -diagram we do not yet understand why a number of objects do become O and Of-stars and do not show WR spectra.

One star, namely BD + 30°3639 lies so far to the left in the $T_{\text{eff}}-g$ -diagram (i.e. it occurs in such an extremely unstable region) that one has the feeling that something must be wrong with the determination of g for this object. It seems probable that the luminosity of this object has been considerably overestimated. The difficulty could be avoided by assuming that the mass of the nebula surrounding BD + 30°3639 is considerably smaller than that of other planetary nebulae. This would lead to a smaller $R(M)$ and consequently to a smaller distance.

Note added in proof

Computations using quantum defect absorption coefficients for the higher ions of C, N, O and Ne have been carried out in the meantime by Böhm (*Z. Astrophys.*, in press). Hidalgo, Hummer and Mihalas (private communication) have calculated model atmospheres for central stars using new absorption coefficients for the high ions of C, N, O and Ne calculated by M. B. Hidalgo (*Astrophys. J.*, in press). Both calculations show that for stars with $T_{\text{eff}} > 10^5 \text{ K}$ the absorption jumps in $F_{\nu}(0)$ due to the high ions of C, N, O and Ne are smaller than originally predicted by Böhm and Deinzer (1966). On the other hand, it is correct that the ultraviolet flux is really strongly suppressed by these ions.

I am most grateful to Dr. D.G. Hummer for informing me about the work by Hidalgo, Hummer and Mihalas in advance of publication.

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DISCUSSION

Underhill: There are plausible reasons for believing that the spectra of classical Wolf-Rayet stars are the result of collisional excitation, probably from streams of protons or α -particles, rather than of purely radiative processes. This is probably true also for the WR spectra of central stars of planetary nebulae, especially where O VI emission is present as well as C III and O III emission. A few classical Wolf-Rayet stars are known with O VI emission, but the appearance of O VI emission does not correlate uniquely with WC or WN type.

In 1958 I pointed out, using rather rough theory, that the precise shape of the continuous spectrum at 228 Å resulting from the C, O, and N continua would strongly affect the ionization ratio in the outer atmosphere and the relative intensities of lines from the C, N and O ions.

The widths and strengths of the stellar emission lines of WR central stars are sufficient to show that this part of the spectrum (WR lines) are formed in a fairly dense region ($N \simeq 10^{13}$ – 10^{14}) where chaotic motions may be large.

Some early work of mine on the effects of allowing for the curvature of a stellar atmosphere indicate that absorption features become sharper and a tendency to emission occurs. Probably this effect on the predicted spectra from models available at present is not negligible.

Böhm: It is true that the computations indicate a considerable extension of some atmospheres, but

the extension does not seem large enough to lead to considerable changes if the effects of spherical symmetry are included.

Underhill: In the work reported, the electron scattering assumes non-coherence. This may make a significant change in the shape of the predicted UV spectrum, especially in the case of the C, N, O continua which are sufficiently close together to be handled, in some cases, as broad lines, rather than as continuous features. Taking account of the non-coherence of electron scattering is an improvement in the theory which is greatly to be desired.