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Undesirable growth in a model with capital accumulation and environmental assets

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ABSTRACT. The expansion of private production erodes the quality of commonly owned assets, thereby forcing individuals to rely increasingly on private goods to satisfy their needs. In the face of this deterioration, households increase their work effort and accumulate more capital in order to buy more consumer goods both in the present and in the future. By so doing, each household contributes to an increase in production and thus has a detrimental—though negligible—impact on commonly owned assets. Hence, the economy converges to a long-run equilibrium level of production that is higher than the level associated with the Pareto-efficient path.

Key words: Common property, Defensive expenditure, Market failure, Undesirable growth, Environmental assets.

1. Introduction

In this paper, we present a neoclassical growth model in which both the time devoted to market activities and the accumulation of productive assets (physical and/or human capital, knowledge...) are increased by the decumulation (depletion) of environmental assets. In our model, indeed, there is a renewable resource that enters positively individual welfare but it is subject to negative externalities: even if each productive activity causes only negligible damage to the quality of the resource, the aggregate effect of these activities on environment quality is not negligible. Faced with the deterioration of this environmental asset, households are increasingly forced to rely on substitutes that are produced and sold on the market. The increase in production and consumption that follows generates further deterioration in the environmental asset, thus feeding the growth process.

The implications of the fact that economic growth causes negative externalities via its impact on natural resources are widely studied by

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environmental economists. In contrast, there has been no attempt to model the idea that the deterioration of these resources may be a force driving the growth process by inducing defensive behaviour which enhances market production.[1] Indeed, households react to the deterioration of the environmental resource by raising labor supply and accumulation, so as to be able to consume (both in the present and in the future) an increased amount of the market good that can substitute for the resource. In its turn, this brings about a further deterioration of the environmental resource. As a result, the economy converges to a long-run equilibrium path characterized by an inefficiently high level of market production and by an inefficiently low stock of environmental resource.

According to the explanation proposed by the endogenous growth theory, in a market system individuals unintentionally generate increasing returns through positive externalities. This gives rise to a self-reinforcing mechanism whereby growth causes externalities and externalities causes growth. We offer a complementary explanation, this too based on the unintended effects of individual actions, but with the difference that in our case the externalities under consideration are negative. Moreover, our growth model is like that of Solow in the sense that the growth process is the convergence to a steady state where per-capita output remains constant over time,[2] but it differs from the Solow paradigm because a commonly owned resource enters the households' utility function. Finally, the mechanism that we model here is open to a sociological interpretation of the process that we describe. According to this interpretation, the expansion of market activities undermines the institutional and immaterial bases of communitarian organizations of life on which individual welfare depends. In its turn, the weakening of these institutions induces the individuals to rely more extensively on the market in order to satisfy their needs, thus enlarging the sphere of market relations.

It is assumed here that what matters for the welfare of agents is not only the goods they are able to purchase but also goods that they do not purchase, and whose endowment is negatively affected by the expansion of the marketplace correlated to growth. As a consequence, in an economy in which those who maintain their purchasing power unchanged experience a lower standard of life because of the deterioration of commonly owned resources, there will be strong individual incentives to increase their income to buy more goods. Hence, the core of this mechanism is that growth works as a substitution process based on the destruction of non-

[1] For an early treatment of the idea of growth as a substitution process between free and costly goods, see Bartolini (1993). The first model (an evolutionary game) including this mechanism appeared in Antoci and Bartolini (1997), which obtained results that are analogous to those contained in this paper. The fact that the same results are obtained either with boundedly rational or optimizing agents supports the logical robustness of the idea that growth can be fed by negative externalities.

[2] Bartolini and Bonatti (1999) show how negative externalities may generate endogenous growth (perpetual growth) by causing a progressive depletion of commonly owned asset.

market goods. Growth is described as a process of market expansion in the sense that along a path converging to the steady state, welfare increasingly depends on what is transacted on the market. This signifies that free resources are increasingly substituted in households' consumption bundles by costly goods.[3] In other words, growth is driven by its own destructive power.

The paper is organized as it follows. Section 1 discusses its motivations, namely to provide an analytical framework for a huge body of literature and knowledge which extends well beyond the bounds of economics. Section 2 presents the model, section 3 derives the equilibrium path of the economy under *laissez faire*, section 4 compares the *laissez-faire* equilibrium path to the Pareto-optimal path of the economy, and section 5 summarizes the results of the paper.

2. Motivations

Growth as a substitution process

As growth proceeds, agents increasingly derive welfare from private rather than common consumption. This conclusion may strike sociologists as familiar, since they often associate growth with the 'creation of new needs' and with a 'change in patterns of consumption'. These expressions tend to be interpreted in terms of an endogenous change in preferences. In our model, the creation of new needs and change in consumption patterns constitute the engine of growth, but in a context of invariant preferences. This is because new needs are viewed as increases in demand for substitute goods generated by a diminution in free consumption, while changes in patterns of consumption concerns the passage from common (free) goods to private (costly) ones. Consequently, the traditional view that increasing quantities of goods become available as growth proceeds may be incomplete. The image conveyed is one of luxury goods which become standard goods for the next generation, and absolute needs for the one that follows thereafter. Our model suggests that this is only partially true, since free goods are also involved whose endowment and quality are progressively reduced. The point is an obvious one in an environmental interpretation of the concept of 'free resource': meadows, woods, clean beaches, unpolluted air and water, silence, and so on, are all examples of free goods which may deteriorate or become scarce. It is often the case in advanced economies that, in order to enjoy what could be obtained for free 30 or 40 years ago, agents must now purchase a house in an exclusive area in the countryside or at the seaside, or buy an expensive holiday in some tropical paradise, etc. However, a sociological interpretation of free goods is also possible, given that many of them relate to social relations and seemingly grow scarcer with growth. With this broader interpretation in mind, the concept of 'substitute' may help to explain changes in lifestyles, as well as in patterns of consumption.

According to Hirsch (1976), growth in advanced economies is largely

[3] Henceforth the term 'free consumption' will be used synonymously with environmental asset (or resource), while 'costly consumption' will be used synonymously with substitute consumption.

due to an increase in defensive consumption: that is, consumption induced by the negative externalities produced by growth, which is similar to the concept used here of substitute consumption. After Hirsch, the notion of defensive consumption was taken up by the debate on corrections to GNP in order to improve it as an index of welfare. The literature on defensive consumption contains a large number of interesting examples, but the idea that seems to inspire all authors, and Hirsch in particular, is that reactions to a situation of general decay may be very general.[4] Individuals may compensate for the deterioration in everything that is public with a concern for everything that is private, giving rise to the contrast typical of 'affluent societies' (Galbraith's well-known observation).

On growth and working time in the long run

By endogenizing the decision on the time spent working, our model makes it possible to deal with a crucial issue in long-term growth: the allocation of productivity gains between leisure and consumption. Growth models in which the labor supply is exogenous assume precisely what they should explain: that increases in productivity are utilized mainly to augment output, and only marginally to reduce working time. If the reverse occurs, growth (in the sense of increased per-capita output) may not take place. As a matter of fact, the tendency for labor supply to decrease in response to productivity advances is weak, and it displays very important exceptions.[5] Obviously, satisfactory assessment of the extent to which total working time reacts to productivity improvements also requires analysis of how productivity changes affect home work. This is especially true in the light of the historical trend toward increased female labor-market participation, distinctive of the advanced countries during the twentieth century. However, the fact that the production of certain services is no longer confined to the family is part of the general and progressive weakening of communitarian modes of life that has accompanied modern economic growth. But, in any case, the perception widespread in contemporary societies is that people suffer from a shortage of time in the midst of affluence.[6]

Growth models which include the labor/leisure choice as a control variable (see, for example, Barro and Sala-i-Martin, 1995) explain the low long-term elasticity of per-capita working time with respect to pro-

[4] For a growth model including defensive expenditure, see Beltratti (1996). In this model, however, the presence of defensive expenditures cannot generate a self-propelled process of growth, since (i) a rise in defensive expenditures does not increase the level of economic activity by inducing individuals to work harder, and (ii) the flow of use of the environmental asset is fixed (it is not affected by the level of production).

[5] In the USA, labor input per head of population (hours) was 710 and labor productivity (GDP per hour worked) was 8.64 in 1938. Analogous figures were 756 and 12.66 in 1950, and 741 and 29.10 in 1992 (see Maddison, 1995). According to Schor (1993), in 1987 Americans worked for around one month per year more than they did in 1969 (+163 hours).

[6] Among the attempts to explain this 'famine of time', see Linder (1970), Hirschmann (1973), Cross (1993).

ductivity advances by assuming that in the long run the income effect only weakly predominates over the substitution effect induced by the increased remuneration of labor. That is to say, standard explanations rely on the peculiarities of individual preferences. The symptoms of widespread discontent with the excessive role played by work in people's lives should prompt a search for complementary explanations as to why productivity gains are not massively transformed into leisure increases.

The explanation proposed in this paper rests on the need of individuals to substitute for diminishing free resources. The presence of negative externalities caused by the growth process is an incentive for individuals to devote productivity increases mainly to the production of substitutes (to the point of increasing the labor supply) because their uncoordinated efforts do not take account of the social cost of increased production, thus fuelling the mechanism whereby increases in output cause a deterioration in the free resources which stimulates increases in output. Indeed, the decline in the per-capita endowment of commonly owned resource raises—*ceteris paribus*—the value of private consumption relatively to the value of time.

On growth and thrift in the long run

The erosion of communitarian institutions plays a role in the formation of attitudes toward thrift essential for a population's saving propensity and capital formation.[7] As an example of how the gradual erosion of these institutions in favour of more individualistic lifestyles may influence the saving rate, one may cite the major effect on saving propensity exerted by the declining importance of the family in providing support to the elderly which typically accompanies the evolution of an advanced society.[8]

In our framework, social wealth also includes those commonly owned assets whose progressive degradation accompanies the accumulation of privately owned assets. In the model, the tendency of the saving rate to decline as private wealth accumulates is damped by the households' anticipation that they will increasingly rely on private goods in order to satisfy their needs because of the degradation of the environmental asset in the course of the growth process. Hence, the impoverishment due to the declining stock of commonly owned resources counterbalances, at least partially, the negative effect on the saving rate due to the increase in private wealth. In other words, the concern for non-market goods

[7] In the case of the Asian Tigers, the very high saving rate was one of the crucial factors of their take-off.

[8] Pay-as-you-go pension schemes—which have replaced the family as the principal source of support for the elderly in most Western societies—still incorporate an important solidaristic element, since they are widely used to perform intragenerational and intergenerational tranfers. The current trend toward reducing their role in favour of more individualistic ways to provide for the elderly could be interpreted as another step in the erosion of communal institutions. Those who advocate this reduction expect it to boost aggregate saving.

that enter the utility function affects the incentive to acquire private wealth.[9]

On congestion and population pressure on environmental and social resources
The model predicts that a larger population size and a greater impact of a given level of production on commonly owned resources tend to boost the steady-state level of per capita output, but at the cost of a declining steady-state level of households' welfare. In fact, everything that exerts greater pressure on free assets and accelerates their decline induces individuals to react by working harder and accumulating more. Thus, according to the model, policies that reduce population growth and the environmental impact of productive activities restrain the steady-state level of per capita output.

It is also worth noting that the prediction that increases in population density will raise the long-run equilibrium level of per capita output is consistent with the predictions made by models of endogenous technological change concerning the impact of population increase on the steady-state rate of growth of the economy (see Grossman and Helpman, 1991; Aghion and Howitt, 1992; Kremer, 1993). However, our model has normative implications regarding the desirability of population increases which are at odds with those stressed by models of endogenous technological change, since our prediction depends on the increase in negative externalities due to congestion (increased pressure on environmental and social assets), rather than on positive externalities due to scale effects.[10]

On growth, discount rate and long-term welfare
One prediction of the model is that the long-term welfare of individuals tends to decline as they discount the future less heavily: the greater the concern of living individuals about the future, the more they worsen the prospects of future generations. This apparent paradox stems from the fact that rational individuals more anxious about the future are inclined to save more in order to safeguard their welfare (or the welfare of their descendants) in anticipation of a deterioration in the free resources. In doing so, they lead the economy to converge to a long-run equilibrium path characterized by higher production and less environmental quality, thereby reducing their long-term well-being (and the well-being of their descendants). This is because the increased availability of produced goods does

[9] Also in Cole, Mailath, and Postlewaite (1992) the concern for non-market goods affects individuals' saving decisions. However, the stylized behavior captured by Cole, Mailath, and Postlewaite is entirely different from the mechanism that we emphasize here, since they intend to stress the importance of caring for one's relative wealth (and, thus, for one's status) as a motivation to save.

[10] In models of technological change an increase in population spurs technological change and economic growth by increasing the size of the market, because the cost of inventing a new technology is independent of the number of people who use it. According to Kuznets (1960) an increase in population boosts technological progress by favouring intellectual contacts among people and labor specialization. In this way, greater population density can explain the disproportionally larger number of innovations in cities.

not compensate for the poorer quality of the environmental assets. More intense and uncoordinated efforts by individuals to safeguard their future welfare in the face of environmental degradation may reduce the long-term welfare of all agents as an unintended outcome of their defensive strategies.

This result reverses the conventional environmentalist explanation that the problems of sustainability depend on the selfishness of the present generation—that is, on its too high discount rate (see Pearce, 1993). This explanation can be inconsistent: one cannot argue that economic growth depends on the accumulation of productive assets—which is boosted by a low rate of time preference—while also claiming that the problems of social and environmental sustainability—which may be exacerbated by high accumulation rates—are made more serious by a high discount rate (see Vercelli, 1992). In our approach, the problem is not intergenerational conflict, but the inability of individuals belonging to the same generation to coordinate their efforts.

On the desirability of growth

It is worth pointing out that we use the term 'growth' in the usual sense: the increase of per-capita output. Here, this implies that growth is measured only with respect to one of the goods entering the welfare of the households: the output. In contrast, economic growth tends to be negatively correlated with the other two goods on which people's well-being depends: leisure and commonly owned resources. This leads to a well-known problem of mismeasurement of the impact of growth on welfare, which is systematically overestimated, to the point that growth may generate net losses of welfare.

It should be noted that even in an economy in which growth involves the further deterioration of free resources and the progressive increase of the time devoted to market activities some growth may lead to greater welfare. Although growth is based on a destructive process, it may generate Pareto-improvements. This is particularly true at earlier stages of the growth process. It seems plausible, in fact, that the social and environmental costs of industrialization are more than offset by, for instance, the decline in child mortality or by increased life expectancy. However, as growth is due to the substitution of declining free resources, it tends to exceed the threshold beyond which its destructive effects predominate over its beneficial impact on welfare. As a result of a market failure, growth 'goes too far', bringing about an excessive use of labor and deterioration of free resources.

Our model describes a world of individuals whose uncoordinated efforts to improve their position may give rise to a general worsening of individual positions. This might be a factor in explaning the 'broken promises of growth': dissatisfaction with the world created by the advanced economies, which people perceive as stressful, fraught with economic difficulties, and characterized by the deterioration of the social and natural environment. Analyses of subjective data, like the perceptions by individuals of their own welfare, conclude that the positive correlation between growth and well-being seems, in the most optimistic of evaluations, 'very slight':[11] in advanced societies higher levels of income do not make people feel significantly better. Since the neoclassical concept of welfare is founded on subjective perceptions, this evidence on the widespread discontent fed by the negative aspects of economic 'progress' cannot be easily reconciled with standard growth models, which generally imply that individual welfare improves as more output is available for consumption. In contrast, our model may give some insights to answer the question puzzling Oswald (1997, p. 1828): 'How can it be, one might ask, that money buys little well-being and yet we see individuals around us constantly striving to make more of it?'

3. The model

We consider an economy in discrete time with an infinite horizon. Identical households and firms operate in this economy.

The households

For simplicity and without loss of generality, it is assumed that population is constant and that each household contains one adult, working member of the current generation. Thus, there is a fixed (and large) number J of adults who take account of the welfare and resources of their actual and perspective descendants. Indeed, following Barro and Sala-i-Martin (1995) we model this intergenerational interaction by imaging that the current generation maximizes utility and incorporates a budget constraint over an infinite future. That is, although individuals have finite lives, we consider immortal extended families ('dynasties').[12] The current adults expect the size of their extended family to remain constant. Indeed, expectations are rational (in the sense that they are consistent with the true processes followed by the relevant variables). In this framework in which there is no source of random disturbances, this implies perfect foresight.

The representative household maximizes its discounted sequence of utilities, as given by

$$\sum_{t=0}^{\infty} \theta^t U_t, \; 0 < \theta < 1 \tag{1}$$

where θ is a time-preference parameter and U_t is the period utility function. The period utility function of the representative household is the following

$$U_t = \beta \ln(X_t) + \gamma \ln(C_{2t}) + (1 - \beta - \gamma)\ln(L_t), \; \beta > 0, \gamma > 0, \beta + \gamma < 1 \tag{2}$$

[11] The expression is Oswald's (1997), who makes the most optimistic evaluation of the data on individuals' perceptions of their own happiness. More pessimistic is Easterlin (1974, 1995), for whom happiness is the same in rich and poor countries, and growth does not increase well-being.

[12] As Barro and Sala-i-Martin (1995, p. 60) point out, 'this setting is appropriate if altruistic parents provide transfers to their children, who give in turn to their children, and so on. The immortal family corresponds to finite-lived individuals who are connected via a pattern of operative intergenerational transfers that are based on altruism'.

where X_t is the amount of services generated by some consumer activity, C_{2t} is the amount of the single good produced in the economy that is consumed in t as a basic good, and L_t is leisure. For simplicity and without loss of generality, the technology adopted by households to produce the services positively entering their utility function is assumed to be linear

$$X_t = R_t + \delta C_{1t}, \delta > 0 \tag{3}$$

where R_t is the stock of a resource to which all households have access, for free, in every period, and C_{1t} is the amount of the produced good that is used in t as a perfect substitute for the resource. Note that there is nonrivalry in the consumers' use of the resource R_t, from which no consumer can be excluded: it has the nonexclusive nature typical of a resource of common property that cannot be produced. Moreover, δ may be interpreted either as a strictly technological parameter prescribing the quantity of C_{1t} required—given R_t—to produce a certain level of X_t, or as a parameter reflecting the consumers' preferences and indicating the quantity of C_{1t} required—given R_t, C_{2t}, and L_t—to realize a certain level of utility.[13]

The total amount of time available to each household in every period is normalized to be one. Thus

$$L_t + H_t \leq 1 \tag{4}$$

where H_t are the units of time spent working in period t by the representative household.

The period budget constraint of the representative household is the following

$$C_{1t} + C_{2t} + I_t \leq W_t H_t + r_t K_t + \pi_t \tag{5}$$

where the single produced good is the numeraire of the system, I_t is the (gross) investment in productive assets (capital), W_t is the wage rate per unit of time, r_t is the rental rate on capital, K_t is the capital held by the representative household (and rent to the firms), and π_t is the share of total (net) profits distributed to each household. For simplicity, we assume that the claims on profits are evenly distributed as households' initial endowments, and—given that both households and firms are identical—we can ignore the possibility that these claims are traded among agents. Similarly, we assume that the initial endowment of capital is evenly distributed among the households.

The motion of the capital stock owned by the representative household is governed by

$$K_{t+1} = I_t + (1 - \sigma)K_t, 0 < \sigma \leq 1, K_0 \text{ given} \tag{6}$$

where σ is a capital depreciation parameter.

[13] As in the applications of the household production function approach to measuring the demand for environmental attributes (see Kerry Smith, 1991), the quality of a consumer's personal environment is treated as a function of the quality of the collective environment and the use of averting inputs (which are also known as defensive or self-protection inputs). In the literature following this approach, one can find examples of these averting inputs which include air filters, water purifiers, noise insulation, and other means of mitigating personal impacts of pollution.

The firms

Again, for simplification and without loss of generality, we assume that the large number of perfectly competitive firms is J, as is the number of households. Each firm produces the single good Y_t according to the technology

$$Y_t = K_t^{1-\alpha} (H_t N_t)^{\alpha}, 0 < \alpha < 1 \tag{7}$$

where N_t is the number of workers employed in t.

The representative firm maximizes its net profits π_t, as given by

$$\pi_t = Y_t - r_t K_t - W_t H_t N_t \tag{8}$$

The resource

The resource R_t is subject to a spontaneous flow of renewal but it is damaged by the volume of economic activity

$$R_{t+1} = A + \xi R_t - \rho J Y_t, A > 0, 0 < \xi < 1, \rho > 0, R_0 \text{ given} \tag{9}$$

By interpreting R_t as an indicator of the quality in t of some environmental resource to which all households have access for free in every period, we can think of firms as freely disposing of their polluting waste because of the lack of property rights on the natural resource. Although a single firm's productive activity has a negligible impact on the environmental quality, the aggregate effect of firms' production in period t on R_{t+1} is not negligible and depends on both the technological parameter ρ and the number of producers J. The waste accumulated during the productive process is disposed of at the end of the period and damages the environment in the next period. In other words, the negative externality caused by each single firm is only intertemporal: (9) captures a productive technology whose negative inpact on the environment is not immediate. According to this interpretation, the level of economic activity remaining equal, a larger R_t in the present entails better environmental quality in the future.

Alternatively, one could propose a sociological interpretation of the concepts of resource and negative externalities: the expansion of market activities undermines the institutional and immaterial bases of a communitarian organization of life on which depends the individual welfare.

Market equilibrium conditions

Equilibrium in the product market and in the capital market implies, respectively

$$Y_t = C_{1t} + C_{2t} + I_t \tag{10a}$$

$$K_t^d = K_t^s \tag{10b}$$

For equilibrium in the labor market, one needs

$$H_t^d = H_t^s \tag{10c}$$

$$J N_t^d = J \tag{10d}$$

since all the households actively participate in the labor market.

4. The equilibrium path under *Laissez-faire*

Firms' optimality conditions

In each period t, the representative firm must choose the combination of K_t, H_t, and N_t that maximizes its net profits (8) subject to (7). Using (10d) to eliminate N_t, we obtain the conditions that firms must satisfy along an equilibrium path

$$W_t = \alpha\left(\frac{K_t}{H_t}\right)^{1-\alpha} \tag{11}$$

$$r_t = (1-\alpha)\left(\frac{H_t}{K_t}\right)^{\alpha} \tag{12}$$

Note that these optimality conditions imply that $\pi_t = 0$.

Households' optimal plan

In each period t, the representative household must choose the combination of C_{1t}, C_{2t}, I_t and H_t that maximizes its discounted sequence of utilities (1) subject to (3)–(6). One can solve this decision problem by maximizing the Hamiltonian

$$\sum_{t=0}^{\infty}\theta^t\{\beta\ln[R_t + \delta(W_tH_t + \pi_t + r_tK_t - I_t - C_{2t})] + \gamma\ln(C_{2t}) + \\ + (1-\beta-\gamma)\ln(1-H_t) - \lambda_t\,[K_{t+1} - (1-\sigma)K_t - I_t]\}$$

with respect to I_t, C_{2t}, H_t and K_{t+1}, and then by eliminating the multipliers λ_t and λ_{t+1}. Hence, an equilibrium path must satisfy the transversality condition

$$\lim_{t\to\infty}\frac{\theta^tK_t\,\beta\delta}{R_t + \delta[W_tH_t + \pi_t + r_tK_t - I_t - C_{2t}]} = 0 \tag{13}$$

the optimality conditions

$$\frac{W_t\beta\delta}{R_t + \delta[W_tH_t + \pi_t + r_tK_t - I_t - C_{2t}]} = \frac{(1-\beta-\gamma)}{1-H_t} \tag{14}$$

$$\frac{\beta\delta}{R_t + \delta[W_tH_t + \pi_t + r_tK_t - I_t - C_{2t}]} = \frac{\gamma}{C_{2t}} \tag{15}$$

and the Euler equation

$$\frac{\theta[(1-\sigma) + r_{t+1}]}{R_{t+1} + \delta[W_{t+1}H_{t+1} + \pi_{t+1} + r_{t+1}K_{t+1} - I_{t+1} - C_{2t+1}]} = \\ = \frac{1}{R_t + \delta[W_tH_t + \pi_t + r_tK_t - I_t - C_{2t}]} \tag{16}$$

Equilibrium path

Together with (6) and (9)–(12), (14)–(16) can be used to obtain the system of difference equations in the labor–capital ratio D_t, in K_t and in R_t governing the equilibrium path of this market economy

$$\Psi(D_{t+1}, K_{t+1}, D_t, K_t) = \frac{\theta[(1-\sigma)+(1-\alpha)D_{t+1}^{\alpha}]}{D_{t+1}^{\alpha-1}(1-D_{t+1}K_{t+1})} - \\ - \frac{1}{D_t^{\alpha-1}(1-D_tK_t)} = 0,\ D_t \equiv \frac{H_t}{K_t} \tag{17}$$

$$\Phi(K_{t+1}, D_t, K_t, R_t) = K_{t+1} - (1-\sigma)K_t - \frac{(R_t + \delta D_t^{\alpha}K_t)}{\delta} + \\ + \frac{\alpha(\beta+\gamma)D_t^{\alpha-1}(1-D_tK_t)}{1-\beta-\gamma} = 0 \tag{18}$$

$$\Omega(R_{t+1}, D_t, K_t, R_t) = R_{t+1} - A - \xi R_t + \eta D_t^{\alpha} K_t = 0,\ \eta \equiv \rho J \tag{19}$$

where the equilibrium values of H_t, C_{1t}, C_{2t} and I_t can be written as functions of D_t, K_t and R_t

$$H_t = H(D_t, K_t) = D_tK_t \tag{20}$$

$$C_{1t} = C_1(D_t, K_t, R_t) = \frac{\delta\beta[D_t^{\alpha}K_t - I(D_t, K_t, R_t)] - \gamma R_t}{\delta(\beta+\gamma)} \tag{21}$$

$$C_{2t} = C_2(D_t, K_t, R_t) = \frac{\delta\gamma[D_t^{\alpha}K_t - I(D_t, K_t, R_t)] + \gamma R_t}{\delta(\beta+\gamma)} \tag{22}$$

$$I_t = I(D_t, K_t, R_t) = \frac{(R_t + \delta D_t^{\alpha}K_t)}{\delta} - \frac{\alpha(\beta+\gamma)D_t^{\alpha-1}(1-D_tK_t)}{1-\beta-\gamma} \tag{23}$$

Solving the system (17)–(19) for $D^* = D_{t+1} = D_t$, $K^* = K_{t+1} = K_t$ and $R^* = R_{t+1} = R_t$, we obtain the steady state (D^*, K^*, R^*). These steady-state values of D_t, K_t and R_t are given, respectively, by

$$D^* = \left\{\frac{1-\theta(1-\sigma)}{\theta(1-\alpha)}\right\}^{1/\alpha} \tag{24}$$

$$K^* = \frac{f(D^*)}{g(D^*,\eta)},\ f(D^*) = (\beta+\gamma)\alpha\delta(D^*)^{\alpha-1} - (1-\beta-\gamma)\,A/(1-\xi), \\ g(D^*,\eta) = (\beta+\gamma)\alpha\delta\,(D^*)^{\alpha} + \\ + (1-\beta-\gamma)\left[\left(\delta - \eta/(1-\xi)\right)(D^*)^{\alpha} - \sigma\delta\right] \tag{25}$$

and

$$R^* = \frac{A - \eta K^*(D^*)^{\alpha}}{1-\xi} \tag{26}$$

where the parameters' values are assumed to satisfy $f(D^*) > 0$, $g(D^*, \eta) > 0$, $A > \eta K^* (D^*)^\alpha$ and $\delta\beta K^*[(D^*)^\alpha - \sigma] > \gamma R^*$. One may check for some combination of parameters' values satisfying these inequalities that the system obtained by linearizing (17)–(19) around (D^*, K^*, R^*) exhibits saddle-path stability (see the Appendix).

Steady-state effects of changes in the discount rate and in the magnitude of the negative externalities caused by the economic activity

To raise the rate of return on capital investment one needs a larger labor–capital ratio. Hence, $\frac{\partial D^*}{\partial \theta} < 0$: a larger θ lowers the steady-state level of D because the rate of return required by the households to invest in capital diminishes as they discount the future less heavily. As a consequence, $\frac{\partial \left(Y^* / H^* \right)}{\partial \theta} > 0$, since $\frac{Y^*}{H^*} = (D^*)^{\alpha - 1}$: at the steady state, labor productivity is raised because of the lower labor–capital ratio.

Given that both the numerator and denominator of (25) are assumed to be strictly positive, it is easy to check that $\frac{\partial K^*}{\partial \theta} > 0$ and $\frac{\partial Y^*}{\partial \theta} > 0$, where $Y^* = K^*(D^*)^\alpha$: both capital accumulation and the steady-state level of economic activity are boosted as the households become less impatient. This entails $\frac{\partial R^*}{\partial \theta} < 0$: in the long run environmental quality deteriorates as the discount rate is lower (in the Appendix, one can find an example showing that this result may hold even if R_t positively affects productivity). Moreover, one has $\frac{\partial H^*}{\partial \theta} > 0$[14]: on the one hand, the larger capital stock leads the firms to increase their labor demand at any wage level; on the other hand, the deterioration of R^* due to the higher level of market production induces the households to increase their labor supply at any wage level.

The individual tendency to react to a deterioration of the free resource by increasing their defensive use of the produced good is reflected by the fact that $\frac{\partial \left(C_1^* / C_2^* \right)}{\partial \theta} > 0$: as a higher steady-state level of economic activity is made possible by the increased capital accumulation, consumers devote a larger proportion of their total expenditure to offset the negative impact on X_t due to the deterioration of the free resource. In other words, the households' expenditure in C_1^* is a larger share of total consumers' expenditure within an economy whose steady-state output is larger.

It is often argued that the welfare of future generations would be increased if the present generations gave less weight to their current well-being, so as to save more and to leave their descendants with more productive assets. Typically, this is not generally the case in the set-up proposed here, where the steady-state level of utility may decrease as the future is discounted less heavily by the households: in the Appendix, one can find the condition that the parameters values must satisfy for having

$\frac{\partial U^*}{\partial \theta} < 0$. It should be emphasized that the lowering of the steady-state level of utility goes together with a higher steady-state level of economic activity: the beneficial effect on individual welfare due to the availability of more output to be used in consumption can be more than offset by the negative effect on individual well-being due to the increased work effort and to the deterioration of the free resource. Moreover, as the households care less for the present, their awareness that individual efforts to improve future well-being will result in a deterioration of the free resource, reinforces their desire to accumulate more, so as to have additional income in the future and to buy more of the good that can substitute for the deteriorated resource. Capital accumulation is fed by individuals' anticipation that uncoordinated efforts on the part of all households to guarantee a better future to themselves and their descendants will exacerbate the welfare problem due to the negative externalities caused by the productive activities.

As this welfare problem becomes more serious, either because individual production is more detrimental for the free resource (ρ is larger) or because the number J of households and producing firms is larger, both steady-state output increases and steady-state utility decreases. Indeed, one can see by inspecting (25) and by considering (7), (20), and (24) that $\frac{\partial K^*}{\partial \eta} > 0$ and $\frac{\partial H^*}{\partial \eta} > 0$, entailing $\frac{\partial Y^*}{\partial \eta} > 0$. Furthermore, one can check that $\frac{\partial U^*}{\partial \eta} < 0$ holds (see the Appendix). Technologies that have a larger impact on the free resource are associated with a higher steady-state level of per-capita output, since they induce the households to save more and to work harder in order to counterbalance the more accentuated deterioration of the free resource. The same is true as total population increases: the prediction that a larger population leads to a higher steady-state level of per-capita output is obtained by the model, but without relying on economies of agglomeration or on the presence of a larger number of agents, each of them producing a negligible externality in favor of all the others. As a result of the individual efforts to escape the negative effects of the deterioration of R_t by saving more and working harder, the resource will be even more harmed and the steady-state utility of the representative agent is lowered in spite of the increased consumption of the produced good.

5. The Pareto-optimal path

Derivation of the steady state consistent with the Pareto-optimal path

To derive the path selected by a benevolent planner which takes into account the impact of the productive activities on the free resource, one can maximize the following Hamiltonian

[14] One can check that $\frac{\partial K^*}{\partial D} \frac{D^*}{K^*} < -1$ is satisfied, thus entailing $\frac{\partial H^*}{\partial \theta} > 0$, since $H^* = K^*D^*$

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$$\sum_{t=0}^{\infty} \theta^t \{\beta \ln[R_t + \delta(K_t^{1-\alpha} H_t^{\alpha} - I_t - C_{2t})] + \gamma \ln (C_{2t}) + (1 - \beta - \gamma) \ln (1 - H_t) - \\ - \lambda_t[K_{t+1} - (1 - \sigma) K_t - I_t] - - \mu_t[R_{t+1} + \eta K_t^{1-\alpha} H_t^{\alpha} - A - \xi R_t]\}$$

with respect to H_t, I_t, C_{2t}, K_{t+1} and R_{t+1}, and then one can eliminate the multipliers λ_t, λ_{t+1}, μ_t and μ_{t+1}. Hence, an optimal path must satisfy the transversality conditions

$$\lim_{t\to\infty} \theta^t K_t \frac{\beta\delta}{R_t + \delta(K_t D_t^{\alpha} - I_t - C_{2t})} = 0 \tag{27}$$

$$\lim_{t\to\infty} \theta^t R_t \left\{ \frac{\alpha\beta\delta\,(1 - D_t K_t) - D_t^{1-\alpha}\,[R_t + \delta(K_t D_t^{\alpha} - I_t - C_{2t})](1 - \beta - \gamma)}{\eta\alpha\,(1 - D_t K_t)\,[R_t + \delta(K_t D_t^{\alpha} - I_t - C_{2t})]} \right\} = 0 \tag{28}$$

the optimality condition

$$\frac{\beta\delta}{R_t + \delta(K_t D_t^{\alpha} - I_t - C_{2t})} = \frac{\gamma}{C_{2t}} \tag{29}$$

and the Euler equations

$$\frac{\theta\beta\delta(1 - \sigma)}{R_{t+1} + \delta(K_{t+1} D_{t+1}^{\alpha} - I_{t+1} - C_{2t+1})} + \frac{(1 - \beta - \gamma)\,(1 - \alpha)\,\theta D_{t+1}}{\alpha(1 - D_{t+1} K_{t+1})} = \\ = \frac{\beta\delta}{R_t + \delta(K_t D_t^{\alpha} - I_t - C_{2t})} \tag{30}$$

$$\frac{\theta\beta(\eta + \xi\delta)}{\eta(R_{t+1} + \delta(K_{t+1} D_{t+1}^{\alpha} - I_{t+1} - C_{2t+1}))} - \frac{(1 - \beta - \gamma)\theta\xi D_{t+1}^{1-\alpha}}{\alpha\eta(1 - D_{t+1} K_{t+1})} = \\ = \frac{\beta\delta}{\eta(R_t + \delta(K_t D_t^{\alpha} - I_t - C_{2t}))} - \frac{(1 - \beta - \gamma)\,D_t^{1-\alpha}}{\alpha\eta(1 - D_t K_1)} \tag{31}$$

It is straightforward from (29) that along an optimal path

$$C_{2t} = \frac{\delta\gamma(D_t^{\alpha} K_t - I_t) + \gamma R_t}{\delta(\beta + \gamma)} \tag{32}$$

from which we can obtain that for optimality

$$C_{1t} = \frac{\delta\beta(D_t^{\alpha} K_t - I_t) - \gamma R_t}{\delta(\beta + \gamma)} \tag{33}$$

Solving the system (6), (9) and (29)–(32) for $D^{\circ} = D_{t+1} = D_t$, $K^{\circ} = K_{t+1} = = K_t$, $R^{\circ} = R_{t+1} = R_t$ and $I^{\circ} = I_{t+1} = I_t$, we obtain the steady state (D°, K°, R°, I°), where

$$D^{\circ} = D^{*} F^{1/\alpha},\ F \equiv \frac{\delta}{\left(\delta - \theta\eta/(1 - \theta\xi)\right)} \tag{34}$$

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$$K^\circ = \frac{(\beta+\gamma)\,\alpha(D^\circ)^{\alpha-1}\left(\delta - \theta\eta\big/(1-\theta\xi)\right) - (1-\beta-\gamma)\,A\big/(1-\xi)}{(1-\beta-\gamma)\left((D^\circ)^\alpha\left(\delta - \eta\big/(1-\xi)\right) - \sigma\delta\right) + (\beta+\gamma)\alpha\delta(D^*)^\alpha} \quad (35)$$

$$R^\circ = \frac{A - \eta K^\circ (D^\circ)^\alpha}{1-\xi} \quad (36)$$

$$I^\circ = \sigma K^\circ \quad (37)$$

Comparison between the 'laissez-faire' and the Pareto-optimal solutions
By inspecting (34) and by comparing (25) and (35), one can see that $D^\circ >> D^*$ and $K^\circ < K^*$: the steady state consistent with the optimal plan exhibits a higher labor–capital ratio (hence, a lower labor productivity) and a smaller capital stock than the steady state associated with *laissez-faire*. Hence, labor productivity is lower in the presence of a benevolent planner. This notwithstanding, one can easily check that $H^\circ = D^\circ K^\circ < H^* = D^* K^*$: the planner would choose an allocation of resources such that the individuals work less than in a pure market economy. Obviously, the combination of a reduced capital accumulation and a shorter working time chosen by the planner entails $Y^\circ = K^\circ(D^\circ)^\alpha < Y^* = K^*(D^*)^\alpha$: steady-state output is larger under *laissez-faire*. Capital accumulation and individual working time are reduced because the planner would endogenize the intertemporal externalities caused by the productive activities of the households: it would prefer to raise the lifetime well-being of households by safeguarding the free resource, whose sustainable level (its steady-state level) is kept higher than under *laissez-faire*. In other words, as the decision process is decentralized, the long-run equilibrium path of the economy is characterized by too high levels of capital and work effort. Therefore, the Pareto-efficient path is characterized by a lower steady-state level of economic activity than the equilibrium path obtained under *laissez-faire*.

As individuals become less impatient (θ increases), the level of activity associated with the Pareto-efficient steady state tends to become a smaller fraction of the steady-state level of economic activity under *laissez-faire*: in the Appendix, one can find a sufficient condition for $\frac{\partial\left(Y^*\big/Y^\circ\right)}{\partial\theta} > 0$. An increase of θ determines a tendency of the differential between Y^* and Y° to increase more than proportionally than Y^*. This is because, as individuals care more for the future, they can react in a *laissez-faire* regime only by accumulating more capital, thus boosting future production; while in the presence of a benevolent planner the increased concern for the future is also reflected in a more 'conservationist' use of the free resource. Indeed, the fact that the benevolent planner can overcome the market failure due to the lack of coordination among the individual actions prevents the steady-state utility of the representative household from decreasing with the planner's degree of concern for the future. This can be seen by considering that

$$\lim_{\theta \to 1} U^{\circ} = U^{GR}$$

where U^{GR} is the 'golden rule' level of utility, i.e. the utility level obtainable by the representative individual when his/her steady-state utility is maximized (see the Appendix for the derivation of U^{GR}).

6. Conclusion

We summarize here the main results of the paper.

In the pure-market economy we have modeled, growth goes 'too far'. Under *laissez faire*, indeed, the long-run equilibrium of the economy is characterized by: (i) an inefficiently high level of production, (ii) an excessive portion of households' time devoted to market activities, and (iii) an inefficiently low stock of environmental resource.

Other implications of the model are the following:

1. Under *laissez-faire*, a lower discount rate worsens the well-being of future generations. This is because agents who are more concerned about the future protect themselves (and their descendants) against the anticipated deterioration of the environmental asset by accumulating more capital, thus contributing to maintaining a high level of production and to reducing the quality of the environment. Hence, the agents' attempt to safeguard their future welfare on an individual basis causes a reduction of all agents' long-term well-being as an unintended result of their defensive strategies. It should be noted that this result reverses the traditional environmentalist explanation of the problems of sustainability based on the selfishness of the present generation, that is, on the too high level of the discount rate.
2. The use of technologies with a less detrimental impact on the environmental asset leads, in the long run, to a lower level of production and consumption, since the commonly owned resource will be less depleted and households will be less dependent on market goods in order to satisfy their needs. As a result, exogenous technological progress reduces the negative incidence of productive activities on the environmental asset and raises the steady-state welfare of households.
3. An increase in population tends to raise per-capita output. Indeed, as population pressure on natural (or social) assets becomes stronger, households are induced to rely more on market goods in order to satisfy their needs, thus raising the steady-state level of economic activity. The associated steady-state level of households' welfare will be lower.

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Appendix: Numerical example in which the laissez-faire economy exhibits saddle-path stability

Let $\alpha = 2/3$, $A = \sigma = 0.25$, $\delta = 1.2$, $\xi = 0.5$, $\beta = \gamma = \eta = 1/3$, and $\theta = 0.8$. Given these parameters' values, $D^* = 1.8371173$, $K^* = 0.278067$, $R^* = = 0.221933$, and the system obtained by linearizing (17)–(19) around the steady state (D^*, K^*, R^*) characterizes a unique path that converges to it starting from initial values of K_t and R_t in a neighborhood of their steady-state values K^* and R^*. Indeed, with these parameters' values, the characteristic roots of the linearized system

$$D_{t+1} - D^* =$$

$$= \frac{(\Psi_{K_{t+1}} \Phi_{D_t} - \Psi_{D_t})(D_t - D^*) + (\Psi_{K_{t+1}} \Phi_{K_t} - \Psi_{K_t})(K_t - K^*) + \Psi_{K_{t+1}} \Phi_{R_t} (R_t - R^*)}{\Psi_{D_{t+1}}}$$

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$$K_{t+1} - K^* = -\Phi_{D_t}(D_t - D^*) - \Phi_{K_t}(K_t - K^*) - \Phi_{R_t}(R_t - R^*)$$
$$R_{t+1} - R^* = -\Omega_{Dt}(D_t - D^*) - \Omega_{Kt}(K_t - K^*) - \Omega_{R_t}(R_t - R^*)$$

are the following: $\rho_1 = 2.0749$, $\rho_2 = 0.601067 + 0.15216i$ and $\rho_3 = 0.601067 - 0.15216i$. These values imply that along the unique saddle path converging to $(D_t = D^*, K_t = K^*, R_t = R^*)$ the linearized system exhibits damped stepped fluctuations.

Example showing that the steady-state level of R_t may decrease as θ rises even if R_t positively affects productivity

Suppose that R_t entered the production function by affecting total factor productivity: for example, one may think that environmental improvements reduce workers' sickness (see Smulders, 2000). Hence, (7) can be rewritten as

$$Y_t = R_t^{\mu} K_t^{1-\alpha}(H_t N_t)^{\alpha},\ 0 < \alpha < 1,\ 0 < \mu < 1 \tag{A1}$$

Given (A1), the steady state of the *laissez-faire* economy becomes $(D^\bullet, K^\bullet, R^\bullet)$, where

$$D^\bullet = D^*(R^\bullet)^{-\mu/\alpha} \tag{A2}$$

$$K^\bullet = \frac{(\beta+\gamma)\alpha\delta(D^*)^{\alpha-1}(R^\bullet)^{\mu/\alpha} - (1-\beta-\gamma)R^\bullet}{\delta(1-\beta-\gamma)[(D^*)^{\alpha} - \sigma] + \alpha\delta(\beta+\gamma)(D^*)^{\alpha}} \tag{A3}$$

$$v(R^\bullet, \theta) = R^\bullet(1-\xi) - A + \frac{\eta(\beta+\gamma)\alpha\delta(D^*)^{\alpha-1}(R^\bullet)^{\mu/\alpha} - \eta(1-\beta-\gamma)R^\bullet}{\delta(1-\beta-\gamma)[1-\sigma(D^*)^{-\alpha}] + \alpha\delta(\beta+\gamma)} = 0 \tag{A4}$$

and D^* is given by (24). Given (A4), one can check that $\frac{\partial R^\bullet}{\partial \theta} = -\frac{\frac{\partial v(.)}{\partial \theta}}{\frac{\partial v(.)}{\partial R^\bullet}} < 0$

Necessary (and sufficient) condition for having $\frac{\partial U^*}{\partial \theta} < 0$

Given (14) and (15), the steady-state utility function can be written as

$$U^* = \ln(1 - D^*K^*) - (\beta+\gamma)[(1-\alpha)\ln(D^*) + \ln(1-\beta-\gamma) - \ln(\alpha)] + \\ + \beta\ln(\beta\delta) + \gamma\ln(\gamma) \tag{A5}$$

Hence

$$\frac{\partial U^*}{\partial \theta} = -\frac{\partial D^*}{\partial \theta}\frac{1}{(1-D^*K^*)D^*} \times \\ \times \left\{(1-\alpha)(\beta+\gamma) + [1-(1-\alpha)(\beta+\gamma)]D^*K^* + \frac{\partial K^*}{\partial D^*}(D^*)^2\right\} \tag{A6}$$

For having $\frac{\partial U^*}{\partial \theta} < 0$ one needs $(1-\alpha)(\beta+\gamma) + [1-(1-\alpha)(\beta+\gamma)]D^*K^* +$

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$+ \frac{\partial K^*}{\partial D^*}(D^*)^2 < 0$. This inequality holds if and only if

$$(1-\alpha)(\beta+\gamma)\left[\left(\delta - \eta/(1-\xi)\right)(D^*)^\alpha - \sigma\delta\right] < \sigma\delta D^* K^* + (1-\alpha)D^* \times \\ \times (1-\beta-\gamma)\, A/(1-\xi)\left[1 + \frac{\sigma\delta}{g(D^*,\eta)}\right] \tag{A7}$$

It should be noticed that (A7) is satisfied if—other things being equal—the impact of production on the resource (which is measured by η) is sufficiently large. For instance, one can check that (A7) is satisfied with $\alpha = 2/3$, $A = \sigma = 0.25$, $\delta = 1.2$, $\xi = 0.5$, $\beta = \gamma = \eta = 1/3$ and $\theta = 0.8$, entailing $\frac{\partial U^*}{\partial\theta} < 0$.

Proof that $\frac{\partial U^*}{\partial\eta} < 0$

Given (A5), it is the case that $\frac{\partial U^*}{\partial\eta} = -\frac{\partial K^*}{\partial\eta}\frac{D^*}{-(1-D^*K^*)} < 0$, since $\frac{\partial K^*}{\partial\eta} > 0$.

Sufficient condition for having $\frac{\partial\left(Y^*/Y^\circ\right)}{\partial\theta} > 0$.

Sufficient condition for having $\frac{\partial\left(Y^*/Y^\circ\right)}{\partial\theta} > 0$ is that

$$\left[\frac{\eta(\beta+\gamma)\alpha\delta(D^*)^{\alpha-1}}{(1-\theta\xi)^2\left(\delta - \theta\eta/(1-\theta\xi)\right)} - \frac{\eta(1-\beta-\gamma)\,A/(1-\xi)}{(1-\theta\xi)^2\left(\delta - \theta\eta/(1-\theta\xi)\right)}\right] \times$$

$$\times\left[\frac{A(1-\beta-\gamma)}{(1-\xi)} + (1-\alpha)(\beta+\gamma)\delta(D^*)^{\alpha-1}F^{-1/\alpha}\right] >$$

$$> \frac{A(1-\beta-\gamma)(\beta+\gamma)\delta}{D^*(1-\xi)\theta^2}\left(1 - F^{-1/\alpha}\right).$$

Derivation of the 'golden rule' level of utlity U^{GR}

The 'golden rule' level of utlity $U^{GR} = U(H^{GR}, K^{GR})$ can be obtained by solving $\max_{H,K}(U(H,K)$, where $U(H,K) = (\beta+\gamma)\ln\left(\delta(K^{1-\alpha}H^\alpha - \sigma K) + A/(1-\xi) - \eta K^{1-\alpha}H^\alpha/(1-\xi)\right) + (1-\beta-\gamma)\ln(1-H) + V$, and V is a constant whose value depends on the parameters. The values of H and K solving this maximization problem are such that $K^{GR} = \lim_{\theta\to1} K^\circ$ and $H^{GR} = K^{GR}D^{GR}$, where $D^{GR} = \lim_{\theta\to1} D^\circ$.

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