

BELIEF-TWISTING SHOCKS AND THE MACROECONOMY

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I study the role of shocks to beliefs combined with Bayesian learning in a standard equilibrium business cycle framework. In particular, I examine how a prior belief arising from the Great Depression may have influenced the macroeconomy during the last 75 years. In the model, households hold twisted beliefs concerning the likelihood and persistence of recession and boom states that are affected by the Great Depression. These initial beliefs are substantially different from the true data generating process and are only gradually unwound during subsequent years. Even though the driving stochastic process for technology is unchanged over the entire period, the nature of macroeconomic performance is altered considerably for many decades before eventually converging to the rational expectations equilibrium. This provides some evidence of the lingering effects of beliefs-twisting events on the behavior of macroeconomic variables.

Keywords: Bayesian Learning, Business Cycles, Non-Linear Dynamics

1. INTRODUCTION

In a provocative analysis of asset pricing puzzles, Cecchetti et al. (2000) showed that if households' beliefs about the driving stochastic process are "twisted" in a particular way, an otherwise standard asset pricing model could be consistent with asset pricing facts. Cogley and Sargent (2008b) extended the analysis of Cecchetti et al. (2000) adding learning. They suggested that the Great Depression was a "beliefs-twisting event" citing Friedman and Schwartz (1963), who suggested that the Great Depression "shattered" beliefs in the future of capitalism. Cogley and Sargent (2008b) captured this shattering of beliefs as a particular representation of a transition probability matrix in a Bayesian learning version of Mehra–Prescott. They found that they could match the asset pricing facts as Cecchetti et al. (2000) did, but that the equilibrium dynamics would eventually

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converge to rational expectations and thus that, in particular, the equity premium would converge to the negligible rational expectations value. However, this process took decades, according to their analysis. They suggested that this might provide an interesting part of the explanation of the equity premium puzzle in the post-war U.S. data.

In this paper, I study the twisted beliefs idea of Cecchetti et al. (2000) and Cogley and Sargent (2008b) in a standard dynamic stochastic general equilibrium macroeconomic context. If beliefs were shattered, then one would expect the behavior of the private sector to change and that this should affect all aspects of the evolution of the economy. Further, the slow convergence described by Cogley and Sargent (2008b) may suggest that these effects would be very persistent. The goal of this paper is to investigate these ideas.

The core idea is to consider a standard equilibrium business cycle framework under twisted beliefs and Bayesian learning. In the paper, productivity follows an observable exogenous stochastic regime-switching process. In contrast to the standard model, I assume that households have subjective beliefs about the distribution of productivity that may not coincide with the true data generating process. Agents learn by starting with initial beliefs and updating them according to Bayes' law. When existing beliefs are "shattered" agents have to learn beginning with their new priors. Without twisted beliefs, this economy would deliver the equilibrium business cycle properties as described by Aruoba et al. (2006). I study the effects of a one-time "shattering" of beliefs on this economy similar to the one studied by Cogley and Sargent (2008b) in the Mehra–Prescott partial equilibrium asset pricing problem. I stress that, while I am studying a particular beliefs-twisting event, the core idea would apply equally well to any such event. I compare how the behavior of the economy with twisted beliefs and Bayesian learning differs from the rational expectations version.

The main findings indicate that for a sufficiently large shock to the beliefs of the agents, the macroeconomic impact can be quantitatively important. In addition, these effects can be very persistent, taking many decades to play out through the macroeconomy. This is because (i) it takes time to correct the pessimistic beliefs induced by the depression event through the observation of macroeconomic data, and (ii) the general equilibrium makes decisions taken with incorrect beliefs to affect the state of the economy longer. This suggests that belief-twisting events may have long-lasting impacts on the macroeconomy through a channel not studied in the previous literature. Many writers since the 1930s have argued informally that the Great Depression created a "depression generation" that behaved in a way that affected the macroeconomy for decades after the depression ended.¹ This conjecture is borne out by the quantitative analysis in this paper.

This paper is related to an emerging literature on the effects of learning on the economy in the standard real business cycle framework. Eusepi and Preston (2011) use an adaptive learning approach in a standard RBC model. They consider specifications in which agents learn about reduced-form equilibrium laws of motion. Eusepi and Preston (2011) allow for multi-period-ahead forecasts and show

that the quantitative effects of adaptive learning in dynamic general equilibrium models can be significant.

The influence of beliefs on the economy has been studied extensively in the asset pricing literature. Barro (2006) studies the effect of a non-zero probability of a “disaster” state on agents’ subjective expectations in the model without learning. Weitzman (2007) shows that in an asset pricing model with Bayesian updating of unknown structural parameters, subjective prior beliefs play important and persistent role in determination of asset prices. Pintus and Suda (2013) consider the effects of beliefs about financial shocks in the model with collateral constraint and learning.

Malmendier and Nagel (2011, 2016) present empirical evidence of experiential learning bias and individual macroeconomic belief formation. Malmendier and Nagel (2011) show that investors who experienced the Great Depression are more pessimistic about stock returns than (younger) investors who did not. Malmendier and Nagel (2016) find that differences in life-time experiences strongly predict differences in subjective inflation expectations.

The paper is organized as follows. The next section describes the environment. Section 3 presents main results and the last section concludes.

2. ENVIRONMENT

I consider a standard equilibrium business cycle model. The core idea is to use a completely standard macroeconomic model, in which the only addition is to twist beliefs and to allow agents to learn via Bayesian methods. Ultimately, the households will again learn the rational expectations equilibrium following the shock to beliefs. Once this convergence occurs, the economy will behave exactly as the standard results suggest. During the transition, however, the economy may depart from the rational expectations norm, and I will present results illustrating the nature of this departure.²

The stochastic process for productivity is exogenous and does not depend on any action taken by agents. Therefore, there is no incentive for “active” learning with agents taking action that would allow them to understand the stochastic process better.

2.1. Preferences and Endowments

The representative household has preferences over stochastic stream of consumption, c , and leisure, $1 - l$, with utility at time t given by

$$U_0 = \hat{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{[c_t^\theta (1 - l_t)^{1-\theta}]^{1-\tau}}{1 - \tau}, \quad (1)$$

where β is the discount factor, τ control intertemporal elasticity of substitution of consumption–leisure bundles, and θ governs intratemporal elasticity of substitu-

tion between consumption and leisure. \hat{E}_t is the subjective expectation operator. Rational expectations can be considered as a special case where the subjective probability distribution coincides with the true data generating process.

In each period, the representative household has one unit of time which it allocates between labor and leisure. The household is also endowed with initial stock of capital k_0 , which can be augmented through investment, x_t . The law of motion for the capital is then given by $k_{t+1} = (1 - \delta)k_t + x_t$, where δ is the net depreciation rate of the existing capital stock.

A representative firm operates the standard stochastic production technology to produce output, y_t , with capital, k_t , and labor, l_t , $y_t = A_t k_t^\alpha l_t^{1-\alpha}$. The variable A_t follows a stochastic process modeled as $A_t = e^{z_t}$ with z_t representing the level of technology relative to a balanced growth path.

The level of technology follows a two-state Markov switching process, $z_t \in \{z_L, z_H\}$, modeled as $z_t = z_H S_t + z_L(1 - S_t)$, where $z_H > z_L$, with $S_t = 1$ denoting an ‘‘expansion’’ state and $S_t = 0$ being a ‘‘recession’’ state. States follow a Markov switching process with the transition probability matrix

$$\Pi = \begin{pmatrix} q & 1 - q \\ 1 - p & p \end{pmatrix}, \tag{2}$$

where $p = \text{Prob}(S_{t+1} = 1 | S_t = 1)$ and $q = \text{Prob}(S_{t+1} = 0 | S_t = 0)$.

2.2. Information and Beliefs

In the model, I allow for agents not having the full knowledge of the data generating process for productivity. I assume agents know productivity is governed by a two-state Markov regime switching process, know the growth rate in each state (i.e., know z_H and z_L) but do not know the probability transition matrix Π . Agents are Bayesian learners: they start with prior beliefs, Π_0 , summarizing their perception of the economy, and update them as they observe the actual states.

Assume agents’ prior beliefs are beta distributed with $q \sim \text{Beta}(u_{00}, u_{01})$ and $p \sim \text{Beta}(u_{11}, u_{10})$. Agents update their beliefs according to Bayes’ law and after observing the actual sequence of states, S^t . The posterior distribution of transition probability matrix, Π_t , is given by

$$f(p, q | S^t) \propto q^{u_{00} + m_{00,t} - 1} (1 - q)^{u_{01} + m_{01,t} - 1} p^{u_{11} + m_{11,t} - 1} (1 - p)^{u_{10} + m_{10,t} - 1}, \tag{3}$$

implying beta posterior distributions of p and q

$$q_t \sim \text{Beta}(u_{00} + m_{00,t}, u_{01} + m_{01,t}), \quad p_t \sim \text{Beta}(u_{11} + m_{11,t}, u_{10} + m_{10,t}).$$

Here, $m_{ij,t}$ denotes number of times the process transitioned from state i to state j in the sequence S_t . According to Bayes’ consistency theorem, the posterior distribution will converge to the data generating process.

The distribution of beliefs and the updating procedure is summarized by counters $n_{ij,t} = u_{ij} + m_{ij,t}$, which are sufficient statistics for the beta distribution.

Under the distributional assumptions, the expected probabilities of transition are

$$p_t^e = E_t(p) = \frac{n_{11,t}}{n_{11,t} + n_{10,t}}, \quad q_t^e = E_t(q) = \frac{n_{00,t}}{n_{00,t} + n_{01,t}}, \quad (4)$$

and their updates are summarized by the evolution of the counters.

If $\Pi_t(S^t)$ is the posterior probability given prior beliefs and sequence of states S^t , it can be represented with $n_t = (n_{00,t}, n_{01,t}, n_{10,t}, n_{11,t})$.³ We can consider n_t as a state variable as, together with S_t , it describes the current state of the beliefs in the economy. The transition equation for $n_{t+1} = (n_{00,t+1}, n_{01,t+1}, n_{10,t+1}, n_{11,t+1})$ is as follows:

$$\begin{aligned} n_{t+1} &= n_t + (1, 0, 0, 0), & \text{if } S_t = 0, S_{t+1} = 0, \\ n_{t+1} &= n_t + (0, 1, 0, 0), & \text{if } S_t = 0, S_{t+1} = 1, \\ n_{t+1} &= n_t + (0, 0, 1, 0), & \text{if } S_t = 1, S_{t+1} = 0, \\ n_{t+1} &= n_t + (0, 0, 0, 1), & \text{if } S_t = 1, S_{t+1} = 1. \end{aligned}$$

In such a formulation, the sufficient statistic, n , governs both the beliefs about transition matrix and the precision of these beliefs. To see this, consider two probability transition matrices represented by $n^l = (4, 1, 1, 4)$ and $n^k = (40, 10, 10, 40)$. They feature the same probabilities of expansion and recession, but after observing a series of states, beliefs given by the n^k vector will be less influenced by the incoming data relative to the one given by n^l . In this sense, the beliefs represented by n^k are more dogmatic than those represented by n^l . In the analysis below, much will depend on the moment at which beliefs are shattered and the counters that are used by the representative household to describe the new beliefs following the beliefs-twisting event.⁴

In this paper, we consider the case of adaptive learning with agents treating their current state of beliefs about distribution of stochastic process as the true one. They do not take into account future updating of their beliefs.⁵

2.3. Recursive Competitive Equilibrium

The benevolent social planner chooses the sequences for consumption, labor supply, and the capital stock to maximize household’s utility in (1) subject to the prior beliefs, the initial level of capital stock, technology, and the sequence of resource constraints.⁶ The planner’s problem can be cast in a recursive fashion. The state variables in dynamic programming formulation are the state of the economy, and the capital stock $\vartheta_t = (s_t, k_t)$. Conditional on agents’ perception of stochastic process, the recursive competitive equilibrium consists of value function v , policy function $c(\vartheta)$, $l(\vartheta)$, and $x(\vartheta)$ for household, and price functions $w(\vartheta)$ and $r(\vartheta)$, such that these functions are consistent with (a) the representative household’s problem, (b) the firm’s maximization problem, and (c) the resource constraint, $c + x = y, \forall (s, k)$. The dynamic programming problem can be written in terms

of the Bellman equation:

$$\begin{aligned}
 v(s, k) &= \max_{c,x,l} \{u(c, l) + \beta \hat{E}[v(s', k')|s]\} \\
 \text{s.t. } c + x &\leq r(s, k)k + w(s, k)l, \\
 k' &= (1 - \delta)k + x, \\
 c \geq 0, 0 \leq l &\leq 1, k_0.
 \end{aligned}
 \tag{5}$$

The time-varying expectations \hat{E} are taken with respect to probabilities which are updated as described earlier. The changing decision rules introduce a new source of variability in the macrovariables. The fluctuations in macrovariables are now the result of both stochastic productivity and the changes in the decision rule.

2.4. Beliefs and Calibration

The optimal decision depends on expectations of future productivity. Under our assumptions, expectations are changing over time and at any date t depend on initial beliefs and the actual sequence of observed states S^t . The true data generating process for productivity, z_t , is exogenous. The posterior beliefs, however, reflect subjective perceptions embodied in the prior along with agents’ observations of the stochastic process driving the evolution of the economy.

In the paper, I study the effects of “shattered” beliefs—events that change households’ perceptions about the stochastic process driving economy. Since households in the paper are Bayesian learners, eventually they will learn the true process. However, in the meantime, the beliefs-twisting event has clear effects on actual household behavior. One logical choice for the twist in beliefs is the Great Depression.⁷ However, the Great Depression does not have to be the only possible beliefs-twisting event. The spirit of this paper is to find a generic description of the behavior following any such event. There may be many other cases, especially outside the post-war G7 economies.

I calibrate the model at a quarterly frequency and follow standard parameterization. The utility function parameters are set to $\beta = 0.9896$, $\tau = 2$, and $\theta = 0.357$ implying steady state values for annual interest rate of 4% and labor supply of 31% of available time. The technology parameters are set to $\alpha = 0.4$ and $\delta = 0.0196$.

I chose the parameters of the stochastic process for productivity to match the stochastic characteristics of the AR(1) process for Solow residual of the U.S. economy in $z_t = \rho z_{t-1} + \varepsilon_t$ with $\rho = 0.95$ and $\sigma_\varepsilon = 0.007$. For the baseline calibration, I follow Bullard and Singh (2012) assuming symmetric probabilities $p = q = 0.975$ and symmetric regimes $z_H = -z_L = 0.0225$.⁸

3. TWISTED BELIEFS IN THE MODEL

The purpose of this paper is to study the case where the initial beliefs about the data generating process for productivity disagree with the true transition probabilities and agents learn and update their beliefs as time passes. The degree of disagreement

TABLE 1. Baseline priors

Process	Counters				Probabilities			
	u_{11}	u_{10}	u_{01}	u_{00}	p	q	$Pr(S_t = 1)$	$Pr(S_t = 0)$
True process	39	1	1	39	0.975	0.975	0.50	0.50
Baseline prior	2	2	2	28	0.500	0.933	0.12	0.88

will determine how far away the agents' initial perception of the economy is from the truth.

For the baseline calibration, I simulate the model with priors that represent pessimistic "twisted" beliefs represented in Table 1. I endow agents with twisted beliefs that differ substantially from the data generating process for productivity in three dimensions. First, agents see productivity as governed by an asymmetric process with expansions lasting on average 2 quarters and recessions lasting on average almost 4 years. This is in stark contrast from the true data generating process according to which both states last on average 10 years.⁹ Second, given the true data generating process, agents have a relatively uninformative prior concerning expansions. Lastly, agents are under-estimating the persistence of both states. These priors are consistent with agents taking NBER dates on recessions and expansions for the period of 1929:2–1933:3. In particular, this is what agents would use based on counters taking the beginning of "new era" as 1929:2 at the end of 1933. This is just a baseline case—I will study different sets of priors and an alternative data generating process for productivity in the following section.

In following subsections, I compare how the evolution of the economy populated by Bayesian agents with twisted beliefs differs from the economy with rational expectations' agents.

For each simulation, a sequence of 400 productivity shocks is drawn from the true distribution. When forming expectations, rational expectations agents use the true transition probability matrix Π . In contrast, Bayesian agents start with initial priors Π_0 and update their beliefs with realizations of the stochastic process. The stock of capital in the economy, k_0 , is initialized at the deterministic steady state level and agents make their optimal decisions.¹⁰ I compare how decisions concerning consumption, investment, labor supply, and the evolution of other macroeconomic variables differ under the assumption of Bayesian learning as compared to the assumption of full information, rational expectations.

3.1. Evolution of Variables

I compare percentage deviations from the steady state under rational expectations and under Bayesian learning. Figures 1 and 2 portray the evolution of macroeconomic variables for two individual simulations. These two figures differ with respect to the realization of the draw of stochastic productivity and, accordingly, the speed of convergence of learning.

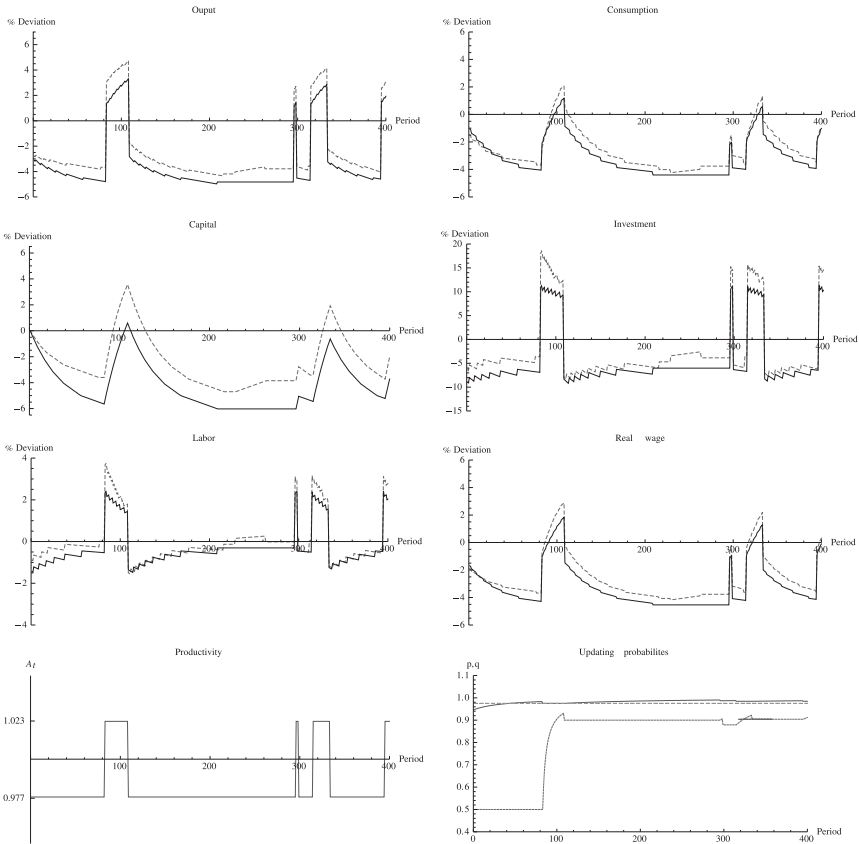


FIGURE 1. Individual simulation: slow convergence. Dashed line represents deviations from steady state under learning; solid line is deviations from steady state under rational expectations.

Figure 1 presents a simulation with slow convergence of probabilities and, as a result, with slow convergence of decision rules under learning to decision rules under rational expectations. As agents observe a sequence of draws mostly from the recession state, they cannot update their pessimistically twisted beliefs about persistence of expansion state. Agents expect expansions to be short, quickly followed by a recession, and they choose to accumulate more capital comparing to the full information case. Moreover, agents over-estimate the persistence of the recession state distorting decisions even more.

In contrast, Figure 2 presents a simulation with relatively fast convergence to rational expectations. As agents observe a long sequence of draws from the expansion state, they can update their mistaken beliefs more readily. However, agents' initial behavior is still distinctively different from the full information case. As they expect the recession state to occur more often than the true probability

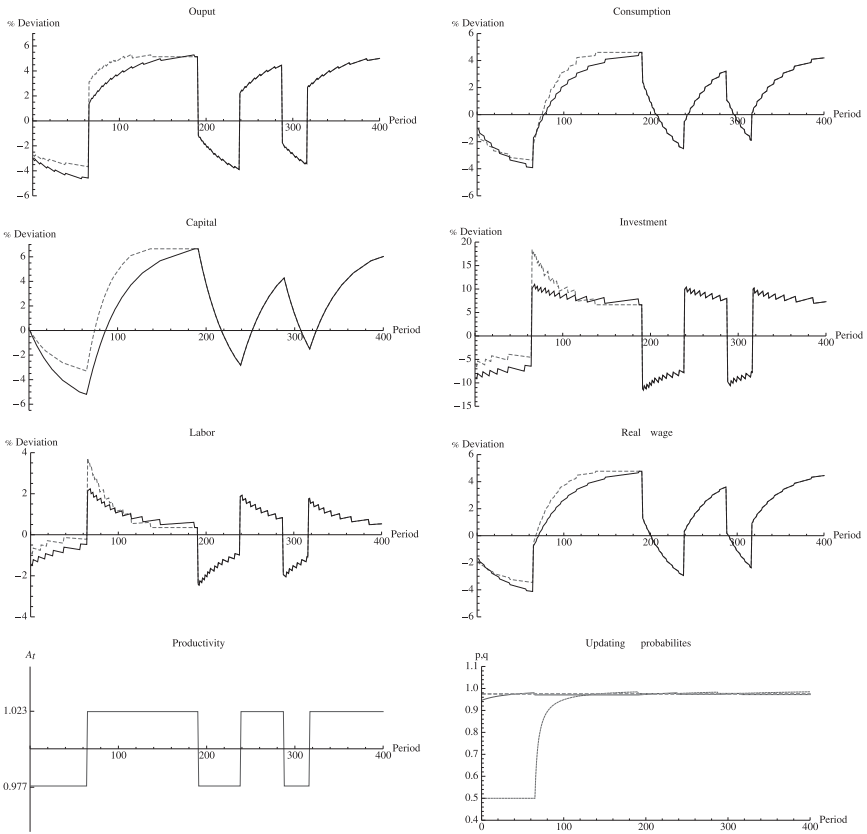


FIGURE 2. Individual simulation: fast convergence. Dashed line represents deviations from steady state under learning; solid line is deviations from steady state under rational expectations.

indicates, once they are in the depression state they dissave, consume, and work less than under rational expectations. Similarly, once they are in expansion state which they expect to last for short period of time, Bayesian agents move more aggressively in their investment behavior than RE agents.

Although in the simulation portrayed in Figure 2 the discrepancy under learning and RE vanishes within 200 periods, Figure 1 depicts a more persistent case. The realization of the random process makes updating of beliefs a long and slow process, even after 400 periods decisions regarding investment and labor supply as well as overall state of the economy under learning can be very different than under rational expectations. For example, the deviation of capital from steady state under learning in period 400 is 54% smaller than under rational expectations (−2% vs. −3.7%) which, in combination with differences in labor supply, translate to

71% larger deviation of output from the steady state under learning in that period in that simulation (3.1% vs. 1.8%).¹¹

Both Figures 1 and 2 illustrate how the degree of dogmatism of priors affects the speed of convergence of beliefs. The bottom panels contain the paths of p_t and q_t . Recall that prior beliefs for p in terms of counters are based on only four observations, $p_0 = \frac{2}{4}$. This prior is initially quickly updated once the productivity is in the expansion state. For example, after only 2 consecutive years (8 periods) of expansion, the updated subjective probability of remaining in expansion state equals $p_t = 0.83$. As a result, the rapidly changing beliefs about transition probability matrix bring substantial revisions of optimal investment, consumption, and labor supply decisions. The corresponding panels in Figures 1 and 2 illustrate these results.

I now turn to a more interesting characterization of the effects of the beliefs shock.

3.2. Average Difference

The average effect of pessimistically “twisted” beliefs on the economy is illustrated in Figure 3. Now, instead of looking at a particular realization of the stochastic productivity sequence, I calculate the percentage deviations of macroeconomic variables under learning from rational expectations.¹² Computed deviations are averaged across all simulations.

Figure 3 approximates impulse response functions to one-time persistent shock to beliefs. Mistakenly believing that, relative to expansions, duration of depression is long, agents choose to initially invest and work more, and consume less relative to what they would do if they had the correct perception of the productivity process. With these beliefs, on average, agents would invest over 5% more, supply over 1.5% more labor, and consume 1% less under learning relative to rational expectations. These differences stem from initial beliefs, regarding the persistence of expansions and recessions, that are substantially different from the true data generating process. However, as agents update their beliefs about stochastic productivity, their perception of the distribution, Π_t , changes, getting closer to the true distribution Π . This evolution of beliefs is reflected in the evolution of decisions made by agents and evolution of macroeconomic variables. Eventually, the convergence of beliefs occurs, and agents use the same decisions as under the rational expectations assumption. This transition can be seen in Figure 3. As agents update their beliefs, the average difference between learning and RE decreases. The investment and labor supply under learning are first to converge to their rational expectations values with consumption and output following.

Despite this convergence, the effects of sufficiently “twisted” beliefs are not only quantitatively important but also long-lasting. Even though the effects of one-time shock to beliefs are temporary, the transition period may be long. In this calibration, even after 40 periods (or 10 years) of learning and updating their

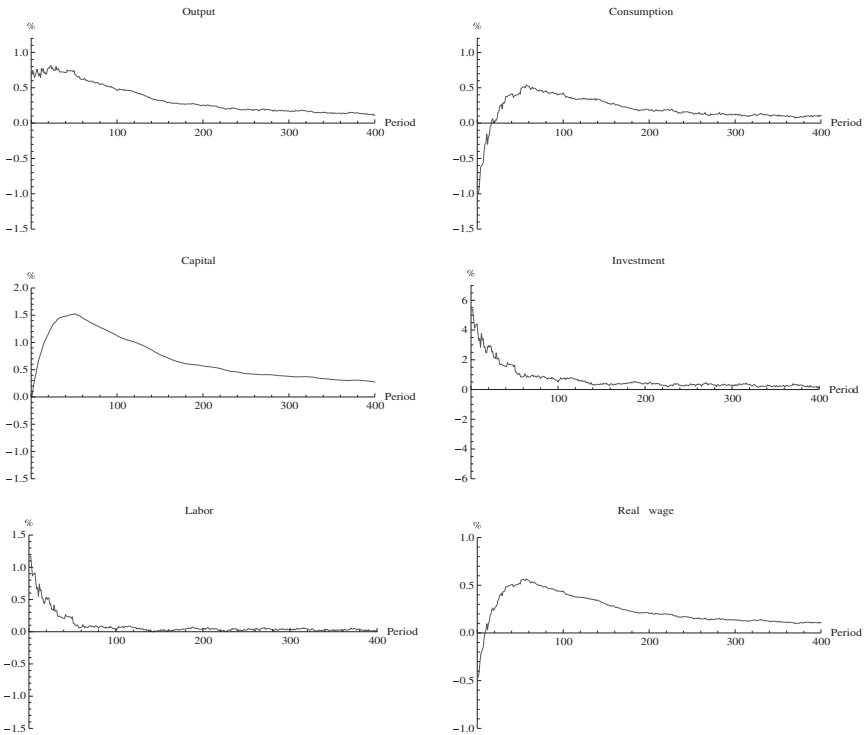


FIGURE 3. Average percentage difference of learning from RE. On average agents save more under learning relative to rational expectations because of pessimistically twisted beliefs.

beliefs, agents would invest almost 2% more and consume 0.4% more under learning relative to rational expectations, and the level of capital is at least 1% above rational expectations case for almost 30 years.¹³

Two elements are responsible for such sizeable and long-lasting effects of incorrect prior beliefs under learning. First, the “twist” in the beliefs and the difference between priors and the true data generating process need to be large in order to generate substantial difference in the beginning. If the perceived persistence and relative frequencies of expansion and recessions are relatively close to the true data generating process, the initial difference is considerably smaller. Second, the combination of learning and the general equilibrium framework is responsible for the persistence of the effects. Figure 4 presents evolution of beliefs in terms of p_t and q_t . It shows that agents, having observed realizations of shocks drawn from the true data generating process, update their beliefs quickly, and even if they started with pessimistic view of the world, they quickly revise such “misperception.” However, given that agents’ investment decisions accumulate over time and affect the capital stock, the long-lasting effects of incorrect prior beliefs bring those

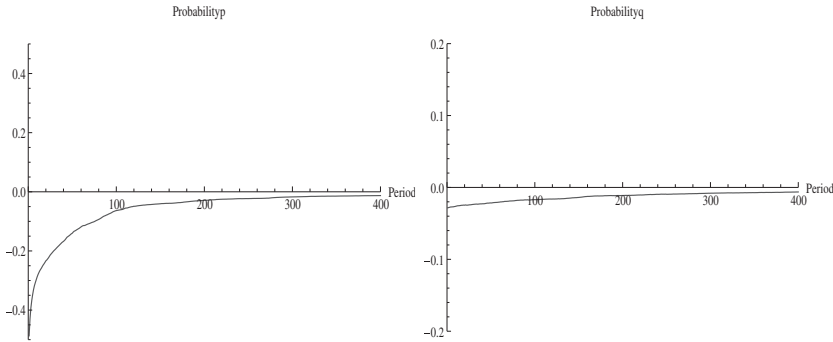


FIGURE 4. Beliefs updating: average difference between twisted beliefs and RE.

choices under “twisted” beliefs resulting in a different state (in terms of capital stock) in the economy under learning.

Figure 3 is the illustration of the theoretical result of the time-varying decision rule in the dynamic programming problem under adaptive learning. The optimal intertemporal decision is given by the following equation:

$$u_c(c, l) = \beta \hat{E}[u_c(c', l')(r' + 1 - \delta)|s], \tag{6}$$

with expectations \hat{E} at time t taken with respect to subjective probability distribution Π_t . As beliefs are updated, Π_t changes, implying changes in intertemporal decision rule given state (s, k) . This introduces time-varying paths for macroeconomic variables. For both the non-learning case and the rational expectations case, agents use time-invariant probability distributions, Π_0 and Π , respectively, which imply time-invariant decision rules.

The time-varying decision rule under learning implies that the volatility of the macroeconomic variables might be not only the result of the stochastic fluctuations of the productivity process but also due to changes in actual decisions. To examine whether updating probabilities in case of twisted beliefs generates additional volatility in the economy, for each variable, I compute the ratio of standard deviation under learning to standard deviation under rational expectations. Deviations are taken with respect to the steady state under the true data generating process.

The overall volatility for the case of pessimistic initial beliefs and learning is higher than for the case of rational expectations. Standard deviations of macroeconomic variables under learning are persistently above standard deviations under rational expectations for the same realization of stochastic productivity. In the case of output, the average volatility under learning is initially 11% higher and remains at least 5% higher for the subsequent 25 years. The differences in volatilities between learning and rational expectations are very persistent. Even after 50 years from the shock in the beliefs, on average, the investment standard deviation is 10% higher and the labor supply standard deviation is almost 20% higher

under learning compared to rational expectations. However, gradual convergence of beliefs implies gradual moderation of volatilities under learning.¹⁴

4. CONCLUSIONS

I studied the effects of “shattered beliefs” in a dynamic stochastic general equilibrium model with Bayesian learning. The main point is that a beliefs-twisting event is likely to alter agents’ behavior even if the underlying processes governing the economy remain unchanged. This is because the perceived distribution of the driving stochastic process differs substantially from the true data generating mechanism.

The learning guarantees that any effects will be temporary, yet the effects of a beliefs-twisting event like the Great Depression are found to be substantial and long-lasting. Even after 50 years, the decisions made under subjective expectations may be markedly different from the ones taken under rational expectations. This is because even though the observation of macroeconomic data eventually corrects the twisted beliefs, the consequences of earlier decisions are long-lasting. This mirrors the findings of Cogley and Sargent (2008b) in their partial equilibrium asset pricing framework.

If a beliefs-twisting event can have large effects on the economy, it may be of interest to study other such events. In particular, one might expect larger and more frequent beliefs-twisting events in developing countries.

This framework can be also used to analyze the behavior of an economy and agents’ beliefs in the case of a process-twisting event. Any changes in the stochastic processes driving the economy are often not directly observable causing subjective and “correct” expectations to differ. As this paper shows, for sufficiently large differences it may take a long time for agents to learn the new process governing the economy.

NOTES

1. For example, Danthine and Donaldson (1999) note:

“Yet, it is not unreasonable to think, for example, that the experience of the Great Depression continues to have a significant influence on the behavior of those who experienced it directly or indirectly, even though it has not recurred in sixty-five years” (p. 608).

2. In this paper, I consider a one-time shock to beliefs in a representative agent economy. Mallemier and Nagel (2011, 2016) find evidence of history- and experience-dependent heterogeneity in agents’ beliefs. While exploration of how such heterogeneity could arise in a model economy and how it would translate to aggregate behavior of macroeconomic variables is very interesting, it is left for future research.

3. This representation of beliefs is used in Cogley and Sargent (2008a).

4. See Suda (2018) for the analysis of the behavior of macroeconomy under alternative set of beliefs.

5. Cogley and Sargent (2008a) show that the consumption and investment choices under fully Bayesian (i.e., internalizing future updating) and adaptive learning behavior are very similar for low values of the coefficient of relative risk aversion. Similarly, Bullard and Suda (2016) show that Bayesian learning schemes, while more sophisticated, do not alter the standard expectational stability conditions

in a class of linear expectational models. They do show, however, that the transitional dynamics could be different. While the questions how the assumption of adaptive learning affects the economy in this model is important and interesting, we leave it for future research.

6. I assume social planner and the representative household have the same prior beliefs.

7. According to Friedman and Schwartz (1963) the Great Depression of 1930s persistently changed the perception about the nature of processes governing economy:

“The contraction after 1929 shattered beliefs in a ‘new era’ . . . The contraction instilled instead an exaggerated fear of continued economic instability, of the danger of stagnation, of the recurrent unemployment.” (p. 673)

8. Bullard and Singh (2012) set $z_H = -z_L = 0.0035$.

9. Given that the probability that economy stays in an expansion equals p , the average duration of expansion state can be then computed as $\frac{1}{1-p}$.

10. One may want to consider the situation where initial capital is not at steady state. Starting away from steady state can be considered but since I consider deviations of rational expectations from learning such an approach would generate the same results.

11. For some variables, for example real wage or consumption, the absolute difference in a variable under learning and rational expectation may be smaller.

12. For example, for output I compute $100 \cdot (y_t^{\text{Learn}} - y_t^{\text{RE}})/y_t^{\text{RE}}$.

13. Modigliani (1986) remarks: “Not only was oversaving seen as having played a major role in the Great Depression, but, in addition, there was widespread fear that the problem might come back to haunt the post-war era. (. . .) These concerns were at the base of the “stagnationist” school which was prominent in the 40s and early 50s.” (p. 151)

14. In Suda (2018), I address some alternative specification and explore the robustness of the main findings. I also examine how the economy would behave if the sequence of productivity shocks corresponds to the one consistent with post-war U.S. experience.

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