

# First Preliminary Fast Static Ambiguity Resolution Results of Medium-Baseline with Triple-Frequency Beidou Wavebands

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Fast high precision relative Global Navigation Satellite System (GNSS) positioning is very important to various applications and ambiguity resolution is a key requirement. It has been a continuing challenge to determine and fix GNSS carrier-phase ambiguity, especially for medium- and long-distance baselines. In past research, with dual-frequency band Global Positioning System (GPS), it is almost impossible for fast ambiguity resolution of medium- and long-distance baselines mainly due to the ionospheric and tropospheric effects. With the launch of the BeiDou system, triple-frequency band GNSS observations are available for the first time. This research aims to test the ambiguity resolution performance with BeiDou triple-frequency band observations. In this research, two mathematical models are compared: zenith tropospheric delay as an unknown parameter versus corrected tropospheric delay. The ambiguity resolution performance is investigated in detail with BeiDou observations. Different distance baselines are tested: 45 km, 70 km and 100 km and the performances are investigated with different elevation cut-off angles. Also the performance with BeiDou alone and combined BeiDou and GPS are compared. Experimental results clearly show that with practical observations of triple-frequency bands, ambiguity of medium- or long-distance baselines can be fixed. The results also show that: the performance of ambiguity resolution with an elevation cutoff angle of 20° is much better than that of 15°; The performance with tropospheric effect corrected is slightly better than that with tropospheric effect as an estimated parameter; Dual-frequency band GPS observations will benefit ambiguity resolution of integrated BeiDou and GPS.

## KEYWORDS

1. BeiDou.
2. Ambiguity resolution.
3. Medium- and long-distance baselines.
4. Ionosphere.
5. Troposphere.

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1. INTRODUCTION. Fast high precision relative Global Navigation Satellite System (GNSS) positioning is very important to various applications, such as navigation, surveying, geodesy and geophysics. Ambiguity resolution is the key to achieve these purposes. It has been a continuing challenge to determine and fix the GNSS carrier-phase ambiguity, especially for medium- and long-distance baselines. With dual-frequency Global Positioning System (GPS) bands, it is almost impossible to achieve fast ambiguity resolution of medium- and long-distance baselines mainly due to the ionospheric and tropospheric effects in the past research.

As an important error source, the ionospheric effect can vary rapidly from a few metres to more than twenty metres within one day and it is difficult to model the ionospheric effects due to complicated physical interactions among the geomagnetic field and solar activities (Xu, 2003). However, as the ionosphere is a dispersive medium, its first-order effect can be corrected with observations of multiple frequency bands and the remaining high-order effect can be generally neglected.

Unlike the ionosphere, the troposphere is a non-dispersive medium at GPS or BeiDou carrier frequencies. In past research, several tropospheric models have been established, such as the Modified Saastamoinen Model (Saastamoinen, 1972; 1973), the Modified Hopfield Model (Hopfield, 1969; 1970; 1972). These can correct up to 90% of the tropospheric effect. But for ambiguity resolution of medium- and long-distance baselines, the remaining part cannot be neglected and should be taken into account. Generally, in the mathematical models, the tropospheric delay in the zenith direction will be treated as an unknown parameter and will be assumed to be stable over a short time, for example, within two hours. A mapping function will be used to combine the delay in the zenith direction and those in the signal-transmitting path.

With the modernization of GPS and the launch of the BeiDou system, observations of triple-frequency bands will be available. Therefore, ionospheric effect will not be a serious problem for ambiguity resolution while tropospheric effect becomes the main obstacle to ambiguity resolution of medium- and long-distance baselines.

In the past 15 years, the three-dimensional numerical weather prediction (NWP) model has become an important research aspect (Schüler, 2006; Jensen and Ovstedal, 2008; Ibrahim and Rabbany, 2009). Recently, several NWP models have been established in many countries and areas around the world, such as North America (Ibrahim and Rabbany, 2009), Japan and Germany (Ghoddousi-Fard et al., 2009; Böhm et al., 2010), Poland (Wielgosz et al., 2008), Australia (Fu, 2008), USA (Gutman and Benjamin, 2001; Ahn et al., 2006; Byun and Bar-Sever, 2009) and this can provide much more accurate corrections compared to conventional models, such as the Modified Hopfield Model and the Modified Saastamoinen Model. The accuracy of the predicted zenith path delay will depend on the resolution of the model and it can reach up to 3 cm level with a horizontal resolution of  $1^\circ \times 1^\circ$  and a vertical resolution of 23 pressure layers up to 20 mbar (Andrei and Chen, 2009). So after correction with the NWP model, the remaining tropospheric effect may be able to be neglected after double-differencing.

With the launch of the BeiDou system, triple-frequency band GNSS observations are available for the first time. This research aims to test the ambiguity resolution performance with BeiDou triple-frequency band observations for medium- and long-distance baselines: 45 km, 70 km and 100 km. For the tropospheric effect, two different mathematical models are compared. First, the zenith tropospheric delay is estimated as an unknown parameter. Second, the tropospheric effect is corrected

Table 1. BeiDou observation codes.

Frequency band	Frequency (MHz)	Observation codes			
		Pseudo-range	Carrier phase	Doppler	Signal strength
B1	1561.098	C2I	L2I	D2I	S2I
B2	1207.14	C7I	L7I	D7I	S7I
B3	1268.52	C6I	L6I	D6I	S6I

and the remaining part is neglected. In our experiment, as there is no NWP model available, the zenith tropospheric delay is estimated with the Precise Point Processing (PPP) technique and used to correct the tropospheric effect.

In addition, as BeiDou observations may have obvious multipath effects and large noise at low elevation angles, even around or higher than 15° (Cheng et al., 2013), the ambiguity resolution performance is investigated with different cut-off elevation angles. At the end, the ambiguity resolution performance is tested with combined Beidou and GPS.

**2. BEIDOU SYSTEM.** The BeiDou Navigation Satellite System (BDS) is a Chinese satellite navigation system. It consists of two separate satellite constellations – a limited test system that has been operating since 2000, and a full-scale global navigation system, that is currently under construction.

The second generation of the system officially called the BeiDou Satellite Navigation System and also known as COMPASS or BeiDou-2, is under construction as of January 2013. BeiDou-2 will be a constellation of 35 satellites, which include five geostationary Earth orbit satellites (GEO) for backward compatibility with BeiDou-1, and 30 non-geostationary satellites (27 in medium earth orbit (MEO) and three in inclined geosynchronous orbit (IGSO)), that will offer complete coverage of the globe (BDS, 2012).

The BDS signals are code division multiple access (CDMA) signals similar to those of GPS and Galileo. Table 1 lists the available observation codes of BeiDou, which includes three frequency bands: B1, B2 and B3 (RINEX 3.02, 2013).

During the experimental period of this research around 19 April 2013, 14 satellites could be observed, including five GEO (PRN: 01, 02, 03, 04, 05), five IGSO (PRN: 06, 07, 08, 09, 10) and four MEO (PRN: 11, 12, 13, 14).

The China Geodetic Coordinate System 2000 (CGCS2000) is a geocentric coordinate system associated with an earth ellipsoid defined slightly differently from the GRS80 (Geodetic Reference System 1980) and WGS84 (World Geodetic System 1984). CGCS2000 is referred to ITRF97 (International Terrestrial Reference Frame 1997) at the epoch of 2000.0 (Wei, 2008; Cheng et al., 2009).

CGCS2000 is compatible to WGS84 and is the same as WGS84 in origin, scale, orientation and time evolution. Among the four defining parameters of the CGCS2000 ellipsoid, semi-major axis  $a$ , flattening  $f$ , Earth's gravitational constant  $GM$  and angular velocity of the Earth  $\omega$ ,  $f$  and  $\omega$  are slightly different from that of WGS84. For WGS84,  $f$  is 1/298.257223563. For CGCS2000,  $f$  is 1/298.257222101. For WGS84,  $\omega$  is 7.2921158553E-5, for CGCS2000,  $\omega$  is 7.2921150E-5 (Wei, 2008; Cheng et al., 2009).

The BDS time reference (BeiDou System Time), named BDT, is based on atomic time. Similar to GPS time, BDS time is a continuous time scale, which does not introduce any leap seconds. The BDS timing system starts from UTC 00:00:00, 1 January 2006 which is 14 seconds different from GPS time (i.e. GPST = BDT +14) (Dong et al., 2007; BDS, 2012).

3. MATHEMATICAL MODELS. The code and carrier phase measurements from a BeiDou satellite  $p$  to receiver  $m$ . at epoch  $t_e$  can be formulated as (Leick, 2004):

$$\rho_{m,i}^p(t_e) = \rho_m^p(t_e) + c(\delta t^p(t_e) - \delta t_m(t_e)) + \delta_{ion,m,i}^p(t_e) + F_m^p(t_e)Z_{trop,m}(t_e) + \varepsilon_{p,m}^p(t_e) \tag{1}$$

$$\varphi_{m,i}^p(t_e) = \rho_m^p(t_e) + c(\delta t^p(t_e) - \delta t_m(t_e)) + \lambda_i N_{m,i}^p - \delta_{ion,m,i}^p(t_e) + F_m^p(t_e)Z_{trop,m}(t_e) + \varepsilon_{p,m}^p(t_e) \tag{2}$$

where  $i=1, 2, 3$ , corresponds to the three frequency bands;  $\rho_{m,i}^p$  is the code measurement;  $\varphi_{m,i}^p$  is the carrier phase measurement in distance;  $\rho_m^p$  is the geometric distance from the satellite  $p$  to the receiver  $m$ ;  $c$  is the light speed;  $\delta t^p$  and  $\delta t_m$  is the clock error of the satellite and the receiver;  $\delta_{ion,m,i}^p$  is the ionospheric delay;  $Z_{trop,m}$  is the zenith tropospheric delay;  $F_m^p$  is the corresponding mapping function of tropospheric delay;  $\varepsilon_{p,m}^p$  and  $\varepsilon_{\varphi,m}^p$  are the other error sources, such as multipath, observation noise and satellite orbital error;  $N_{m,i}^p$  is the integer ambiguity parameter and  $\lambda_i$  is the corresponding wavelength.

After linearization and double-differencing between stations  $m$  and  $n$  and satellites  $p$  and  $q$ , there is (Parkinson and Spilker, 1996):

$$\Delta \nabla \rho_{mn,i}^{pq}(t_e) = a_{mn} \Delta x_{mn} + b_{mn} \Delta y_{mn} + c_{mn} \Delta z_{mn} + \Delta \nabla \delta_{ion,mn,i}^{pq}(t_e) + \Delta \nabla F_{mn}^{pq}(t_e) Z_{trop}(t_e) + \Delta \nabla \varepsilon_{P,mn}^{pq}(t_e) \tag{3}$$

$$\Delta \nabla \varphi_{mn,i}^{pq}(t_e) = a_{mn} \Delta x_{mn} + b_{mn} \Delta y_{mn} + c_{mn} \Delta z_{mn} + \lambda_i \Delta \nabla N_{mn,i}^{pq} - \Delta \nabla \delta_{ion,mn,i}^{pq}(t_e) + \Delta \nabla F_{mn,i}^{pq}(t_e) Z_{trop}(t_e) + \Delta \nabla \varepsilon_{P,mn}^{pq}(t_e) \tag{4}$$

where  $(\Delta x_{mn}, \Delta y_{mn}, \Delta z_{mn})$  is the correction to the coordinate difference between stations  $m$  &  $n$  and  $a_{mn}, b_{mn}$  and  $c_{mn}$  are the corresponding coefficients. Note that, in forming Equations (3) and (4), zenith tropospheric delay at stations  $m$  and  $n$  is assumed to be the same and denoted as  $Z_{trop}$ . For satellites  $p$  and  $q$  and stations  $m$  and  $n$ , the double-differenced observation equations at epoch  $t_e$  will become:

$$\begin{cases} A_{mn}^{pq} X + d_1 \Delta \nabla \delta_{ion,mn}^{pq}(t_e) + \Delta \nabla F_{mn}^{pq}(t_e) Z_{trop}(t_e) = \Delta \nabla \rho_{mn,1}^{pq}(t_e) \\ A_{mn}^{pq} X + d_2 \Delta \nabla \delta_{ion,mn}^{pq}(t_e) + \Delta \nabla F_{mn}^{pq}(t_e) Z_{trop}(t_e) = \Delta \nabla \rho_{mn,2}^{pq}(t_e) \\ A_{mn}^{pq} X + d_3 \Delta \nabla \delta_{ion,mn}^{pq}(t_e) + \Delta \nabla F_{mn}^{pq}(t_e) Z_{trop}(t_e) = \Delta \nabla \rho_{mn,3}^{pq}(t_e) \\ A_{mn}^{pq} X - d_1 \Delta \nabla \delta_{ion,mn}^{pq}(t_e) + \Delta \nabla F_{mn}^{pq}(t_e) Z_{trop}(t_e) + \lambda_1 \Delta \nabla N_{mn,1}^{pq} = \Delta \nabla \varphi_{mn,1}^{pq}(t_e) \\ A_{mn}^{pq} X - d_2 \Delta \nabla \delta_{ion,mn}^{pq}(t_e) + \Delta \nabla F_{mn}^{pq}(t_e) Z_{trop}(t_e) + \lambda_2 \Delta \nabla N_{mn,2}^{pq} = \Delta \nabla \varphi_{mn,2}^{pq}(t_e) \\ A_{mn}^{pq} X - d_3 \Delta \nabla \delta_{ion,mn}^{pq}(t_e) + \Delta \nabla F_{mn}^{pq}(t_e) Z_{trop}(t_e) + \lambda_3 \Delta \nabla N_{mn,3}^{pq} = \Delta \nabla \varphi_{mn,3}^{pq}(t_e) \end{cases} \tag{5}$$

where  $A_{mn}^{pq} = (a_{mn}, b_{mn}, c_{mn})$ ;  $d_i$  ( $i=1, 2, 3$ ) are the coefficients of ionospheric delay and  $d_1=1$  for frequency bands B1;  $X$  is  $(\Delta x_{mn}, \Delta y_{mn}, \Delta z_{mn})$ .

In the above equations, satellite and receiver related errors are cancelled, including the satellite and receiver clock errors and satellite orbital error. For short baselines

(<30 km), the remaining ionospheric and tropospheric delays after double-differencing can be neglected generally. But for medium- or long-distance baselines, they should be taken into account in the observation equations.

Then, all observation equations of epoch  $t_e$  can be denoted as:

$$\begin{cases} \text{AI} + \text{BX} + \text{CT} = L_{\text{code}} \\ \text{AI} + \text{BX} + \text{CT} + \text{DN} = L_{\text{phase}} \end{cases} \quad (6)$$

where  $L_{\text{code}}$  and  $L_{\text{phase}}$  are double-differenced code and carrier phase measurements; I is the double-differenced ionospheric delays; X is  $(\Delta x_{mn}, \Delta y_{mn}, \Delta z_{mn})$ ; T is the zenith tropospheric delay; N is the double-differenced integer ambiguity parameters and A, B, C and D are the corresponding coefficients.

If the tropospheric effect can be corrected with NWP model or other methods, the above equations will become:

$$\begin{cases} \text{AI} + \text{BX} = L_{\text{code}} \\ \text{AI} + \text{BX} + \text{DN} = L_{\text{phase}} \end{cases} \quad (7)$$

For composing the weight matrix, the sigma of carrier phase noise is set to 3 mm same as GPS. As multipath of BeiDou code measurements is obvious (Cheng et al., 2013), the sigma of code noise is set to 0.5 m instead of 0.3 m as GPS.

In order to solve the above equations, ionospheric parameters will be equivalently eliminated every epoch (Xu, 2003). After that, the sequential least-squares adjustment method can be used to obtain the float ambiguity solution and the LAMBDA method (Teunissen, 1995) can be employed to obtain the integer ambiguity candidate corresponding to the minimum quadratic form of the residuals.

In past research, many ambiguity resolution validation methods have been discussed and suggested (Euler and Schaffrin, 1991; Wang et al., 1998; 2000; Leick, 2004; Ji et al., 2010). In this research, the popular R-ratio (Euler and Schaffrin, 1991; Leick, 2004) is used and a threshold value is set to two.

**4. NUMERICAL RESULTS.** To investigate the ambiguity resolution performance of BeiDou medium- and long-distance baselines, experimental data were collected on 1 May 2013 and three Trimble NetR9 receivers were used at stations HK01, HK02 and HK03 with coordinates around 40°N, 100°E. Both GPS and BeiDou observations were collected from 00:00:00 to 23:59:59 (GPS time) and the sample interval was 15 seconds.

Three baselines can be formed, including HK01-HK02 (about 45 km), HK01-HK03 (about 70 km) and HK02-HK03 (about 100 km).

The observed satellites include BeiDou satellites from PRN 1 to 14. But for PRN 13 and 14, only observations of two frequency bands (B1 and B2) were available. No broadcast ephemeris and precise ephemeris (normally provided by Wuhan University) was available for satellites PRN 2 and 5. The BeiDou satellites used in data processing and their observation periods are shown in Figure 1, including: three GEO (PRN 1, 3, 4), five IGSO (PRN 6, 7, 8, 9, 10) and two MEO (PRN 11, 12).

**4.1. Results with cutoff elevation angle 15°.** The data was first processed with the mathematical model of Equation (6) every hour from 01:00:00 to 21:00:00 (GPS time) with a cut-off elevation angle 15° for all the three baselines. If the required time

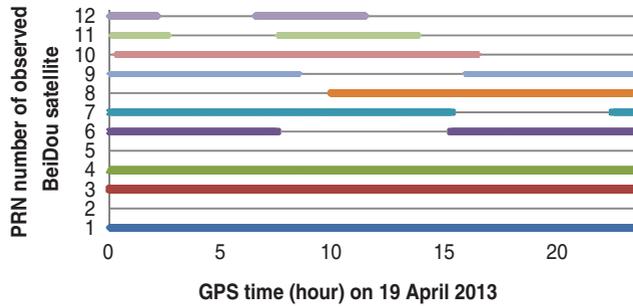


Figure 1. BeiDou satellites used in data processing and their observation time.

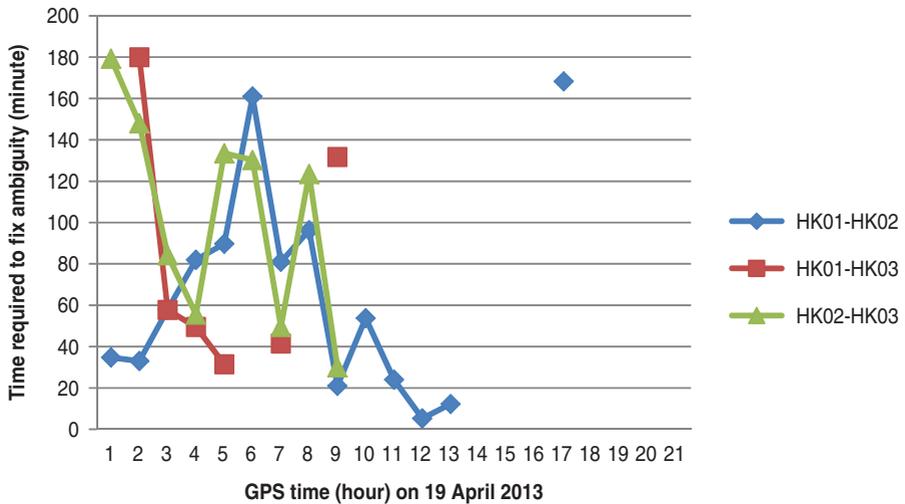


Figure 2. Time required fixing ambiguity with a cut-off elevation angle 15° and without tropospheric parameter.

exceeds four hours, it will be regarded as failed. Figure 2 shows the time required to reach an R-ratio value of two and all ambiguity resolutions are correct.

From the figure, we can see that the required time differs greatly from several minutes to about three hours. From 14:00:00 to 21:00:00 (GPS time), almost all failed except for 17:00:00 for baseline HK01-HK02. Figure 3 shows a sample failed case at 17:00:00 (GPS time) of baseline HK01-HK02 and we can see that the ratio value is only slightly larger than 1.0 for all three hours. The possible reasons for the failed cases may be due to bad observation quality. But there may be another reason, that is, the change of the satellite geometry is not enough as most of them are geostationary.

4.2. Compare different cut-off elevation angles. To investigate the reason for the failed cases, different cut-off elevation angles are tested from 15° to 40° every 5° for baseline HK01-HK03 based on the mathematical model of Equation (6). Table 2 shows the time required to fix ambiguities with different cut-off elevation angles. We can see that with a cut-off elevation angle of 20°, the performance is obviously better than that of 15° and there is no failed case. The performances from 20° to 40°

Table 2. Time required fixing ambiguities with different cut-off elevation angles.  
(unit: minute)

Hour	15°	20°	25°	30°	35°	40°
1	179:25	52	68:25	67:25	40	47:5
2	148	11:5	11:5	11:5	23:25	28
3	84:25	41:75	41:75	41:75	26:75	26:75
4	55:5	8	8	8	17:5	34:25
5	133:5	12:5	12:5	12:5	56:5	58:25
6	130:25	113:5	89:5	80	70:5	55:75
7	49:5	26:75	54:75	79	118:5	20:75
8	123:5	45	45	45	53	52:75
9	30	16:25	16:25	16:25	17	14
10	–	4:75	2:75	0:75	2:75	3:25
11	–	88:75	38	38	15	15
12	–	14:5	14:5	14:5	6:75	6:75
13	–	11:5	11:5	42	115:5	91
14	–	92:5	127:75	127:5	49:25	115:25
15	–	84:75	90	78:25	75:5	80:5
16	–	24:75	36:75	24:75	23:5	23:5
17	–	6:5	6:5	6:5	7	10:25
18	–	68:25	68:25	68:25	38:75	58:75
19	–	32:75	32:75	32:75	22:75	22:75
20	–	10:5	18	18	9:75	9:75
21	–	162:75	156:5	157:25	142	142
Average	103:75	44:26	45:27	46:18	44:36	43:65

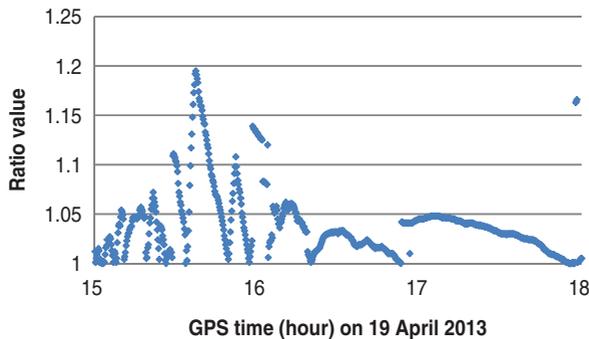


Figure 3. Sample failed case at 15:00:00 of baseline HK01-HK02.

are similar and the average times are all around 45 minutes. As we know, the number of observed satellites will decrease with the increase of the cut-off elevation angle, which will be disadvantageous to final positioning results; a cut-off elevation angle of 20° is the best choice.

4.3. Ambiguity resolution performance with and without tropospheric parameter.

The performances with cut-off elevation angle of 20° are investigated with the two mathematical models of Equations (5) and (6) for all three baselines. Figures 4 and 5 show the time required and Table 3 gives the average required time.

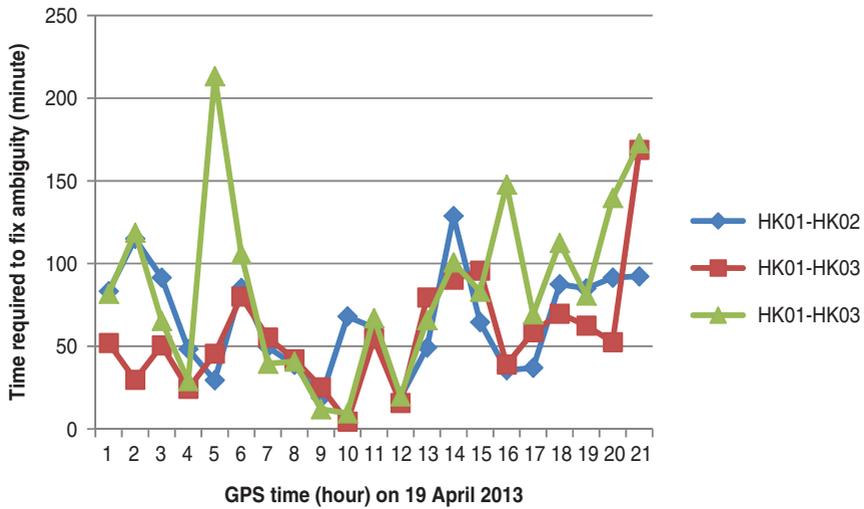


Figure 4. Time required fixing ambiguities for mathematical model of Equation (5) with cut-off elevation angle of 20°.

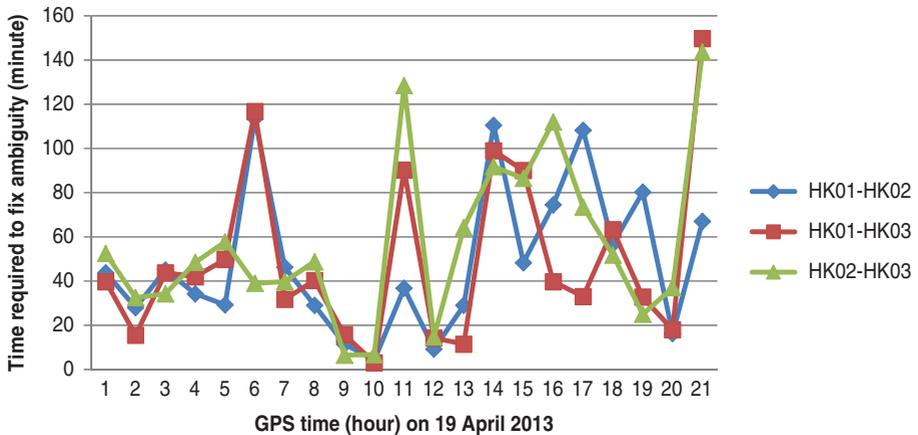


Figure 5. Time required fixing ambiguities for mathematical model of Equation (6) with cut-off elevation angle of 20°.

From the two figures, we can see that the required time still differs greatly from several minutes to about three hours and there is no failed case for all three baselines and both models. Comparing the performance of the three baselines, the time required has no great difference generally and the average required time is also similar for the two models. Comparing the performance of the two models, we can find that the required time of Equation (6) is generally less than that of Equation (5) and the average time required of Equation (6) in Table 3 is also less than that of Equation (5).

4.4. *BeiDou alone vs BeiDou and GPS.* In this section, the performance with combined BeiDou and GPS is investigated and compared to that of BeiDou alone

Table 3. Average time required fixing ambiguities.  
(unit: minute)

Mathematic model	Baseline	Distance	Average time
With tropospheric parameter	HK01-HK02	45 km	65.7
	HK01-HK03	70 km	56.9
	HK02-HK03	100 km	84.5
Without tropospheric parameter	HK01-HK02	45 km	48.6
	HK01-HK03	70 km	49.5
	HK02-HK03	100 km	56.9

Table 4. Time required fixing ambiguities with the cutoff elevation angle 20° and the model of Equation (6).  
(unit: minute)

Hour	BeiDou	BeiDou + GPS
1	52	51
2	11.5	9.5
3	41.75	37
4	8	6
5	12.5	13
6	113.5	101
7	26.75	27.5
8	45	42.5
9	16.25	11.25
10	4.75	4
11	88.75	85
12	14.5	8.5
13	11.5	11
14	92.5	89
15	84.75	25
16	24.75	22.5
17	6.5	7
18	68.25	19
19	32.75	12
20	10.5	8.75
21	162.75	44
Average	44.26	30.21

and only ambiguities of BeiDou satellites are fixed. The cut-off elevation angle is 20°, the mathematical model used is Equation (6) and only baseline HK01-HK03 (70 km) is tested. Table 4 shows the time required for BeiDou alone and BeiDou and GPS. We can see that the required time of BeiDou and GPS is generally less than that of BeiDou alone, which shows that GPS observations are beneficial to the ambiguity resolution of BeiDou. The possible reason may be that GPS observations are helpful to float the BeiDou ambiguity solution.

5. CONCLUSIONS. In this research, ambiguity resolution performance with triple-frequency BeiDou bands is investigated for baselines of 45 km, 70 km

and 100 km. Experimental results clearly show that with observations of triple-frequency bands, ambiguity of medium- or long-distance can be fixed. The results also show that:

- The test results with different cut-off elevation angle show that the performance with 20° is much better than that of 15°;
- The performance with tropospheric effect corrected is slightly better than that with tropospheric effect as an estimated parameter;
- Dual-frequency band GPS observations will benefit ambiguity resolution performance;
- The performances are similar for baselines with different distances;
- The time required for fixing ambiguities differs greatly with at different tested times and the average required time is around 45 minutes.

The results of this research will benefit us for further research to improve the ambiguity resolution performance. For our further work, we need to analyse the reason for the obvious variation of the fixing time required. The possible reasons may be slow geometry change of geostationary satellites as the satellite geometry change is important to ambiguity resolution. Another possible reason may be that the stochastic model used is not optimal. BeiDou and GPS are different in some aspects. BeiDou has various types of satellites, including: GEO, IGSO and MEO. Every type of satellite may have its own characteristics, such as multipath and noise levels. BeiDou and GPS also have different frequency bands. It is for future research to investigate how to improve the ambiguity resolution performance for medium- and long-distance baselines based on the different characteristics of BeiDou.

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