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RECAST THE DICE AND ITS POLICY RECOMMENDATIONS

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The DICE (for Dynamic Integrated model of Climate and the Economy) introduced two important policy aspects to the economic discussion of global warming. First, it argues for a "climate-policy ramp" that deems back-loading of mitigation optimal. Second, it demonstrates that an intergenerational tradeoff is at the heart of the mitigation problem. In this paper we argue that both of these findings rest on contestable assumptions. To demonstrate this a recast DICE is presented. Its outcomes show that DICE's predictions are not robust with higher migitations earlier on and slower temperature increases along the optimal path. The adoption of a baseline scenario in which pollution is a negative externality makes mitigating climate change a Pareto improvement. The alleged sacrifice of present generations vanishes. This strengthens the case for immediate policy action.

Keywords: DICE, Climate Change, Optimal Economic Growth, Externality, Mitigation Policy

1. INTRODUCTION

Economic development of the industrial era implied adverse developments in worldwide greenhouse gas (GHG) emissions. According to recent findings, such as the ones presented in the fourth assessment report of the IPCC [IPCC (2007)], these developments will have severe and lasting negative impacts on the well-being of everyone. Given the strong correlation between climate change and economic development, an economic analysis of global warming is desirable. Over the last two decades, William Nordhaus and others have developed this problem into an entire research project.

The DICE model (for Dynamic Integrated model of Climate and the Economy) of Nordhaus (1992, 1994) and, most recently, Nordhaus (2008) plays a predominant role in the economic analysis of global warming and introduced two aspects to the economic policy of global warming: First, it argues for a "climate-policy ramp" that deems back-loading of mitigation optimal. Second, it demonstrates that an intergenerational tradeoff is at the heart of the mitigation

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problem. Given the eminence of DICE, a clear understanding of its mechanisms is important. For this reason, in the following, DICE-07 is discussed briefly and some of its assumptions are found questionable; the functional form of mitigation technology and the baseline scenario are incompatible with underlying theoretical considerations.

To uncover these facts, DICE's mechanisms need to be put to work in a slightly altered atmosphere. A more parsimonious, recast DICE that omits an explicit description of climate dynamics and introduces credible upper bounds to the atmosphere's capacity to absorb carbon dioxide is presented. Although all other controversial assumptions on the mitigation function and model parameters are retained, the model projections change dramatically. DICE and its policy recommendations are not robust. Whereas DICE suggests an increase in average temperature of 3°C compared to current levels with a slow response in mitigation efforts, the more parsimonious model finds that economic policies should move the economy to an emission-free path within 10 decades so that temperature does not rise more than 1°C further.

More importantly, in its endeavor to establish the opportunity cost of global warming, DICE presents a misleading picture of the social choice problem of mitigation by adopting an illogical and flawed definition of the "baseline" reference path on which no efforts are made to control global warming. By allowing the representative agent to take into account the impact of production and investment on emissions, while constraining her to zero mitigation, the thus-constrained DICE baseline path actually induces a shift of consumption into the early decades of the program. This creates the illusion that an intergenerational tradeoff is at the heart of the problem. By respecifying the status quo or "baseline" as an inefficient allocation with climate change as a negative externality, following Foley (2008), zero mitigation becomes a corollary rather than an ad hoc assumption and mitigating climate change represents a Pareto improvement, further strengthening the case for immediate policy action.

2. DICE-07

DICE is a standard Ramsey–Cass–Koopmans model extended to include GHG dynamics. It solves the intertemporal allocation problem in order to maximize the total sum of discounted utility. The representative agent has the choice to invest or save. Climate change enters the model through a damage function of temperature, which decreases available output. The representative agent has the second choice of abating a certain share of current emissions, thus reducing overall concentration of GHG in the atmosphere. This second choice variable is called the control rate, $\mu(t)$, and represents the share of current emissions that are avoided by mitigation. Producing output creates emissions. These emissions feed through a climate module consisting of atmospheric and upper and lower oceanic carbon stocks. Atmospheric carbon increases global temperature, which regulates environmental damage. Figure 1 provides a complete overview of DICE. Stock

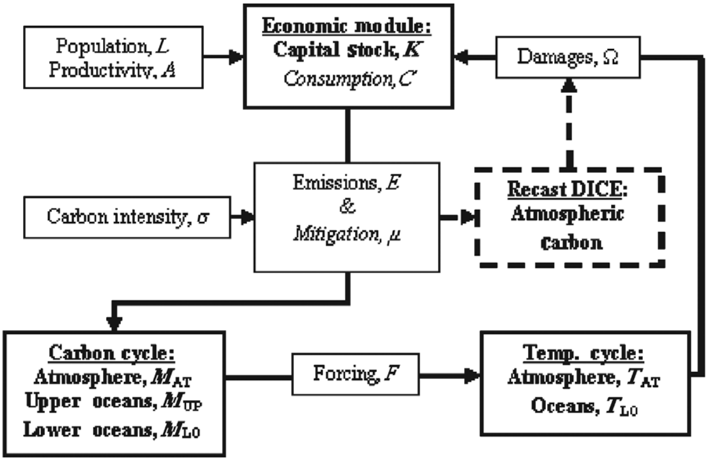

FIGURE 1. Flowchart of the model structure of DICE [its stock variables (bold) and control variables (italics)] and the short-circuited, recast DICE (dashed).

variables are bold and choice variables in italics. The recast DICE presented below short-circuits the explicit description of climate dynamics by making damages dependent on atmospheric GHG concentration directly.

Given the limitations on an article's page count, only a brief discussion of DICE is possible here. The most emphasis is placed on its mitigation and damage properties and its specification of the current state of affairs (the baseline case).[1] First, however, the most important model outcomes are presented.

2.1. Results

Nordhaus (2008) uses DICE-07 to compute a wide range of policy scenarios. We will concentrate on the optimal, the Baseline, and the $\leq 2°\text{C}$ case. In the optimal case, the unconstrained optimal first-order conditions are derived. In the baseline case, mitigation is effectively constrained to zero. Agents are aware of and responding to price signals when emitting carbon, but they have their arms tied behind their backs in terms of mitigation. The only way to avoid climate change in this case is to reduce investment and capital accumulation. This will only occur, however, if the marginal benefit of doing so is large enough—that is, if climate change becomes costly enough. In the $\leq 2°\text{C}$ case, temperature is constrained to increase less than 2°C (from 1900 levels).

Figure 2 reports the equilibrium paths for the selected scenarios. Carbon concentration and temperature are rising significantly in the optimal and baseline cases. In the optimal case the temperature increase peaks at 3.5°C in twenty decades and carbon concentration peaks at slightly below 700 ppmv at around the same time (up from 280 ppmv in 1750 and 380 ppmv today). The carbon price increases steadily from $30 per tonne of carbon to roughly $200 in 10 decades. In

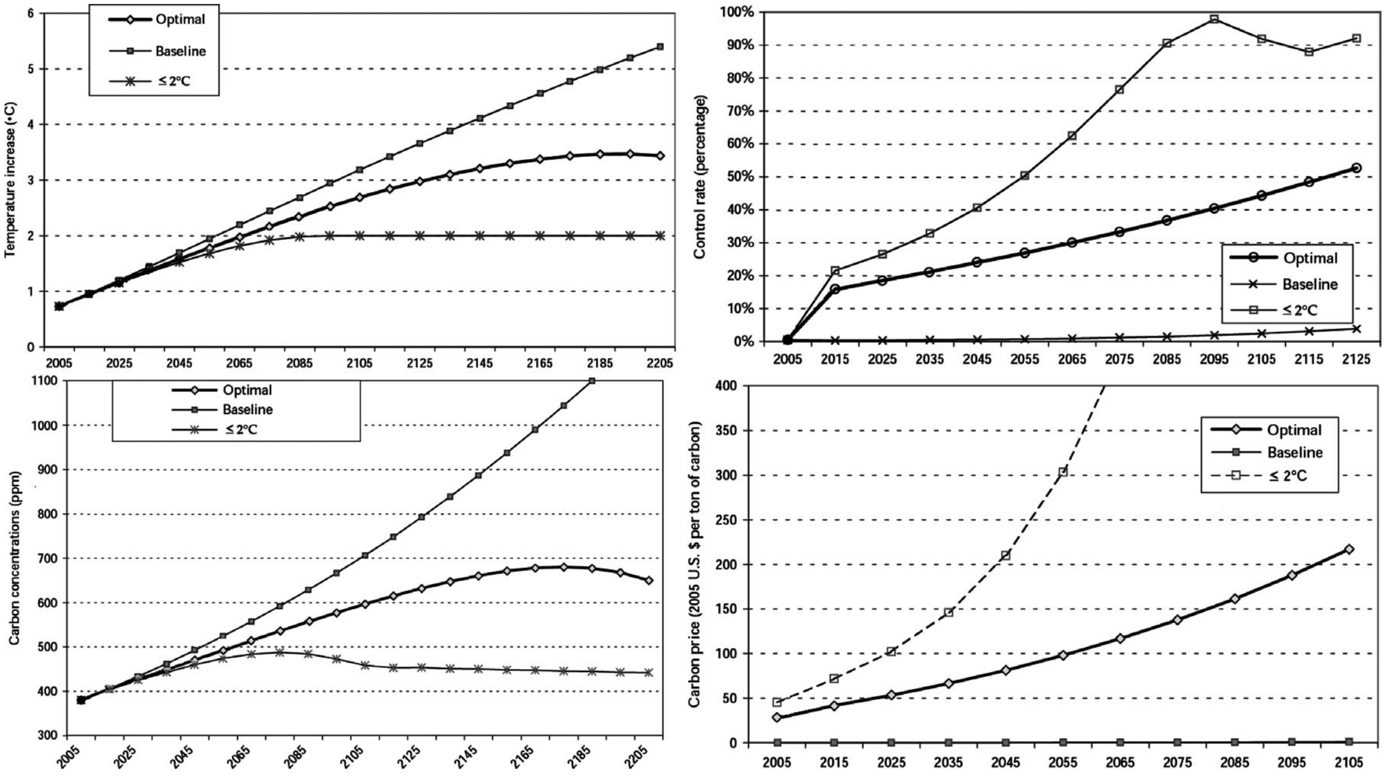

FIGURE 2. DICE equilibrium paths of the carbon price, the control rate, carbon concentration, and global temperature for various regimes. *Source*: Figures 5–8, 5–5, 5–7, and 5–4 from Nordhaus (2008).

the baseline case temperature increases almost linearly by 5.5°C and atmospheric carbon to roughly 1,200 ppmv over the next twenty decades. Given the lack of an upper bound on the damage function, temperature and carbon concentration most likely continue to increase for a significant amount of time beyond the reported period. The emissions control rate (the percentage of new emissions mitigated) stagnates at 0 in the baseline case (by definition) and only ramps up very slowly in the optimal case, reaching 50% in twelve decades. In the ≤ 2°C case, temperature increases to its upper limit over the course of the next ten decades and stays there for the remaining time horizon. Carbon concentration peaks at 490 ppmv in seven decades and then stabilizes around 450 ppmv. The emissions control rate increases faster than in the optimal case due to the binding temperature constraint. It reaches 100% in seven decades and stays in the range from 80% to 100% after that.[2] The carbon price increases rapidly to \$300 per tonne of carbon over the next 50 years.

Nordhaus (2008) also plots the consumption paths for several regimes in Figures 5–9. All curves are moving very closely—virtually indistinguishably, as he notes in the figure caption—over the next ten decades.

2.2. Discussion

DICE is an important effort to create an economic response to climate change and global warming. It also deserves credit for being fully available to the public. However, DICE has some deficiencies that weaken the credibility of its prediction.

The answer to the question of how the world climate responds to unprecedented GHG levels is not known. Likewise there is no answer as to what economic damages will be at CO_2 levels of 1,000, 700, or even 500 ppmv, compared to the current level of 380 ppmv. In this sense, any assumption on the functional form of damages is arbitrary. There exist, however, certain general restrictions one can impose on the shape of the damage function. One such is that there should exist an upper bound at which the world and with it all output is lost. One extreme bound could be a boiling hot average world temperature of 100°C. Clearly there exist other, more restrictive ones.

The cost of mitigating 100%, 50%, or even 10% of GHG emissions is as unsettled as the economic cost of environmental damages. Again, there exist certain fundamentals that can be derived from economic theory that have to be respected by modelers. One of them is the relationship between mitigation efforts and the carbon price that falls out of the optimization. As shown in Rezai et al. (in press), the first-order condition for the control rate $\mu(t)$ establishes a direct relationship between marginal mitigation efforts and the carbon price. Let the shadow price of capital be $\lambda(t)$ and the shadow price of atmospheric carbon concentration $\zeta(t)$; then the carbon price is $\chi(t) = \zeta(t)/\lambda(t)$. With $\Omega[T_{\text{AT}}]$ the multiplicative damage as a function of atmospheric temperature, T_{AT}, and $\Lambda[\mu(t)]$ the abatement cost function, deriving the DICE's Lagrangian with respect to $\mu[\text{t}]$

yields the following first-order condition:[3]

$$\chi(t) = \frac{\Lambda'[\mu(t)]\Omega\,[T_{\mathrm{AT}}]}{\sigma}.$$

The carbon price is a linear function of the effectiveness of marginal mitigation. One way to interpret this relation is as placing important restrictions on the functional form of the mitigation function (and its slope). The most important is that currently observed carbon prices have to be consistent with current mitigation efforts, which are effectively zero. DICE starts out with a zero slope at zero mitigation, which implies a carbon price close to zero at current mitigation efforts. With current carbon prices in the range of \$75 to \$125 per tonne of carbon (based on a range of €15 to €25 per tonne of carbon dioxide in the European Union Emission Trading Scheme), the parametric form of DICE's mitigation function implies mitigation levels in the range of 14% and 19% of current emissions. This would suffice to fulfill the Kyoto Protocol.

Another shortcoming of DICE is its specification of the state of current affairs, the baseline case. Nordhaus (2008, p. 65) defines this case as "...a world in which there are no controls for two and a half centuries. In this scenario, emissions are uncontrolled until 2250, after which a full set of controls is imposed.[4]" This specification implies that the economic agents are fully aware of the effects of climate change, but are not allowed to do anything about it in terms of mitigation. The only choice they have in obviating a severe climate catastrophe is to avoid emissions altogether. This can only be achieved by lowering the capital stock. The basic logic of the baseline case dictates a world economy with decreasing capital stock and near-zero interest rates.[5] High productivity growth only heightens this problematique.[6] A further corollary is that the baseline case cannot reach a steady state because net emissions that are fed into the carbon cycle are always positive. The soft damage function in DICE, which keeps environmental damage low and does not allow for severe climate crises, masks these features. To illustrate this more clearly, DICE is recast.

3. RECAST DICE

DICE incorporates elaborate geophysical dynamics that are supposed to improve the realism of its predictions. From an economic perspective, however, they conceal much of the dynamics of global warming. This section recasts DICE by short-circuiting temperature altogether. Environmental damage is a function of carbon dioxide in the atmosphere directly. In addition, the new damage function features a credible maximal carbon concentration in the atmosphere at which complete output loss occurs. All other parameters, including the dubiously decreasing carbon intensity and the questionable mitigation function, are carried over from DICE-07.

The new model is similar to DICE, yet it differs in details that are crucial to understanding global warming dynamics and to the right policy response. A representative agent still strives to maximize per capita utility and faces the options of consuming, investing in conventional capital, or mitigating. Although capital's state equation is in its standard form, the CD (carbon dioxide) state equation differs from that in DICE. Rather than assuming unlimited sinks in which emitted carbon is deposited, atmospheric carbon is assumed to dissipate at a constant rate ε. ε is calibrated at 0.0036 annually, to match 2007 data.[7] The 280ε-term in the state equation below ensures that CD approaches preindustrial levels rather than zero in the absence of emissions. In this recast DICE, all emitted carbon is atmospheric and affects output directly via the damage function $Z[\mathrm{CD}(t)]$. Emissions, $G[Y(t)]$, are a function of usable (i.e., net of damages) world output $Y(t) = Z[\mathrm{CD}(t)]F[K(t), L(t)]$. Let $I(t)$ be total investment, $C(t)$ total consumption, $(s(t) - m(t))$ the conventional investment share, and $m(t)$ the mitigation investment share. Then, with K_0 and CD_0 as the initial endowments, the maximization problem is

$$\max_{s[t],m[t]} U[C(t)], t] = \sum_{t=1}^{T} \frac{1}{(1+\rho)^{(t-1)}} U\left[\frac{C(t)}{L(t)}\right]$$

subject to

$$K(t+1) = (1-\delta)K(t) + (s(t+1) - m(t+1))Y(t),$$
$$\mathrm{CD}(t+1) = (1-\varepsilon)\mathrm{CD}(t) + G[Y(t)] - M[m(t+1)]Y(t) + 280\varepsilon,$$

and

$$Y(t) = I(t) + C(t) + m(t)Y(t) = [(s(t) - m(t)) + (1 - s(t)) + m(t)]Y(t),$$
$$K(0) = K_0, \mathrm{CD}(0) = \mathrm{CD}_0, \lambda(T)K(T) = 0, \zeta(T)\mathrm{CD}(T) = 0.$$

Note that mitigation here is a function of the output share dedicated to mitigation, yielding a CD reduction per output. DICE's abatement cost function is the inverse of this function. Following DICE, the discount factor is set to 0.15 per decade and $U[c(t)]$ is of isoelastic form, with an elasticity parameter of 2. $F[K(t), L(t)]$ is a Cobb–Douglas production function with $\alpha = 0.3$. Capital depreciates at a decadal rate of 0.65. Population, $L(t)$, total factor productivity, $A(t)$, and carbon intensity, $\sigma(t)$, also take the same time paths as in DICE. Following DICE, emissions are linear in output.[8]

The new damage function deserves consideration, because it has to incorporate both the carbon and the temperature dynamics of DICE. $Z[\mathrm{CD}]$ is assumed to be isoelastic with zero damage at the preindustrial level of 280 ppmv. Complete output loss is assumed to occur at $\mathrm{CD}_{\mathrm{Max}}$. γ defines the severity of damage for given CD levels. Whereas $\gamma = 1$ corresponds to linear damage, lower γ lead to less damage for any given level of CD. This parametric shape is chosen to reflect both climate change optimists' and pessimists' views. At low levels of CD little output is lost; $\mathrm{CD}_{\mathrm{Max}}$, however, introduces an absolute floor with complete

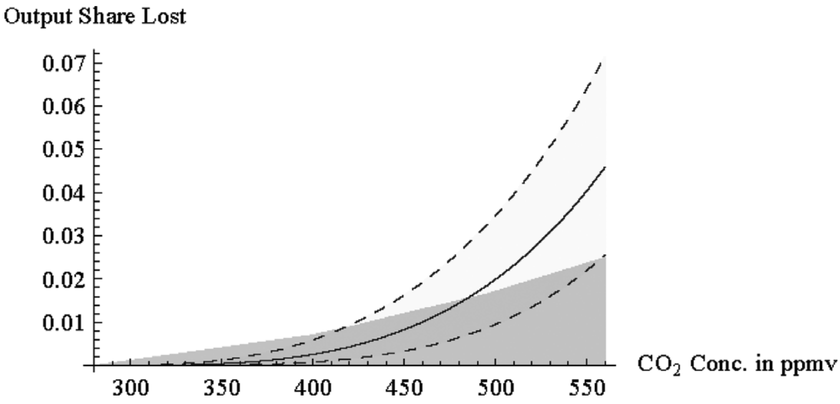

Figure 3. Damage function of DICE (shaded area) and the recast DICE (dark line) with $\gamma = 0.3$ and γ € [0.25, 0.35]-interval (dashed lines).

collapse of output,

$$Z[\mathrm{CD}] = \left(1 - \left(\frac{\mathrm{CD} - 280}{\mathrm{CD}_{\mathrm{Max}} - 280}\right)^{1/\gamma}\right)^{\gamma}.$$

Figure 3 plots the environmental damage (i.e., $1 - Z[\mathrm{CD}]$) for $\gamma = 0.3$ and $\mathrm{CD}_{\mathrm{Max}} = 780$ ppmv and compares it with inferred damages from DICE steady states. The parameter choice for γ and $\mathrm{CD}_{\mathrm{Max}}$ is difficult due to the deep uncertainty about the implications of such high levels of carbon dioxide for the climate and the economy. We choose $\mathrm{CD}_{\mathrm{Max}} = 780$ ppmv which, according to the IPCC (2007, p. 67), is equivalent to an increase of more than 6°C.[9,10] The value for $\mathrm{CD}_{\mathrm{Max}}$ can also be justified in terms of threshold effects and the possibility of catastrophic climate change. It is important to note that the specific parametric values for the damage function are not relevant to the fundamental logic of the equilibrium paths presented below. The new damage function stays below DICE's for $\mathrm{CD} < 480$.

Before the model is solved, a note on the timing and units is in order. All parameters and flow variables are geared to a decadal scale. Output of period $t - 1$ is used in period t. The units of the model are $trillion (at current prices) for economic and ppmv for carbon-related variables.

First, we compute the equilibrium paths for the optimal and constrained-optimal baseline case to see how dropping geophysical dynamics and honing environmental damages affect the solutions.

Figure 4 solves for the intertemporal equilibrium paths of the bare-bones version of DICE laid out above. Several interesting observations can be made. In the optimal case, agents invest in mitigation and avert a climate crisis. The control rate (the share of mitigated emissions) increases to a range of 80%–100% in less than 10 decades and thereby moves to the economy to a zero-emission path. The

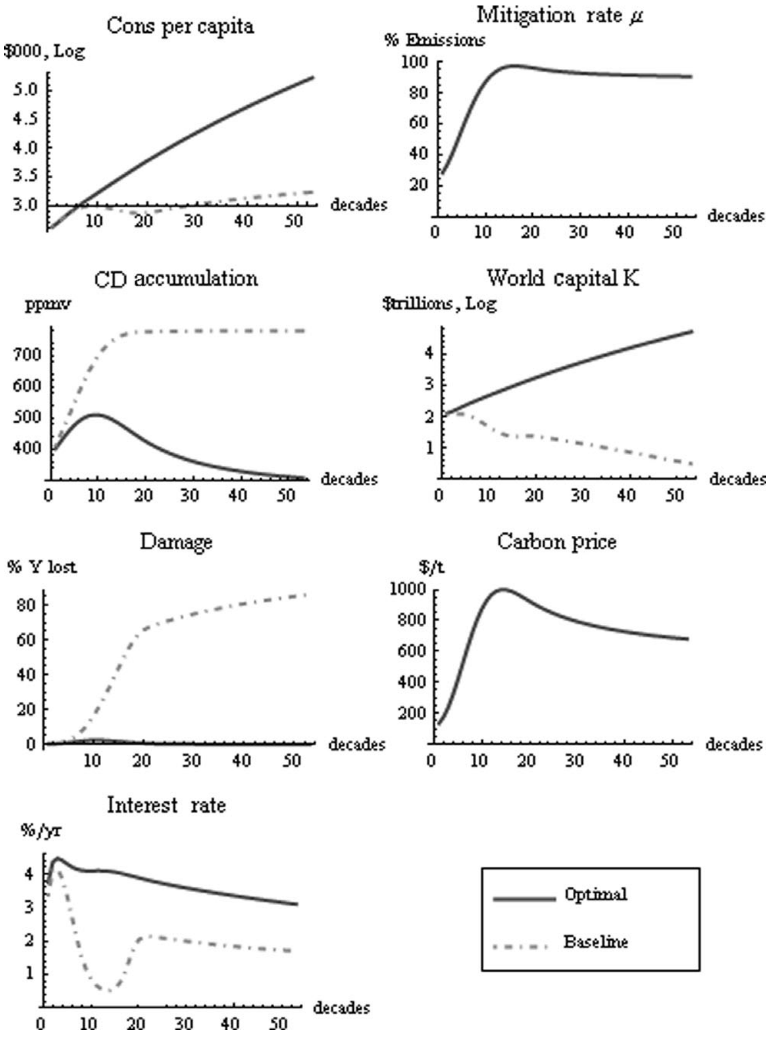

FIGURE 4. Recast DICE: simulations of the optimal and constrained–optimal baseline cases.

overall cost of doing so as derived from the mitigation function is relatively small, with m (the share of output devoted to mitigation) ranging around 1%. Although for the relevant range of CD the new damage function is less harsh than Nordhaus' original, the carbon concentration is significantly lower throughout the equilibrium path. In fact, it appears as if the optimal case of the recast DICE is more similar to Nordhaus' $T < 2^\circ$ scenario, with CD peaking at 500 ppmv and the carbon price (per tonne of carbon) rising through $400 after five decades and peaking below \$1,000 in roughly ten decades. Capital stock and per capita consumption are growing steadily and the interest rate is falling, in line with the usual transition dynamics.

The constrained–optimal baseline case (in which the program is constrained to zero mitigation although optimal otherwise) in Figure 4 appears to be quite close to its DICE original. Carbon concentration in the atmosphere rises rapidly over the first ten decades to around 700 ppmv. Then the original and the emulate diverge as the introduced upper bound of $CD_{\text{Max}} = 780$ ppmv starts to bind.[11] The economy is running into a severe climate crisis and starts losing large shares of output. It is at this point that the consumption paths of the optimal case and the constrained–optimal baseline case diverge. Nordhaus only plots the consumption paths ten decades out into the future. All paths seem to be following the same line with marginal deviations. Given the observation above and his soft damage function, this result is not surprising. The road to climate crisis in DICE-07 might be long, but it is nonetheless a slippery one. As discussed above, the carbon price is the ratio between the shadow prices of capital and carbon dioxide. It is rising quickly to the \$10,000s in light of the ever-increasing deleterious effects of global warming. This causes the interest rate, which is the marginal rate of intertemporal substitution, to become near-zero and induces the agents to emit less carbon by accumulating less and less capital.[12] Because it is total output that causes emissions, stronger productivity growth induces even less capital accumulation in order to keep emissions constant.

The specification of the status quo that Nordhaus presents is ad hoc and implausible economically (why should agents be constrained to zero mitigation when they have full information about the harmful effects of GHG emissions). A more intuitive characterization is to consider such emissions an economic externality. Foley (2008) makes the case that if global warming poses a negative externality, then zero mitigation is a corollary rather than an ad hoc assumption. Theoretically, a state variable creates an externality when it has a real impact on the objective function or constraints, but no institutions exist to enforce the social price on individual agent decisions involving it. Each agent assumes that her decisions will not affect the path of the externality, but when all agents make the same decisions, the path of the externality changes. The difference between the equilibrium path with an uncorrected externality and the optimal path is the fact that the typical agent does not adjust her controls to take account of their effect on the externality. In the presence of an externality the competitive equilibrium is not an optimum. The removal of the externality is a Pareto improvement. Rezai et al. (in press) discuss the implications of externalities for optimal growth models and their application to global warming. The representative agent assumes that her contribution to climate change is negligible. But if all agents assume this, climate change occurs. In effect, global warming is a coordination failure. Let the business-as-usual (BAU) case be the scenario with the externality present; then Figure 5 presents a comparison of all three cases. Note that because of numerical instability, we reduce exogenous productivity growth by about half. This merely reduces the slope off of which the

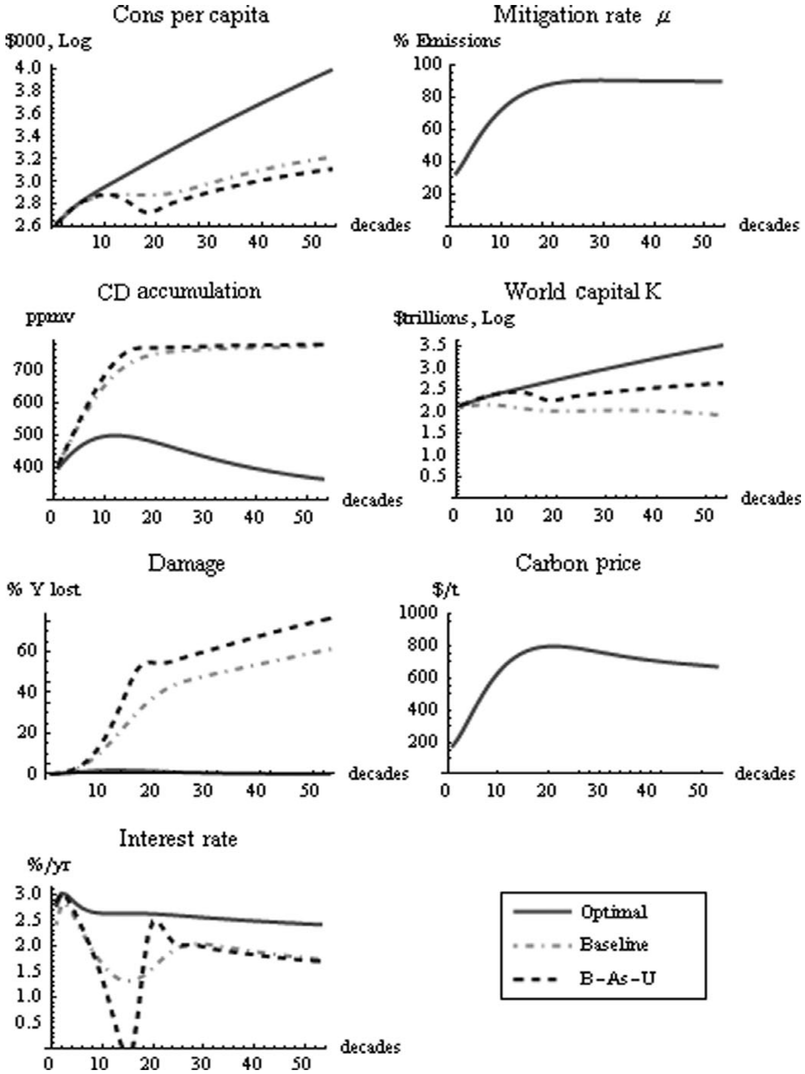

FIGURE 5. Simulations of the optimal, constrained–optimal baseline, and BAU cases with lower productivity growth.

constrained-optimal baseline and business-as-usual cases are slipping into their climate crises.

The slowing of productivity growth does not appear to have had much effect on the optimal case, except for a slowing in consumption and capital growth. The qualitative features of the constrained–optimal baseline case also remain; most importantly, capital stock still falls. Quantitatively, the climate crisis occurs at a later point in time and is less severe. Atmospheric carbon dioxide, CD, increases

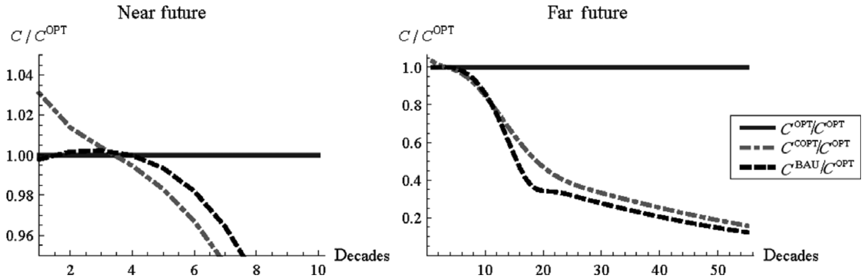

FIGURE 6. Comparison of optimal, constrained-optimal baseline, and BAU consumption paths. Values greater (smaller) than 1 imply higher (lower) consumption levels than under the optimal program.

to levels above 700 ppmv over the first fifteen decades and continues to rise to ultimately 770 ppmv.

In the BAU case, GHG emissions pose an unaccounted externality, and as a result agents want to accumulate high levels of capital. Capital, output, and consumption rise until damages offset net investment at high levels of CD. At this point the climate crisis occurs. As agents still consider their contribution to the problem as negligible, capital stock keeps rising. This thrift, however, does not pay off. Consumption is low in the beginning (due to high saving) and remains low throughout (due to accumulation-related environmental damage). This is the inefficiency of the BAU case that Foley (2008) points out. Using the BAU case as a reference path has important implications for current policy debate around the problem of intergenerational equity. The basic issue is an alleged trade-off in well-being between present and future generations. Specifically, is it advantageous for current generations to divert resources from their own consumption to mitigate global warming?

Figure 6 compares the consumption paths for the optimal, constrained–optimal baseline, and BAU scenarios normalized to the optimal consumption. Consumption in the constrained–optimal baseline case lies above consumption in the optimal case in the first few decades, as agents who are aware of the deleterious effects of global warming wisely choose to accumulate less and consume more. This positive difference in the first few decades forms the basis for the equity discussion because the choice of the appropriate program now depends on its parameterization (most importantly, the discount factor). Nordhaus, among many, has placed much emphasis on the dominance of constrained–optimal baseline consumption over optimal consumption in the first few decades. This implies that current generations would attain lower consumption and utility levels if they started investing in mitigation. It is important to understand that the higher consumption levels can only be sustained by lower investment levels. In the constrained–optimal baseline case, capital stock decreases in Figures 4 and 5 due to productivity growth. Agents are fully aware of the deleterious effects of economic growth and constrain themselves to less wealth, which frees resources for higher initial consumption levels.

The consumption paths of the optimal and BAU (in which the externality is present) scenarios move together in the first few decades, with the optimal dominating the BAU baseline for the complete horizon (except for periods 2 and 3, in which BAU consumption is 0.1% higher than optimal). More importantly, the sum of discounted utility for any period and any (plausible) discount factor is greater on the OPT than on the BAU consumption trajectory. This is due to the inefficiency. Agents are saving and investing at high levels as if their decisions were not altering the climate. Once the climate crisis sets in, global warming–related damage lowers output and consumption levels. As a result, consumption levels are almost always lower than under the optimal path. This implies that there is a net gain to internalizing the externality and mitigating global warming. If one accepts this view of the state of affairs, the whole discussion on discounting and equity may seem to be of secondary importance, valid as it might be. With respect to this debate, however, two fundamental questions remain.

First, are overlapping-generations models with imperfect bequest motives more appropriate in the analysis of intergenerational equity? Although there has been some research on this issue [e.g., Howarth (1998) and Gerlagh and van der Zwaan (2001)] and the welfare effects of environmental taxation [e.g., Bovenberg and Heijdra (1998)], none of these authors studied the welfare effects of climate change as a global negative externality in such a framework.

Second, is it methodologically valid to compare programs based on the sum of discounted welfare of shorter periods than the complete horizon of the model? After all, by definition the constrained optimal path can at most do as well as the unconstrained optimal path when the complete horizon is considered.

Trying to identify generations within a framework of infinitely lived agents makes the welfare analysis difficult. We see our contribution as pointing out that even if one does so on methodologically questionable grounds, the implementation of mitigation policies can represent a Pareto improvement.

4. CONCLUSION

The DICE model of Nordhaus (1992, 1994) and, most recently, Nordhaus (2008) is an important contribution to the economic analysis of global warming. Two important policy implications follow from it. First, DICE biases the optimal path toward delaying mitigation and advocates a slowly increasing policy response (the "climate-policy ramp"). Second, the constrained-optimal baseline scenario presented in Nordhaus (2008) features a world economy with decreasing capital, near-zero interest rates, and geophysical dynamics without a steady state. This can only be seen, however, if DICE is recast with a more credible damage function and without the climate dynamics. On a theoretical basis, the constrained–optimal baseline scenario is not only economically counterintuitive but also ad hoc in its assumptions. When the status quo is respecified as an inefficient allocation with climate change as a negative externality, zero mitigation is a corollary rather than an ad hoc assumption and mitigating climate change can represent a Pareto improvement.

Both of these policy findings are shown to be nonrobust using the recast DICE. The policy ramp is shown to be much steeper with higher mitigation efforts earlier on and numerical simulations show that moving the economy from a BAU reference path to an optimal scenario is, in fact, Pareto-improving, strengthening the case for immediate policy action.

NOTES

1. A more detailed discussion of the model can be found in Rezai (2009).

2. An emission-free economy is easily modeled. Given current production possibilities, however, this does not seem feasible, a point made in Lewis (2007).

3. See Rezai (2009) for the derivation.

4. In the published GAMS code it appears to be a world in which damages are set to 0 for the first 250 years.

5. See Rezai et al. (in press) for a detailed discussion of the first-order conditions.

6. The damage function in DICE, however, is very soft, which counteracts this force.

7. According to the United Nations Energy Statistics, 7 Gt carbon are burnt or 3.37 ppmv carbon dioxide are added per year. The actual increase is only about 2 ppmv; dissipation is 1.37 ppmv. This yields a depreciation factor of 0.0036/year. This is in line with the implied range of depreciation rates of DICE-07. Because in any steady state atmospheric carbon depreciation is equal to net—meaning unmitigated—emissions, one can infer the implied rate of carbon depreciation. For DICE-07 it lies between 0.0025 and 0.0055 annually after all linkages between carbon and temperature are taken into account.

8. In this model only actual output, i.e., potential output net of damages, emits carbon. This assumption decreases emissions slightly and does not change the results significantly. Because CD is rescaled to ppmv, carbon intensity has to be divided by 2.13.

9. Already an increase of 5°C leads to the highest concern in the IPCC's "Burning Ember" figures [IPCC (2007)]. Lynas (2008) highlights the disastrous implications of a 6°C warming.

10. Doubling the permissible CD range by increasing CD_{Max} to 1280 while increasing γ to 0.45 to hold damages under 480 ppmv comparable only increases peak CD by 70 ppmv and slows the policy response slightly. Steady state levels of CD would decrease. More details on changing parameter values are presented in the sensitivity appendix, which is available from the author.

11. The effect of increasing CD_{Max} on the baseline case would merely be a delay of the climate crisis and its stabilizing effect on the economy.

12. In contrast, Nordhaus (2008) defines the interest rate as only the net marginal product of capital.

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