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Plasma expansion dynamics in the presence of a relativistic electron beam

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Abstract

The dynamics of an electron–ion plasma is studied when a mono-energetic relativistic electron beam penetrates the expanding plasma. Combined effects of thermal pressure and ambipolar electrostatic potential are considered for fully relativistic multi-fluids model where the quasi-neutrality assumption is used for a beam–plasma system. Relativistic effects are considered for both density and velocity of the beam fluid. Ion acceleration is depicted through a spike-like structure resulting from non-local charge separation and associated with the beak-down of quasi-neutrality. The beam initial speed is found to enhance the spike amplitude and change its position. Moreover, relativistic effects are particularly found significant for a non-thermal plasma.

Keywords: Electron beam; Relativistic effects; Expansion; Quasi-neutrality

1. INTRODUCTION

Charged particle beams presently have different applications such as in various microwave tubes, cancer therapy, and in fusion devices among many other applications (Molokovsky & Sushkov, 2005; Mitrani *et al*., 2014). A stream of charged particles can be produced by a short, dense electron bunch, which generates a high-gradient accelerating field in plasma that accelerates a second trailing beam to high energies (Vafaei-Najafabadi *et al*., 2014). Because plasmas tolerate as high as 1 GV/cm electric field, which is at least 3 orders higher than radio frequency cavities (Li *et al*., 2013*a*), a beam of particles can be accelerated to 200 MeV over a distance of millimeter (Bingham *et al*., 2004). At this energy, electrons acquire relativistic speeds. Moreover, relativistic electron beam can also be provided by an external traditional electron gun that produces a narrow and collimated beam with a precise kinetic energy. Increasing the current density of the electron beam in the ion trap gives rise to highly ions' charge states and enhances ions' energy which is of great importance for Booster applications (Pikin *et al*., 2013). Recent experimental results demonstrated the production of narrow energy spreading electron beam from compact high-gradient laser plasma accelerator (Faure *et al*., 2006). Those particles beam have a large number of important characteristics and could lead to applications in a large range of fields, including medicine (radiotherapy), chemistry (radiolysis), accelerator physics (Jagher *et al.*, 1988), and also in the ionospheric and astrophysical phenomena, such as the aurora Borealis that occur because of solar wind beam–plasma interaction (Kapetanakos & Hammer, 1973). The production of electron beam can also be achieved using ionization triggered by two transversely colliding laser pulses inside a beam-driven plasma wake (Li *et al*., 2013*b*). The interaction between electron beam and the plasma generates high-energy particle beams that are short, bright, and have a good spatial quality distribution (Mangles *et al*., 2006). In the relativistic regime, particles beam can excite large amplitude plasma waves, which are important in the quest for producing ultra-high acceleration gradients (Bingham *et al*., 2004). When a finite plasma is passed by electron beam with initial velocity exceeding the average thermal velocity of plasma electron, longitudinal electric waves are excited having an exponentially increasing amplitude (Akhieze & Fainberg, 2008). Assuming that the amplitude of the excited wave exceeds some critical value, determined by the beam and plasma parameters, gives rise to a stationery non-linear wave (Kovtun & Rukhadze, 1970). The plasma waves conversion into electromagnetic waves by producing a strong Langmuir turbulence is realized using a high-current relativistic beam of a two-stream instability (Thumm & Arzhannikov, 2014).

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Moreover, in the target normal sheath acceleration regime, the key parameter is the hot electron temperature. It has been demonstrated in many experiments that the final energy of accelerated ions strongly depends on the hot-electron temperature (Kumar & Pukhov, 2008). The control of electron energy distribution function can be achieved by an energy sink which is provided by electron beam injection (Boris *et al.*, 2013). For relativistic charged particles beam moving in a laser-irradiated plasma, the energy loss increases with the beam energy forming a maximum at moderate relativistic factors ($\gamma \leq 2$) (Nersisyan & Deutsch, 2014).

Plasma expansion is a key issue for particles acceleration. When an electron beam penetrates an expanding plasma produced by laser–solid interaction, a strong electrostatic field can develop in the interaction of an electron beam with plasma. Due to many phenomena as radiation and anomalous transport across the field lines, a significant fraction of the beam energy is lost in the processes other than plasma heating. This corresponds to a transfer efficiency of about 25% (Kapetanakos & Hammer, 1973). Thus, the electron beam transfers part of its energy into the plasma and accelerates plasma particles. This acceleration is attributed to the electric charge separation that produces the quasi-static field generated by the ponderomotive or thermal explosion acceleration (Esarey *et al.*, 1996). The electron thermal pressure, in the thermal explosion scenario, drives an expansion of these hot electrons around the solid target, setting up a large-amplitude electrostatic field at the target–vacuum interfaces. This field leads to an efficient acceleration from the target surface over a very short distance (Malka *et al.*, 2008).

When a short electron bunch propagates in a plasma, the plasma electrons are expelled by the space-charge force of the bunch and blown them out of the beam's path (O'Connell *et al.*, 2002). Such an interaction, depends on the interaction length and plasma density, may have an adverse effect on the beam propagation and the resultant spot size (Zhu *et al.*, 2010). If the beam density is much less than the plasma density, the latter is sufficient to neutralize the beam space charge and quasi-neutrality can be then assumed. In fact, the beam duration plays an important role in the physical picture of this interaction. When the beam duration is longer than the growth period of the two-stream instability, a high-amplitude plasma wave is produced which traps the particles and accelerates them (Kovtun & Rukhadze, 1970; Kapetanakos & Hammer, 1973; O'Connell *et al.*, 2002). In the present work, we considered a plasma created by the laser–solid interaction, and focus our study on the expansion dynamics of such a plasma when it is penetrated by a relativistic electron beam. As the energy of electron beam is much more important than the plasma electron energy, the beam particles are considered as a fluid apart governed by fluid equations that count for fully relativistic effects. It was demonstrated that relativistic electron beam–plasma expansion shows different behaviors depending on the relativistic regimes (Benkhelifa & Djebli, 2014).

2. BASIC EQUATIONS AND SELF-SIMILAR APPROACH

Commonly, relativistic effects are included in plasma fluid equations by the replacement of the momentum mv with the relativistic momentum $m\gamma v$; the relativistic factor corresponds to $\gamma = 1/\sqrt{1 - v^2/c^2}$ (c is the speed of light). Following this approach supposes that the density is not affected by relativistic effects. This investigation is motivated by the fact that the relativistic effect does not involve only the momentum but also the density variation (Lee, 2008). Due to that, relativistic effects must be considered in density conservation equation for the electron beam. We studied the expansion of electron–ion plasma penetrated by a relativistic electron beam. For that purpose, the expansion is assumed to hold in one dimension along with the beam streamline. The dynamics of a fully ionized non-relativistic unmagnetized plasma is governed by the set of fluid equations:

$$\frac{\partial n_e}{\partial t} + \frac{\partial n_e v_e}{\partial x} = 0 \tag{1}$$

$$\left(\frac{\partial v_e}{\partial t} + v_e \frac{\partial v_e}{\partial x}\right) + \frac{3T_{e0}}{m_e n_e}\frac{\partial n_e}{\partial x} = \frac{e}{m_e}\frac{\partial \phi}{\partial x} \tag{2}$$

for electron plasma fluid, and:

$$\frac{\partial n_i}{\partial t} + \frac{\partial n_i v_i}{\partial x} = 0 \tag{3}$$

$$\left(\frac{\partial v_i}{\partial t} + v_i \frac{\partial v_i}{\partial x}\right) + \frac{3T_{i0}}{m_i n_i}\frac{\partial n_i}{\partial x} = -\frac{e}{m_i}\frac{\partial \phi}{\partial x} \tag{4}$$

for ions, where m_j, n_j, v_j, and T_j ($j = e(i)$ for electrons (ions)) stand to mass, density, velocity, and temperature, respectively. Electron beam injection in the plasma can be achieved, such as using the setup described by O'Connell *et al.* (2002). Fluid equations for the relativistic beam are:

$$\frac{\partial \gamma_b n_b}{\partial t} + \frac{\partial \gamma_b n_b v_b}{\partial x} = 0 \tag{5}$$

where $\gamma_b = 1/\sqrt{1 - v_b^2/c^2}$ is the beam relativistic factor.

$$\left(\frac{\partial \gamma_b v_b}{\partial t} + v_b \frac{\partial \gamma_b v_b}{\partial x}\right) + \frac{3T_{b0}}{\gamma_b m_b n_b}\frac{\partial n_b}{\partial x} = \frac{e}{m_b}\frac{\partial \phi}{\partial x} \tag{6}$$

The electrostatic potential ϕ rises due to local charge separation. The dynamics of the plasma electrons and ions are tied together that the plasma scales in distances larger than the Debye length $\lambda_D = \sqrt{k_B T_i/4\pi n_i e^2}$, k_B is the Boltzmann constant. Moreover, the plasma density is much larger than the beam density and also due to space charge created by the beam penetration which expels the plasma electron forms the beam's path, then the quasi-neutrality is established and the self-similar approach can be used. Thus,

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quasi-neutrality condition corresponds to:

$$n_i = n_e + \gamma_b n_b \tag{7}$$

This equation is important because it permits, along with the set of differential equations, to calculate self-consistently the electrostatic potential and also includes the relativistic factor. Equations (1)–(7) are normalized according to: $N_e = n_e/n_{i0}$, $N_i = n_i/n_{i0}$, $N_b = n_b/n_{i0}$, $V_j = v_j/c_s$, $V_b = v_b/c$, $\psi_I = eE/\omega_p m_e c_s$. n_{i0} is the ion density at the initial stage of the expansion and ω_p is the plasma frequency.

The set of governing equations is transformed using the self-similar variable $\xi = x/c_s t$, $c_s = \sqrt{T_e/m_i}$ is the acoustic velocity. We note that this self-similar variable is dimensionless. Sometimes $\xi = x/t$, of velocity dimension, is used when the acoustic speed of the plasma is unknown. In fact, the longitudinal electrostatic wave is excited by the beam propagation in the plasma through the Cerenkov resonance. The latter, is one of the unstable modes where the frequency of the electrostatic wave is similar to the plasma mode. In this case, the phase velocity of the wave has to be close to the beam velocity (Liu & Tripathi, 1994). Due to the ion's inertia, the ions are not excited by the electrostatic wave, but they are accelerated by the electric field created by the plasma electrons movement, which are accelerated by getting apart of the beam energy. The expansion front is accelerated and a rarefactive wave propagates in the opposite direction of the expansion with an acoustic velocity. Using Eqs. (1)–(6), in terms of the self-similar variable ξ, yields to a set of ordinary differential equations governing the expansion:

$$(V_e - \xi)\frac{\partial N_e}{\partial \xi} + N_e \frac{\partial V_e}{\partial \xi} = 0 \tag{8}$$

$$(V_i - \xi)\frac{\partial N_i}{\partial \xi} + N_i \frac{\partial V_i}{\partial \xi} = 0 \tag{9}$$

$$(V_b - \xi)\frac{\partial N_i}{\partial \xi} - (V_b - \xi)\frac{\partial N_e}{\partial \xi} + [(N_i(V_b - \xi) + N_e(\xi - V_b))V_b\gamma_b^2 + N_i - N_e]\frac{\partial V_b}{\partial \xi} = 0 \tag{10}$$

$$(V_e - \xi)\frac{\partial V_e}{\partial \xi} + \frac{3}{\delta N_e}\frac{\partial N_e}{\partial \xi} = -\alpha\psi \tag{11}$$

$$(V_i - \xi)\frac{\partial V_i}{\partial \xi} + \frac{3\sigma}{N_i}\frac{\partial N_i}{\partial \xi} = \alpha\delta\psi \tag{12}$$

$$\frac{3\beta}{\gamma_b \alpha (N_e - N_i)}\left(\frac{\partial N_i}{\partial \xi} - \frac{\partial N_e}{\partial \xi}\right) - (V_b - \xi)\frac{\gamma_b}{\alpha}(1 + \gamma_b^2 V_b^2)\frac{\partial V_b}{\partial \xi} = \psi \tag{13}$$

where $\beta = T_{b0}/T_{e0}$, $\delta = m_e/m_i$, $\sigma = T_{i0}/T_{e0}$, and $\alpha = \omega_p c/c_s$. We solved the set of fluid Eqs. (8)–(13) for two values of temperature ratio σ, ($\sigma = 0.4$, 1) and different initial values of the speed of the relativistic beam. The importance of studying the behavior of the expansion dependence on the parameter σ, lies in the fact that the response of a plasma to any external excitation depends on its initial state. We take $\sigma = 0.4$ for an initially non-thermal plasma, and $\sigma = 1$ for a plasma in thermal equilibrium. Initial conditions correspond to the study of the propagation of a highly relativistic and underdense electron bunch through a plasma. Injected electrons are generated via the further ionization of Rb^+ which gives Rb^{+2} (Vafaei-Najafabadi *et al.*, 2014). Initial parameters correspond to $\beta = 10^2$, $\delta = 2.10^{-3}$, $N_{i0} = 1$, for an initial beam density less than the plasma density ($n_i \sim 10^{17}\ \text{cm}^{-3}$).

3. RESULTS AND DISCUSSION

Numerical investigations focus on the role of the electron beam. Therefore, we implement the numerical code and increase gradually the initial values of beam velocity. In Figure 1, the normalized density is plotted for different situations that concern the electron beam initial velocity, in the aim to compare between non-relativistic, weakly relativistic, and relativistic cases. We note that the expansion ends at $\xi \sim 2$ for $\sigma = 0.4$ and $\xi \sim 3.4$ for $\sigma = 1$, whereas, the density is reported to vanish for $\xi \sim 7$ in the case of non-relativistic beam. The ionic density vanishes at lower values of self-similar variable when the beam velocity increases. For the non-relativistic case, we recover the common result that is a density depletion versus the self-similar variable. Increasing the beam velocity gives the same profile when $\sigma > \sigma_c$, σ_c is a critical value $\sigma_c = 0.55$. Thermal plasma evolves with density depletion without any change whatever the beam is relativistic or not. The effect of relativistic beam is depicted through the acceleration of the expanding front. The higher the initial beam velocity is the lower the limited similar parameter is. Such result is expected because the beam particles go through the plasma and are at the head of the front. Therefore, the ambipolar electrostatic potential

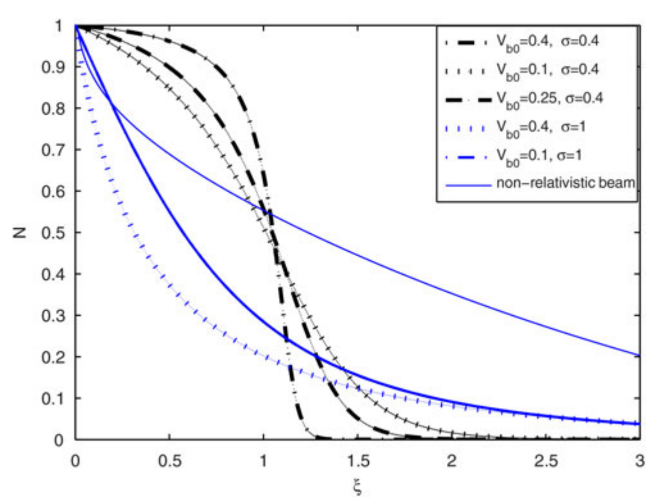

Fig. 1. Normalized ion density versus the self-similar variable with different initial values of beam velocity and temperature ratio σ, $n_{b0}/n_{i0} = 0.01$, and $V_{e0} = 1.2$.

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is more important and turns out to accelerate ions. For the non-thermal plasma, ion thermal velocity is less than the electronic one, which causes a significant delay near the source. This delay makes stronger the ambipolar electrostatic potential and turns out to accelerate ions to preserve quasi-neutrality. However, increasing the beam velocity gives rise to a higher acceleration of the expansion front. Figure 1, also shows two kinds of curves, those corresponds to non-thermal plasma (σ = 0.4) (black), which have a concavity instead of a convexity as it is in the thermal plasma case (σ = 1) (blue). This can be attributed to a depletion of the density near the source, which is very slow due to the small velocity of the ionic front, caused by a small energy gain from the excited plasma-wave after the propagation of the relativistic beam. At the same time, the acceleration of a large number of electrons leaving the target and escaping in vacuum, leads to the generation of very intense space-charge field which drives the acceleration of ions.

In Figure 2 is plotted the normalized velocity versus the self-similar variable. We have almost the same profile given by the self-similar hydrodynamical plasma expansion into vacuum that is a velocity increases $v \rightarrow \infty$ as $\xi \rightarrow \infty$ (Sack & Schamel, 1987). It is clear that beyond $\xi > \xi_l$ (limited value), this result does not make sense because $n \rightarrow 0$, as it is demonstrated by the experimental results of the expanding plasma front, where, it was observed that the velocities of the plasma front reach a steady state after 400 ns (Zhu *et al.*, 2010). The main difference is seen close to the plasma source region where the density is high and the beam did not yet spread out of the plasma bulk to drive the ion motion. For thermal or non-thermal plasma the acceleration of the ion front is more important with higher initial beam velocity. Let us recall that the expansion results from the combination of two effects, thermal pressure gradient and electrostatic potential. The first effect is dominant close to plasma source region where the density is high. Such effect is also a common mechanism to neutral gases expansion. The second, specific to ionized gases, dominates when the local

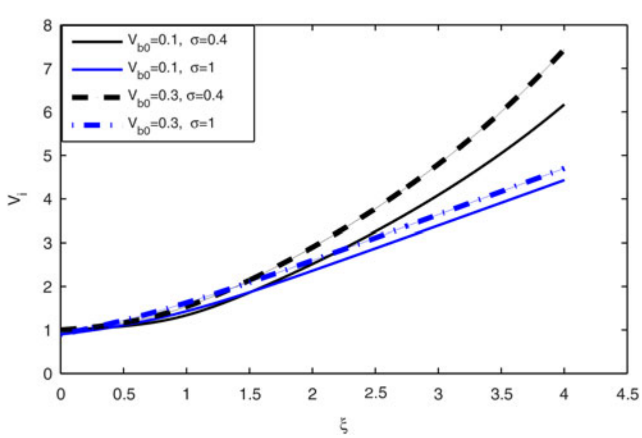

Fig. 2. The same as Figure 1 but for normalized ion velocity versus the self-similar variable.

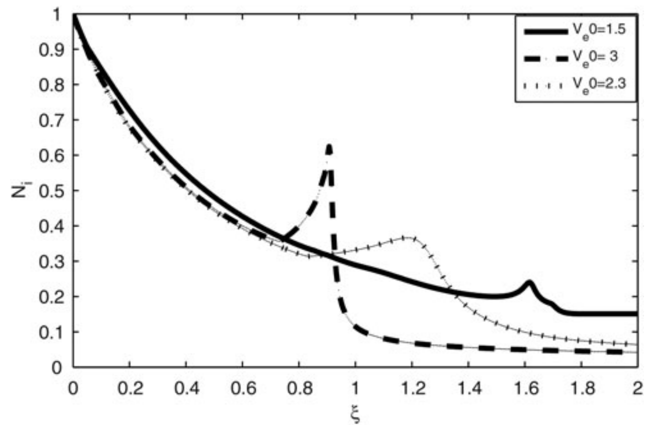

Fig. 3. Normalized ion density versus the self-similar variable with different initial values of normalized plasma electron velocity and ion-to-electron temperature ratio σ = 1.

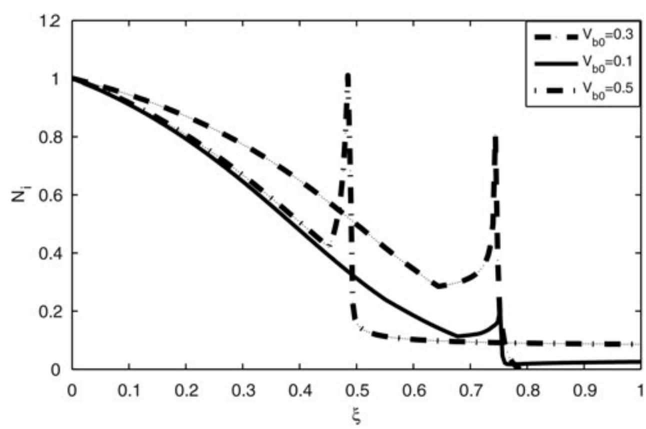

Fig. 4. Normalized ion density versus the self-similar variable with different initial values of electron beam velocity, with $V_{e0} = 2.5$ and σ = 0.4.

charge separation is important. Figures 3 and 4 show the impact of increasing the initial electron fluid velocity combined with different initial beam velocities. In both cases, the density profiles exhibit a spike-like structure associated with the possibility of ion fronts behind which exist ion sound waves caused by a density derivative jump producing large electric fields which indicate the breakdown of quasi-neutrality. Beyond this limit, the expansion ceases to be self-similar (Djebli *et al.*, 2002). Such structures show groups of accelerated ions, associated with density peaks. The amplitude and position of the peaks are depending on the initial beam velocity, higher velocity provides a closer peak to the plasma source.

4. CONCLUSION

The effect of a relativistic electron beam on the expansion of an electron–ion plasma is investigated using fluid equations. Numerical study, conducted through a self-similar approach invoking quasi-neutrality assumption, focused on the effect of the beam initial velocity which can be more important due to rising technology of particles accelerators. In the present study, we assumed that the relativistic beam density is less than the plasma density. Ions are found to be accelerated efficiently when the plasma is non-thermal and the beam has a higher initial velocity. The acceleration can be depicted through a density peak associated with an oscillating front created by a large electric field and indicates the limit of the quasi-neutrality assumption. Relativistic effects are significant for a non-thermal plasma. A critical value for the electron beam–electron plasma temperature, for which the expansion exhibits different behaviors, is also found. Non-thermal plasma shows a significant relativistic effect in comparison with thermal plasma, that is, peaks with higher amplitude.

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