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Self-phase modulation of a laser in self created plasma channel

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Abstract

An analytical formalism of self focusing and self-phase modulation of an intense short pulse laser in a plasma due to relativistic and ponderomotive nonlinearities is developed. In the paraxial ray approximation, the pulse retains its Gaussian radial profile, however, its spot size varies with the distance of propagation in a periodic manner. It is influenced by self focusing. The frequency of the laser undergoes red shift. For a tanhyperbolic temporal profile of pulse, the red-shift is maximum at the foot of the pulse and decreases slowly as one goes to higher and higher intensity portions. The effect of ponderomotive nonlinearity is very significant in this respect. The maximum downshift occurs at a distance at which the laser acquires a minimum spot size. With retarded time normalized axial intensity increases more at $z \sim R_d$ and the radial intensity is also more narrowly peaked at $z \sim R_d$, where $R_d = 2\pi\ r_0^2/\lambda$ is the Rayleigh length, r_0 and λ are the spot size and wavelength of the laser pulse respectively.

Keywords: Self-focusing; Self-phase modulation; Relativistic mass nonlinearity; Ponderomotive nonlinearity

1. INTRODUCTION

Self-phase-modulation (SPM) of laser is an important nonlinear process in a variety of media with intensity dependent index of refraction. Max *et al.* (1974) observed the nonlinear frequency shift of a strong electromagnetic wave in plasma, due to weak relativistic effects. Yablonovitch (1974*a*, 1974*b*) investigated SPM and short pulse generation from laser breakdown plasmas. Tsintsadze *et al.* (1979) obtained the generalized dispersion relation for modulational instabilities due to relativistic electron mass variations. The SPM is accompanied by self focusing/defocusing of the laser beams and is significantly modified by those effects. Gill and Saini (2007) observed the enhancement of Raman scattering in collisional plasma by the interaction of rippled laser beam with upper hybrid mode and also observed the focusing of the upper hybrid wave. Willi *et al.* (2007) presented a novel technique for focusing and energy selection of MeV protons by employing a hollow micro-cylinder, which irradiated at the outer wall by a high intensity ultra-short laser pulse. Purohit *et al.* (2008) studied the excitation of an upper hybrid wave by a relativistic laser beam in the presence of perpendicular static magnetic field and also introduced the relativistic electron mass nonlinearity as well as relativistic self-focusing effect. Liu and Tripathi (2000) have developed an unified formalism of SPM and self defocusing in a tunnel ionizing plasma. The self defocusing limits the frequency upshift of the laser. Liu and Tripathi (2001) studied the frequency downshift of a Gaussian beam due to relativistic self focusing of the laser.

Watts *et al.* (2002) observed the relativistic SPM in the interaction of a high-intensity laser pulse (1 ps, 1.053 μm, 80 J) with the plasma. Singh *et al.* (2001) observed the resonant cross modulation of two laser beams in a semiconductor slab. Saini and Gill (2006) studied the self focusing and SPM of an elliptic Gaussian laser beam in collisionless magnetized plasma, by using the variational approach considering the effects of nonlinearity and diffraction. Gupta and Suk (2007) obtained the electron acceleration by two crossing chirped lasers of same amplitude and frequency, at an arbitrary angle, and causing modulation of laser intensity. Recently, Nickles *et al.* (2007) experimentally observed the interactions of a laser pulse at intensities above 10^{19} W/cm^2, and covered the rear and front side acceleration mechanisms, particle dynamics inside the dense target, proton source characteristics, strong modulations in proton and deuteron emission spectra, and generation of quasi-monoenergetic deuteron bursts.

Liu and Jetendra (2006) studied the self-defocusing/focusing of a right circularly polarized laser, in a preexisting

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density channel, and studied the effect of ponderomotive force driven plasma wave on the growth of modulational instability. Borghesi *et al.* (2007) studied the probing technique to provide the maps of impulsive electrostatic fields with high spatial and temporal resolution by the high-intensity laser matter interactions. The dynamics of ponderomotive channeling in underdense plasmas is also observed by the processes of Debye sheath formation and MeV ion front expansion at the rear of laser-irradiated thin metallic foils. Hafizi *et al.* (2000) studied the relativistic focusing and ponderomotive channeling of intense laser beams—ponderomotive channeling modified the effective potential of the laser spot size—and obtained the envelope equation by using the source-dependent expansion method with Laguerre-Gaussian Eigen functions. Experimental observations of relativistic focusing and ponderomotive channeling have been reported in the literature (Faenov *et al.*, 2007; Torrisi *et al.*, 2008; Chessa *et al.*, 1998; Sun *et al.*, 1987; Kurki-Suonio *et al.*, 1989; Borisov *et al.*, 1990, 1992*a*, 1992*b*; Konar & Manoj, 2005; Abramyan *et al.*, 1992; Chen & Sudan, 1993; Annou *et al.*, 1996; Tzeng & Mori, 1998; Monot *et al.*, 1995).

In this paper, we study the effect of self focusing on SPM of a laser pulse in self-channeled plasma. The self focusing, caused by relativistic mass shift as well as radial ponderomotive force, modifies the electron density. In Section 2, we obtain the nonlinear current density due to a high power laser. In Section 3, we drive coupled equations for amplitude and phase of the laser pulse. In Section 4, we solve these equations in near-axis approximation and by expanding the Eikonal up to second order in radial coordinates r.

2. NONLINEAR CURRENT DENSITY

Consider the propagation of a laser pulse in a plasma of electron density n_0^0 along the z-axis. At $z = 0$, the electric field of the laser is

$$\vec{E} = E_0(\hat{x} + i\hat{y})\exp(-i\omega_0 t),$$
$$E_0^2 = E_{00}^2 \exp(-r^2/r_0^2) g(t), \tag{1}$$

where $g(t)$ is the temporal shape of the pulse For $z > 0$, we may write,

$$\vec{E} = A(\hat{x} + i\hat{y})\exp(-i\phi), \tag{2}$$

where $A(t, z, r)$ is the complex amplitude, $\phi\ (t, z)$ is the fast phase of the wave, and one may take $\omega = \partial\phi/\partial t$ and $k = -\partial\phi/\partial z$.

The laser pulse imparts an oscillatory velocity to the electrons,

$$\vec{v} = \frac{e\vec{E}}{mi\omega\gamma},\ \gamma = \left(1 + \frac{e^2|A|^2}{m^2\omega^2c^2}\right)^{1/2}, \tag{3}$$

where $-e$ and m are the electronic charge and mass, respectively, and c is the velocity of light in vacuum. It also exerts a relativistic ponderomotive force on the electrons $\vec{F}_p = e\nabla\phi_p$,

$$\phi_p = -\frac{mc^2}{e}(\gamma - 1) \approx -\frac{e|A|^2}{m\gamma\omega^2}, \quad \text{for } (eA/m\omega c) < 1. \tag{4}$$

The ponderomotive force expels the electrons away from the region of higher electric field, while the ions remain stationary due to their heavy mass. On the time scale longer than the electron plasma period and shorter than the ion plasma period ($\omega_p^{-1} < t < \omega_{pi}^{-1}$), one may take the static space charge potential, caused by the displacement of electrons, $\phi_s \approx -\phi_p$. Using this in the Poisson equation, $\nabla^2\phi_s = 4\pi\ e(n_e - n_i)$, we get

$$n_e = n_0^0 - \nabla^2\phi_p/4\pi e. \tag{5}$$

The nonlinear electron current density at the frequency of the laser can now be written as,

$$\vec{J} = -n_e e\vec{v} = -\left\{n_0^0 + \frac{1}{4\pi e}\nabla^2\left(\frac{e|A|^2}{m\gamma\omega^2}\right)\right\}\left(\frac{e^2\vec{E}}{mi\omega\gamma}\right). \tag{6}$$

3. COUPLED EQUATIONS FOR AMPLITUDE AND PHASE

The wave equation governing the propagation of the laser in underdense plasma is

$$\nabla^2\vec{E} - \nabla(\nabla.\vec{E}) - (1/c^2)(\partial^2\vec{E}/\partial t^2) = (4\pi/c^2)(\partial\vec{J}/\partial t). \tag{7}$$

We get

$$\nabla^2\vec{E} - \frac{\omega_p^2}{\gamma c^2}\vec{E} - \frac{e}{m\gamma c^2}\nabla^2\left(\frac{e|A|^2}{m\gamma\omega^2}\right)\vec{E} - \frac{1}{c^2}\frac{\partial^2\vec{E}}{\partial t^2} = 0, \tag{8}$$

where $\omega_p^2 = (4\pi n_0^0\ e^2/m)$ is the plasma frequency. The second term on left-hand-side arises due to relativistic mass variation and the third is due to the ponderomotive force.

On substituting $\vec{E}$ from Eq. (2) in the wave equation and assuming the wave amplitude A to be a slowly varying function of t and z (Wentzel-Kramer-Brillouin approximation) we obtain,

$$\omega^2 = \left(\frac{\omega_p^2}{\gamma_0} + k^2c^2\right), \tag{9}$$

$$\begin{aligned} &2ik\frac{\partial A}{\partial z} + \frac{2i\omega}{c^2}\frac{\partial A}{\partial t} + \nabla_\perp^2 A + i\frac{\partial k}{\partial z}A + \frac{i}{c^2}\frac{\partial\omega}{\partial t}A \\ &= \frac{\omega_p^2}{c^2}\left(\frac{1}{\gamma} - \frac{1}{\gamma_0}\right)A + \frac{1}{\gamma}\left(\frac{e}{m\omega c}\right)^2\nabla^2(|A|^2)A, \end{aligned} \tag{10}$$

where $\gamma_0 = \gamma$ at $r = 0$.

Differentiating Eq. (9) with respect to t by use of $\partial k/\partial t = -\partial\omega/\partial z$,

$$\frac{\partial\omega^2}{\partial t} + \vec{v}_g\frac{\partial\omega^2}{\partial z} = -\frac{\omega_p^2}{\gamma_0^3}\frac{e^2}{2mc^2}\frac{\partial}{\partial t}\left(\frac{|A|^2}{\omega^2}\right), \quad (11)$$

where $\vec{v}_g = c(1-\omega_p^2/\omega^2\gamma_0)$ is the group velocity.

We define a function $F = (\omega/\omega_0)^{1/2} A$, $t' = t - z/c$ and $z' = z$, Eqs (10) and (11) become

$$\frac{2i\omega}{c}\frac{\partial F}{\partial z'} + \nabla_\perp^2 F = \frac{\omega_p^2}{c^2}\left(\frac{1}{\gamma}-\frac{1}{\gamma_0}\right)F + \frac{1}{\gamma}\frac{\omega_0}{\omega}\left(\frac{e}{m\omega c}\right)^2\nabla^2(|F|^2)F, \quad (12)$$

and

$$\frac{\partial\omega^2}{\partial z'} = \frac{\omega_p^2}{\gamma_0^2}\frac{e^2}{2\,m\omega_0^2c^2}\frac{\partial}{\partial t'}\left(\frac{|F|^2}{(\omega/\omega_0)^3}\right). \quad (13)$$

We may write $F = F_0\exp(iS)$, where $F_0(t', z', r)$ and $S(t', z', r)$ are real and separate the real and the imaginary parts of Eq. (11),

$$-\frac{2\omega}{c}\frac{\partial S}{\partial z'}F_0 + \frac{\partial^2 F_0}{\partial r^2} + \frac{1}{r}\frac{\partial F}{\partial r} - \left(\frac{\partial S}{\partial r}\right)^2 F_0 = \frac{\omega_p^2}{c^2}\left(\frac{1}{\gamma}-\frac{1}{\gamma_0}\right) + \frac{1}{\gamma}\frac{\omega_0}{\omega}\left(\frac{e}{m\omega c}\right)^2\nabla^2(|F_0|^2)F_0 \quad (14)$$

and

$$\frac{\omega}{c}\frac{\partial F_0^2}{\partial z'} + \left(\frac{\partial^2 S}{\partial r^2} + \frac{1}{r}\frac{\partial S}{\partial r}\right)F_0^2 + \frac{\partial S}{\partial r}\frac{\partial F_0^2}{\partial r} = 0. \quad (15)$$

Equations (13), (14), and (15) are the coupled phase and amplitude equations.

4. FREQUENCY DOWN SHIFT

We solve Eqs (14) and (15) by expanding S as $S = S_0 + S_2 r^2/r_0^2$ and introducing a function $f(z')$ such that $S_2 = (\omega r_0^2/2c)(1/f)(\partial f/\partial z')$. Then Eq. (15), for an initially Gaussian profile of laser intensity, gives

$$F_0^2 = \frac{E_{00}^2}{f^2}\exp\left(-\frac{r^2}{r_0^2 f^2}\right)g(t'), \quad (16)$$

where $(r_0 f)$ is the modified radius of the laser beam and the beam width parameter f has implicit dependence on t'.

Using Eq. (16) in Eq. (14) and collecting the coefficient of r^2, we obtain

$$\frac{\partial^2 f}{\partial\xi^2} + \frac{1}{\Omega}\frac{\partial\Omega}{\partial\xi}\frac{\partial f}{\partial\xi} = \frac{1}{\Omega^2 f^3} - \frac{r_0^2\omega_p^4 v_0^2 g(t')}{2c^4 f^3\Omega^5\gamma_0^2} - \frac{8v_0^2}{f^3\Omega^5 c^2\gamma_0^2} \times\left\{1 - \frac{v_0^2}{2c^2 f^2\Omega^3\gamma_0^2}g(t')\right\}g(t') - \frac{2v_0^4}{c^4 f^7\Omega^8\gamma_0^4}|g(t')|^2 \quad (17)$$

$$\frac{\partial\Omega}{\partial\xi} = \frac{r_0^2\omega_p^2}{c^2\Omega}\frac{v_0^2}{4c^2\gamma_0^2\omega_0}\frac{\partial}{\partial t'}\left(\frac{g(t')}{f^2\Omega^3}\right), \quad (18)$$

where $v_0/c = eE_{00}/m\omega_0\ c$ is the normalized oscillatory velocity and $\xi = z'/R_d$, $\Omega = \omega/\omega_0$, $R_d = (\omega_0/c)r_0^2$.

The boundary conditions at $\xi = 0$ for an initially plane wavefront are $f = 1$, $\partial f/\partial\xi = 0$, $\Omega = 1$, and $S_0 = 0$. The first term on the right-hand-side of Eq. (17) represents the diffraction divergence, the second term gives the beam convergence due to relativistic mass non-linearity, whereas the third and forth terms describe the convergence due to ponderomotive nonlinearity. We choose the temporal profile $g(t') = \tanh\,(t'/\tau)$ for $t' > 0$ and zero otherwise, where τ is the pulse rise time, and solve the coupled Eqs (17) and (18) numerically for the parameters: $r_0\omega_p/c = 5$, $\omega_0\tau = 40$, $v_0/c = 1$, $\gamma_0 = \sqrt{2}$, $T \cong (t - z/c)/\tau = 0 - 3$, $z/R_d = 0 - 3$.

Figure 1 shows the variation of the beam width parameter f with the normalized distance of propagation ξ. At $T = 0$, f increases monotonically with ξ due to diffraction. At higher T, the nonlinear convergence exceeds the diffraction divergence, and f decreases with ξ, i.e., focusing occurs. After certain distance, the electrons are significantly depleted from the axial region, and the nonlinear convergence weakens. At a certain value of ξ, the beam starts to diverge due to the predominance of diffraction divergence. As the beam acquires a large spot size, the nonlinear convergence again predominates beyond $\xi = 0.7$. Figure 2 shows the

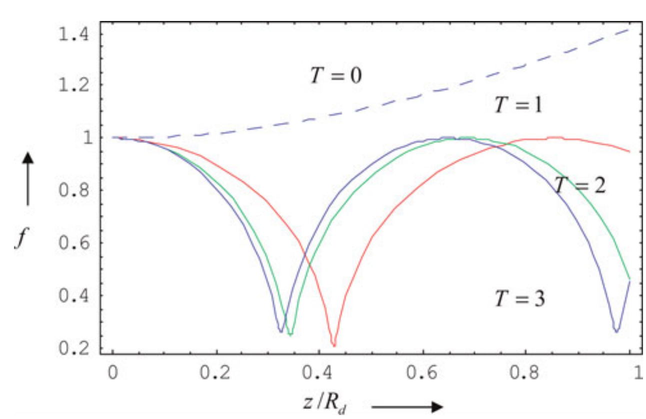

Fig. 1. (Color online) Beam-width parameter f as a function of normalized propagation distance $\xi = z/R_d$ at $T = 0,1,2,3$, where τ is the pulse rise time. The parameters are $r_0\omega_p/c = 5$, $\omega_0\tau = 40$ and $v_0/c = 1$.

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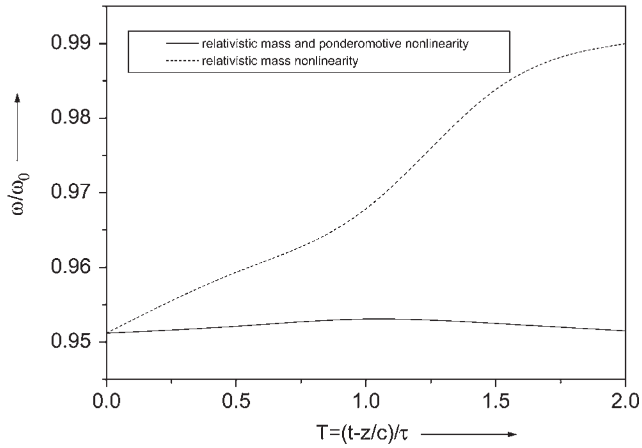

Fig. 2. Normalized frequency of laser as a function of the retarded time $T \cong (t - z/c)/\tau$ at $z/R_d = 1$. The other parameters are the same as in Figure 1.

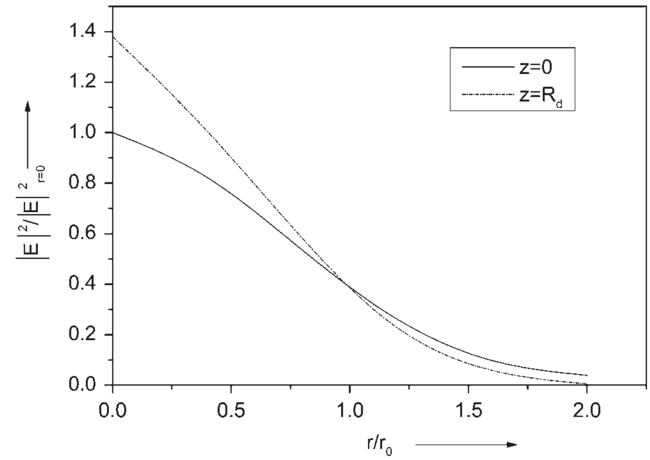

Fig. 4. Radial intensity profile $|E|^2/E^2_{r=0}$ at $z = 0$ and $z = R_d$ for $(t - z/c)/\tau = 1.5$. The other parameters are the same as in Figure 1.

frequency of the laser pulse as a function of retarded time T at $z/R_d = 1$. The front of pulse undergoes frequency downshift at $T = 0$. With increasing T, ω/ω_0 increases monotonically and attains a saturation value. In the simultaneous presence of relativistic mass nonlinearity and ponderomotive nonlinearity, self focusing is stronger; hence red shift remains fairly constant with retarded time throughout the channel. Figure 3 shows the variation of the normalized axial intensity $|E|^2_{r=0}/E^2_{00}$ as a function of T at $z = R_d$ and $z = 0$. At $z = 0$, the normalized axial intensity remains tanh (T) with retarded time, but at $z = R_d$ due to periodic self focusing, it deviates from tanh (T) and increases monotonically with the retarded time. For the normalized retarded time $T = 1.5$, the variation of the radial intensity $|E|^2/|E|^2_{r=0}$ with (r/r_0) at $z = R_d$ and $z = 0$ is shown in Figure 4. Due to periodic self focusing, the radial intensity is also more narrowly peaked at $z \sim R_d$ than at $z = 0$, but its shape remains Gaussian at $z = R_d$ and $z = 0$.

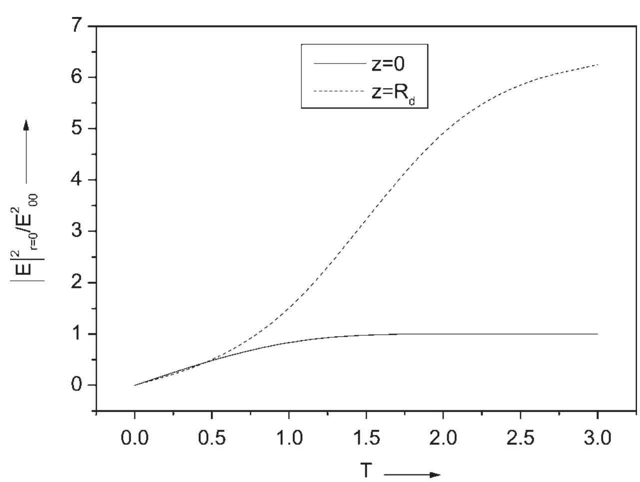

Fig. 3. Normalized axial intensity $|E|^2_{r=0}/E^2_{00}$ plotted as a function of retarded time T at $z = 0$, R_d. The other parameters are the same as in Figure 1.

5. DISCUSSION

A fast rising laser pulse, with tanhyperbolic temporal profile and Gaussian radial profile, propagating through preformed plasma undergoes downshift in frequency. Maximum downshift occurs at $t' = (t - z/v_g) \sim 0$, i.e., at the foot of the pulse. However, self focusing significantly modifies the temporal profile of the pulse, hence influences the frequency shift. When relativistic mass nonlinearity alone is considered, the frequency shift diminishes as one approaches the higher intensity of the pulse. With the inclusion of ponderomotive nonlinearity the frequency downshift continues to be significant up to longer times due to the temporal variation of electron density in the axial regions. Normalized axial intensity deviates from tanh (T) form. The deviation increases with retarded time and is maximum at $z \sim R_d$ (Fig. 3).

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