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Relativistic acceleration of micro-foils with prospects for fast ignition

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Abstract

In this work, it is suggested that the ponderomotive force, induced by a multi-petawatt laser on the interface of a vacuum with solid target, can accelerate a micro-foil to relativistic velocities. The extremely high velocities of the micro-foil can be achieved due to the very short time duration (about a picosecond) of the laser pulse. This accelerated micro-foil is used to ignite a pre-compressed cylindrical shell containing the deuterium tritium fuel. The fast ignition is induced by a heat wave produced during the collision of the accelerated foil with the pre-compressed target. This approach has the advantage of separating geometrically the nanoseconds lasers that compress the target with the picosecond laser that accelerates the foil.

Keywords: Fast-ignition; Micro-foil acceleration; Petawatt laser; Ponderomotive force

INTRODUCTION

Nuclear fusion ignition by inertial confinement induced by lasers is very promising and is on the forefront of research today (Velarde & Carpentero-Santamaria, 2007; Mima *et al.*, 2009). It is expected to be achieved with the mega-joule laser of a few nanoseconds pulse duration by Livermore National Laboratory (Moses, 2009). The target and the driver pulse shape are specially designed in order to start ignition at the center (a spark) of the compressed fuel (Lindl, 1997; Rosen, 1999; Atzeni & Meyer-Ter-Vehn, 2004). The rest of the fuel is heated by alpha particles produced in the deuterium tritium (DT) reactions.

In order to ignite a DT target with less energy, it was suggested (Basov *et al.*, 1992; Tabak *et al.*, 1994) to separate the drivers that compress and heat the target. This idea is called fast ignition (FI). First, the pellet is compressed by a laser system with a few nanoseconds pulse duration; then a second driver, for example, a multi-petawatt laser beam, ignites a small part of the pellet. In this case, the alpha particles produced in the DT reactions heat the rest of the target.

The main problem of FI is that the laser pulse does not penetrate directly into the compressed target since the electron density is on the order of $10^{24}\ \text{cm}^{-3}$. Therefore, many schemes of FI were suggested: (1) the laser energy is converted into electrons that ignite the target (Norreys *et al.*, 2000), (2) the laser energy is converted into protons that ignite the target (Roth *et al.*, 2001; Brenner *et al.*, 2011; Pae *et al.*, 2011; Torrisi *et al.*, 2011). (3) A gold cone was stuck in the spherical pellet (Kodama *et al.*, 2001) in order to solve the preheating problem. (4) FI is induced by plasma jets (Martinez Val & Piera, 1997; Velarde *et al.*, 2005) that are produced by the same laser system that compresses the pellet. (5) The FI is accomplished by the plasma flow created from a thin exploding pusher foil (Caruso & Strangio, 2001; Guskov, 2001; Krasa *et al.*, 2011). (6) Plasma blocks for FI were also suggested (Hora *et al.*, 2005, 2011). (7) The old impact fusion with the help of the cone (Murakami *et al.*, 2006; Azechi *et al.*, 2009) was suggested as an alternative to the petawatt laser ignition. (8) The features of shock wave collision (Jackel *et al.*, 1983) were introduced into the FI with a laser induced strong shock wave (Betti *et al.*, 2007). A novel scheme of combining the FI fusion with a shock wave by a foil impact was recently published (Eliezer & Martinez Val, 2011).

In the next section, we suggest to accelerate micro-foils to relativistic velocities by using a multi-terawatt laser. From momentum and energy conservation a two model equations are obtained and solved analytically in a consistent way. The maximum velocity achieved so far was reported (Azechi *et al.*, 2009) to be about 1000 km/s and used for a DT impact fusion device (Murakami *et al.*, 2006). In this work, velocities by an order of magnitude higher are

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recommended for impact fusion. Due to the significantly higher temperature that one gets in this case, this scheme may be appropriate also for the clean proton-boron11 fusion (Eliezer & Martinez Val, 1998).

After the relativistic acceleration of the micro-foil chapter, in the following section, we propose to use the very high velocities of the micro-foil for impact fusion of a pre-compressed cylinder (Basko *et al.*, 2002; Guskov, 2005; Nakamura *et al.*, 2008; Vauzour *et al.*, 2011). The conclusion of the ideas of this work is discussed in the last section.

RELATIVISTIC ACCELERATION OF MICRO-FOILS

The ponderomotive force per unit volume is related to the gradient of the radiation pressure P_L. If the laser propagates in vacuum with irradiance I_L and hits a sharp boundary, a solid-vacuum in our case, then the radiation pressure is (Eliezer, 2002)

$$P_L = \frac{I_L}{c}(1+R). \tag{1}$$

R is the reflection from the boundary and c is the speed of light. For $I_L >> 10^{16}\ \mathrm{W/cm^2}$ the radiation pressure is the dominant pressure at the solid-vacuum interface. A very high power laser on the order of multi-petawatts with very short pulse duration (about picoseconds) accelerates a solid micro-foil to very high velocities. In this case, the Newton second law in the longitudinal direction (direction of the laser beam) can be described by the following equation

$$\begin{aligned} &(1+R)\left(\frac{I_L}{c}\right)S = m_0\gamma^3\frac{dv}{dt}, \\ &\gamma = \frac{1}{[1-v^2/c^2]^{1/2}}, \end{aligned} \tag{2}$$

S is the cross-section area of a micro-foil with a thickness l and with a density ρ_0, so that the mass is $m_0 = \rho_0 Sl$ and dv/dt is the longitudinal acceleration. The micro-foil moves with a velocity v and γ is the relativistic factor. The momentum conservation of Eq. (2) has to be satisfied together with the energy conservation

$$\begin{aligned} &\eta I_L S\tau_L = \eta W_L = m_0c^2(\gamma_m - 1) \\ &\eta = (1-R)\eta_K \end{aligned} \tag{3}$$

W_L is the laser energy since we have assumed a constant irradiance I_L during the laser pulse duration $0 < t < \tau_L$. 1-R is the laser absorption, η_K is the efficiency from the absorbed laser energy to the kinetic energy of the micro-foil, and γ_m is the relativistic factor at the end of the laser pulse duration τ_L.

We shall solve Eqs. (2) and (3), the model equations, in the following dimensionless units

$$\begin{aligned} &(i)\ \Delta = \frac{1}{(1-\beta^2)^{3/2}}\left(\frac{d\beta}{d\tau}\right), \\ &(ii)\ \eta\left(\frac{\Delta}{1+R}\right) = \frac{1}{\left(1-\beta_m{}^2\right)^{1/2}} - 1, \\ &\tau = \frac{t}{\tau_L},\ \ \beta \equiv \frac{v}{c},\ \ \beta_m = \beta(t=\tau_L), \\ &\Delta \equiv (1+R)\delta,\ \ \delta \equiv \frac{W_L}{m_0c^2} = \frac{I_L\tau_L}{\rho_0 c^2 l}. \end{aligned} \tag{4}$$

In our model Eqs. (4*i*) and (4*ii*) δ and R are given, there are two equations (*i*) and (*ii*) with two unknowns β and η that are solved simultaneously satisfying the constrains

$$\beta < 1,\ \ \eta < 1. \tag{5}$$

In order to get a feeling of the acceleration suggested in this work, the following numerical example is appropriate: a multi-petawatt laser defined by $I_L = 3\times10^{21}\ \mathrm{W/cm^2}$, $\tau_L =$ 1 ps, $W_L = 30$ kJ accelerating a micro-foil with dimension $S = 10^{-5}\mathrm{cm^2}$ (about $30\ \mu\mathrm{m}\times 30\ \mu\mathrm{m}$), $l = 1\ \mu\mathrm{m}$, density about $1\mathrm{g/cm^3}$ implying a value of $\delta \sim 1/3$.

The exact solution of the differential equation 4(*i*) for β is

$$\begin{aligned} &\beta(\tau) = \frac{\tau\Delta}{\sqrt{1+\tau^2\Delta^2}}, \\ &\beta_m = \beta(\tau=1) = \frac{\Delta}{\sqrt{1+\Delta^2}}. \end{aligned} \tag{6}$$

Note that for zero laser energy input the velocity goes to zero, while for infinite energy the velocity goes to the speed of light as expected, namely

$$\begin{aligned} &\Delta \to 0 \Rightarrow \beta \to 0, \\ &\Delta \to \infty \Rightarrow \beta \to 1. \end{aligned} \tag{7}$$

Substituting our solution for β into equation 4(*ii*) we get

$$\eta = (1+R)\left[\frac{\sqrt{\Delta^2+1}-1}{\Delta}\right]. \tag{8}$$

The values of β and η for $R = 0.2$, 0.5, and 0.8 are plotted in Figure 1. Since $\eta < 1$ we get a maximum value of Δ_m for every R

$$\begin{aligned} &\Delta_m = \frac{2(1+R)}{R(2+R)} \quad \text{for R} \neq 0, \\ &\Delta_m = 0 \qquad\qquad \text{for R} = 0. \end{aligned} \tag{9}$$

It is interesting to point out that the relativistic equations of momentum and energy conservation, as given in Eq. (4), have a solution only in a limited domain of energy $\Delta < \Delta_m$.

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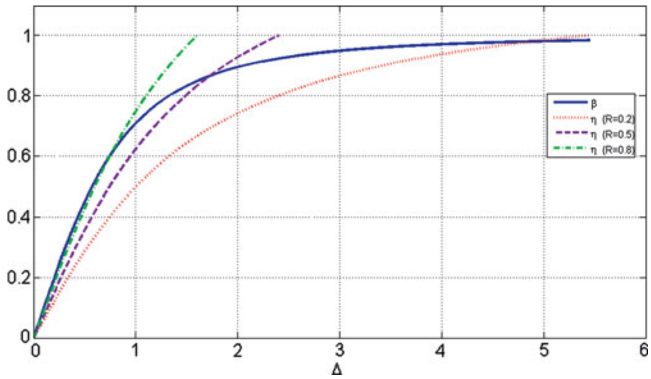

Fig. 1. (Color online) The relativistic velocity β for the micro-foil and the efficiency η for R = 0.2, 0.5 and 0.8 as a function of the dimensionless laser energy Δ.

A FI SCHEME

The high compression of the main fuel is achieved by shock waves and by the accumulation of matter during the stagnation of the implosion of the target shell. We suggest using the high velocity micro-foil to ignite the pre-compressed cylinder shell as described schematically in Figure 2. The initial mass, density, and dimensions of the fuel (DT for example) cylinder shell are accordingly M_0, ρ_0 and radius R_0, thickness ΔR_0 and length L. The cylinder length is related to the final radius of compression R_C by a physical parameter A_L, $L = A_L R_C$, and the thickness shell is fixed by an aspect ratio $R_0/\Delta R_0 = A$. The equality of the initial mass and the final mass (before ignition) of the compressed fuel $M_C = M_0$ implies

$$\frac{\rho_C}{\rho_0} = \left(\frac{2}{A}\right)\left(\frac{R_0}{R_C}\right)^2 \tag{10}$$

where the compressed density is ρ_C.

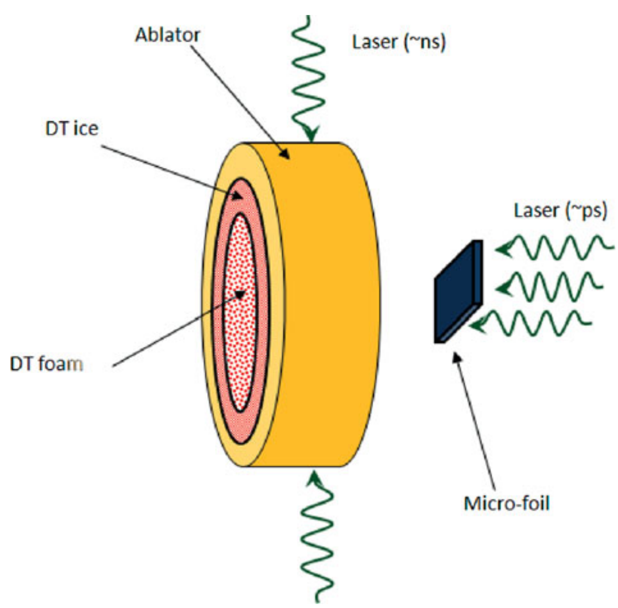

Fig. 2. (Color online) Schematic configurations of impact fusion of a pre-compressed cylinder.

The gain G is given by

$$G = \frac{\phi q_{DT} M_C}{W_d}, \quad W_d = W_{LC} + W_{LI}, \tag{11}$$

W_d is the driver energy composed of two terms: the compression laser energy W_{LC} and the ignition energy W_{LI} given by the laser energy W_L that accelerates the micro-foil defined in Eq. (3) and discussed in the previous section. $q_{DT} = 3.39 \times 10^8$ kJ/g is the energy output per gram and ϕ is the burning fraction in the DT fusion given by

$$\phi \approx \frac{H_C}{H_C + 7[g/cm^2]}; \quad H_C \equiv \rho_C R_C = \left(\frac{\rho_C{}^2 M_C}{A_L \pi}\right)^{1/3} = \left(\frac{2\rho_0\rho_C}{A}\right)^{1/2} R_0. \tag{12}$$

In order to compress and heat the main fuel, defined by its pressure P_C and temperature T_C one requires driver energy W_{LC} given by the following energy conservation (Eliezer *et al.*, 2007)

$$\eta_C W_{LC} = 3\left(\frac{M_C}{2.5m_p}\right)(K_B T_C) = \frac{3}{5}\left(\frac{M_C}{2.5m_p}\right)\alpha_C \varepsilon_F, \tag{13}$$

η_C is the efficiency from laser to thermal energy, ε_F is the Fermi energy, and α is related to the electron degenerate quantum pressure P_{deg} (Eliezer & Ricci, 1991; Eliezer *et al.*, 2002) for DT according to the relations

$$\varepsilon_F = \frac{(3\pi^2)^{2/3}\ h^2 \eta_e{}^{2/3}}{8\pi^2 m_e} \simeq 2.25 \cdot 10^{-11} \rho_C{}^{2/3}, \tag{14}$$

$$P_{deg} = \frac{2}{5}\eta_e \varepsilon_F = \frac{P_C}{\alpha_C}. \tag{15}$$

The numerical factor in Eq. (14) is given for using *c.g.s.* units. n_e is the electron density, h is the Planck constant, m_e is the electron mass and the effective temperature T_C and pressure P_C are defined for the DT plasma via $P_C = 2n_e k_B T_C$. Simulations show that α_C equals about 3 is a reasonable number (Rosen, 1999). Using Eqs. (13) and (15) for the DT fusion case one gets

$$W_{LC}[KJ] \simeq 32.3\left(\frac{\alpha_C}{\eta_C}\right)\left(\frac{\rho_C}{1000g/cm^3}\right)^{2/3}\left(\frac{M_C}{mg}\right). \tag{16}$$

For example, from Eqs (10)–(12) with $\phi = 0.3$, if one has $\rho_C = 1000$ g/cm^3 (Azechi *et al.*, 1991), $\rho_0 = 0.2$ g/cm^3, $A = 20$, $A_L = 10$ then $L = 0.0447R_0$, $\rho_C R_C = 3$g/cm^2, $R_0 = 0.474$ cm, and $M_C = 0.848$ mg. Using Eq. (16) we get $W_{LC} = 0.82$ MJ and for these set of parameters one gets from Eq. (11) a gain of G ~ 100 can be achieved.

We discuss now the ignition physics. The ignition in our scheme is not caused by an induced shock wave. A shock wave ignition will require an accelerating laser pulse with energy about two orders of magnitude or more larger than suggested above. This can be easily seen from the high pressure (P) shock wave thickness (d) estimated from the equality of pressures at the interface between the flyer and the compressed target. Since $P \sim \rho_0 u_0^2 \sim \rho_C u_C^2$ and the shock wave transition time in the flyer with a thickness l is $t = l/u_0$ one gets a shock wave thickness d of the order $d \sim (\rho_0/\rho_C)^{1/2}$ l. In the above scheme, $(\rho_0/\rho_C) \sim 0.001$ and $l \sim 1$ μm implying $d \sim 0.03$ μm. The ignition criterion is based on the requirement that the alpha particles created in the DT reaction are reabsorbed in the hot spot implying a "ρR" value larger than 0.3 g/cm^2 for a temperature about 10 keV and larger values for higher temperatures. In the shock wave ignition "ρR" $= \rho_C d \sim l(\rho_0\rho_C)^{1/2} \sim 0.003$ g/cm^2 for our case that is two order of magnitude too low. From this ignition criterion in the hot spot, we need to increase the foil thickness by two orders of magnitude that imply the undesired result of increasing the accelerating driver energy by two orders of magnitude.

Our scheme is based on heat wave ignition. The local absorption of the flyer energy is associated with the creation of a temperature gradient and a thermal flux to transport the absorbed energy. If the interaction of the impact is very short, on the order of 1 ps or less, the hydrodynamic motion does not have time to develop, and therefore in these cases the heat transport is dominant. Electrons or X-rays may serve as heat carriers. For nonlinear transport coefficient, like the electron conductivity, we expect that the energy transport is caused by a heat wave. This heat wave plays the important role in the inertial confinement fusion during the ignition process.

The nonlinear heat transport equation and the energy conservation are described accordingly (Zeldovich & Raizer, 1966; Eliezer, 2002)

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial}{\partial x}\left(T^n \frac{\partial T}{\partial x}\right), \tag{17}$$

$$\int T(x, t)dx = \frac{W_s}{S\rho_C C_V}, \tag{18}$$

T is the temperature, x is the position, t is time, W_S [erg] is the deposited energy into the hot spot area S [cm^2], ρ_C is the density of the compressed target, C_V [erg/(g·K)] is the specific heat at constant volume and, a is a constant related to the thermal diffusivity $\chi[\mathrm{cm}^2/\mathrm{s}] = aT^n$. The solution of these equations for $n = 5/2$ is

$$T(x, t) = \begin{cases} T_0(t)\left(1 - \dfrac{x^2}{x_f(t)^2}\right)^{2/5} & x < x_f \\ 0 & x > x_f \end{cases}. \tag{19}$$

Solution of this equation for three different times $t_1 > t_2 > t_3$ are shown in Figure 3. T_0 is taken as 100 keV, 20 keV, and 10 keV for the appropriate times $t_1 > t_2 > t_3$. The first T_0 was chosen from the energy relation $(1/2)\rho_0 u_0^2 \sim 3(k_B T)\ \rho_C/$ (2.5 m_p) where m_p is the proton mass. This implies an initial temperature T[keV] $\sim 10^6(\rho_0/\rho_C)\beta^2$. For $\rho_0/\rho_C \sim 0.001$ and $\beta = 0.3$ we get $T \sim 100$ keV. The energy W_s and the mass in the hot spot M_s are

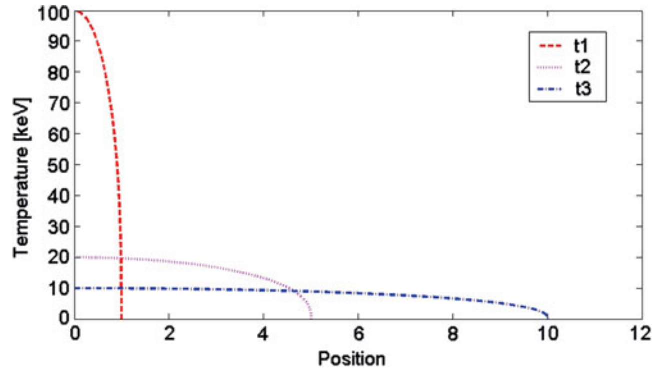

Fig. 3. (Color online) Solution of the heat wave equation for nonlinear coefficient $n = 5/2$ at three different times $t_1 > t_2 > t_3$.

$$\begin{aligned} M_s &= S x_1 \rho_C; \quad x_1 = x_f(t_1), \\ W_s &= 3k_B T_s\left(\frac{M_s}{2.5m_p}\right), \\ W_{LI} &= \frac{W_s}{\eta_s}. \end{aligned} \tag{20}$$

W_{LI} is the laser driver that accelerates the foil, η_s is the efficiency from the laser to the thermal energy of the hot spot, and T_s is the hot spot temperature changing with time as can be seen from Figure 3. Ignition requires $\rho x_f =$ "ρR" ≈ 0.3 g/cm^2 and 2 g/cm^2 for $T_s = 10$ keV and 100 keV accordingly, and since the density is constant during the heat wave transition we require $x_f = 3$ μm for a temperature of 10 keV or a larger x_f in agreement with the alpha mean free path.

Using the ignition criterion and Eq. (20) for $T = 100$ keV and $T = 10$ keV we need accordingly: $M_s(100\text{ keV}) = 1.8 \times 10^{-5}$g, $W_s(100\text{ keV}) = 5.7 \times 10^5$ J and $M_s(10\text{ keV}) = 0.27 \times 10^{-5}$g, $W_s(10\text{ keV}) = 8.5 \times 10^3$ J. From Figure 3 one can see that during the change in temperature from 100 keV to 10 keV the x_f changes by about a factor of 10 without change in the density. Therefore, we claim that the criterion for the 10 keV case is the relevant one and using this value we need a laser driver for accelerating the foil of about 85 kJ assuming that the efficiency from the laser to the thermal energy of the hot spot η_s is 10%.

DISCUSSION

In this work, the acceleration of a micro-foil to relativistic velocities is suggested. The accelerating force is the ponderomotive force induced by a multi-petawatt laser on the interface of a vacuum with the micro-foil solid target. The extremely high velocities of the micro-foil can be achieved

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due to the very short time duration (picoseconds) of the laser pulse. In these cases, the ablation pressure is negligible relative to the ponderomotive pressure. The model equations for this acceleration are based on momentum and energy conservation as given in equations (4*i*) and (4*ii*). The two unknown of these equations are the micro-foil velocity β (in units of the speed of light) and the energy efficiency η transferred from the laser to the micro-foil. The model is subjected to constrain given in Eq. (5). The solution of this model is given in Eqs. (6) and (8) and described in Figure 1. The equations of our model have a solution only for a dimensionless energy (Δ) smaller than a value dependent on the laser reflection *R* as calculated in Eq. (9). This effect can be understood from the model since both the acceleration force and the kinetic energy given to the micro-foil are linear function with Δ while the relativistic formulas are different nonlinear functions in β.

Our model is using the acceleration before the hydrodynamic instabilities start their destructive action. Similarly, all high pressure shock wave experiments are based on the fact that diagnostics is very fast and the experiment ends before the foil-target disassemble. Taking into account the material strength, the shock wave transient time through the foil and the relative motion between the fluid particles there should be a relaxation time τ_0 before the hydrodynamic instabilities break the foil. It is out of the scope of this paper to analyze this problem in depth, however using a simple dimensional analysis we estimate

$$\tau_o = \left(\frac{G}{\rho}\right)^{-1/2}\left(\frac{l}{100}\right) \tag{21}$$

G is the shear modulus and ρ is the density of the accelerated matter and *l* is foil thickness and we assume that 1% deformation will trigger a foil breaking. For example, for aluminum 6061one has $G = 0.276$ Mbar, $\rho = 2.70\ \mathrm{g/cm^3}$ and taking a micro-foil with $l = 1\ \mu$m one gets $\tau_0 = 3.1$ ps. Although this is a very crude approximation, we suggest in this case that acceleration during a time of about 3 ps will keep the micro-foil intact even for extremely high acceleration like the one suggested in this paper.

For a further order of magnitude check, the Rayleigh-Taylor instabilities were calculated taking into account the shear stresses of the matter (Piriz *et al.*, 2005). It is interesting to point out that this calculation for the accelerations of 10^{20} $\mathrm{cm/s^2}$ and 10^{21} $\mathrm{cm/s^2}$ yields a τ_0 on the order of one picosecond, consistent with our rough estimation of Eq. (19).

This accelerated micro-foil is used to ignite a pre-compressed cylindrical shell containing the imploded plasma. Our scheme is based on heat wave ignition. This approach has the advantage of separating geometrically the nanoseconds lasers that compress the target with the picosecond laser that accelerates the foil. In this paper extremely high velocities are recommended for impact fusion. Due to the significantly higher temperature that one gets in this case this scheme may be appropriate also for the clean proton-boron11 fusion. The present model suggests that nuclear fusion by a micro-foil impact ignition could be attained with existing present technology.

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APPENDIX

One has to solve exactly equation (4*i*) as has been done in the text of this paper. Nevertheless we show here (for pedagogical reasons) the solution of equation (4*i*) with the approximation often used for short laser pulses, namely $d\beta/dt \sim \beta/\tau_L$. This approximation leads to a cubic equation in β^2:

$$z^3 + a_2 z^2 + a_1 z + a_0 = 0$$
$$z \equiv \beta^2;\ \ a_2 = -3;\ \ a_1 = 3 - \Delta^{-2};\ \ a_0 = -1$$

The three roots of this cubic equation are ($\beta_1{}^2$, $\beta_+{}^2$, $\beta_-{}^2$):

$$\beta_1{}^2 = -\frac{a_2}{3} + (S_+ + S_-)$$
$$\beta_\pm{}^2 = -\frac{a_2}{3} - \frac{1}{2}(S_+ + S_-) \pm \frac{i\sqrt{3}}{2}(S_+ - S_-)$$
$$-\frac{a_2}{3} = 1$$

$$S_\pm = \left[r \pm (q^3 + r^2)^{1/2}\right]^{1/3} = \left(\frac{1}{2\Delta^2}\right)^{1/3} \left[1 \pm \left(1 - \frac{4}{27\Delta^2}\right)^{1/2}\right]^{1/3}$$

$$q = \frac{1}{3}a_1 - \frac{1}{9}a_2{}^2 = -\frac{1}{3\Delta^2};\ \ r = \frac{1}{6}(a_1 a_2 - 3a_0) - \frac{1}{27}a_2{}^3 = \frac{1}{2\Delta^2}$$

For $q^3 + r^2 > 0$ one has a real root and a pair of complex conjugate roots for β^2. This case is not physical in our problem since the real root is larger than 1 ($\beta > 1$). If $q^3 + r^2 = 0$ all roots are real and two are equal, while for $q^3 + r^2 < 0$ all

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roots (of β^2!) are real. In the last case one gets

$$\Delta^2 < \frac{4}{27} \Rightarrow \Delta = (1+R)\delta \leq \frac{2}{3\sqrt{3}} \simeq 0.385$$

The three roots of our cubic equation in this case are

$$\beta_1{}^2 = 1 + 2\left(\frac{1}{3\sqrt{3}}\right)^{1/3}\left(\frac{1}{\Delta}\right) \cos\left[\frac{1}{3}\arcsin\left(\sqrt{1-\left(\frac{27\Delta^2}{4}\right)}\right)\right]$$

$$\beta_\pm{}^2 = 1 - \left(\frac{1}{3\sqrt{3}}\right)^{1/3}\left(\frac{1}{\Delta}\right) \left\{\cos\left[\frac{1}{3}\arcsin\left(\sqrt{1-\left(\frac{27\Delta^2}{4}\right)}\right)\right] \pm \left(3\sqrt{3}\right)^{1/3}\sin\left[\frac{1}{3}\arcsin\left(\sqrt{1-\left(\frac{27\Delta^2}{4}\right)}\right)\right]\right\}$$

$$\frac{4}{27} > \Delta^2$$

The β solution that satisfy equations 4(*i*), 4(*ii*) and Eq. (5) is

$$\beta = \begin{cases} \beta_- \text{ if } \left(\frac{4}{27}\right)^{1/2} > \Delta > \left(1-\beta_-{}^2\right)^{-1/2} - 1 \\ 0 \text{ if } \left(\frac{4}{27}\right)^{1/2} < \Delta \text{ or } \Delta < \left(1-\beta_-{}^2\right)^{-1/2} - 1 \end{cases}$$

It is interesting to point out that the relativistic equations of momentum and energy conservation in this approximation have a solution only in a limited dimensionless energy (Δ) domain.

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