J. Fluid Mech. (2012), *vol.* 709, *pp.* 289–312.
doi:10.1017/jfm.2012.335

Linear water waves: the horizontal motion of a structure in the time domain

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(Received 5 April 2012; revised 21 June 2012; accepted 28 June 2012; first published online 10 August 2012)

The framework of the linearized theory of water waves in the time domain is used to examine the horizontal motion of an unrestrained floating structure. One of the principal assumptions of the theory is that an infinitesimal disturbance of the rest state will lead to an infinitesimal motion of the fluid and structure. It has been known for some time that for some initial conditions the theory predicts an unbounded horizontal motion of the structure that violates this assumption, but the possibility does not appear to have been examined in detail. Here some circumstances that lead to predictions of large motions are identified and, in addition, it is shown that not all non-trivial initial conditions lead to violations of the assumptions. In particular, it is shown that the horizontal motion of a floating structure remains bounded when it is initiated by the start up of a separate wave maker. The general discussion is supported by specific calculations for a vertical circular cylinder.

Key words: surface gravity waves, wave scattering, wave–structure interactions

1. Introduction

This paper is concerned with the linearized theory of water waves and structures in the time domain, in which it is assumed that an infinitesimal disturbance of the rest state leads to motions that remain infinitesimal for all time. This is true, for example, for the purely vertical motion of an unrestrained floating structure that is given a small vertical displacement and released from rest. In this case, the hydrostatic spring acts as a natural restoring force and the structure ultimately returns to its equilibrium position (Ursell 1964). However, for horizontal motion, and for yaw motion about a vertical axis, there is no natural restoring force and there is the possibility that the linearized theory will predict unbounded displacements of an unrestrained structure, in violation of one of the initial assumptions of the theory. The possibility of unbounded motions in modes without a natural restoring force has been alluded to in the literature, for example by John (1949) and Cummins (1962), and more recently by Hazard & Loret (2007) and Fitzgerald & Meylan (2011) in the context of generalized eigenfunction expansions, but it does not appear to have been investigated in detail. This is done in the present work, where attention is restricted to purely horizontal motions, and conditions are identified under which the linear theory predicts that an infinitesimal forcing results in the unbounded displacement of an unrestrained structure. Specifically, the occurrence of an apparent steady translation at large times is investigated. Further,

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and perhaps more importantly, the conditions are discussed under which there is no such steady translation so that the linearized theory remains valid for all time.

The approach used here is to take a Fourier transform in time and to examine the equation of motion for the structure in the frequency domain. The key observation is that a component of the solution for the large-time motion of the structure will be a steady translation if the frequency-domain displacement $x_1(\omega)$ has a double pole at the origin, when viewed as a function of complex frequency ω (McIver & McIver 2011, § 5). The form of $x_1(\omega)$ is fairly simple and identification of its pole structure at $\omega = 0$ proves to be straightforward. The possible motions of a structure are discussed in general and calculations are presented for a vertical circular cylinder extending throughout the depth of the fluid.

In § 2, consideration is given to an isolated, unrestrained structure that is set in horizontal motion by an impulse applied directly to the structure, and/or a pressure impulse applied to the free surface. The structure is assumed to be symmetric about the vertical plane containing the direction of the impulse. When the motion is caused solely by an impulse applied directly to the structure, which is equivalent to the specification of a non-zero initial velocity as long as the impulse imparted to the fluid is properly accounted for, the linear theory formally suggests that the structure continues to move in the direction of the impulse and that at large times the structure has a non-zero steady velocity, so that the assumptions of the linearized theory are violated. The release from rest of a structure in an existing ambient wave field can be interpreted as an initial-value problem in which the wave field is switched on at time zero and an additional impulse is applied to the structure to give an initial velocity of zero, and again there is an apparent non-zero steady velocity at large times. However, when the structure is set in motion indirectly by a pressure impulse there is no such non-zero asymptotic velocity and, even though the structure has a non-zero initial velocity, the assumptions of the theory are not violated.

The application of a pressure impulse to the free surface is unlikely to be directly relevant to problems of practical interest. Thus, in § 3, results are given for a structure that is set in motion indirectly by the action of a separate wave maker starting up from rest. This situation is mathematically similar to the application of a pressure impulse to the free surface, and again it is found that there is no violation of the linearizing assumptions.

This article is structured so that the principal points of the time-domain behaviour are discussed in the main body, while the frequency-domain solutions required for calculations are described in the appendices.

2. An isolated structure

2.1. *Formulation*

A structure with mass M floats in water of depth h above a horizontal bed S_{bed}. The fluid is assumed to be inviscid and incompressible, the flow to be irrotational, and all motions are assumed to be sufficiently small for the linearized theory of water waves to be applicable. In particular, this means that the boundary condition on the structure is applied on its initial wetted surface S_B (it is possible to choose a different reference position for the structure as long as its motion remains suitably small relative to that position). Cartesian coordinates (x, y, z) are chosen with z directed vertically upwards from the undisturbed free surface S_F. For simplicity, the structure is assumed to be symmetric about the vertical plane $x = 0$, and free to move in the x-direction only.

The fluid and structure are at rest for times $t < 0$, and the motion is initiated at $t = 0$ by a prescribed distribution of potential and elevation on the free surface, and/or a horizontal impulse I applied directly to the structure in the direction of the x-axis. The subsequent motion of the fluid is described by a velocity potential $\Phi(\boldsymbol{x}, z, t)$, and the motion of the structure is described by its horizontal displacement and velocity that are denoted by $X_1(t)$ and $V_1(t)(\equiv\dot{X}_1(t))$, respectively. The initial-value problem for the potential is as follows:

$$\nabla^2\Phi = 0 \quad \text{in } \mathscr{D}, \tag{2.1}$$

$$\frac{\partial^2\Phi}{\partial t^2} + g\frac{\partial\Phi}{\partial z} = 0 \quad \text{on } S_F, \tag{2.2}$$

$$\frac{\partial\Phi}{\partial n} = V_1(t)n_1 \quad \text{on } S_B, \tag{2.3}$$

$$\frac{\partial\Phi}{\partial n} = 0 \quad \text{on } S_{bed}, \tag{2.4}$$

$$|\nabla\Phi| \to 0 \quad \text{as } |\boldsymbol{x}| \to \infty, \tag{2.5}$$

and

$$\Phi(\boldsymbol{x}, 0, 0) = \Phi_0(\boldsymbol{x}), \quad \frac{\partial\Phi}{\partial t}(\boldsymbol{x}, 0, 0) = -g\mathscr{H}_0(\boldsymbol{x}), \quad \boldsymbol{x} \in S_F, \tag{2.6}$$

where $\mathscr{D}$ is the fluid domain, g is the acceleration due to gravity and $\mathscr{H}_0(\boldsymbol{x})$ is the initial elevation of the free surface. For $\boldsymbol{x} \in S_F$, $-\rho\Phi_0(\boldsymbol{x})$ may be interpreted as a pressure impulse applied to the free surface at $t = 0$ (Batchelor 1967, § 6.10); here ρ is the fluid density. Throughout this paper the normal coordinate n is directed out of the fluid domain and n_1 denotes the x-component of this normal. The velocity potential Φ and the displacement X_1 are also related through the equation of motion for the structure which, for $t > 0$, is

$$M\dot{V}_1(t) = -\rho\iint_{S_B}\frac{\partial\Phi}{\partial t}(\boldsymbol{x}, z, t)n_1\,\mathrm{d}S - c_{11}X_1(t) - d_{11}V_1(t), \tag{2.7}$$

where c_{11} and d_{11} are respectively spring and damper coefficients for any moorings (for most of what follows, $c_{11} = d_{11} = 0$).

The initial impulses applied to the fluid and structure give discontinuous changes in the velocity potential Φ and the velocity $V_1(t)$ of the structure (which are both zero for $t < 0$) that are related by

$$\Phi(\boldsymbol{x}, z, 0) = \Phi_S(\boldsymbol{x}, z, 0) + V_1(0)\Omega_1(\boldsymbol{x}, z) \tag{2.8}$$

(see McIver & McIver 2011, equation (34)). Here $\Phi_S(\boldsymbol{x}, z, t)$ is the solution to the above initial-value problem when the structure is held fixed, and $\Omega_1(\boldsymbol{x}, z)$ is the solution to the 'high-frequency radiation problem'

$$\nabla^2\Omega_1 = 0 \quad \text{in } \mathscr{D}, \tag{2.9}$$

$$\Omega_1 = 0 \quad \text{on } S_F, \tag{2.10}$$

$$\frac{\partial\Omega_1}{\partial n} = n_1 \quad \text{on } S_B, \tag{2.11}$$

$$\frac{\partial\Omega_1}{\partial n} = 0 \quad \text{on } S_{bed}, \tag{2.12}$$

and

$$\Omega_1 \to 0 \quad \text{as } |\boldsymbol{x}| \to \infty. \tag{2.13}$$

When the instantaneous change in the momentum of the structure at $t = 0$ is equated to the impulses acting upon it, this gives

$$MV_1(0) = -\rho \iint_{S_B} \Phi(\boldsymbol{x}, z, 0)\, n_1\, \mathrm{d}S + I = -\widetilde{\Phi}_{S,1} - V_1(0)\mu_{11}(\infty) + I, \tag{2.14}$$

where

$$\widetilde{\Phi}_{S,1} = \rho \iint_{S_B} \Phi_S(\boldsymbol{x}, z, 0)\, n_1\, \mathrm{d}S, \tag{2.15}$$

and

$$\mu_{11}(\infty) = \rho \iint_{S_B} \Omega_1(\boldsymbol{x}, z)\, n_1\, \mathrm{d}S \tag{2.16}$$

is the high-frequency limit of the added-mass coefficient for the structure. It follows from (2.14) that the initial velocity of the structure

$$V_1(0) = \frac{I - \widetilde{\Phi}_{S,1}}{M + \mu_{11}(\infty)}. \tag{2.17}$$

2.2. *The frequency-domain problem*

The frequency-domain problem is obtained by taking a Fourier transform in time t. For a function $F(t)$ that is piecewise smooth and satisfies

$$F(t) = 0, \quad t < 0, \tag{2.18}$$

its Fourier transform is defined as

$$f(\omega) = \int_0^\infty F(t)\, \mathrm{e}^{\mathrm{i}\omega t}\, \mathrm{d}t \equiv \mathscr{F}\{F(t)\}, \quad \operatorname{Im}\omega = \upsilon > 0, \tag{2.19}$$

and the corresponding inversion formula is

$$F(t) = \frac{1}{2\pi} \int_{-\infty+\mathrm{i}\upsilon}^{\infty+\mathrm{i}\upsilon} f(\omega)\, \mathrm{e}^{-\mathrm{i}\omega t}\, \mathrm{d}\omega \equiv \mathscr{F}^{-1}\{f(\omega)\} \tag{2.20}$$

(see Stakgold 2000, § 5.6). The positive imaginary part of ω is required to ensure the existence of the Fourier transform in cases where F grows algebraically as t tends to infinity.

Fourier transformation of the equation of motion (2.7) yields

$$\begin{aligned} M[-\mathrm{i}\omega v_1(\omega) - V_1(0)] \\ = -\rho \iint_{S_B} [-\mathrm{i}\omega\phi(\boldsymbol{x}, z, \omega) - \Phi(\boldsymbol{x}, z, 0)]\, n_1\, \mathrm{d}S - c_{11}x_1(\omega) - d_{11}v_1(\omega), \end{aligned} \tag{2.21}$$

where lower-case letters are used to denote the Fourier transforms of the corresponding time-domain quantities. It is convenient for the following discussion to make the decomposition

$$\phi(\boldsymbol{x}, z, \omega) = \phi_S(\boldsymbol{x}, z, \omega) + v_1(\omega)\phi_1(\boldsymbol{x}, z, \omega), \tag{2.22}$$

where the frequency-domain scattering potential ϕ_S satisfies

$$\nabla^2\phi_S = 0 \quad \text{in } \mathscr{D}, \tag{2.23}$$

$$\frac{\partial \phi_S}{\partial z} = \frac{\omega^2}{g}\phi_S - \frac{\mathrm{i}\omega}{g}\Phi_0 - \mathscr{H}_0 \quad \text{on } S_F, \tag{2.24}$$

$$\frac{\partial \phi_S}{\partial n} = 0 \quad \text{on } S_B, \tag{2.25}$$

and

$$\frac{\partial \phi_S}{\partial n} = 0 \quad \text{on } S_{bed}, \tag{2.26}$$

while the radiation potential ϕ_1 satisfies

$$\nabla^2 \phi_1 = 0 \quad \text{in } \mathscr{D}, \tag{2.27}$$

$$\frac{\partial \phi_1}{\partial z} = \frac{\omega^2}{g}\phi_1 \quad \text{on } S_F, \tag{2.28}$$

$$\frac{\partial \phi_1}{\partial n} = n_1 \quad \text{on } S_B, \tag{2.29}$$

and

$$\frac{\partial \phi_1}{\partial n} = 0 \quad \text{on } S_{bed}. \tag{2.30}$$

Causality requires that each of ϕ_S and ϕ_1 must also satisfy an appropriately formulated radiation condition. The potential ϕ_1 is a conventional radiation potential for time-harmonic motions but, in general, ϕ_S is not a scattering potential for time-harmonic motions because of the appearance of initial values in (2.24). With the coordinates chosen so that $X_1(0) = 0$, and hence so that the frequency-domain displacement $x_1(\omega) = v_1(\omega)/(-\mathrm{i}\omega)$, the equation of motion can be solved to give

$$x_1(\omega) = \frac{[M + \mu_{11}(\infty)]\, V_1(0) + \widetilde{\Phi}_{S,1} + \xi_1(\omega)}{c_{11} - \mathrm{i}\omega d_{11} - \omega^2 [M + \mu_{11}(\omega) + \mathrm{i}\nu_{11}(\omega)]} \tag{2.31}$$

(this is equivalent to the reduction to one mode of motion of equation (40) in McIver & McIver 2011) or, after substitution for the initial velocity given in (2.17),

$$x_1(\omega) = \frac{I + \xi_1(\omega)}{c_{11} - \mathrm{i}\omega d_{11} - \omega^2 [M + \mu_{11}(\omega) + \mathrm{i}\nu_{11}(\omega)]}. \tag{2.32}$$

Here, the exciting force

$$\xi_1(\omega) = \mathrm{i}\omega\rho \iint_{S_B} \phi_S(\boldsymbol{x}, z, \omega)\, n_1 \,\mathrm{d}S, \tag{2.33}$$

and the added-mass and damping coefficients, μ_{11} and $\omega\nu_{11}$ respectively, follow from

$$\mu_{11}(\omega) + \mathrm{i}\nu_{11}(\omega) = \rho \iint_{S_B} \phi_1(\boldsymbol{x}, z, \omega)\, n_1 \,\mathrm{d}S. \tag{2.34}$$

From McIver & McIver (2011, equation (46)), the time-domain exciting force $\Xi_1(t)$ is related to the frequency-domain exciting force $\xi_1(\omega)$ by

$$\Xi_1(t) = \mathscr{F}^{-1}\left\{\xi_1(\omega) + \widetilde{\Phi}_{S,1}\right\}, \tag{2.35}$$

and from properties of Fourier integrals (Bleistein & Handelsman 1986, § 3.2)

$$\lim_{|\omega|\to\infty} \xi_1(\omega) = -\widetilde{\Phi}_{S,1}. \tag{2.36}$$

A pole in $\xi_1(\omega)$ at $\omega = 0$ would imply that there is a non-oscillatory component of $\Xi_1(t)$ that does not decay to zero as $t \to \infty$ (McIver & McIver 2011, § 5); it will be assumed in the following that there is no such pole. (For a given geometry, the absence of such a pole may be demonstrated by calculation.)

The possible low-frequency asymptotic forms, that is as $\omega \to 0$, of $\mu_{11}(\omega) + \mathrm{i}\nu_{11}(\omega)$ have been extensively investigated and a review is given by McIver (1994*a*). In particular, it is known that $\nu_{11}(0) = 0$, and that $\mu_{11}(0)$ is finite and positive.

2.3. *Large-time asymptotics*

The large-time asymptotics of $X_1(t)$ can be deduced from the form of $x_1(\omega)$ as $\omega \to 0$. Specifically, from the results given by McIver & McIver (2011, § 5), if $x_1(\omega) \sim -V_\infty/\omega^2$ as $\omega \to 0$, where V_∞ is independent of ω, then $X_1(t) \sim V_\infty t$ as $t \to \infty$. A non-zero value of V_∞ violates the assumptions of the linearized theory as the predicted displacement of the structure is no longer appropriately small for all time, and hence the boundary condition (2.3) cannot be applied on the initial surface S_B. If $x_1(\omega)$ does not have a double pole at $\omega = 0$ then $V_\infty = 0$. This is the case when the structure is moored so that $c_{11} \neq 0$ and/or $d_{11} \neq 0$. The presence of a spring provides a restoring force that constrains the motion of the structure to a neighbourhood of its original position, while with $c_{11} = 0$, but $d_{11} \neq 0$, $x_1(\omega)$ may have a simple pole at $\omega = 0$ which corresponds to a finite asymptotic displacement of the structure. From now on it will be assumed that the structure is unrestrained so that $c_{11} = d_{11} = 0$ and then, from (2.17), (2.31) and (2.32)

$$V_\infty = \frac{[M + \mu_{11}(\infty)]\, V_1(0) + \widetilde{\Phi}_{S,1} + \xi_1(0)}{M + \mu_{11}(0)} = \frac{I + \xi_1(0)}{M + \mu_{11}(0)}. \tag{2.37}$$

These results may also be obtained from consideration of the change in the momentum of the structure over the time interval $(0, \infty)$.

A number of different scenarios are discussed in the following subsections in which numerical calculations are compared with asymptotic results. The procedure used for the numerical evaluation of the time-domain displacement $X_1(t)$ by inverse Fourier transform is described in appendix A. It is noted there that the required inverse transforms may be expressed in terms of inverse Fourier cosine or sine transforms; for the particular cases described in this section evaluation of the Fourier integrals using an inverse Fourier sine transform is computationally advantageous.

2.3.1. *Motion initiated by an applied impulse*

The simplest case is when there is no scattered wave field (that is $\Phi_0(\boldsymbol{x}) = \mathscr{H}_0(\boldsymbol{x}) \equiv 0$) and the motion is initiated by a non-zero impulse I applied to the structure, which is equivalent to prescribing a non-zero initial velocity (as long as the impulse imparted to the fluid is correctly accounted for). From (2.37), $V_\infty \neq 0$ and the assumptions of the linearized theory are violated. A sample computation for a vertical circular cylinder of radius a that extends throughout the depth is shown in figure 1; in this, and all subsequent computations, lengths are scaled by a, and time by $\sqrt{h/g}$. The required hydrodynamic coefficients $\mu_{11}(\omega)$ and $\nu_{11}(\omega)$ are readily computed from the well-known solution for the radiation of waves by the horizontal oscillations of a vertical cylinder; see, for example, Dean & Dalrymple (1991, § 6.4). For all of the computations reported in this paper, the mass M of the cylinder is taken to be $\rho\pi a^2 h$. As the values of $\mu_{11}(0)$ and $\mu_{11}(\infty)$ do not differ greatly, it follows from the first of (2.37) that $V_\infty/V_1(0)$ is close to unity.

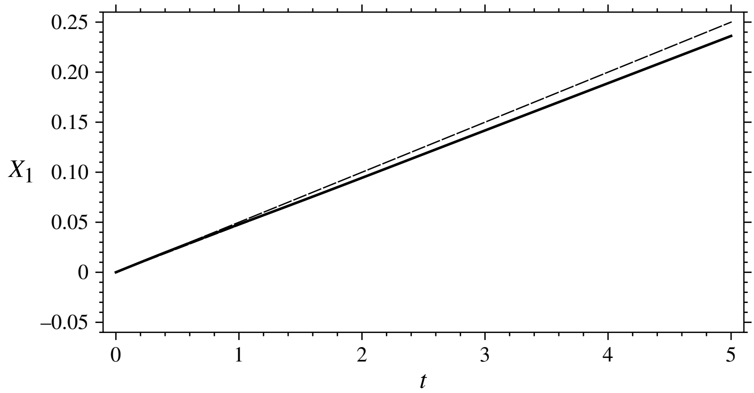

FIGURE 1. The displacement X_1 versus time t of a cylinder of radius $a = 0.1h$ given an initial velocity $V_1(0) = 0.05a\sqrt{g/h}$. The dashed line has slope $V_1(0)$.

2.3.2. *Release from rest in an ambient wave field*

If the structure is unrestrained and has an initial velocity $V_1(0) = 0$ then, from the first of (2.37),

$$V_\infty = \frac{\widetilde{\Phi}_{S,1} + \xi_1(0)}{M + \mu_{11}(0)}, \tag{2.38}$$

which is non-zero provided that $\widetilde{\Phi}_{S,1} + \xi_1(0) \neq 0$. From (2.17), $V_1(0) = 0$ occurs when the applied impulse I negates exactly the impulse $\widetilde{\Phi}_{S,1}$ arising from the pressure impulse on the free surface. From the viewpoint of times t strictly greater than zero, so that the impulses that initiate the motion are not considered, this is equivalent to the structure being released from rest within an existing wave field. 'Switching on' the ambient wave field produces a pressure impulse on the structure which, to ensure that $V_1(0) = 0$, is negated by an equal and opposite direct impulse. In view of the great variety of ambient wave fields that might be prescribed, it is difficult to make general statements about this situation but an illustrative example is now described.

Suppose that for times $t \geqslant 0$ a time-domain scattered wave field has the form

$$\Phi_S(\boldsymbol{x}, z, t) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left[-\frac{\mathrm{i}gA(\omega')}{\omega'}\right]\psi_S(\boldsymbol{x}, z, \omega')\,\mathrm{e}^{-\mathrm{i}\omega' t}\,\mathrm{d}\omega', \tag{2.39}$$

where for $\omega \geqslant 0$

$$\psi_S(\boldsymbol{x}, z, \omega) = \mathrm{e}^{\mathrm{i}kx}\frac{\cosh k(z+h)}{\cosh kh} + \psi_D(\boldsymbol{x}, z, \omega), \tag{2.40}$$

$\psi_S(\boldsymbol{x}, z, -\omega) = \overline{\psi_S(\boldsymbol{x}, z, \omega)}$, and $\psi_D(\boldsymbol{x}, z, \omega)$ satisfies a radiation condition that specifies outgoing waves. The corresponding initial values of the free-surface potential Φ_0 and elevation $\mathscr{H}_0$ follow from (2.39). Here k is the positive real root of

$$\omega^2 = gk \tanh kh, \tag{2.41}$$

and $A(\omega)$ is prescribed with $A(-\omega) = \overline{A(\omega)}$ and $A(0) = 0$. The potential $\psi_S(\boldsymbol{x}, z, \omega)$ describes a time-harmonic scattered wave field corresponding to a long-crested incident wave with wavenumber k that propagates in the x-direction.

For the time-domain potential in (2.39), Fourier transformation using the definition (2.19) yields the corresponding frequency-domain potential

$$\phi_S(\boldsymbol{x}, z, \omega) = -\frac{g}{2\pi} \int_{-\infty}^{\infty} \frac{A(\omega')\, \psi_S(\boldsymbol{x}, z, \omega')}{\omega'(\omega' - \omega)}\, \mathrm{d}\omega', \quad \operatorname{Im} \omega > 0, \tag{2.42}$$

so that the frequency-domain exciting force

$$\xi_1(\omega) = \mathrm{i}\omega\rho \iint_{S_B} \phi_S(\boldsymbol{x}, z, \omega)\, n_1\, \mathrm{d}S = -\frac{\mathrm{i}\omega}{2\pi\mathscr{A}} \int_{-\infty}^{\infty} \frac{A(\omega')\, \chi_1(\omega')}{\omega'(\omega' - \omega)}\, \mathrm{d}\omega', \tag{2.43}$$

where

$$\chi_1(\omega) = \mathrm{i}\omega\rho \iint_{S_B} \left(\frac{-\mathrm{i}g\mathscr{A}}{\omega}\right) \psi_S(\boldsymbol{x}, z, \omega)\, n_1\, \mathrm{d}S = \rho g \mathscr{A} \iint_{S_B} \psi_S(\boldsymbol{x}, z, \omega)\, n_1\, \mathrm{d}S \tag{2.44}$$

is the conventional exciting force for an incident time-harmonic wave of amplitude $\mathscr{A}$. In addition

$$\widetilde{\Phi}_{S,1} \equiv \rho \iint_{S_B} \Phi_S(\boldsymbol{x}, z, 0)\, n_1\, \mathrm{d}S = -\frac{\mathrm{i}}{2\pi\mathscr{A}} \int_{-\infty}^{\infty} \frac{A(\omega')\, \chi_1(\omega')}{\omega'}\, \mathrm{d}\omega' \tag{2.45}$$

so that

$$\widetilde{\Phi}_{S,1} + \xi_1(0) = -\frac{\mathrm{i}}{2\pi\mathscr{A}} \int_{-\infty}^{\infty} \frac{A(\omega')\, \chi_1(\omega')}{\omega'}\, \mathrm{d}\omega' = \frac{1}{\pi\mathscr{A}} \operatorname{Im} \int_{0}^{\infty} \frac{A(\omega')\, \chi_1(\omega')}{\omega'}\, \mathrm{d}\omega'. \tag{2.46}$$

It is possible to choose $A(\omega)$ to give $\widetilde{\Phi}_{S,1} + \xi_1(0) = 0$ but, in general, $\widetilde{\Phi}_{S,1} + \xi_1(0) \neq 0$ so that the asymptotic velocity $V_\infty \neq 0$ and the assumption of small motion is violated.

A time-domain ambient wave field that is time harmonic with frequency $\hat{\omega}$ is recovered by taking

$$A(\omega) = \pi\mathscr{A}[\delta(\omega + \hat{\omega}) + \delta(\omega - \hat{\omega})], \tag{2.47}$$

where δ is the Dirac delta function, so that (2.39) and (2.46) become, respectively,

$$\Phi_S(\boldsymbol{x}, z, t) = \operatorname{Re}\left[-\frac{\mathrm{i}g\mathscr{A}}{\hat{\omega}} \psi_S(\boldsymbol{x}, z, \hat{\omega})\, \mathrm{e}^{-\mathrm{i}\hat{\omega}t}\right], \tag{2.48}$$

and

$$\widetilde{\Phi}_{S,1} + \xi_1(0) = \frac{\operatorname{Im} \chi_1(\hat{\omega})}{\hat{\omega}}. \tag{2.49}$$

The exciting force χ_1 for time-harmonic waves is readily computed for many structures. For example, in the case that S_B is a vertical cylinder of radius a extending throughout the depth (Linton & McIver 2001, § 2.4.1)

$$\widetilde{\Phi}_{S,1} + \xi_1(0) = -\frac{4\rho\hat{\omega}\mathscr{A}\, \mathrm{Y}_1'(\hat{k}a)}{\hat{k}^3\, |\mathrm{H}_1'(\hat{k}a)|^2} \tag{2.50}$$

where Y_1 is a Bessel function, H_1 a Hankel function of the first kind, the primes denote differentiation with respect to the function argument, and $\hat{k}$ is the positive real root of $\hat{\omega}^2 = g\hat{k} \tanh \hat{k}h$. In general $\widetilde{\Phi}_{S,1} + \xi_1(0) \neq 0$, and hence $V_\infty \neq 0$, and as $t \to \infty$ the solution is a translation with a superimposed steady oscillation of frequency $\hat{\omega}$. However, whenever $\hat{k}a$ is a zero of Y_1' the translation is eliminated. It might be anticipated that other structures will display a similar behaviour with no asymptotic steady translation at isolated wave frequencies.

2.3.3. *Motion initiated by a pressure impulse*

Motion initiated by a pressure impulse applied to the free surface is now investigated, both with and without an impulse applied directly to the structure. Again, calculations are presented for a vertical cylinder of radius a extending throughout the depth. The free surface is chosen to be initially flat, so that $\mathscr{H}_0(\boldsymbol{x}) = 0$ for $\boldsymbol{x} \in S_F$, and the initial free-surface potential is taken as

$$\Phi_0(\boldsymbol{x}) = \sqrt{\frac{g}{\gamma^3}}\,\mathrm{K}_1(\gamma r)\cos\theta, \quad r > a, \tag{2.51}$$

where (r, θ) are horizontal polar coordinates with origin on the axis of the cylinder, K_1 is a modified Bessel function, and γ is a constant. The solution to the scattering problem is obtained in appendix B and, in particular, the exciting force $\xi_1(\omega)$ is given in (B 9). In this case $\xi_1(0) = 0$ and hence, from the second of (2.37), there is no asymptotic steady translation unless an impulse is applied directly to the structure. Furthermore, from (B 14), $\widetilde{\Phi}_{S,1} \neq 0$ and hence, in general, the structure has a non-zero initial velocity.

The displacement of the structure is shown in figure 2 when there is both a pressure impulse and a direct impulse I (the latter is obtained by the equivalent process of specifying an initial velocity). In this case, the theory predicts that the displacement of the structure settles to a decaying oscillation about $X_1 = V_\infty t$.

Figure 3 shows a calculation when there is a pressure impulse but no impulse I applied directly to the structure so that, from (2.37), $V_\infty = 0$ and the large-time behaviour is a decaying oscillation about $X_1 = 0$. There seems to be no reason to think that $\xi_1(0) = 0$ is atypical for motions initiated by a pressure impulse and therefore, despite the fact that the initial velocity $V_1(0) \neq 0$, there is no asymptotic steady translation when the structure moves solely as a result of fluctuations in the fluid pressure. This can be understood by decomposing the motion of the structure into two components, namely: (i) that part arising solely from the initial pressure impulse; and (ii) that part, in $t > 0$, arising from the evolution of the scattered wave field. From (2.14), in the equation of motion for the structure the pressure impulse is equivalent to a directly applied impulse $I = -\widetilde{\Phi}_{S,1}$ and, by the second of (2.37), this contributes an amount $-\widetilde{\Phi}_{S,1}/[M+\mu_{11}(0)]$ to the asymptotic velocity. The initial impulse accounts for the non-zero value of $V_1(0)$ and hence, in the present decomposition, the motion of the structure arising solely from the scattered field in $t > 0$ has $V_1(0) = 0$. Then, from the first of (2.37) with $\xi_1(0) = 0$, the corresponding contribution to the asymptotic velocity is $\widetilde{\Phi}_{S,1}/[M + \mu_{11}(0)]$, which exactly cancels that arising from the initial impulse.

To shed more light on motions generated by a pressure impulse in a more practical setting, the motion initiated by a wave maker in the presence of a floating structure is investigated in the following section.

3. Response to a wave maker

3.1. *Formulation*

The problem is now modified by introducing into the fluid domain a wave maker that performs small motions in the x-direction, with a prescribed velocity $V_W(t)$, about the wetted surface S_W. The geometrical form of this wave maker need not be specified at this stage, although figure 4 illustrates the geometry and coordinate systems when

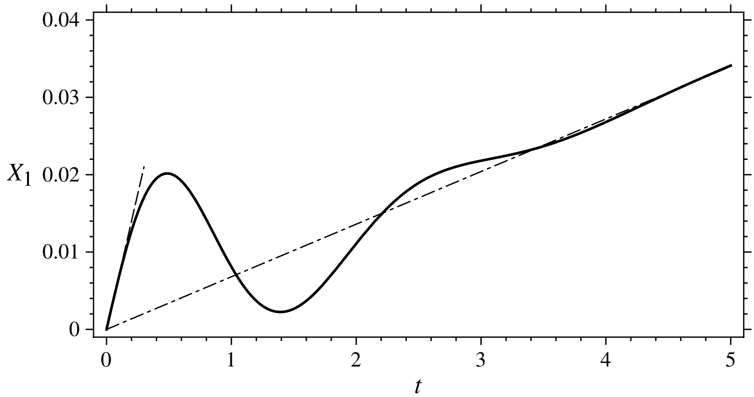

FIGURE 2. The displacement X_1 versus time t of a cylinder of radius $a = 0.1h$ given an initial velocity $V_1(0) = 0.07a\sqrt{g/h}$ and for an initial free-surface potential (2.51) with $\gamma a = 1$. The dashed line has slope $V_1(0)$ and the dot-dash line has slope V_∞.

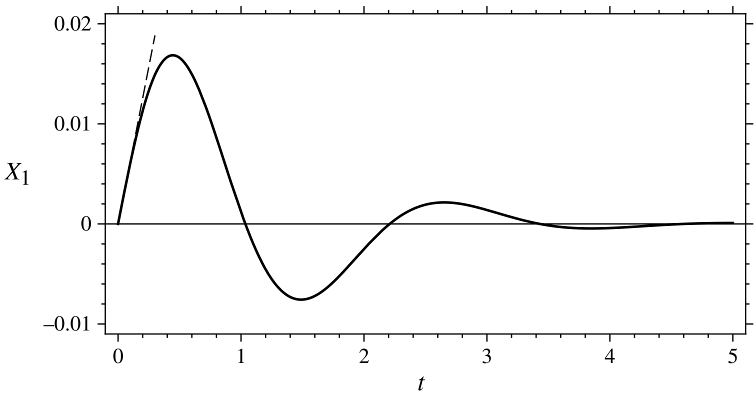

FIGURE 3. The displacement X_1 versus time t of a cylinder of radius $a = 0.1h$ for an initial free-surface potential (2.51) with $\gamma a = 1$. The dashed line has slope $V_1(0)$.

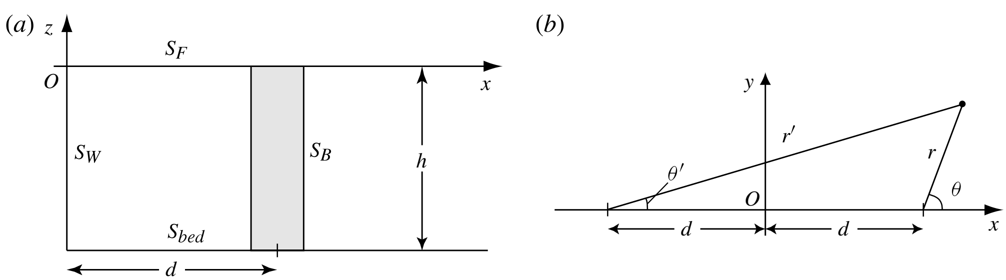

FIGURE 4. Coordinate systems for § 3 illustrated in (a) a vertical plane and (b) a horizontal plane.

S_W is a vertical wall and the structure S_B is a vertical cylinder, both extending throughout the depth. As before, the motion of the fluid is described by a velocity potential $\Phi(\boldsymbol{x}, z, t)$, and the motion of the structure by its horizontal displacement $X_1(t)$, measured from its initial position, and by its velocity $V_1(t)$. The fluid and structures are at rest for $t < 0$ and the motion is initiated solely by the motion of the wave maker. The initial-value problem for the potential is as follows:

$$\nabla^2 \Phi = 0 \quad \text{in } \mathscr{D}, \tag{3.1}$$

$$\frac{\partial^2 \Phi}{\partial t^2} + g\frac{\partial \Phi}{\partial z} = 0 \quad \text{on } S_F, \tag{3.2}$$

$$\frac{\partial \Phi}{\partial n} = V_W(t) n_1 \quad \text{on } S_W, \tag{3.3}$$

$$\frac{\partial \Phi}{\partial n} = V_1(t) n_1 \quad \text{on } S_B, \tag{3.4}$$

$$\frac{\partial \Phi}{\partial n} = 0 \quad \text{on } S_{bed}, \tag{3.5}$$

$$|\nabla \Phi| \to 0 \quad \text{as } |\boldsymbol{x}| \to \infty \quad \text{within } \mathscr{D}, \tag{3.6}$$

and

$$\Phi(\boldsymbol{x}, 0, 0) = \frac{\partial \Phi}{\partial t}(\boldsymbol{x}, 0, 0) = 0, \quad \boldsymbol{x} \in S_F. \tag{3.7}$$

The Fourier transform $\phi(\boldsymbol{x}, z, \omega)$ of the time-domain potential $\Phi(\boldsymbol{x}, z, t)$ is written as

$$\phi(\boldsymbol{x}, z, \omega) = v_W(\omega)\phi_W(\boldsymbol{x}, z, \omega) + v_1(\omega)\phi_1(\omega) \tag{3.8}$$

where again lower-case letters are used to denote the Fourier transforms of the corresponding time-domain quantities. The potential ϕ_1 describes the forced oscillations of the structure when the wave maker is held fixed, and solves the problem for ϕ_1 given in § 2 with the addition of

$$\frac{\partial \phi_1}{\partial n} = 0 \quad \text{on } S_W. \tag{3.9}$$

The potential ϕ_W describes the forced oscillations of the wave maker when the structure is held fixed, and solves

$$\nabla^2 \phi_W = 0 \quad \text{in } \mathscr{D}, \tag{3.10}$$

$$\frac{\partial \phi_W}{\partial z} = \frac{\omega^2}{g}\phi_W \quad \text{on } S_F, \tag{3.11}$$

$$\frac{\partial \phi_W}{\partial n} = n_1 \quad \text{on } S_W, \tag{3.12}$$

and

$$\frac{\partial \phi_W}{\partial n} = 0 \quad \text{on } S_B \cup S_{bed}, \tag{3.13}$$

together with a radiation condition.

The start up of the wave maker may generate a pressure impulse which instantaneously causes the structure to move with a non-zero velocity. In a similar fashion to (2.8), this results in an initial potential

$$\Phi(\boldsymbol{x}, z, 0) = V_W(0)\Omega_W(\boldsymbol{x}, z) + V_1(0)\Omega_1(\boldsymbol{x}, z), \tag{3.14}$$

where Ω_1 solves the same problem as the Ω_1 that appears in § 2, with the addition of a homogeneous Neumann condition on S_W, while Ω_W solves a similar problem that includes the conditions

$$\frac{\partial \Omega_W}{\partial n} = n_1 \quad \text{on } S_W, \tag{3.15}$$

and

$$\frac{\partial \Omega_W}{\partial n} = 0 \quad \text{on } S_B. \tag{3.16}$$

The change in momentum of the structure induced by the start up of the wave maker is

$$MV_1(0) = -\rho \iint_{S_B} \Phi(\boldsymbol{x}, z, 0)\, n_1 \,\mathrm{d}S = -V_W(0)\widetilde{\Phi}_{W,1} - V_1(0)\mu_{11}(\infty), \tag{3.17}$$

where

$$\widetilde{\Phi}_{W,1} = \rho \iint_{S_B} \Omega_W(\boldsymbol{x}, z)\, n_1 \,\mathrm{d}S \tag{3.18}$$

and $\mu_{11}(\infty)$ is given by (2.16), and hence the initial velocity of the structure is

$$V_1(0) = -\frac{V_W(0)\widetilde{\Phi}_{W,1}}{M + \mu_{11}(\infty)}. \tag{3.19}$$

The equation of motion for the structure is identical in form to that in (2.7). Fourier transformation in time and introduction of the decomposition (3.8) yields, with the aid of (3.14) and (3.19),

$$x_1(\omega) = \frac{v_W(\omega)\xi_1(\omega)}{c_{11} - \mathrm{i}\omega d_{11} - \omega^2\,[M + \mu_{11}(\omega) + \mathrm{i}\nu_{11}(\omega)]}, \tag{3.20}$$

where

$$\xi_1(\omega) = \mathrm{i}\omega\rho \iint_{S_B} \phi_W(\boldsymbol{x}, z, \omega)\, n_1 \,\mathrm{d}S. \tag{3.21}$$

The notation here is chosen to mimic that used in § 2; thus $\widetilde{\Phi}_{W,1}$ is proportional to the fluid impulse on S_B due to the start up of the wave maker, and $\xi_1(\omega)$ is proportional to the exciting force on S_B due to unit oscillations of the wave maker. An alternative notation is in terms of conventional added-mass and damping coefficients that describe the forces on the structure due to the motion of the wave maker. It follows from (3.18) and (3.21) that

$$\widetilde{\Phi}_{W,1} = \lim_{\omega\to\infty} \frac{\xi_1(\omega)}{\mathrm{i}\omega}. \tag{3.22}$$

As discussed in § 2.3, for there to be a steady translation of the structure at large times the frequency-domain displacement $x_1(\omega)$ must have a double pole at $\omega = 0$. From (3.20), when the structure is unrestrained so that $c_{11} = d_{11} = 0$, this will occur provided $v_W(0)\xi_1(0) \neq 0$. The Fourier transform $v_W(\omega)$ of the wave-maker velocity may indeed be non-zero at $\omega = 0$; for example, the velocity

$$V_W(t) = \mathscr{V}_0 \cos\hat{\omega}t\, \mathrm{e}^{-\beta t}, \quad \beta > 0, \tag{3.23}$$

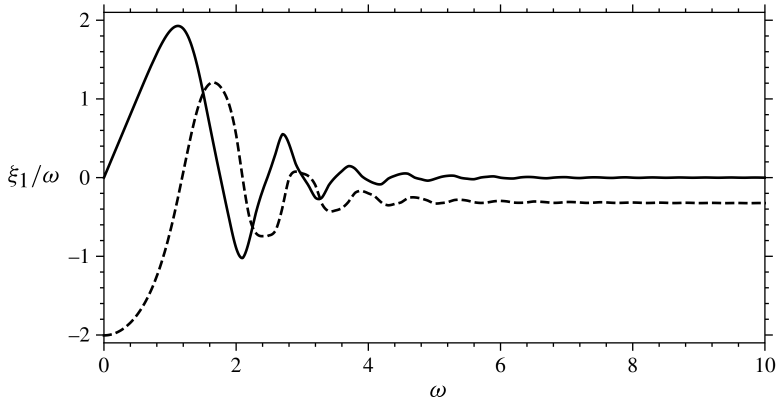

FIGURE 5. The real (solid line) and imaginary (dashed line) parts of the ratio of exciting force to frequency $\xi_1(\omega)/\omega$ versus frequency ω for a cylinder of radius $a = 0.1h$ a distance $d = h$ from a wave maker.

has the Fourier transform

$$v_W(\omega) = \frac{\mathscr{V}_0(\beta - \mathrm{i}\omega)}{(\beta - \mathrm{i}\omega)^2 + \hat{\omega}^2} \sim \frac{\mathscr{V}_0\beta}{\beta^2 + \hat{\omega}^2} \quad \text{as } \omega \to 0. \tag{3.24}$$

The existence of a double pole in $x_1(\omega)$ then depends on $\xi_1(0)$ which may be calculated for a given geometry. In the next subsection, the solution is investigated for a vertical cylinder excited by the motion of a vertical wall.

3.2. *A vertical cylinder excited by the motion of wave maker*

An unrestrained vertical cylinder of radius a extending throughout the depth is initially a distance d from a wave maker which occupies that part of the plane $x = 0$ in $-h < z < 0$. The geometry and coordinate systems are sketched in figure 4, and methods for the calculation of the required hydrodynamic quantities are described in appendix C. From the discussion at the end of § 3.1, the value of the zero-frequency exciting force $\xi_1(0)$ is key to understanding the large-time behaviour of the cylinder. Typical behaviour of $\xi_1(\omega)/\omega$ is illustrated in figure 5 and it is clear that $\xi_1(0) = 0$ so that there is no double pole at $\omega = 0$ in the frequency-domain displacement $x_1(\omega)$, and hence no asymptotic steady translation of the cylinder.

A calculation of the cylinder displacement $X_1(t)$ for the decaying wave-maker velocity (3.23) is given in figure 6. In this case, $x_1(\omega)$ has a simple pole at $\omega = 0$ and, from the residue at that pole,

$$X_1(t) \to X_\infty \equiv \frac{v_W(0)}{M + \mu_{11}(0)} \lim_{\omega \to 0} \frac{\mathrm{i}\xi_1(\omega)}{\omega} \quad \text{as } t \to \infty \tag{3.25}$$

(McIver & McIver 2011, § 5) so that the cylinder is displaced a distance X_∞ from its initial position. It may be shown from the reciprocity relation (C 29) that, in fact, $X_\infty = v_W(0)$ so that the asymptotic displacement depends only on the motion of the wave maker. (The method of calculation for $X_1(t)$ is described in appendix A. For both of the calculations presented in this section the inverse cosine transform is appropriate.)

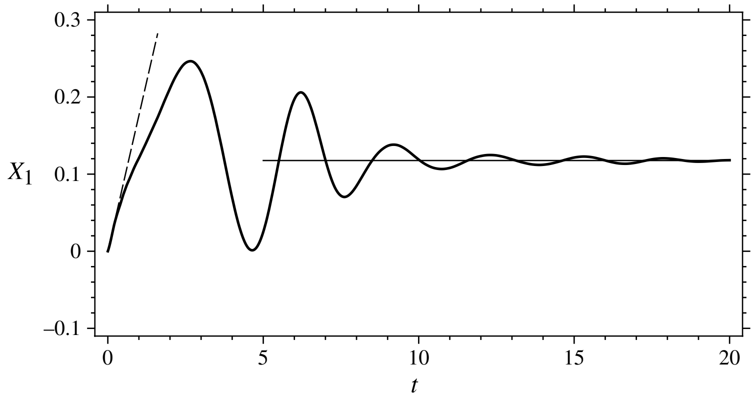

FIGURE 6. The displacement X_1 versus time t of a cylinder of radius $a = 0.1h$ a distance $d = h$ from a wave maker which has velocity (3.23), where $\hat{\omega} = 2\sqrt{g/h}$, $\beta = \sqrt{g/h}$ and $\mathscr{V}_0 = a\sqrt{g/h}$. The dashed line has slope $V_1(0)$ and the solid line shows the asymptotic displacement X_∞.

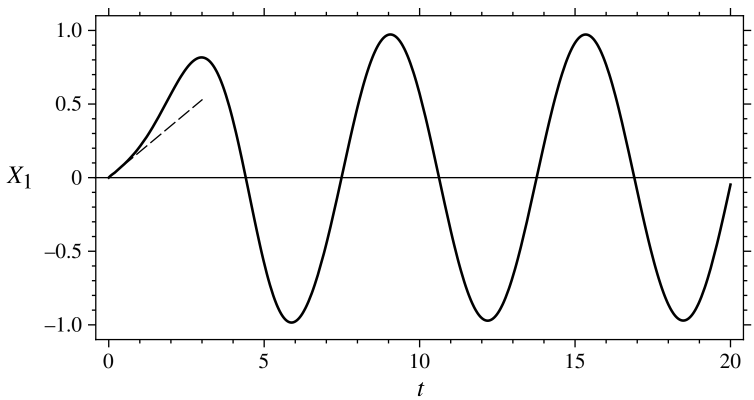

FIGURE 7. The displacement X_1 versus time t of a cylinder of radius $a = 0.1h$ a distance $d = h$ from a wave maker which has velocity (3.23), where $\hat{\omega} = \sqrt{g/h}$ and $\mathscr{V}_0 = a\sqrt{g/h}$. The dashed line has slope $V_1(0)$.

The final calculation, shown in figure 7, is for the oscillatory wave-maker velocity (3.23)

$$V_W(t) = \mathscr{V}_0 \cos \hat{\omega} t \tag{3.26}$$

for which the Fourier transform

$$v_W(\omega) = \frac{\mathrm{i}\omega\mathscr{V}_0}{\omega^2 - \hat{\omega}^2} \sim -\frac{\mathrm{i}\omega\mathscr{V}_0}{\hat{\omega}^2} \quad \text{as } \omega \to 0. \tag{3.27}$$

Now $x_1(\omega)$ has no pole of any order at $\omega = 0$ and the cylinder settles to an oscillation about its initial position with frequency $\hat{\omega}$; this is an example of the 'limiting amplitude principle' which is proved for the case of an oscillatory forcing applied directly to a structure by Vullierme-Ledard (1987). In neither of the cases illustrated in figures 6 and 7 are the assumptions of the linear theory violated.

3.3. *An integro-differential equation*

Cummins (1962) obtains an integro-differential equation for the time-domain motion of a structure with coefficients in terms of frequency-domain quantities. The derivation is based on the responses to impulsive motions, but a direct derivation of such equations using the inverse Fourier transform is used, for example, by Yu & Falnes (1995), Meylan & Sturova (2009) and McIver & McIver (2011), and this approach is followed here.

When expressed in terms of the frequency-domain velocity $v_1(\omega)$, and with the aid of (3.19), (3.20) may be written as

$$\begin{aligned}&[M+\mu_{11}(\infty)]\,[-\mathrm{i}\omega v_1(\omega)-V_1(0)]\\ &\quad+[\mu_{11}(\omega)-\mu_{11}(\infty)+\mathrm{i}\nu_{11}(\omega)]\,[-\mathrm{i}\omega v_1(\omega)-V_1(0)]\\ &\quad+l_{11}(\omega)V_1(0)+c_{11}x_1(\omega)+d_{11}v_1(\omega)\\ &=\left[l_{W,1}(\omega)-l_{W,1}(\infty)\right][\mathrm{i}\omega v_W(\omega)+V_W(0)]+\left[l_{W,1}(\omega)-l_{W,1}(\infty)\right]V_W(0)\\ &\quad+l_{W,1}(\infty)\,[\mathrm{i}\omega v_W(\omega)+V_W(0)]\,,\end{aligned} \tag{3.28}$$

where

$$l_{W,1}(\omega)=\frac{\xi_1(\omega)}{\mathrm{i}\omega}\quad\text{and}\quad l_{W,1}(\infty)=\lim_{\omega\to\infty}l_{W,1}(\omega)\equiv\widetilde{\varPhi}_{W,1}. \tag{3.29}$$

Inverse Fourier transform and application of the convolution theorem then gives, for times $t\geqslant 0$,

$$\begin{aligned}&[M+\mu_{11}(\infty)]\,\dot V_1(t)+\int_0^t L_{11}(t-\tau)\dot V_1(\tau)\,\mathrm{d}\tau\\ &\quad+L_{11}(t)V_1(0)+c_{11}X_1(t)+d_{11}V_1(t)=\varXi_1(t),\end{aligned} \tag{3.30}$$

where

$$L_{11}(t)=\mathscr{F}^{-1}\{\mu_{11}(\omega)-\mu_{11}(\infty)+\mathrm{i}\nu_{11}(\omega)\}=\frac{2}{\pi}\int_0^\infty \nu_{11}(\omega)\sin\omega t\,\mathrm{d}\omega, \tag{3.31}$$

$$\varXi_1(t)=-\int_0^t L_{W,1}(t-\tau)\dot V_W(\tau)\,\mathrm{d}\tau-L_{W,1}(t)V_W(0)-l_{W,1}(\infty)\dot V_W(t) \tag{3.32}$$

and

$$L_{W,1}(t)=\mathscr{F}^{-1}\left\{l_{W,1}(\omega)-l_{W,1}(\infty)\right\}=\frac{2}{\pi}\int_0^\infty \mathrm{Im}\left[\frac{\xi_1(\omega)}{\mathrm{i}\omega}\right]\sin\omega t\,\mathrm{d}\omega. \tag{3.33}$$

Equation (3.30) is an integro-differential equation for the displacement $X_1(t)$ of the structure.

Here L_{11} and $L_{W,1}$ are so-called impulse response functions, that is inverse Fourier transforms of time-harmonic quantities. It is known that an impulse response function may be non-causal, so that it is non-zero for most times $t<0$, in situations where the exciting force is used as the output and the wave elevation at some reference location is used as the input (Falnes 1995). This lack of causality arises because the chosen input is not the true cause of the output; the actual cause is whatever mechanism generates the waves. The impulse response functions used here, and in particular $L_{W,1}$ which governs the excitation, are causal as the input is the motion of the wave maker.

Although no new calculations using (3.30) are presented here, it has been verified that it reproduces the results presented in figures 6 and 7. Although for these particular examples, computations are more straightforward with a direct application of the inverse Fourier transform, (3.30) may be advantageous for complicated forcing functions $V_W(t)$.

4. Conclusion

The aim of this paper is to shed light on the circumstances under which the linearized theory of water waves predicts motions which violate the assumptions of the theory. Specifically, initial conditions which lead to apparently unbounded horizontal motions of an unrestrained floating structure have been identified. Initial conditions which involve an impulse directly applied to the structure will, in general, lead to a motion that at large times is dominated by a steady translation. The simplest such case is when an horizontal impulse is applied to a structure residing in a quiescent fluid, but the same phenomenon is, in general, observed when a structure is released from rest within an existing wave field. In the latter case, a direct impulse is required to negate the non-zero initial velocity that arises from 'switching on' the ambient waves. As far as the author is aware, the correct long-time behaviour of the impulsively started free motion of an unrestrained structure in water with a free surface has not been investigated. In contrast to this, the fluid motion arising from the *prescribed* motion of a structure started impulsively from rest has been studied extensively (see, for example, Joo, Schultz & Messiter 1990).

Unlike the motion of a structure initiated by a direct impulse, motions initiated indirectly by a pressure impulse do not display an asymptotic translation, and hence the assumptions of the theory are not violated. Two particular examples of this are the application of a pressure impulse to the free surface, and the pressure impulse generated by the start up of a wave maker. This is significant as it confirms that the standard formulation of the linearized theory of water waves may be used for a structure in a wave tank when the motions of both the fluid and the structure are started from rest by the action of the wave maker. It would be useful to extend this work to include nonlinear effects.

Although this paper has been concerned with unrestrained horizontal motions, it can be anticipated that similar conclusions apply to yaw motions for which, again, there is no natural restoring force.

Appendix A. Numerical evaluation of the inverse Fourier transform

Here the numerical evaluation of the inverse Fourier transform

$$X_1(t) = \frac{1}{2\pi}\int_{-\infty+\mathrm{i}v}^{\infty+\mathrm{i}v} x_1(\omega)\,\mathrm{e}^{-\mathrm{i}\omega t}\,\mathrm{d}\omega \tag{A 1}$$

is considered in the case that

$$x_1(\omega) = -\frac{V_\infty}{\omega^2} + \frac{\mathrm{i}X_\infty}{\omega} + O(1) \quad \text{as } \omega \to 0, \tag{A 2}$$

and

$$x_1(\omega) \sim \frac{x_0}{\omega - \hat{\omega}} \quad \text{as } \omega \to \hat{\omega} \tag{A 3}$$

with $\hat{\omega} \in \mathbb{R}$. From the definition of the Fourier transform equation (2.19), $x_1(-\overline{\omega}) = \overline{x_1(\omega)}$ and hence there must also be a pole of $x_1(\omega)$ at $\omega = -\hat{\omega}$ with

$$x_1(\omega) \sim \frac{-\overline{x_0}}{\omega + \hat{\omega}} \quad \text{as } \omega \to -\hat{\omega}. \tag{A 4}$$

It is assumed that all other singularities of $x_1(\omega)$ are in the lower half of the complex-ω-plane.

The inverse transform equation (A 1) is written as

$$\begin{aligned} X_1(t) &= \frac{1}{2\pi}\int_{-\infty+\mathrm{i}v}^{\infty+\mathrm{i}v}\left[x_1(\omega) + \frac{V_\infty}{\omega^2} - \frac{\mathrm{i}X_\infty}{\omega} - \frac{x_0}{\omega - \hat{\omega}} + \frac{\overline{x_0}}{\omega + \hat{\omega}}\right]\mathrm{e}^{-\mathrm{i}\omega t}\,\mathrm{d}\omega \\ &\quad - \frac{1}{2\pi}\int_{-\infty+\mathrm{i}v}^{\infty+\mathrm{i}v}\left[\frac{V_\infty}{\omega^2} - \frac{\mathrm{i}X_\infty}{\omega} - \frac{x_0}{\omega - \hat{\omega}} + \frac{\overline{x_0}}{\omega + \hat{\omega}}\right]\mathrm{e}^{-\mathrm{i}\omega t}\,\mathrm{d}\omega \\ &= \frac{1}{2\pi}\int_{-\infty}^{\infty}\left[x_1(\omega) + \frac{V_\infty}{\omega^2} - \frac{\mathrm{i}X_\infty}{\omega} - \frac{x_0}{\omega - \hat{\omega}} + \frac{\overline{x_0}}{\omega + \hat{\omega}}\right]\mathrm{e}^{-\mathrm{i}\omega t}\,\mathrm{d}\omega \\ &\quad + V_\infty t + X_\infty + 2\mathrm{Im}\,\{x_0\,\mathrm{e}^{-\mathrm{i}\hat{\omega}t}\}, \end{aligned} \tag{A 5}$$

where in the first integral the path of integration is moved onto the real axis as the integrand is now free of singularities, and the second integral is evaluated by closing the integration path using a semicircle at infinity in the lower half-plane.

From the definition (2.19), for a function $F(t)$ the real and imaginary parts of its Fourier transform $f(\omega)$ for $\omega \in \mathbb{R}$ are respectively

$$\mathrm{Re} f(\omega) = \int_0^\infty F(t)\cos\omega t\,\mathrm{d}t \quad \text{and} \quad \mathrm{Im} f(\omega) = \int_0^\infty F(t)\sin\omega t\,\mathrm{d}t. \tag{A 6}$$

Thus, $F(t)$ may be recovered from either the inverse cosine transform of $\mathrm{Re} f(\omega)$, or from the inverse sine transform of $\mathrm{Im} f(\omega)$; that is

$$F(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} f(\omega)\,\mathrm{e}^{-\mathrm{i}\omega t}\,\mathrm{d}\omega = \frac{2}{\pi}\int_0^\infty \mathrm{Re} f(\omega)\cos\omega t\,\mathrm{d}\omega = \frac{2}{\pi}\int_0^\infty \mathrm{Im} f(\omega)\sin\omega t\,\mathrm{d}\omega. \tag{A 7}$$

In numerical computations, the inverse cosine or sine transform can be chosen according to which of $\mathrm{Re} f(\omega)$ and $\mathrm{Im} f(\omega)$ decays most rapidly as $\omega \to \infty$. This approach to the numerical evaluation of Fourier integrals was suggested to the author by Dr M. Meylan (private communication).

Appendix B. The scattering problem for a vertical cylinder

Solutions to scattering problems in the presence of a vertical cylinder of radius a may be obtained using the modified version of Weber's integral theorem given by Hunt & Baddour (1980, equation (54)), and used in McIver (1994*b*). The theorem states that provided $r^{1/2} f(r)$ is integrable over (a, ∞) then

$$f(r) = \int_0^\infty \frac{C_n(qr) q\,\mathrm{d}q}{\left|\mathrm{H}_n'(qa)\right|^2}\int_a^\infty C_n(qR) f(R) R\,\mathrm{d}R, \tag{B 1}$$

where

$$C_n(qr) = \mathrm{J}_n(qr)\mathrm{Y}_n'(qa) - \mathrm{Y}_n(qr)\mathrm{J}_n'(qa), \tag{B 2}$$

J_n and Y_n are Bessel functions, and H_n is a Hankel function of the first kind. An important property of C_n is that $C_n'(qa) = 0$.

Attention will be restricted to the case that the free surface is flat initially, so that $H_0(\boldsymbol{x}) \equiv 0$ in (2.24), and it will be that assumed that the initial free-surface potential has a Fourier series expansion

$$\Phi_0(\boldsymbol{x}) = \sum_{n=0}^{\infty} [\alpha_n(r)\cos n\theta + \beta_n(r)\sin n\theta], \quad \boldsymbol{x} \in S_F, \tag{B 3}$$

where α_n and β_n are known functions of r that decay to zero as $r \to \infty$ sufficiently quickly for the application of Weber's integral theorem ($\beta_0 \equiv 0$, but is included above for convenience). A form for the scattering potential that satisfies identically the required equations, with the exception of the free-surface and radiation conditions, is

$$\phi_S(\boldsymbol{x}, z, \omega) = \sum_{n=0}^{\infty} \int_{C_\omega} [A_n(q)\cos n\theta + B_n(q)\sin n\theta]\, C_n(qr)\cosh q(z+h)\,\mathrm{d}q, \tag{B 4}$$

where C_ω is a contour that passes along the positive real-q-axis in a way to be specified shortly. Substitution of the ansatz (B 4) into the free-surface condition (2.24) with the above $\Phi_0(\boldsymbol{x})$, followed by applications of orthogonality properties of the trigonometric functions in θ, and of Weber's integral theorem, gives

$$\begin{Bmatrix} A_n(q) \\ B_n(q) \end{Bmatrix} \left[q\sinh qh - \frac{\omega^2}{g}\cosh qh \right] = -\frac{\mathrm{i}\omega}{g} \frac{q}{\left|\mathrm{H}_n'(qa)\right|^2} \int_a^{\infty} C_n(qR) \begin{Bmatrix} \alpha_n(R) \\ \beta_n(R) \end{Bmatrix} R\,\mathrm{d}R. \tag{B 5}$$

From the forms for A_n and B_n it is apparent that the integrand in the right-hand side of (B 4) has poles at the roots of $q\tanh qh = \omega^2/g$, and in particular at $q = k$ where k is real and positive. The radiation condition of outgoing waves is satisfied by choosing the contour C_ω to ensure that ϕ_S behaves appropriately as $r \to \infty$. Examination of the residue at the pole at $q = k$ shows that C_ω must pass beneath the pole at $q = k$ when $\omega > 0$, but above the pole when $\omega < 0$.

The specific initial condition

$$\Phi_0(\boldsymbol{x}) = \sqrt{\frac{g}{\gamma^3}}\,\mathrm{K}_1(\gamma r)\cos\theta, \quad \boldsymbol{x} \in S_F, \tag{B 6}$$

where $\gamma > 0$ is a constant, is now examined in detail. From the result

$$\int_a^{\infty} C_1(qR)\mathrm{K}_1(\gamma R) R\,\mathrm{d}R = -\frac{2\gamma \mathrm{K}_1'(\gamma a)}{\pi q(q^2+\gamma^2)} \tag{B 7}$$

(Abramowitz & Stegun 1965, equation (11.3.29)) it follows that the scattering potential

$$\phi_S(\boldsymbol{x}, z, \omega) = -\frac{2\mathrm{i}\omega\sqrt{g}\,\mathrm{K}_1'(\gamma a)\cos\theta}{\pi\sqrt{\gamma}} \int_{C_\omega} \frac{C_1(qr)\cosh q(z+h)\,\mathrm{d}q}{\left|\mathrm{H}_1'(qa)\right|^2 (q^2+\gamma^2)(\omega^2 - W^2)\cosh qh}, \tag{B 8}$$

and the exciting force defined in (2.33) is

$$\xi_1(\omega) = -\frac{4\omega^2\rho\sqrt{g}\,\mathrm{K}_1'(\gamma a)}{\pi\sqrt{\gamma}} \int_{C_\omega} \frac{\tanh qh\,\mathrm{d}q}{q^2\left|\mathrm{H}_1'(qa)\right|^2 (q^2+\gamma^2)(\omega^2 - W^2)}, \tag{B 9}$$

where

$$W^2 = gq\tanh qh. \tag{B 10}$$

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(A Wronskian relation for Bessel functions has been used to obtain $C_1(qa) = 2/\pi qa$; see Abramowitz & Stegun (1965), equation (9.1.16).)

By inverse Fourier transformation, the time-domain scattering potential

$$\Phi_S(\boldsymbol{x}, z, t) = \frac{1}{2\pi} \int_{-\infty+\mathrm{i}v}^{\infty+\mathrm{i}v} \phi_S(\boldsymbol{x}, z, \omega)\, \mathrm{e}^{-\mathrm{i}\omega t}\, \mathrm{d}\omega \tag{B 11}$$

where now $\operatorname{Im}\omega = v > 0$ (McIver & McIver 2011, equation (14)). A consequence of a non-zero imaginary part to ω is to move the pole at $q = k$ in the integrand in (B 8) off the real-q-axis; for $\omega > 0$ it moves in to the upper half of the complex-q-plane, and for $\omega < 0$ it moves into the lower half-plane. As a result of these pole movements the q-integration can now be taken strictly along the positive part of the real-q-axis. For $t > 0$ the ω integration is carried out by closing the contour in the lower half of the complex-ω-plane, to obtain

$$\int_{-\infty+\mathrm{i}v}^{\infty+\mathrm{i}v} \frac{\omega\, \mathrm{e}^{-\mathrm{i}\omega t}}{\omega^2 - W^2}\, \mathrm{d}\omega = -2\pi\mathrm{i} \cos Wt, \tag{B 12}$$

and the time-domain potential corresponding to (B 8) is therefore

$$\Phi_S(\boldsymbol{x}, z, t) = -\frac{2\sqrt{g}\, \mathrm{K}_1'(\gamma a) \cos\theta}{\pi\sqrt{\gamma}} \int_0^\infty \frac{C_1(qr) \cosh q(z+h) \cos Wt\, \mathrm{d}q}{\left|\mathrm{H}_1'(qa)\right|^2 (q^2+\gamma^2) \cosh qh}. \tag{B 13}$$

From (2.15)

$$\widetilde{\Phi}_{S,1} = \frac{4\rho\sqrt{g}\, \mathrm{K}_1'(\gamma a)}{\pi\sqrt{\gamma}} \int_0^\infty \frac{\tanh qh\, \mathrm{d}q}{q^2 \left|\mathrm{H}_1'(qa)\right|^2 (q^2+\gamma^2)}, \tag{B 14}$$

which is equal to $\lim_{|\omega|\to\infty}[-\xi_1(\omega)]$, as required by (2.36).

Appendix C. Response to a wave maker: frequency-domain solution

Here methods of solution are given for the frequency-domain solution used in the calculations reported in § 3. The structure S_B is a vertical cylinder of radius a extending throughout the depth, and the forcing is provided by the motion about $x = 0$ of a vertical wave maker. The geometry and coordinate systems are illustrated in figure 4.

C.1. *Forced motion of the cylinder*

The governing equations for the radiation potential ϕ_1 that describes the fluid response to the forced oscillations of the cylinder while the wave maker is held fixed are

$$\nabla^2\phi_1 = 0 \quad \text{in } \mathscr{D}, \tag{C 1}$$

$$\frac{\partial\phi_1}{\partial r} = \cos\theta \quad \text{on } S_B, \tag{C 2}$$

$$\frac{\partial\phi_1}{\partial z} = \frac{\omega^2}{g}\phi_1 \quad \text{on } S_F, \tag{C 3}$$

and

$$\frac{\partial\phi_1}{\partial n} = 0 \quad \text{on } S_{bed} \cup S_W, \tag{C 4}$$

together with a radiation condition specifying outgoing waves as $kr \to \infty$ for $\theta \in (-\pi/2, \pi/2)$.

The form

$$\phi_1 = \sum_{n=0}^{\infty} \psi_n(z) \sum_{m=0}^{\infty} B_{mn} Z_{mn} \left[\mathrm{K}_m(k_n r) \cos m\theta + (-1)^m \mathrm{K}_m(k_n r') \cos m\theta' \right], \tag{C 5}$$

satisfies all of the governing equations except for the non-homogeneous boundary condition (C 2) on the surface of the cylinder. This last condition is satisfied by appropriate choice of the series coefficients B_{mn}, and the factor

$$Z_{mn} = \frac{\mathrm{I}'_m(k_n a)}{\mathrm{K}'_m(k_n a)} \tag{C 6}$$

has been introduced for convenience. The vertical eigenfunctions are

$$\psi_n(z) = \frac{\cos k_n(z+h)}{N_n}, \quad n = 0, 1, 2, \ldots, \tag{C 7}$$

where

$$N_n = \sqrt{\frac{1}{2}\left(1 + \frac{\sin 2k_n h}{2k_n h}\right)}, \tag{C 8}$$

k_n is a root of

$$\omega^2 + g k_n \tan k_n h = 0, \tag{C 9}$$

$k_0 = -\mathrm{i}k$ where the real wavenumber $k > 0$, and the positive real numbers k_n, $n = 1, 2, 3, \ldots$ are arranged in increasing order. With these definitions, the vertical eigenfunctions satisfy the orthogonality conditions

$$\frac{1}{h} \int_{-h}^{0} \psi_m(z) \psi_n(z) \, \mathrm{d}z = \delta_{mn}, \quad m, n = 0, 1, 2, \ldots, \tag{C 10}$$

where δ_{mn} is the Kronecker delta.

For $r < 2d$, Graf's addition theorem in the form given by Linton & McIver (2001, equation (6.20)) allows (C 5) to be rewritten as

$$\phi_1 = \sum_{n=0}^{\infty} \psi_n(z) \sum_{m=0}^{\infty} \cos m\theta \left\{ B_{mn} Z_{mn} \mathrm{K}_m(k_n r) + \frac{1}{2} (-1)^m \epsilon_m \mathrm{I}_m(k_n r) \sum_{l=0}^{\infty} B_{ln} Z_{ln} (-1)^l \left[\mathrm{K}_{l-m}(2k_n d) + \mathrm{K}_{l+m}(2k_n d) \right] \right\}. \tag{C 11}$$

With the aid of (C 10) and the orthogonality of the trigonometric functions, the boundary condition (C 2) then yields a system for each n of the form

$$B_{mn} + \frac{1}{2} (-1)^m \epsilon_m \sum_{l=0}^{\infty} B_{ln} Z_{ln} (-1)^l \left[\mathrm{K}_{l-m}(2k_n d) + \mathrm{K}_{l+m}(2k_n d) \right] = \frac{\delta_{1m} \sin k_n h}{k_n^2 h N_n \mathrm{I}'_m(k_n a)}, \quad m = 0, 1, 2, \ldots, \tag{C 12}$$

and this is readily solved by truncation.

As first noted by Linton & Evans (1990), in problems of this type calculations of the force on the cylinder are simplified by substituting (C 12) back into (C 11) to obtain

$$\phi_1 = \sum_{n=0}^{\infty} \psi_n(z) \left\{ \frac{\sin k_n h}{k_n^2 h N_n} \frac{\mathrm{I}_1(k_n r)}{\mathrm{I}_1'(k_n a)} \cos\theta + \sum_{m=0}^{\infty} B_{mn} \left[\mathrm{K}_m(k_n r) Z_{mn} - \mathrm{I}_m(k_n r) \right] \cos m\theta \right\}, \tag{C 13}$$

so that the force coefficient

$$\begin{aligned} \mu_{11}(\omega) + \mathrm{i}\nu_{11}(\omega) &\equiv -\rho \int_{S_B} \phi_1 \cos\theta \, \mathrm{d}S \\ &= -\rho a \pi \sum_{n=0}^{\infty} \frac{\sin k_n h}{k_n N_n} \left\{ \frac{\sin k_n h}{k_n^2 h N_n} \frac{\mathrm{I}_1(k_n a)}{\mathrm{I}_1'(k_n a)} + B_{1n} \left[\mathrm{K}_1(k_n a) Z_{1n} - \mathrm{I}_1(k_n a) \right] \right\}. \end{aligned} \tag{C 14}$$

C.2. *Forced motion of the wave maker*

In the absence of the cylinder S_B, so that the fluid occupies the whole of $x > 0$, $-h < z < 0$, the fluid response to the motion of the wave maker is described by the potential

$$\phi_I = \sum_{n=0}^{\infty} A_n \mathrm{e}^{-k_n x} \psi_n(z) \tag{C 15}$$

where

$$A_n = -\frac{1}{k_n h} \int_{-h}^{0} \psi_n(z) \, \mathrm{d}z \tag{C 16}$$

(Linton & McIver 2001, § 2.2.1). The governing equations for the potential ϕ_S that describes the fluid response to the forced motion of the wave maker when the cylinder is held fixed are

$$\nabla^2 \phi_S = 0 \quad \text{in } \mathscr{D}, \tag{C 17}$$

$$\frac{\partial \phi_S}{\partial r} = -\frac{\partial \phi_I}{\partial r} \quad \text{on } S_B, \tag{C 18}$$

$$\frac{\partial \phi_S}{\partial z} = \frac{\omega^2}{g} \phi_S \quad \text{on } S_F, \tag{C 19}$$

and

$$\frac{\partial \phi_S}{\partial n} = 0 \quad \text{on } S_{bed} \cup S_W, \tag{C 20}$$

together with a radiation condition specifying outgoing waves as $kr \to \infty$ for $\theta \in (-\pi/2, \pi/2)$. The forcing on the cylinder S_B is obtained by using equation (9.6.34) of Abramowitz & Stegun (1965) to write

$$\mathrm{e}^{-k_n x} = \mathrm{e}^{-k_n d} \mathrm{e}^{k_n r \cos(\pi - \theta)} = \mathrm{e}^{-k_n d} \sum_{m=0}^{\infty} (-1)^m \epsilon_m \mathrm{I}_m(k_n r) \cos m\theta \tag{C 21}$$

so that

$$\frac{\partial \phi_I}{\partial r} = -\sum_{n=0}^{\infty} k_n A_n \psi_n(z) \mathrm{e}^{-k_n d} \sum_{m=0}^{\infty} (-1)^m \epsilon_m \mathrm{I}_m'(k_n a) \cos m\theta \quad \text{on } S_B. \tag{C 22}$$

Here ϵ_m is the Neumann symbol defined by $\epsilon_0 = 1$, $\epsilon_m = 2$ for $m \geqslant 1$.

The procedure for obtaining the solution for ϕ_S is similar to that used above to obtain ϕ_1 and it is found that

$$\phi_S = -\phi_I + \sum_{n=0}^{\infty} \psi_n(z) \sum_{m=0}^{\infty} B_{mn} \left[\mathrm{K}_m(k_n r) Z_{mn} - \mathrm{I}_m(k_n r)\right] \cos m\theta, \tag{C 23}$$

where the series coefficients for each n now satisfy

$$B_{mn} + \tfrac{1}{2} (-1)^m \epsilon_m \sum_{l=0}^{\infty} B_{ln} Z_{ln} (-1)^l \left[\mathrm{K}_{l-m}(2k_n d) + \mathrm{K}_{l+m}(2k_n d)\right] = -A_n \mathrm{e}^{-k_n d} (-1)^m \epsilon_m,$$
$$m = 0, 1, 2, \ldots . \tag{C 24}$$

The force on the cylinder due to the fluid motion generated by the wave maker is

$$\xi_1(\omega) \equiv -\mathrm{i}\omega\rho \int_{S_B} (\phi_I + \phi_S) \cos\theta \, \mathrm{d}S = -\mathrm{i}\omega\rho a \pi \sum_{n=0}^{\infty} \frac{\sin k_n h}{k_n N_n} B_{1n} \left[\mathrm{K}_1(k_n a) Z_{1n} - \mathrm{I}_1(k_n a)\right]. \tag{C 25}$$

C.3. *A reciprocity relation*

Reciprocity relations are well known in water waves (see, for example, Linton & McIver 2001, § 1.4); here a relation is obtained that yields the force ξ_1 on the cylinder arising from the forced motion of the wave maker, in terms of the potential ϕ_1 that describes the fluid response to the forced motion of the cylinder. Green's theorem applied over the fluid domain to the harmonic potentials $\phi_I + \phi_S$ and ϕ_1 yields

$$\begin{aligned} 0 &= \int_{S_W \cup S_B \cup S_\infty} \left[(\phi_I + \phi_S) \frac{\partial \phi_1}{\partial n} - \phi_1 \frac{\partial}{\partial n} (\phi_I + \phi_S) \right] \mathrm{d}S \\ &= \int_{S_W} \phi_1 U(z) \, \mathrm{d}S + \int_{S_B} (\phi_I + \phi_S)(-\cos\theta) \, \mathrm{d}S \\ &\quad + \int_{S_\infty} \left[(\phi_I + \phi_S) \frac{\partial \phi_1}{\partial n} - \phi_1 \frac{\partial}{\partial n} (\phi_I + \phi_S) \right] \mathrm{d}S, \end{aligned} \tag{C 26}$$

where S_∞ is a closing vertical half-cylinder at infinity occupying $\theta \in (-\pi/2, \pi/2)$. The standard radiation condition satisfied by both ϕ_S and ϕ_1 gives

$$\int_{S_\infty} \left[\phi_S \frac{\partial \phi_1}{\partial n} - \phi_1 \frac{\partial \phi_S}{\partial n} \right] \mathrm{d}S = 0, \tag{C 27}$$

while a routine application of the method of stationary phase yields

$$\int_{S_\infty} \left[\phi_I \frac{\partial \phi_1}{\partial n} - \phi_1 \frac{\partial \phi_I}{\partial n} \right] \mathrm{d}S = 0, \tag{C 28}$$

so that from (C 25) and (C 26)

$$\xi_1 = -\mathrm{i}\omega\rho \int_{S_W} \phi_1 \, U(z) \, \mathrm{d}S. \tag{C 29}$$

In the calculation of ξ_1 from this expression the form of ϕ_1 in (C 5) simplifies because, on the wave maker S_W, $r = r'$ and $\theta = \pi - \theta'$ so that, after taking account of the definition (C 16),

$$\xi_1 = 4\mathrm{i}\omega\rho dh \sum_{n=0}^{\infty} k_n A_n \sum_{m=0}^{\infty} B_{mn} Z_{mn} (-1)^m \mathscr{C}_m(k_n d) \tag{C 30}$$

where

$$\mathscr{C}_m(\alpha) = \int_0^{\pi/2} \mathrm{K}_m(\alpha \sec\theta') \cos m\theta' \sec^2\theta' \, \mathrm{d}\theta'. \tag{C 31}$$

For a non-negative integer m and real $\alpha > 0$, the binomial series and Gradshteyn & Ryzhik (1980, equation (6.596.3)) allow this integral to be rewritten as

$$\begin{aligned}
\mathscr{C}_m(\alpha) &= \mathrm{Re} \int_0^{\infty} \mathrm{K}_m\left(\alpha\sqrt{1+u^2}\right) \left(\mathrm{e}^{\mathrm{i}\arctan u}\right)^m \mathrm{d}u \\
&= \mathrm{Re} \int_0^{\infty} \mathrm{K}_m\left(\alpha\sqrt{1+u^2}\right) \left(\frac{1+\mathrm{i}u}{\sqrt{1+u^2}}\right)^m \mathrm{d}u \\
&= \mathrm{Re} \sum_{k=0}^{m} \binom{m}{k} \mathrm{i}^k \int_0^{\infty} \frac{\mathrm{K}_m\left(\alpha\sqrt{1+u^2}\right)}{\left(\sqrt{1+u^2}\right)^m} u^k \, \mathrm{d}u \\
&= \mathrm{Re} \sum_{k=0}^{m} \binom{m}{k} \frac{\mathrm{i}^k\, 2^{(k-1)/2}}{\alpha^{(k+1)/2}} \Gamma\left(\frac{k+1}{2}\right) \mathrm{K}_{m-k/2-1/2}(\alpha) \\
&= \sum_{k=0}^{[m/2]} \binom{m}{2k} \frac{(-1)^k\, 2^{k-1/2}}{\alpha^{k+1/2}} \Gamma\left(k+\tfrac{1}{2}\right) \mathrm{K}_{m-k-1/2}(\alpha),
\end{aligned} \tag{C 32}$$

where $[\cdot]$ denotes the integer part. The Bessel functions $\mathrm{K}_{q-1/2}(\alpha)$, $q = 0, 1, 2, \ldots$, are readily evaluated using the identities

$$\mathrm{K}_{-1/2}(\alpha) = \mathrm{K}_{1/2}(\alpha) \quad \text{and} \quad \mathrm{K}_{q+1/2}(\alpha) = \sqrt{\frac{\pi}{2\alpha}}\, \mathrm{e}^{-\alpha} \sum_{k=0}^{q} \frac{(q+k)!}{k!(q-k)!\,(2\alpha)^k}, \quad q = 0, 1, 2, \ldots \tag{C 33}$$

(Gradshteyn & Ryzhik 1980, equation (8.468)). Using the above formulae, the author has evaluated $\mathscr{C}_m(\alpha)$ for many integers m and in every case found that

$$\mathscr{C}_m(\alpha) = \frac{\pi \mathrm{e}^{-\alpha}}{2\alpha}, \quad \alpha \neq 0. \tag{C 34}$$

However, the author has not been able to demonstrate that this holds for all m. Symbolic and numerical calculations using Mathematica suggest that (C 34) also holds when α is pure imaginary.

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