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Anthropogenic climate change in a descriptive growth model

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ABSTRACT. This paper studies the effects of global warming in a descriptive model of endogenous growth. It is assumed that deviations from the pre-industrial global surface temperature negatively affect aggregate output. The paper studies the effects of varying the tax rate and of different abatement activities on the emission of greenhouse gases and on the growth rate. We study both effects for the long-run balanced growth rate and for the growth rate of GDP on the transition path. Using simulations, it is demonstrated that higher abatement activities may both reduce greenhouse gas emissions and lead to higher growth. Further, the second-best abatement share is computed and the corresponding growth rate as well as the social optimum.

1. Introduction

The emission of greenhouse gases (GHGs), like carbon dioxide (CO_2) or methane (CH_4), has drastically increased in the twentieth century and still continues to rise, leading to higher concentrations of GHGs in the atmosphere. Higher GHG concentrations generate a rise in the average global surface temperature and make extreme weather events more likely. According to the Intergovernmental Panel on Climate Change (IPCC) it is very likely[1] that the 1990s was the warmest decade and 1998 the warmest year since 1861 (IPCC, 2001, p. 26). In addition, it is likely that statistically significant increases in heavy and extreme weather events have occurred in many mid and high latitude areas, primarily in the Northern Hemisphere.[2]

In the economics literature numerous studies analyze the impact of environmental degradation on economic growth using endogenous growth models (for a survey see, for example, Smulders, 1995; Hettich, 2000; or Smulders, 2000). Generally, these studies are rather abstract because they intend to derive general results. It is assumed that economic activities lead to environmental degradation and, as a consequence, reduce utility and/or production possibilities. The goal of these studies often is to analyze how

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[1] Very likely (likely) means that the level of confidence is between 90–99 (66–90) per cent.

[2] More climate changes are documented in IPCC (2001, p. 34).

public policy affects environmental conditions as well as the growth rate and welfare of economies.

However, as far as I know there do not exist economic studies which incorporate climate models in a growth model and study the effects of different time paths of GHG emissions on the growth rate of economies. Instead, economic studies dealing with global warming are mostly cost–benefit analyses which take the growth rate of economies as an exogenous variable. These studies then compute the discounted cost of reducing GHG emissions and confront them with the discounted benefit of a lower increase in GHG concentration and, as a consequence, of a smaller increase in average global surface temperature (see, for example, Nordhaus, 2000; or Tol, 2001, and for a survey IPCC, 1996).[3]

A great problem in studying the economic consequences of global warming is the uncertainty as concerns the damages caused by a change in the Earth's climate. Nevertheless, there are analyses doing this. For example, the IPCC estimates that a doubling of CO_2, which goes along with an increase of the global average surface temperature between 1.5 and 4.5 degrees Celsius, reduces world GDP by 1.5–2 per cent (see IPCC, 1996, p. 218). This damage is obtained for the economy in steady state and comprises both market and non-market impacts, where non-market impacts are direct reductions in peoples' welfare resulting from a climate change.

In this paper we intend to integrate a simple climate model in a descriptive model of endogenous growth in order to analyze the effects of GHG emissions and of abatement policies on economic growth. We study a descriptive growth model because we want to analyze both the balanced growth path and the economy on the transition path. Assuming a utility-maximizing individual, however, would result in a dynamic system, which is in general not asymptotically stable but a saddle point. Therefore, transition dynamics are often studied by assuming that the economy jumps to the stable manifold (the so-called jump variable technique). This implies that the level of consumption must perform a discontinuous jump at time $t = 0$ which does not seem to give a realistic description of economies.

The rest of the paper is organized as follows. In section 2.1 we present our general descriptive growth model. Further, we model GHG emissions and changes in average surface temperature using a simple energy balance model (EBM). Section 2.2 introduces the balanced growth path and section 2.3 presents simulation studies analyzing the effects of different tax rates and abatement spending on GHG emissions. In addition, we study the effects of global warming as regards the growth rate for both the balanced growth path and for the economy on the transition path. Section 3.1 introduces a specialization of the growth model giving the so-called *AK* growth model which is linear in the capital stock and compares this model with the one presented in section 2. In section 3.2 we derive the second-best abatement share and in section 3.3 we study the problem of the social planner, who determines both the first-best investment share and the first-best abatement share. Section 4, finally, concludes the paper.

[3] We do not go into the details of these studies. The interested reader is referred to the IPCC report (see IPCC, 1996).

2. A descriptive model of endogenous growth

2.1. Structure of the model

We assume that aggregate production takes place according to the following aggregate production function

$$\bar{Y}(t) = A\bar{K}(t)^{\alpha}(H(t)L(t))^{1-\alpha}D(T(t) - T_o) \tag{1}$$

with $\bar{Y}(t)$ aggregate production, A a positive constant, $H(t)$ human capital or a stock of knowledge which is formed as a by-product of aggregate investment, and $L(t)$ labour input. $\bar{K}$ is aggregate physical capital, $\alpha \in (0, 1)$ is the capital share, and t gives the time which will be deleted in the following if no ambiguity results. $D(T(t) - T_o)$ is the damage function giving the damage resulting from deviations of actual temperature from pre-industrial temperature, T_o. It should be mentioned that the assumption of a continuous damage function is only justified provided the temperature increase does not exceed a certain threshold. This holds because for higher increases of the temperature catastrophic events may occur, going along with extremely high economic costs, which cannot be even evaluated. Just one example is the break down of the Gulf Stream, which would dramatically change the climate in Europe. Therefore, the analysis assuming a damage function only makes sense for temperature increases within certain bounds.

Per capita production is obtained by dividing both sides of (1) by L as[4]

$$Y = AK^{\alpha}H^{1-\alpha}D(\cdot) \tag{2}$$

The income identity in per capita variables in the economy is given by

$$Y - X = I + C + B \tag{3}$$

with $X = \tau Y, \tau \in (0, 1)$, the (per capita) tax revenue, I investment, C consumption, and B abatement activities. This means that national income after tax is used for investment, consumption, and abatement. As to abatement activities we assume that this variable is expressed as the ratio to total tax revenue X

$$B = \tau_b X = \tau_b \tau Y \tag{4}$$

with $\tau_b \in (0, 1)$ the ratio of abatement spending to the tax revenue.

As to the damage function from $D(T - T_o)$ we assume that it is C^2 and satisfies

$$D(T - T_o)\begin{cases} = 1, & \text{for } T = T_o \\ < 1, & \text{for } T \neq T_o \end{cases} \tag{5}$$

with derivative

$$\frac{\partial D(\cdot)}{\partial T} \equiv D'(\cdot)\begin{cases} > 0, & \text{for } T < T_o \\ < 0, & \text{for } T > T_o \end{cases} \tag{6}$$

[4] In the following we omit the time argument.

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The per capita capital accumulation function is given by[5]

$$\dot{K} = AK^{\alpha}H^{1-\alpha}D(\cdot)(1-\tau) - C - B - (\delta+n)K$$
$$= AK^{\alpha}H^{1-\alpha}D(\cdot)(1-\tau(1+\tau_b) - c(1-\tau)) - (\delta+n)K \quad (7)$$

with $n \in (0,1)$ the growth rate of labour input and $\delta \in (0,1)$ is the depreciation rate of physical capital. Further, we express consumption as ratio to GDP after tax, i.e. $C = cY(1-\tau), c \in (0,1)$. It should be noted that the parameters must be such that $\tau(1+\tau_b) + c(1-\tau) \in (0,1)$ holds.

As mentioned above we assume that gross investment in physical capital is associated with positive externalities, which build up a stock of knowledge capital, which positively affects labour productivity. Knowledge per capita evolves according to

$$\dot{H} = \varphi I - (\eta+n)H = \varphi(AK^{\alpha}H^{1-\alpha}D(\cdot)(1-\tau) - C - B) - (\eta+n)H$$
$$= \varphi(AK^{\alpha}H^{1-\alpha}D(\cdot)(1-\tau(1+\tau_b) - c(1-\tau))) - (\eta+n)H \quad (8)$$

with $\varphi > 0$ a coefficient determining the external effect associated with investment and $\eta \in (0,1)$ depreciation of knowledge.

Next, we describe the interrelation between economic activities and the change in the average global surface temperature. The simplest method of considering the climate system of the earth is in terms of its global energy balance, which is done by so-called energy balance models (EBM). According to an EBM the change in the average surface temperature on earth is described by[6]

$$\frac{dT(t)}{dt}c_h \equiv \dot{T}(t)c_h = S_E - H_E(t) - F_N(t) + \beta_1(1-\xi)\,6.3\ln\frac{M}{M_o}, T(0)$$
$$= T_o \quad (9)$$

with $T(t)$ the average global surface temperature measured in Kelvin[7] (K), c_h the heat capacity[8] of the earth with dimension $J\,m^{-2}K^{-1}$ (Joule per square meter per Kelvin)[9], which is considered a constant parameter, S_E is the solar input, $H_E(t)$ is the non-radiative energy flow, and $F_N(t) = F\uparrow(t) - F\downarrow(t)$ is the difference between the outgoing radiative flux and the incoming radiative flux. S_E, $H_E(t)$, and $F_N(t)$ have the dimension Watt per square meter (Wm^{-2}). $F\uparrow$ follows the Stefan–Boltzmann–Gesetz which is

$$F\uparrow = \epsilon\sigma_T T^4 \quad (10)$$

with ϵ the emissivity which gives the ratio of actual emission to blackbody emission. Blackbodies are objects which emit the maximum amount of

[5] The dot over a variable gives the derivative with respect to time.
[6] This part follows Roedel (2001, chap. 10.2.1 and chap. 1). See also Henderson (1987), and Gassmann (1992). A more complex presentation can be found in Harvey (2000).
[7] 273 Kelvin are 0 degree Celsius.
[8] The heat capacity is the amount of heat that needs to be added per square meter of horizontal area to raise the surface temperature of the reservoir by 1K.
[9] 1 Watt is 1 Joule per second.

radiation and which have $\epsilon = 1$. For the earth ϵ can be set to $\epsilon = 0.95$. σ_T is the Stefan–Boltzmann constant which is given by $\sigma_T = 5.67\,10^{-8} Wm^{-2} K^{-4}$. Further, the ratio $F\uparrow/F\downarrow$ is given by $F\uparrow/F\downarrow = 109/88$. The difference $S_E - H$ can be written as $S_E - H_E = Q(1-\alpha_1)\alpha_2/4$, with $Q = 1367.5 Wm^{-2}$ the solar constant, $\alpha_1 = 0.3$ the planetary albedo, determining how much of the incoming energy is reflected by the atmosphere and $\alpha_2 = 0.3$ that part of the energy which is not absorbed by the surface of the Earth.

The effect of emitting GHGs is to raise the concentration of GHGs in the atmosphere which increases the greenhouse effect of the Earth. This is done by calculating the so-called radiative forcing which is a measure of the influence of GHG, like CO_2 or CH_4, has on changing the balance of incoming and outgoing energy in the Earth-atmosphere system. The dimension of the radiative forcing is Wm^{-2}. For example, for CO_2 the radiative forcing, which we denote as F, is given by

$$F \equiv 6.3 \ln(M/M_o) \tag{11}$$

with M the actual CO_2 concentration, M_o the pre-industrial CO_2 concentration and ln the natural logarithm (see IPCC, 2001, pp. 52–53).[10] For other GHGs other formulas can be given describing their respective radiative forcing and these values can be converted into CO_2 equivalents. β_1 is a feedback factor which captures the fact that a higher CO_2 concentration affects, for example, atmospheric water vapour which has effects for the surface temperature on Earth. β_1 is assumed to take values between 1.1 and 3.4. The parameter ξ, finally, captures the fact that $\xi = 0.3$ of the warmth generated by the greenhouse effect is absorbed by the oceans, which transport the heat from upper layers to the deep sea. In equilibrium, i.e. for $\dot{T} = 0$, (9) gives a surface temperature of about 288.4 Kelvin which is about 15 degree Celsius for the pre-industrial GHG concentration, i.e. for $M = M_o$.

The heat capacity of the Earth, c_h, is largely determined by the oceans since most of the Earth's surface is covered by seawater. Therefore, the heat capacity of the oceans is used as a proxy for that of the earth. c_h is then given by $c_h = \rho_w\, c_w\, d 0.7$, with ρ_w the density of seawater (1027 $m^{-3}kg$), c_w the specific heat of water (4186 $J\, kg^{-1} K^{-1}$), and d the depth of the mixed layer which is set to 70 meters. The constant 0.7 results from the fact that 70 per cent of the Earth is covered with seawater. Inserting the numerical values, assuming a depth of 70 meters, and dividing by the surface of the Earth gives $c_h = 0.1497$.

Setting $\beta_1 = 1.1$ and assuming a doubling of CO_2 implies that in equilibrium the average surface temperature rises from 288.4 to 291.7 Kelvin, implying a rise of about 3.3 degree Celsius. This is in the range of IPCC estimates[11] which yield increases between 1.5 and 4.5 degree Celsius as a consequence of a doubling CO_2 concentration (IPCC, 2001, p. 67).

[10] The CO_2 concentration is given in parts per million (ppm).

[11] IPCC results are obtained with more sophisticated Atmosphere–Ocean General Circulation Models.

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Summarizing this discussion the EBM can be rewritten as

$$\dot{T}(t)\,c_h = \frac{1367.5}{4}\,0.21 - 0.95\,(5.67\,10^{-8})\,(21/109)\,T^4 + 4.851\ln\frac{M}{M_o},\ T(0) = T_o \tag{12}$$

The concentration of GHGs M evolves according to the following differential equation

$$\dot{M} = \beta_2 E - \mu M, M(0) = M_o \tag{13}$$

E denotes emissions and μ is the inverse of the atmospheric lifetime of CO_2. As to the parameter μ we assume a value of $\mu = 0.1$.[12] β_2 captures the fact that a certain part of GHG emissions are taken up by oceans and do not enter the atmosphere. According to IPCC $\beta_2 = 0.49$ for the time period 1990 to 1999 for CO_2 emissions (IPCC, 2001, p. 39).

As concerns emissions of GHGs we assume that these are a by-product of production and expressed in CO_2 equivalents. So, emissions are a function of per capita output relative to per capita abatement activities. This implies that higher production goes along with higher emissions for a given level of abatement spending. This assumption is frequently encountered in environmental economics (see, for example, Smulders, 1995). It should also be mentioned that the emission of GHGs does not affect production directly but only indirectly by affecting the climate of the Earth which leads to a higher surface temperature and to more extreme weather situations. Formally, emissions are described by

$$E = \left(\frac{a\,Y}{B}\right)^{\gamma} \tag{14}$$

with $\gamma > 0$ and $a > 0$ constants. The parameter a can be interpreted as a technology index describing how polluting a given technology is. For large values of a a given production (and abatement) goes along with high emissions implying a relatively polluting technology and vice versa. It should also be mentioned that our definition (14) implies that emissions are a public good, or, more appropriately, a public bad.

The economy is completely described by equations (7), (8), (12), and (13), with emissions given by (14).

2.2. *The balanced growth path*

The balanced growth path (BGP) is defined as follows:[13]

Definition 1
A balanced growth path (BGP) is a path such that $\dot{T} = 0, \dot{M} = 0$ *and* $\dot{K}/K = \dot{H}/H$ *hold, with* $M \geq M_o$.

[12] The range of μ given by IPCC is $\mu \in (0.005, 0.2)$, see IPCC1 (2001, p. 38).
[13] In the following, steady state is used equivalently to balanced growth path.

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This definition contains several aspects. First, we require that the temperature and the GHG concentration must be constant along a BGP. This is a sustainability aspect. Second, the growth rate of per capita capital equals that of per capita knowledge and is constant. It should be noted that this implies that the growth rates of per capita GDP and of per capita consumption are constant, too, and equal to that of capital and knowledge. Third, we only consider balanced growth paths with a GHG concentration which is larger than or equal to the pre-industrial level. This requirement is made for reasons of realism. Since GHG concentration has been rising monotonically over the recent decades it is not necessary to consider a situation with declining GHG concentration. Proposition 1 shows that there exists a unique BGP for this economy.

Proposition 1
For the model economy there exists a unique BGP which is asymptotically stable.

Proof: See appendix.

This proposition shows that any solution starting in the vicinity of the BGP will converge to this path in the long run. The balanced growth rate of the economy is given by $(7)/K$ as

$$g \equiv A(k^\star)^{\alpha-1} D(\cdot)((1-\tau)(1-c) - \tau\tau_b) - (\delta + n), \tag{15}$$

with $k^\star$ the value of k on the BGP, where k is defined as $k \equiv K/H$. However, it cannot be excluded that the BGP goes along with a negative growth rate because the sign of the balanced growth rate depends on the concrete numerical values of the parameters. The question of whether there exists a BGP with a positive growth rate, i.e. a non-degenerate BGP, for a certain parameter constellation is addressed in the next subsection. In this section we make the assumption that a non-degenerate BGP exists, and analytically study growth effects of varying the tax rate and abatement spending.

In a next step we analyze how the balanced growth rate reacts to changes in the income tax rate τ and to different values of the ratio τ_b. To do so we differentiate g with respect to τ. This gives

$$\begin{aligned}\frac{\partial g}{\partial \tau} = {} & AD(\cdot)(k^\star)^{\alpha-1}(-1)(1-c+\tau_b) \\ & + (\alpha-1)(k^\star)^{\alpha-2}\frac{\partial k}{\partial \tau} AD(\cdot)((1-\tau)(1-c)-\tau\tau_b) \\ & + D'(\cdot)\frac{\partial T}{\partial \tau}(k^\star)^{\alpha-1} A((1-\tau)(1-c)-\tau\tau_b) >< 0 \end{aligned} \tag{16}$$

with * denoting values on the BGP. From (12) and (13) it is easily seen that $\partial k^\star/\partial \tau < 0$ and $\partial T^\star/\partial \tau < 0$, for $T > T_o$. To see this one uses that $T^\star$ positively depends on $M^\star$, which, for its part, negatively depends on τ. The latter is seen by calculating $M^\star$ from $\dot{M} = 0$ and using (14). $\partial k^\star/\partial \tau < 0$ is obtained from implicitly differentiating $\dot{k}/k = \dot{K}/K - \dot{H}/H$.

The second inequality in (16) results from the fact that in our model an increase in the tax revenue raises abatement activities since we assume a fixed ratio of abatement activities to tax revenue. The first inequality states that a rise in the tax rate reduces the ratio of physical to human capital. This shows that an increase in the tax rate has both positive and negative partial growth effects. On the one hand, a higher tax rate reduces investment because more resources are spent for abatement. On the other hand, a higher tax rate reduces the increase in average global surface temperature and, as a consequence, the damage resulting from T_o. This raises aggregate production which has a positive growth effect. Further, it should be mentioned that the decrease in $K/H = k$ has also a positive growth effect since a lower ratio of physical to human capital goes along with higher growth. This means that economies with small physical capital stocks and high stocks of knowledge are likely to show large growth rates. This, for example, was the case for Germany and Japan after the Second World War. So, the analytical model does not allow to answer the question of whether a higher tax rate reduces or increases the long-run balanced growth rate.

Next, we analyze the effects of an increase in abatement activities implying a higher value of τ_b. The derivative of g with respect to τ_b is given by

$$\begin{aligned}\frac{\partial g}{\partial \tau_b} &= AD(\cdot)(k^\star)^{\alpha-1}(-\tau) \\ &\quad + AD(\cdot)(\alpha-1)(k^\star)^{\alpha-2}\frac{\partial k}{\partial \tau_b}((1-\tau)(1-c)-\tau\tau_b) \\ &\quad + AD'(\cdot)\frac{\partial T}{\partial T}(k^\star)^{\alpha-1}((1-\tau)(1-c)-\tau\tau_b) >< 0 \qquad (17)\end{aligned}$$

As for the tax rate we see that higher abatement activities may raise or lower economic growth. The reason is as above. On the one hand, more abatement activities reduce investment spending. On the other hand, higher abatement activities have positive indirect growth effects by reducing the temperature increase, and thus the damage, and by reducing the value $k = K/H$.

To get further insights, we undertake simulations in the next subsection.

2.3. Numerical examples

We consider one time period to comprise one year. The population growth rate is assumed to be $n = 0.02$ and the depreciation rate of capital is $\delta = 0.075$. The pre-industrial level of GHGs is normalized to one, i.e. $M_o = 1$, and we set $\gamma = 0.9$. This is motivated by an OECD study which runs regressions with emissions per capita as the dependent variable which is explained among others by GDP per capita and which obtains a value of about 0.9 (see OECD, 1995). β_1 and ξ are set to $\beta_1 = 1.1$ and $\xi = 0.3$ (see section 2). $c = 0.8$ and the tax share is set to $\tau = 0.2$ which is about equal to the tax share in Germany in 1996 (see Sachverständigenrat, 2001). The capital share is $\alpha = 0.35$ and B is set to $B = 2.9$. As to τ_b we consider the values $\tau_b = 0.0075, 0.01, 0.0125$. For example, in Germany the ratio of abatement spending to prevent air pollution to total tax revenue was 0.01 in 1996 (see Sachverständigenrat, 2001, table 30 and table 65). a is set to $a = 0.00075$. This implies that GHGs double for $\tau_b = 0.01$.

Table 1. *Varying the abatement share between 0.0075 and 0.0125 with* $a_2 = 0.05$, $\phi = 0.05$

τ_b	$T^\star$	$M^\star$	g	g_Y
0.0075	293.0	2.63	0.0185	0.0191
0.01	291.8	2.03	0.0198	0.0203
0.0125	290.8	1.66	0.0207	0.0211

An important role is played by the damage functions $D(\cdot)$. This will be introduced now. As to $D(\cdot)$ we assume the function

$$D(\cdot) = (a_1(T - T_o)^2 + 1)^{-\phi} \tag{18}$$

with $a_1 > 0, \phi > 0$. As to the numerical values of the parameters in (18) we assume $a_1 = 0.05$ and $\phi = 0.05$ and $a_1 = 0.025$ and $\phi = 0.025$. ($a_1 = 0.05, \phi = 0.05$) implies that an increase of the surface temperature by 1 (2, 3) degree(s) leads to a decrease of aggregate production by 0.2 (0.9, 1.8) per cent. The combination ($a_1 = 0.025, \phi = 0.025$) implies that an increase of the surface temperature by 1 (2, 3) degree(s) leads to a decrease of aggregate production by 0.06 (0.2, 0.5) per cent. Comparing these values with the estimates published in (IPCC, 1996), mentioned in the Introduction, we see that the values we choose yield a damage which is a bit lower than the one reported by (IPCC, 1996).

In the following tables we report the results of our numerical studies where we report the temperature on the BGP and the GHG concentration on the BGP. Further, we report the balanced growth rate denoted by g. g_Y is the average growth rate of per capita GDP on the transition path for the next 100 years, where initial conditions are set to $T(0) = 289$, $M(0) = 1.13$ and $k(0) = 8.1$. The growth rate of output is given by

$$\frac{\dot{Y}}{Y} = \alpha \frac{\dot{K}}{K} + (1 - \alpha)\frac{\dot{H}}{H} + \frac{D'(\cdot)}{D(\cdot)}\dot{T}.$$

In table 1 we vary the abatement share between $\tau_b = 0.0075$ and $\tau_b = 0.0125$.

This table shows that an increase in the ratio of abatement spending to tax revenue, and also to GDP, leads to both higher growth rates and to a smaller increase in GHG emissions and, as a consequence, to a smaller increase in average global surface temperature. This holds both for the long-run balanced growth rate as well as for the transition path for the next 100 years. In this case, the decline in investment caused by more abatement spending is compensated for by the higher production resulting from smaller damage since the temperature increase is smaller with higher abatement. The maximum growth rate is obtained then there is no increase in the average temperature, implying that the damage is zero. This is achieved for τ_b about $\tau_b = 0.02$. This outcome, of course, depends on the specification of the damage function. This is shown in the next table, where we set $a_2 = 0.025$ and $\phi = 0.025$.

Table 2 shows that the growth rate first rises when abatement spending is increased but then declines when abatement is further increased, although

Table 2. *Varying the abatement share between 0.0075 and 0.0125 with $a_2 = 0.025$, $\phi = 0.025$*

τ_b	$T^\star$	M^{star}	g	g_Y
0.0075	293.0	2.63	0.0218	0.0222
0.01	291.8	2.03	0.022	0.0223
0.0125	290.8	1.66	0.0219	0.0222

Table 3. *Varying the tax share between 0.15 and 0.25 with $a_2 = 0.05$, $\phi = 0.05$*

τ	$T^\star$	$M^\star$	g	g_Y
0.15	293.0	2.63	0.0266	0.0274
0.2	291.8	2.03	0.0198	0.0203
0.25	290.8	1.66	0.0124	0.0127

the growth effects are very small. This is due to the smaller damage caused by the increase in average global surface temperature.

Next, we consider the case of varying the tax share between 15 and 25 per cent. The results for ($a_2 = 0.05, \phi = 0.05$) are shown in table 3.

Table 3 shows that raising the tax rate reduces GHG emissions but also the balanced growth rate and the GDP growth rate on the transition path. It is true that a higher tax revenue raises economic growth and abatement spending since the latter are always in fixed proportion to the tax revenue. However, the negative direct growth effect of a higher tax share clearly dominates the positive growth effect of smaller damage due to less GHG emissions and a smaller increase in temperature.

We also studied the effects of varying abatement activities for different values of γ. To do so we set $\gamma = 0.5$ and $\gamma = 1.5$.[14] From a qualitative point of view the results are the same as for $\gamma = 0.9$. The only difference is that the quantitative growth effects are a bit different. We do not report the outcome of these studies here but they are available on request.

In the next section we present a special form of the growth model by assuming that physical capital and the stock of knowledge can be summarized in one variable. This gives the so-called *AK* model of endogenous growth.

3. The *AK* endogenous growth model

Assuming that physical capital and human capital evolve at the same rate, i.e. $\dot{K} = \dot{H}$ and $K(0) = H(0)$ hold, allows to rewrite the aggregate per capita production function as follows

$$Y = AK\,D(\cdot) \tag{19}$$

which is linear in capital. In the economics literature this simplifying assumption is frequently made and in the following we study this model.

[14] To get temperature increases which are compatible with IPCC estimates we set $a = 0.00035$ and $a = 0.0011$ respectively.

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Table 4. *Varying the abatement share between 0.0075 and 0.02 with $\tau = 0.2$*

τ_b	$T^\star$	$M^\star$	g
0.0075	293.0	2.63	0.0197
0.01	291.8	2.03	0.0208
0.0125	290.8	1.66	0.0216
0.018	289.3	1.19	0.022
0.02	288.8	1.08	0.0219

First, we consider the descriptive growth model and then we analyze the second-best solution where abatement activities are chosen optimally.

3.1. The descriptive growth model

The differential equation describing the evolution of capital is now given by

$$\dot{K} = AK\,D(\cdot)(1 - \tau(1 + \tau_b) - c(1 - \tau)) - (\delta + n)K \tag{20}$$

where we assume again $B = \tau_b X$ and $C = cY(1 - \tau)$ as in section 2. The economy then is completely described by (20), (12), and (13), with emissions given by (14) and the balanced growth path is

$$g = AD(\cdot)(1 - \tau(1 + \tau_b) - c(1 - \tau)) - (\delta + n) \tag{21}$$

The balanced growth rate is independent from the capital stock but only depends on the average global surface temperature T. This implies that a BGP is as in defined in section 2 with the only difference that we only have to consider the equations $\dot{M}$ and $\dot{T}$.[15]

From (21) it is immediately seen that variations in τ_b and in τ affect the balanced growth rate both directly as well as indirectly by affecting the temperature on the balanced growth path. As in section 2, there is a positive indirect growth effect and a negative direct growth effect going along with changes in τ_b and in τ the overall effect, however, cannot be determined for the general model. Therefore, we will present the results of our numerical examples without presenting the results for the analytical model.

The parameters are as in section 2, i.e. $n = 0.02, \delta = 0.075, \tau = 0.2, M_o = 1, c = 0.8, \gamma = 0.9, \beta_1 = 1.1, \xi = 0.3, a = 0.00075$, and $\alpha = 0.35$. The only different parameter is the value of A, which we set to $A = 0.75$ in order to get plausible growth rates. The tax share is set to $\tau = 0.2$ and we consider for τ_b the values $\tau_b = 0.0075, 0.01, 0.0125$. a_2 and ϕ are again set to $a_2 = 0.05$ and $\phi = 0.05$.

Table 4 and table 5 present the results of varying abatement spending and of variations of the tax share with $\tau = 0.2$ and $\tau_b = 0.01$ respectively.[16]

Table 4 and table 5 largely confirm the results of section 2. That is, a rise in the tax share reduces the balanced growth rate and also GHG emissions. The growth rates in the AK model differ a bit from those of section 2 but

[15] Of course, the AK model is also asymptotically stable.

[16] In this section we only consider the balanced growth path.

Table 5. *Varying the tax share 0.15 and 0.25 with $\tau_b = 0.01$*

τ	$T^\star$	$M^\star$	g
0.15	293.0	2.63	0.0269
0.2	291.8	2.03	0.0208
0.25	290.8	1.66	0.0142

the change is about the same. However, in contrast to section 2, the growth rate is maximized for τ_b such that the temperature is larger than the pre-industrial temperature T_o. Next, we consider the second-best solution.

3.2. The second-best solution

To derive the second-best solution we assume that the government takes private consumption and the tax share as given and sets abatement such that welfare is maximized. As to welfare we assume as usual that it is given by the discounted stream of per capita utility times the number of individuals over an infinite time horizon. As concerns utility we assume a logarithmic function. More concretely, the government solves the following optimization problem

$$\max_{\tau_b} \int_0^\infty e^{-(\rho-n)t} L(0) \ln(c(1-\tau)AK\,D(\cdot))\,dt \tag{22}$$

subject to (20), (13), (12) with $c(1-\tau)AK\,D(\cdot) = C$ per capita consumption. ln denotes the natural logarithm and ρ is the discount rate. In the following we normalize $L(0) \equiv 1$.

To find necessary optimality conditions we formulate the current-value Hamiltonian as

$$\begin{aligned} \bar{H}(\cdot) = {} & \ln(c(1-\tau)AK\,D(\cdot)) + \lambda_1(AK\,D(\cdot)(1-\tau(1+\tau_b)) - c(1-\tau)) \\ & - (\delta+n)K) + \lambda_2\left(\beta_2\left(\frac{a}{\tau_b\tau}\right)^\gamma - \mu M\right) \\ & + \lambda_3(c_h)^{-1}\left(\frac{1367.5}{4}0.21 - (5.67\,10^{-8})(19.95/109)T^4\right. \\ & \left. + \beta_1(1-\xi)\,6.3\ln\frac{M}{M_o}\right) \end{aligned} \tag{23}$$

with $\lambda_i, i = 1,2,3$ the shadow prices of K, M, and T respectively and $E = a^\gamma Y^\gamma A^{-\gamma}$ emissions. Note that λ_1 is positive, while λ_2 and λ_3 are negative.

The necessary optimality conditions are obtained as

$$\frac{\partial\bar{H}(\cdot)}{\partial\tau_b} = \lambda_1 AK\,D(\cdot)(-\tau) - \lambda_2\beta_2(a/\tau)^\tau(-\gamma)\tau_b^{-\gamma-1} = 0, \tag{24}$$

$$\dot{\lambda}_1 = (\rho+\delta)\lambda_1 - K^{-1} - \lambda_1 AD(\cdot)(1-\tau(1+\tau_b) - c(1-\tau)), \tag{25}$$

$$\dot{\lambda}_2 = (\rho-n)\lambda_2 + \lambda_2\mu - \lambda_3(1-\xi)\beta_1\,6.3\,c_h^{-1}M^{-1}, \tag{26}$$

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$$\dot{\lambda}_3 = (\rho - n)\lambda_3 - \frac{D'(\cdot)}{D} - \lambda_1 AK\, D'(\cdot)(1 - \tau(1+\tau_b) - c(1-\tau)) \\ + \frac{\lambda_3 (5.67\, 10^{-8}(19.95/109)4T^3)}{c_h} \tag{27}$$

Further, the limiting transversality condition $\lim_{t\to\infty} e^{-(\rho+n)t}(\lambda_1 K + \lambda_2 T + \lambda_3 M) = 0$ must hold.

From (24) we get the second-best optimal abatement activities (as ratio to the tax revenue) as

$$\tau_b^o = \left(\frac{-\lambda_2 \beta_2 \gamma (a/\tau)^\gamma}{AD(\cdot)K\lambda_1 \tau} \right)^{1/(1+\gamma)} \tag{28}$$

(28) shows that τ_b^o is higher the more polluting the technology in use, which is modelled in our framework by the coefficient a. This means that economies with less clean production technologies have a higher optimal abatement share than economies with a cleaner technology. However, this does not mean that economies with a cleaner technology have higher emissions. This holds because, on the one hand, the higher abatement share may not be high enough to compensate for the more polluting technology. On the other hand, the second-best pollution tax rate also depends on λ_1, λ_2, and K. Further, from the expression for τ_b^o one realizes that the higher the absolute value of the shadow price of GHG concentrations, $|\lambda_2|$, the higher the abatement share has to be set.

For the second-best solution a balanced growth path is defined similar to section 2.

Definition 2
For the second-best solution a balanced growth path is a path such that $\dot{T} = \dot{M} = \dot{\lambda}_2 = \dot{\lambda}_3 = 0$ *and* $\dot{K}/K = -\dot{\lambda}_1/\lambda_1$ *hold, with* $M \geq M_o$.

Unlike for the descriptive versions of our growth models we cannot give results as to the existence and stability of a balanced growth model for the analytical model. Therefore, and in order to get an impression about the quantitative results of the second-best solution we again make simulations with the same parameters of section 3.1, with $\tau = 0.2$ and a discount rate of 5 per cent, i.e. $\rho = 0.05$. For the numerical values of the parameters it can be shown that there exists a unique BGP which, however, is not asymptotically stable but a saddle point. This is the contents of proposition 2.

Proposition 2
For the second-best model economy there exists a unique BGP which is a saddle point for the numerical parameter values of section 3.1.

Proof: See appendix.

Table 6 gives the balanced growth rate and the values of T, M, B/Y, and τ_b^o on the BGP.

Table 6 shows that for the AK growth model there exists an optimal share for abatement spending which maximizes utility and also the balanced growth rate. This share is about 1.7 per cent of the tax revenue for

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Table 6. *The second-best solution*

a	τ_b^o	$B^\star/Y^\star$	$T^\star$	$M^\star$	g
$7.5 \cdot 10^{-4}$	0.017	0.0034	289.5	1.26	0.0221
$5 \cdot 10^{-4}$	0.012	0.0024	289.2	1.17	0.0229

$a = 7.5 \cdot 10^{-4}$, implying a share of abatement spending per GDP, $B^\star/Y^\star$, of 0.34 per cent. Further, it is seen that in a world with a more polluting technology (higher a) the second-best pollution tax rate is larger but, nevertheless, emissions and the increase in temperature are also higher. So, economies with a more polluting production technology should have a higher pollution tax rate. But, in spite of this, the emissions in the economy with the more polluting technology are higher because the higher abatement share is not enough to compensate for the more polluting technology.

Thus, our model is in part consistent with the literature which postulates that an environmental Kuznets curve exists, where emissions first rise with an increase in GDP (when the technology in use is relatively polluting), but decline again when a certain level of GDP is reached and the technology becomes cleaner (see, for example, the contribution by Stokey, 1998).[17] But it should be kept in mind that our result is obtained for second-best government policies and it may be doubted that in reality governments pursue optimal policies.

It should also be mentioned that the choice of the discount rate ρ affects the optimal τ_b^o and also the balanced growth rate. However, neither the growth rate nor optimal abatement react sensitively to changes in ρ. For example, setting $\rho = 0.02$ or $\rho = 0.1$ basically leaves unchanged both optimal abatement spending and the growth rate.

In the next section we study the problem of the social planner. The difference to the optimization problem faced by the government in this section is that the social planner decides on both the consumption share and on the abatement share instead of taking consumption as given.

3.3. *The social optimum*

As mentioned at the end of the last section the social planner can decide both on the investment share and on the abatement share. The optimization problem, then, is given by

$$\max_{c_s, b} \int_0^\infty e^{-(\rho-n)t} L(0) \ln(c_s AK D(\cdot))\, dt \tag{29}$$

with c_s the consumption share and b the abatement share in the social optimum respectively. Again, ln denotes the natural logarithm, ρ is the discount rate, and we normalize $L(0) \equiv 1$. The constraints are (12) and (13), where $b \equiv B/Y$, and the differential equation $\dot{K}$. The latter is now given by $\dot{K} = AK D(\cdot)(1 - c_s - b) - (\delta + n)K$. Again, we formulate the current-value

[17] In the model by Stokey the pollution intensity is a choice variable while it is a parameter in our model.

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Hamiltonian which is

$$\bar{H}(\cdot) = \ln(c_s AK\, D(\cdot)) + \lambda_4 \dot{K} + \lambda_5 \dot{M} + \lambda_6 \dot{T} \quad (30)$$

with $\lambda_i, i = 4, 5, 6$, the shadow prices of K, M, and T in the social optimum. As in the previous section, λ_4 is positive and λ_5 and λ_6 are negative.

The necessary optimality conditions now are

$$\frac{\partial \bar{H}(\cdot)}{\partial c_s} = c_s^{-1} - \lambda_4 AK\, D(\cdot) = 0 \quad (31)$$

$$\frac{\partial \bar{H}(\cdot)}{\partial b} = -\lambda_4 AK\, D(\cdot) - \lambda_5 \beta_2 \gamma b^{-\gamma-1} a^{\gamma} = 0 \quad (32)$$

$$\dot{\lambda}_4 = (\rho + \delta)\,\lambda_4 - K^{-1} - \lambda_4\, AD(\cdot)(1 - c_s - b), \quad (33)$$

$$\dot{\lambda}_5 = (\rho - n + \mu)\,\lambda_5 - \lambda_6 (1 - \xi)\beta_1\, 6.3\; c_h^{-1} M^{-1}, \quad (34)$$

$$\dot{\lambda}_6 = (\rho - n)\,\lambda_6 - \frac{D'(\cdot)}{D} - \lambda_4 AK\, D'(\cdot)(1 - c_s - b) + \frac{\lambda_6 (5.67\, 10^{-8} (19.95/109)\, 4T^3)}{c_h} \quad (35)$$

The transversality condition is as in the last section.

From (31) and (32) we get the first-best consumption share and abatement share (relative to GDP respectively) as

$$c_s = (\lambda_4 AK\, D(\cdot))^{-1} \quad (36)$$

$$b = \left(\frac{-\lambda_5 \beta_2 \gamma a^{\gamma}}{AD(\cdot) K \lambda_4} \right)^{1/(1+\gamma)} \quad (37)$$

One immediately realizes that (37) is similar to (28) and the interpretation is basically the same as in section 3.2. (36) shows that the first-best optimal consumption share negatively depends on both physical capital and on its shadow price. That means the higher the stock of capital and the higher its 'price' the smaller is the consumption share. Of course, as for the second-best solution, consumption grows while the consumption share is constant on the BGP. Further, the higher the damage caused by the temperature increase the smaller the consumption share in the economy. This outcome is due to the fact that the temperature increase negatively affects aggregate production, Y, and, thus, consumption which is equal to $c_s \cdot Y$.

As to the question of whether there exists a BGP for the social optimum we again resort to simulations. Here, we can state proposition 3.

Proposition 3

For the social optimum there exists a unique BGP, as defined in definition 2, which is a saddle point for the numerical parameter values of section 3.1.

Proof: See appendix.

In the following we direct our attention to the abatement share and to the temperature increase in the social optimum compared with the second-best economy. Table 7 gives the values of T, M, and B/Y on the BGP for

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Table 7. *The social optimum*

a	$B^\star/Y^\star$	$T^\star$	$M^\star$
$7.5 \cdot 10^{-4}$	0.0041	288.65	1.05
$5 \cdot 10^{-4}$	0.0028	288.57	1.04

$a = 7.5 \cdot 10^{-4}$ and $a = 5 \cdot 10^{-4}$ respectively. The other parameters are as in section 3.2.

Table 7 demonstrates that, in the social optimum, economies with a less polluting technology (smaller a) have a smaller abatement share than economies with a more polluting technology. But, nevertheless, the economies with the cleaner technology have a smaller level of GHG emissions and, consequently, a smaller increase in temperature. This result is equivalent to the one obtained for the second-best solution. That means that the higher abatement share cannot compensate for the less cleaner production technology. One also realizes that the abatement share in the social optimum is higher compared with the second-best solution and, as a consequence, the temperature increase is smaller.

4. Conclusions

In this paper we have studied the interrelation between anthropogenic global warming and economic growth assuming a simple descriptive model of endogenous growth. Using simulations, we have seen that increases in abatement spending may yield a win–win situation. That means a rise in abatement activities both reduces GHG emissions and raises economic growth. This holds for both the balanced growth rate and for the growth rate of GDP on the transition path. Further, we have seen that a situation may exist where maximum growth is obtained if the average global surface temperature is reduced to its pre-industrial level. This outcome, however, depends on the growth model employed. So, in the *AK* model maximum growth was obtained for an average global surface temperature which is higher than the pre-industrial level.

Assuming a logarithmic utility function we computed the second-best value for the share of abatement spending and we have seen that economies with a cleaner production technology have a smaller temperature increase compared with economies with more polluting technologies, although the latter should spend a higher share for abatement. The same outcome has been obtained for the social optimum. Further, the abatement share in the social optimum is higher than in the second-best solution.

Of, course the result that a win–win situation may be observed crucially depends on the damage caused by the temperature increase. But, the damage function we used was well in line with the damage reported by IPCC studies so that our outcome cannot be dismissed as purely academic. Nevertheless, additional research is necessary to get further insight into the impact of global warming on economic growth. So, more elaborate economic models should be constructed using climate models in order to achieve additional results.

Further, our paper assumed that the change in temperature and the damage caused is the same for all regions in the world. However, in reality different regions will be affected differently. This could be taken into account by resorting to partial differential equations where the variables are functions of time and of the relative position on Earth.

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Appendix

Proof of proposition 1

First, we define $k = K/H$ giving $\dot{k}/k = \dot{K}/K - \dot{H}/H$.[18] To show uniqueness of the steady state we solve $\dot{k}/k = 0$, (12) = 0 and (13) = 0 with respect to k,

[18] Since k is raised to a negative power in (7)/K, $k = 0$ is not feasible and we therefore consider the equation $\dot{k}/k$ in the rate of growth.

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T, and M. Setting (12) = 0 gives

$$T_{1,2} = \pm 99.0775\,(71.7935 + 4.851\ \ln(M/M_o))^{1/4}$$
$$T_{3,4} = \pm 99.0775\sqrt{-1}\,(71.7935 + 4.851\ln(M/M_o))^{1/4}$$

Clearly $T_{3,4}$ are not feasible. Further, since $M \geq M_o$ only the positive solution of $T_{1,2}$ is feasible. Uniqueness of M is immediately seen. The equation $\dot{k}/k$ is given by

$$f(k,\cdot) \equiv \frac{\dot{k}}{k} = Ak^{\alpha-1}D(\cdot)(1-\varphi k)((1-\tau)(1-c)-\tau\tau_b)-(\delta-\eta)$$

with $\partial f(k,\cdot)/\partial k < 0$ and $\lim_{k\to 0} f(k,\cdot) = +\infty$ and $\lim_{k\to\infty} f(k,\cdot) = -\infty$ for a given T. This shows that there exists a unique k which solves $\dot{k}/k = 0$.

To study the local dynamics we calculate the Jacobian matrix J corresponding to this dynamic system which is obtained as

$$J = \begin{pmatrix} \partial\dot{k}/\partial k & D'(\cdot)A(k^\star)^\alpha(1-\varphi k^\star)((1-\tau)(1-c)-\tau\tau_b) & 0 \\ 0 & -(79.8(5.67\,10^{-8})(T^\star)^3)/(109c_h) & 4.851/(c_h M^\star) \\ 0 & 0 & -\mu \end{pmatrix}$$

with * denoting steady state values and the parameter values as in section 2. The eigenvalues of J are given by

$$e_1 = -\mu, \quad e_2 = -(79.8(5.67\,10^{-8})(T^\star)^3)/(109c_h) \quad \text{and} \quad e_3 = \partial\dot{k}/\partial k$$

Since $\partial f(k,\cdot)/\partial k < 0, \partial\dot{k}/\partial k < 0$ also holds. Thus, proposition 1 is proved. □

Proofs of propositions 2 and 3

To prove proposition 2 we define $\Lambda \equiv K \cdot \lambda_1$ giving $\dot{\Lambda}/\Lambda = \dot{K}/K + \dot{\lambda}_1/\lambda_1$. Setting $\dot{\Lambda}/\Lambda = 0$ gives $\Lambda^\star = (\rho - n)^{-1}$. Inserting $\Lambda^\star$ in (28) and the resulting expression as well as $\Lambda^\star$ in equations (13), (12), (26), and (27) gives an autonomous system of differential equations which depends on M, T, λ_2, and λ_3. A rest point of this system yields a BGP. With the numerical parameters the solution is given by $M^\star = 1.25625, T^\star = 289.50603$, $\lambda_2^\star = -0.75023$, and $\lambda_3^\star = -0.00378$.

The eigenvalues of the Jacobian are given by

$$e_1 = 6.75544, \quad e_2 = -6.75544 \quad e_3 = 0.19010 \quad e_4 = -0.19010$$

Thus, proposition 2 is proved. For $a = 5 \cdot 10^{-4}$ the numerical values are different, but the qualitative result remains unchanged.

The proof of proposition 3 proceeds in complete analogy to that of proposition 2. Therefore, we do not mention it in detail. Again, the value of a affects the numerical values but leaves unchanged the qualitative outcome.