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Adaptive trajectory tracking control of a differential drive wheeled mobile robot

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SUMMARY
This paper presents an adaptive trajectory tracking controller for a non-holonomic wheeled mobile robot (WMR) in the presence of parametric uncertainty in the kinematic and dynamic models of the WMR and actuator dynamics. The adaptive non-linear control law is designed based on input–output feedback linearization technique to get asymptotically exact cancellation for the uncertainty in the given system parameters. In order to evaluate the performance of the proposed controller, a non-adaptive controller is compared with the adaptive controller via computer simulation results. The results show satisfactory trajectory tracking performance by virtue of SPR-Lyapunov design approach. In order to verify the simulation results, a set of experiments have been carried out on a commercial mobile robot. The experimental results also show the effectiveness of the proposed controller.

KEYWORDS: Adaptive feedback linearization; Parametric uncertainty; Trajectory tracking; Non-holonomic WMR.

1. Introduction

Wheeled mobile robot (WMR) is one of the most attractive research areas. The problem of motion control of WMRs has attracted a great deal of attention over past decades for the sake of autonomous motion capabilities.[1–5] The motion control of WMRs using kinematic and dynamic model is frequently reported in the literature.[2–14] A survey on various motion control problems of such non-holonomic systems can be found in the research paper of Kolmanovsky.[8] Among these attractive problems, trajectory tracking is concerned with the design of a controller to force a WMR to track a geometric path with an associated timing law. A variety of control algorithms for trajectory tracking problem is developed in the literature.[14–28] Because of the challenging non-linear model of WMRs, the feedback linearization technique is one of the successful design tools to solve this problem. d'Andrea-Novel *et al.*[13] applied the linearization technique to achieve tracking control of mobile robots. Yun *et al.*[11] showed that non-holonomic systems are not input-state linearizable. But if a proper set of output equations are chosen, then these systems may be input–output linearizable. There are many works[3–16] that propose tracking controllers based on feedback linearization for WMRs, but they use exact kinematic and dynamic model of mobile robots. In practical situations, the physical parameters are most often not precisely known. Feedback linearization is based on cancellation of non-linear terms. Therefore, in presence of uncertainty in WMR parameters, this cancellation may not be achieved perfectly and it will be a motivation for adaptive version of feedback linearization technique. However, WMRs are multiple-input and multiple-output (MIMO) non-linear underactuated mechanical systems and therefore, development of adaptive feedback linearization for WMRs is more cumbersome. The first attempt of applying an adaptive version of this technique to trajectory tracking problem of WMRs is covered in this paper and there is no similar work to this paper in the literature. Since the actuator dynamic is ignored based on the assumption of wheel torques as the input of the robot system in most of the previous researches, it is more reasonable and practical to take into account the actuator input voltages as the control inputs. However, the commercial WMRs may be commanded by velocities and they may not accept the actuator voltages as the input.[27–31] Das *et al.*[30] proposed a neuron-based adaptive controller for a non-holonomic WMR including actuator dynamics. Their proposed control law provides voltage signals as the input and may not be applicable for commercially available WMRs. Martins *et al.*[27] proposed an adaptive controller for kinematic and dynamic models of a differential drive WMR including actuator dynamics. Their proposed controller provides velocities as the input for a commercial WMR based on the presented model by De La Cruz and Carelli.[31]

The main contributions of this paper are listed as follows: (1) An adaptive tracking controller is designed based on input–output feedback linearization technique to compensate for a significant uncertainty in kinematic and dynamic parameters and actuators parameters. Most of previously

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presented works[15–30] design individual controllers for kinematic and dynamic models. However, this paper proposes a unified tracking controller for an integrated kinematic and dynamic model of the WMR. (2) An experimental result is presented to evaluate the tracking performance of the proposed adaptive controller on a commercial WMR which is named robuLAB 10. For the purpose of the implementation of the controller on robuLAB 10 WMR, the presented model in the work of Martins and De La Cruz is used.[27–31]

The rest of the paper is arranged as follows: Section 2 presents a kinematic and dynamic model of WMR including actuator dynamics. Section 3 proposes an adaptive version of input–output feedback linearization controller in absence of knowledge about parameters. Simulation results are presented to show the performance of the proposed controller in Section 4. Some experiments are presented in Section 5 in order to show the effectiveness of the proposed control law. Finally, Section 6 concludes the paper.

2. Kinematic and Dynamic Model of a WMR

In this section, a mathematical formulation of a non-holonomic differential drive mobile robot moving on a planar surface is presented. Figure 1 shows the configuration of the non-holonomic WMR. It is assumed that WMR has two motorized wheels on an axis that independently drive the robot.

The centre of mass of the robot is located in $P_C = (x_C, y_C)$. The point $P_0 = (x_O, y_O)$ is the origin of the local coordinate frame that is attached to the WMR body. The point $P_L = (x_L, y_L)$ is a virtual reference point on x-axis of the local frame at a distance L (look-ahead distance) of P_O. The other parameters of the WMR are summarized in Table I. The pose of the robot (position and orientation) in global coordinate frame is specified by vector $X_R = [x_O, y_O, \varphi]^T$. By considering assumptions in the works of Sarkar[2] and Coelho,[3] n generalized coordinates, q, are assumed to describe the WMR model.

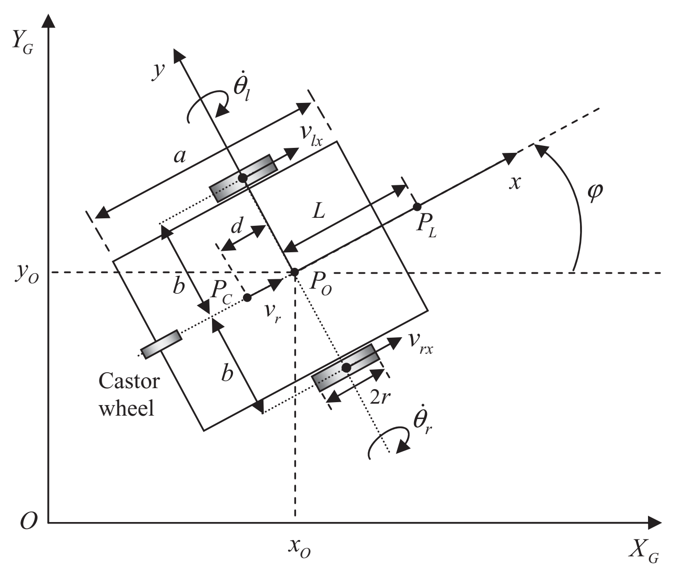

Fig. 1. Non-holonomic differential drive mobile robot.

Table I. Model parameters of non-holonomic WMR.

Parameter	Description
r	Driving wheels radius
$2b$	Distance between two wheels
d	Distance point P_C from point P_O
L	Distance point P_O from point P_L
m_C	The mass of the platform without the driving wheels and the rotors of the DC motors
m_W	The mass of each driving wheel plus the rotor of its motor
I_C	The moment of inertia of the platform without the driving wheels and the rotors of the motors about a vertical axis through P_C
I_W	The moment of inertia of each wheel and the motor rotor about the wheel axis
I_m	The moment of inertia of each wheel and the motor rotor about a wheel diameter

Suppose that WMR has m holonomic and non-holonomic velocity constraints ($m < n$) in Pfaffian form:

$$g_j(q, \dot{q}) = \sum_{i=1}^{n} g_{ji}(q) \cdot \dot{q}_i = 0, \quad j = 1, \ldots, m, \tag{1}$$

where all of them may be written in the following form:

$$A(q) \cdot \dot{q} = 0, \tag{2}$$

where $A(q) \in R^{m\times n}$ is a full-rank matrix. Assume that $S(q) = [s_1(q), \ldots, s_{n-m}(q)]^T$ is a full-rank matrix that is made up of a set of smooth and linearly independent vector fields in the null space of the $A(q)$ (see Sarkar's paper[2] for more details). Therefore, it may be written as

$$A(q) \cdot S(q) = 0. \tag{3}$$

Assuming that the velocity of P_0 is in the direction of x-axis of the local frame and there is no side slip, and considering $q = [x_O, y_O, \varphi]^T$, the following constraint with respect to P_O is obtained:

$$\dot{y}_O \cos\varphi - \dot{x}_O \sin\varphi = 0. \tag{4}$$

By writing Eq. (4) in matrix form (2), matrices $A(q)$ and $S(q)$ that satisfy Eq. (3) are given by

$$A(q) = [-\sin\varphi \quad \cos\varphi \quad 0], \quad S(q) = \begin{bmatrix} \cos\varphi & 0 \\ \sin\varphi & 0 \\ 0 & 1 \end{bmatrix}. \tag{5}$$

According to Eqs. (2) and (3), it is possible to write the kinematic equation of WMR motion in terms of pseudo-velocities vector $v(t) \in R^{n-m}$ as

$$\dot{q} = S(q) \cdot v(t), \tag{6}$$

where $v(t) = [v_r(t), \omega_r(t)]^T$ is made up of linear and angular velocities. The WMR dynamic model is derived by Lagrangian mechanics. First, the Lagrangian L of the system must be calculated. Because of the planar motion, the potential energy of the robot is zero. Therefore, the Lagrangian is only equal to kinetic energy:

$$L = \frac{1}{2}\sum_{i=1}^{n_i}\left[v_i^T m_i v_i + \omega_i^T I_i \omega_i\right]. \tag{7}$$

Then, one may use Euler–Lagrange equation incorporating velocity constraints in the following form:

$$\frac{\mathrm{d}}{\mathrm{d}t}\left\{\frac{\partial L}{\partial \dot{q}_i}\right\} - \frac{\partial L}{\partial q_i} = F_G, \tag{8}$$

where F_G denotes the generalized forces. After calculating Eq. (8), the dynamic model of WMR may be written as follows:

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} = B(q).\tau - A(q)^T\lambda. \tag{9}$$

where $M(q) \in R^{n\times n}$ is the inertia matrix; $C(q,\dot{q}) \in R^{n\times n}$ is a matrix which denotes the Coriolis and centripetal forces; $B(q) \in R^{n\times(n-m)}$ is the input transformation matrix; $\tau \in R^{(n-m)\times 1}$ is the torque vector which is generated by wheels actuators; and $\lambda \in R^{m\times 1}$ is the vector of constraint forces. These matrices are expressed as follows:

$$\left.\begin{aligned} M(q) &= \begin{bmatrix} m & 0 & m_C d\sin\varphi \\ 0 & m & -m_C d\cos\varphi \\ m_C d\sin\varphi & -m_C d\cos\varphi & I \end{bmatrix}, \\ C(q,\dot{q}) &= \begin{bmatrix} 0 & 0 & m_C d\dot{\varphi}\cos\varphi \\ 0 & 0 & m_C d\dot{\varphi}\sin\varphi \\ 0 & 0 & 0 \end{bmatrix}, \\ B(q) &= \frac{1}{r}\begin{bmatrix} \cos\varphi & \cos\varphi \\ \sin\varphi & \sin\varphi \\ b & -b \end{bmatrix}, \end{aligned}\right\} \tag{10}$$

where $m = m_C + 2m_w$ and $I = I_C + 2I_m + m_C d^2 + 2m_w b^2$, and parameters m_C, m_w, I_C and I_m are defined in Table I. This model may also be derived by applying the change of state variables: $[x_O, y_O] = [x_C + d\cos\varphi,\ y_C + d\sin\varphi]$ and ignoring I_w in the presented model in the work of Sarkar.[2] To include actuator dynamic in Eq. (9), it is assumed that the robot wheels are driven by two brush DC motors with mechanical gears. Figure 2 shows the simplified drive system.

The electrical equation of the motor armature is written as follows:

$$u_a = L_a\frac{\mathrm{d}i_a}{\mathrm{d}t} + R_a i_a + K_b\dot{\theta}_M, \tag{11}$$

where K_b is the back electromotive force (EMF) constant. By ignoring the inductance of armature circuit, and considering the relation between torque and armature current (i.e. $\tau_M = K_\tau.i_a$) and relations between torque and velocity before and after gears (i.e. $\tau = n.\tau_M$ and $\dot{\theta}_M = n\cdot\dot{\theta}$), the delivered torque to the right and left wheels by actuators is given by[30]

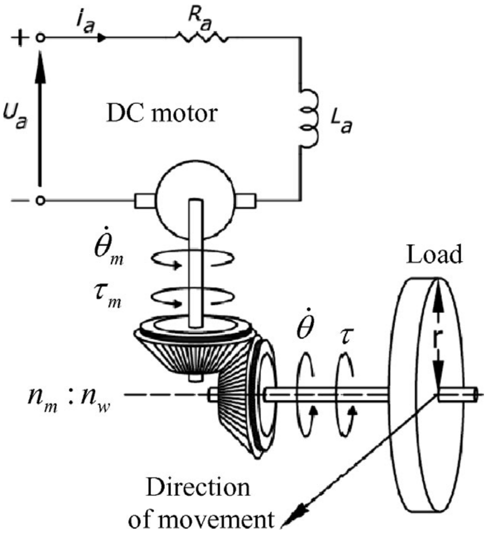

Fig. 2. Drive system for each wheel.

$$\begin{bmatrix}\tau_r \\ \tau_l\end{bmatrix} = K_1\begin{bmatrix}u_{ar} \\ u_{al}\end{bmatrix} - K_2\begin{bmatrix}\dot{\theta}_r \\ \dot{\theta}_l\end{bmatrix}, \tag{12}$$

where $K_1 = (nK_\tau/R_a)$, $K_2 = n\cdot K_b K_1$, n is gear ratio and K_τ is torque constant of the motor. Considering the relation between angular velocities of wheels and pseudo-velocities, we have

$$\begin{bmatrix}\tau_r \\ \tau_l\end{bmatrix} = K_1\begin{bmatrix}u_{ar} \\ u_{al}\end{bmatrix} - K_2 X\begin{bmatrix}v_r \\ \omega_r\end{bmatrix}, \quad X = \begin{bmatrix}\frac{1}{r} & \frac{b}{r} \\ \frac{1}{r} & -\frac{b}{r}\end{bmatrix}, \tag{13}$$

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} = B(q)\cdot(K_1\cdot u_a - K_2 X v) - A(q)^T\lambda. \tag{14}$$

For controller design purposes, the state space representation can be derived by taking time derivative of the kinematic model (6):

$$\ddot{q} = \dot{S}(q)\cdot v + S(q)\cdot\dot{v}. \tag{15}$$

Next, by replacing Eqs. (6) and (15) in Eq. (14) and multiplying the result by S^T and considering Eq. (3), we obtain

$$\bar{M}\dot{v}(t) + \bar{C}(\dot{q})\cdot v(t) = K_1\bar{B}\cdot u_a, \tag{16}$$

where

$$\left.\begin{aligned} \bar{M} = S^T M S, \quad \bar{C}(\dot{q}) = S^T M\dot{S} + S^T C S + K_2\bar{B}X, \\ \bar{B} = S^T B. \end{aligned}\right\} \tag{17}$$

The kinematic model (6) and dynamic equation shown by Eq. (16) can be integrated into the following state space

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representation in companion form:

$$\dot{x} = \begin{bmatrix} Sv \\ -\bar{M}^{-1}\bar{C}v \end{bmatrix} + \begin{bmatrix} 0 \\ K_1\bar{M}^{-1}\cdot\bar{B} \end{bmatrix} u_a, \tag{18}$$

where $x = [q^T, v^T]^T$ is the state vector. This representation allows us to apply the differential geometric control theory for trajectory tracking problem.

Remark 1. In the derived dynamic model, the un-powered castor wheel is ignored to reduce the complexity of the model. However, it is more reasonable to take the free wheel dynamic into account to avoid the poor performance of the proposed controller in experimental results.

3. Adaptive Feedback Linearization Control

A trajectory tracking control law can be designed based on adaptive feedback linearization technique for non-holonomic system given in Eq. (18). In order to design the adaptive controller, the WMR system must be exactly linearly parameterized. Therefore, the matrices $\bar{M}$, $\bar{C}$ and $\bar{B}$ in Eq. (17) are expressed as

$$\left.\begin{aligned} &\bar{M} = \begin{bmatrix} m & 0 \\ 0 & I \end{bmatrix}, \quad \bar{C} = \begin{bmatrix} \dfrac{2K_2}{r^2} & m_C d\dot{\varphi} \\ -m_C d\dot{\varphi} & \dfrac{2b^2K_2}{r^2} \end{bmatrix}, \\ &\bar{B} = \begin{bmatrix} \dfrac{1}{r} & \dfrac{1}{r} \\ \dfrac{b}{r} & -\dfrac{b}{r} \end{bmatrix}. \end{aligned}\right\} \tag{19}$$

After substituting these matrices in Eq. (18) and simplification of the resulting equation, we obtain

$$\dot{x} = \begin{bmatrix} v_r\cos\varphi \\ v_r\sin\varphi \\ \omega_r \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ -\theta_1 v_r - \theta_2\omega_r^2 \\ \theta_3\omega_r v_r - \theta_4\omega_r \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ \theta_5 & \theta_5 \\ \theta_6 & -\theta_6 \end{bmatrix} u, \tag{20}$$

where the parameters $\theta_i, i = 1, \ldots, 6$ are bounded and are defined as follows:

$$\left.\begin{aligned} &\theta_1 = \frac{2K_2}{mr^2}, \quad \theta_2 = \frac{m_C d}{m}, \quad \theta_3 = \frac{m_C d}{I}, \\ &\theta_4 = \frac{2b^2K_2}{Ir^2}, \quad \theta_5 = \frac{K_1}{mr}, \quad \theta_6 = \frac{K_1 b}{Ir}. \end{aligned}\right\} \tag{21}$$

The presented system in Eq. (20) might be summarized as the following affine MIMO non-linear model:

$$\dot{x} = f(x) + q(x,\theta) + g(x,\theta)u_a \tag{22}$$

where $x \in R^n$ and $f(x), q(x,\theta)$ and $g(x,\theta)$ are smooth vector fields on R^n with $g(0,\theta) \neq 0$:

$$\left.\begin{aligned} &f(x) = \begin{bmatrix} Sv \\ 0 \end{bmatrix}, \quad q(x,\theta) = \begin{bmatrix} 0 \\ Q(x,\theta) \end{bmatrix}, \\ &g(x,\theta) = K_1\begin{bmatrix} 0 \\ G(\theta) \end{bmatrix}, \end{aligned}\right\} \tag{23}$$

$$Q(x,\theta) = \begin{bmatrix} -\theta_1 v_r - \theta_2\omega_r^2 \\ \theta_3\omega_r v_r - \theta_4\omega_r \end{bmatrix}, \quad G(\theta) = \begin{bmatrix} \theta_5 & \theta_5 \\ \theta_6 & -\theta_6 \end{bmatrix}. \tag{24}$$

The following output variables are chosen to track a desired trajectory based on look-ahead control method:[2]

$$y = h(x) = [h_1(q), \quad h_2(q)]^T = [x_O + L\cos\varphi, \quad y_O + L\sin\varphi]^T. \tag{25}$$

Remark 2. Based on the study of non-holonomic WMR dynamics shown in Eq. (18), the following results might be summarized:

(1) The system is controllable and its equilibrium point $x_e = 0$ can be made Lyapunov stable, but can not be made asymptotically stable by a smooth state feedback.[24]
(2) The internal dynamics of a WMR is stable, when the mobile robot moves forwards, but unstable when it moves backwards.[25]
(3) If at least one constraint is non-holonomic, it has been shown that the WMR system is not input-state linearizable. But if we choose a proper set of output equations, it may be input–output linearizable.[2–12]

Definition 1. Given a smooth bounded reference trajectory $y_r(t) = h(q_r(t))$ with bounded derivatives which is generated by a reference mobile robot, and supposing that q_r satisfies the non-holonomic constraints $A(q_r)\cdot\dot{q}_r = 0$, then the trajectory tracking problem is to design a feedback control such that it satisfies:

$$\lim_{t\to\infty}(y(t) - y_r(t)) = 0. \tag{26}$$

The basic approach to obtain a linear input–output relation is to repeatedly differentiate the outputs so that they are explicitly related to inputs.[9] After differentiating Eq. (25), we obtain:

$$\dot{y}_j = L_f h_j + L_q h_j + L_g h_j u_a = J_{h_j} Sv, \quad j = 1, 2, \tag{27}$$

$$\ddot{y}_j = L_f^2 h_j + L_f L_q h_j + L_q L_f h_j + L_q^2 h_j + (L_g L_f h_j + L_g L_q h_j).u_a. \tag{28}$$

As a result,

$$\ddot{y} = L_f^2 h(x) + L_q L_f h(x) + L_g L_f h(x).u_a, \tag{29}$$

where

$$L_f^2h(x) = \begin{bmatrix} \frac{\partial}{\partial q}(J_{h1}(q)\cdot S(q)v)\cdot S(q)v \\ \frac{\partial}{\partial q}(J_{h2}(q)\cdot S(q)v)\cdot S(q)v \end{bmatrix}, \qquad (30)$$

$$\left.\begin{aligned} L_qL_fh(x) &= J_h(q)S(q)Q(x,\theta), \\ L_gL_fh(x) &= J_h(q)S(q)G(\theta), \end{aligned}\right\} \qquad (31)$$

where $J_h(q) = [J_{h1}(q),\ J_{h1}(q)]^T$ denotes Jacobian matrix and $L_gL_fh(x) = D(x)$ is a decoupling matrix. Assuming that $\det(D(x)) \neq 0$, the system (29) is input–output linearizable. The following non-linear feedback

$$u_a = D^{-1}(x)\cdot(\eta - L_f^2h(x) - L_qL_fh(x)) \qquad (32)$$

linearizes and decouples the system in the following form:

$$\begin{bmatrix} \ddot{y}_1 \\ \ddot{y}_2 \end{bmatrix} = \begin{bmatrix} \eta_1 \\ \eta_2 \end{bmatrix}, \qquad (33)$$

where $\eta_j,\ j = 1, 2$ represents the external inputs. The parameters mass (m), moment of inertia (I), wheel radius (r), distance between two wheels ($2b$) and actuator parameters are supposed to have uncertainties. The following theorem is presented to solve the trajectory tracking problem in presence of parametric uncertainty. But before proposing the tracking controller, the following assumptions are made:

Assumption 1. Measurements of all states, i.e. $x = [q^T, v^T]^T$, are available in real time.

Assumption 2. Pseudo-velocities, i.e. $v(t) = [v_r(t),\ \omega_r(t)]^T$, are bounded for all time $t > 0$.

Theorem 1. Provided that the reference trajectory $y_r(t)$ is selected to be bounded for all times $t > 0$, and under assumptions 1 and 2, the following adaptive tracking controller guarantees that all signals in the closed-loop system are bounded and the tracking error $e(t) = y(t) - y_r(t)$ converge to zero as $t \to \infty$.

$$\left.\begin{aligned} u_a &= \hat{D}^{-1}(x)\cdot\left(\eta - L_f^2h - L_q\hat{L}_fh\right), \\ \eta &= \ddot{y}_r + \beta_1(\dot{y}_r - \dot{y}) + \beta_2(y_r - y), \\ \dot{\hat{\theta}} &= \Gamma W^T E_1, \end{aligned}\right\} \qquad (34)$$

where W is the regression matrix, E_1 is a vector of error signals and Γ is a symmetric and positive definite matrix as the adaptive gain. β_1 and β_2 are diagonal matrices which denote derivative and proportional gains of the linear control law, respectively.

Proof. According to certainty equivalence principle, we need to replace $D(x)$ and $L_qL_fh(x)$ by their estimates in decoupling control law (32):

$$u_a = \hat{D}^{-1}(x)\cdot\left(\eta - L_f^2h - L_q\hat{L}_fh\right), \qquad (35)$$

where

$$\hat{D}(x) = L_{\hat{g}}L_fh(x), \quad L_q\hat{L}_fh = L_{\hat{q}}L_fh. \qquad (36)$$

By substituting Eq. (35) in Eq. (29), we have

$$\ddot{y} = L_f^2h(x) + L_qL_fh(x) + D(x)\cdot\hat{D}^{-1}(x) \cdot\left(\eta - L_f^2h - L_q\hat{L}_fh\right). \qquad (37)$$

After some manipulation, Eq. (37) may easily be written in the following form

$$\ddot{y} = \eta + L_q\tilde{L}_fh(x) + \tilde{D}(x)\hat{D}^{-1}(x)\cdot\left(\eta - L_f^2h - L_q\hat{L}_fh\right). \qquad (38)$$

where

$$L_q\tilde{L}_fh(x) = L_qL_fh(x) - L_q\hat{L}_fh, \quad \tilde{D}(x) = D(x) - \hat{D}(x). \qquad (39)$$

Considering Eqs. (23) and (31), the terms $\tilde{D}(x)\hat{D}^{-1}(x)$ and $L_q\tilde{L}_fh(x)$ are easily computed as

$$\tilde{D}(x)\hat{D}^{-1}(x) = \begin{bmatrix} \tilde{\theta}_5\hat{\theta}_5^{-1}\cos^2\varphi + \tilde{\theta}_6\hat{\theta}_6^{-1}\sin^2\varphi & \left(\tilde{\theta}_5\hat{\theta}_5^{-1} - \tilde{\theta}_6\hat{\theta}_6^{-1}\right)\cos\varphi\sin\varphi \\ \left(\tilde{\theta}_5\hat{\theta}_5^{-1} - \tilde{\theta}_6\hat{\theta}_6^{-1}\right)\cos\varphi\sin\varphi & \tilde{\theta}_5\hat{\theta}_5^{-1}\sin^2\varphi + \tilde{\theta}_6\hat{\theta}_6^{-1}\cos^2\varphi \end{bmatrix}, \qquad (40)$$

$$L_q\tilde{L}_fh(x) = \begin{bmatrix} -\tilde{\theta}_1\, v_r\cos\varphi - \tilde{\theta}_2\omega_r^2\cos\varphi - \tilde{\theta}_3\omega_r\, v_rL\sin\varphi + \tilde{\theta}_4\omega_r\, L\sin\varphi \\ -\tilde{\theta}_1\, v_r\sin\varphi - \tilde{\theta}_2\omega_r^2\sin\varphi + \tilde{\theta}_3\omega_r\, v_rL\cos\varphi - \tilde{\theta}_3\omega_rL\cos\varphi \end{bmatrix}. \qquad (41)$$

By replacing Eqs. (40) and (41) into Eqs. (38), the parametric model may be readily derived:

$$\ddot{y} = \eta + W.\tilde{\theta}, \qquad (42)$$

where $\tilde{\theta} = [\tilde{\theta}_1, \tilde{\theta}_2, \tilde{\theta}_3, \tilde{\theta}_4, \tilde{\theta}_5, \tilde{\theta}_6]^T$ and the matrix $W \in R^{(n-m)\times 6}$ is defined as

$$\left.\begin{aligned} W &= \begin{bmatrix} w_{11} & w_{12} & w_{13} & w_{14} & w_{15} & w_{16} \\ w_{21} & w_{22} & w_{23} & w_{24} & w_{25} & w_{26} \end{bmatrix}, \\ w_{11} &= -v_r\cos\varphi, \\ w_{12} &= -\omega_r^2\cos\varphi, \\ w_{13} &= -\omega_r\, v_rL\sin\varphi, \\ w_{14} &= \omega_r\, L\sin\varphi \\ w_{15} &= \hat{\theta}_5^{-1}(\Phi_1\cos^2\varphi + \Phi_2\sin\varphi\cdot\cos\varphi), \\ w_{16} &= \hat{\theta}_6^{-1}(\Phi_1\sin^2\varphi - \Phi_2\sin\varphi\cdot\cos\varphi), \\ w_{21} &= -v_r\sin\varphi, \\ w_{22} &= -\omega_r^2\sin\varphi, \\ w_{23} &= \omega_r\, v_rL\cos\varphi, \\ w_{24} &= -\omega_r\, L\cos\varphi, \\ w_{25} &= \hat{\theta}_5^{-1}(\Phi_1\sin\varphi\cdot\cos\varphi + \Phi_2\sin^2\varphi), \\ w_{26} &= \hat{\theta}_6^{-1}(-\Phi_1\sin\varphi\cdot\cos\varphi + \Phi_2\cos^2\varphi). \end{aligned}\right\} \qquad (43)$$

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In Eqs. (43), Φ_1 and Φ_2 are defined as

$$\left.\begin{aligned}\Phi_1 &= \eta_1 - \frac{\partial}{\partial q}(J_{h1}.Sv).Sv - w_{11}\hat{\theta}_1 - w_{12}\hat{\theta}_2\\ &\quad - w_{13}\hat{\theta}_3 - w_{14}\hat{\theta}_4,\\ \Phi_2 &= \eta_2 - \frac{\partial}{\partial q}(J_{h2}.Sv).Sv - w_{21}\hat{\theta}_1 - w_{22}\hat{\theta}_2\\ &\quad - w_{23}\hat{\theta}_3 - w_{24}\hat{\theta}_4.\end{aligned}\right\} \tag{44}$$

Now, the adaptive law may be derived by SPR-Lyapunov design approach which is motivated from the textbooks on adaptive control of Sastry,[22] Ioannou[23] and the work of Craig.[26] Assume that the external control input η_j in Eq. (42) is chosen so that *j*th output, $y_j(t)$, tracks the desired output, $y_{jr}(t)$:

$$\eta_j = \ddot{y}_{jr} + \beta_{1j}(\dot{y}_{jr} - \dot{y}_j) + \beta_{2j}(y_{jr} - y_j), \quad j = 1, 2. \tag{45}$$

This yields the following error equation,

$$\ddot{e}_j + \beta_{1j}\dot{e}_j + \beta_{2j}e_j = W_j\tilde{\theta}, \tag{46}$$

where $W_j = [w_{j1}, w_{j2}, w_{j3}, w_{j4}, w_{j5}, w_{j6}]$, $j = 1, 2$.

Remark 3. The determinant of decoupling matrix in control law in Eq. (34) is $\det(\hat{D}(x)) = -L\,\hat{\theta}_5\,\hat{\theta}_6$. Hence, prior bounds on the parameters θ_5 and θ_6 are sufficient to guarantee non-singularity of the decoupling matrix. As implied in the work of Sastry,[9] several techniques exist in the literature for this purpose (e.g. see the textbooks on adaptive control[22–23]). This remark and Assumption 2 also guarantee that $W_j \in L_\infty$. For purposes of adaptation, one may use the following filtered error signal for *j*th output:

$$\varepsilon_j = \dot{e}_j + \alpha_j e_j. \tag{47}$$

Note that since $\dot{e}_j = \dot{y}_j - \dot{y}_{jr}$ is known as a function of measured states by considering Eq. (27), therefore ε_j is available. The parameter α_j is chosen such that the following transfer function is strictly positive real (SPR):

$$T_j(s) = \frac{s + \alpha_j}{s^2 + \beta_{1j}s + \beta_{2j}}. \tag{48}$$

This means that $T_j(s)$ is analytic in the closed right half plane and $Re(T_j(j\omega)) > 0$. Then, by positive real lemma,[23] there exist the positive definite matrices P_j and Q_j such that

$$\left.\begin{aligned}A_j^T P_j + P_j A_j &= -Q_j,\\ P_j B_j &= C_j^T,\end{aligned}\right\} \tag{49}$$

where matrices A_j, B_j and C_j are defined by minimal state space realization of Eqs. (46) and (47) in the following form:

$$\left.\begin{aligned}\dot{X}_j &= A_j X_j + B_j W_j\tilde{\theta},\\ \varepsilon_j &= C_j X_j,\end{aligned}\right\} \tag{50}$$

where $X_j = [e_j,\ \dot{e}_j]^T$ is the state variable and

$$A_j = \begin{bmatrix} 0 & 1\\ -\beta_{2j} & -\beta_{1j}\end{bmatrix}, \quad B_j = \begin{bmatrix}0\\1\end{bmatrix}, \quad C_j = \begin{bmatrix}\alpha_j & 1\end{bmatrix}. \tag{51}$$

As a result, the entire system error equations may be written as

$$\left.\begin{aligned}\dot{X} &= AX + BW\tilde{\theta},\\ E_1 &= CX,\end{aligned}\right\} \tag{52}$$

where A, B and C are block diagonal matrices:

$$\left.\begin{aligned}A = diag(A_1, A_2), \quad B = diag(B_1, B_2),\\ C = diag(C_1, C_2),\end{aligned}\right\} \tag{53}$$

and $X = [X_1^T, X_2^T]^T$. The Lyapunov equation (49) is also written for entire system as follows:

$$\left.\begin{aligned}A^T P + PA &= -Q,\\ PB &= C^T,\end{aligned}\right\} \tag{54}$$

where,

$$P = diag(P_1, P_2), \quad Q = diag(Q_1, Q_2). \tag{55}$$

Now, one may define the following Lyapunov function to derive adaptive law.

$$V(X, \tilde{\theta}) = X^T PX + \tilde{\theta}^T\Gamma^{-1}\tilde{\theta}. \tag{56}$$

By taking time derivative of the proposed function and applying Eqs. (52) and (54), we may easily write

$$\dot{V}(X, \tilde{\theta}) = -X^T QX + 2\tilde{\theta}^T(W^T E_1 + \Gamma^{-1}\dot{\tilde{\theta}}). \tag{57}$$

One may choose

$$\dot{\tilde{\theta}} = -\Gamma W^T E_1 \tag{58}$$

to be assured that the derivative of Lyapunov function is negative definite. Since θ is a constant parameter, then $\dot{\tilde{\theta}} = -\dot{\hat{\theta}}$ and the adaptive law in Eq. (34) is readily obtained.

As a result, we have

$$\dot{V}(X, \tilde{\theta}) = -X^T QX \le -\lambda_{\min}(Q)\,\|X\|^2. \tag{59}$$

This means that $X \in L_2$ and thanks to the Lyapunov theory, we have $X, \tilde{\theta} \in L_\infty$. Therefore, $E_1 = CX \in L_2$ and $E_1 \in L_\infty$. By considering adaptive law, we have $|\dot{\hat{\theta}}| \le \|\Gamma\|.|W^T|.|E_1|$ which together with $E_1 \in L_2$ and $W \in L_\infty$ implies that $\dot{\hat{\theta}} \in L_2$. Since $X, \tilde{\theta} \in L_\infty$, then it follows that $\dot{X} = AX + BW\tilde{\theta} \in L_\infty$ and $\dot{E}_1 = C \cdot \dot{X} \in L_\infty$; therefore, together with $E_1 \in L_2$, we conclude that $E_1 \to 0$ as $t \to \infty$, which, in turn, implies that $\dot{\hat{\theta}} \to 0$ as $t \to \infty$. This result shows that the tracking errors e_j and $\dot{e}_j$ are asymptotically stable. However, from this analysis, we only conclude that the parameter estimation error remains bounded. As it is standard in the literature, in order that the parameter error converges to zero exponentially fast, the following *Persistency of*

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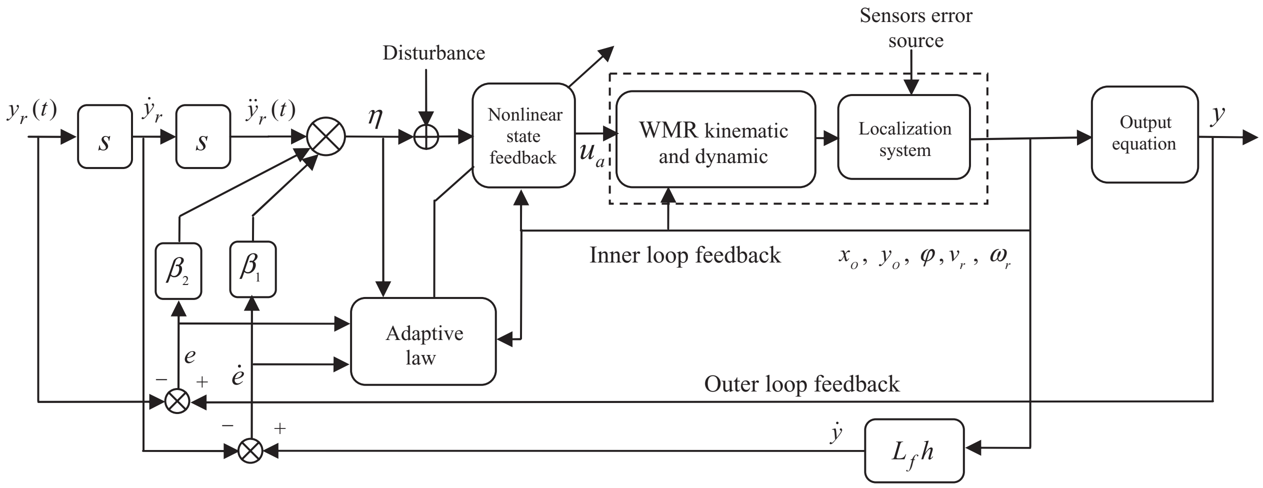

Fig. 3. Adaptive feedback linearizing control system for the trajectory tracking of WMR.

Excitation (PE) condition must be satisfied over any interval of time of length T_0 :

$$\alpha I \leq \int_t^{t+T_0} W(\tau)^T W(\tau)\, d\tau \leq \beta I, \qquad (60)$$

where α is the level of excitation and $\beta > 0$ is a constant parameter. Note that if $W \notin L_\infty$, a similar adaptive scheme may be derived by using normalization techniques to cope with unbounded regression matrix. The block diagram of the adaptive feedback linearizing controller is shown in Fig. 3.

Remark 4. Sometimes, parameters convergence is not satisfactory due to lack of PE condition. One may provide sufficient excitation for the reference signal by appending a bounded excitation signal $d(t)$ to $y_r(t)$ (e.g. see the work of Adetola[19]). A good candidate for $d(t)$ is a linear combination of sinusoidal functions with $\bar{n}$ distinct frequencies:

$$d(t) = \sum_{i=1}^{\bar{n}} a_i(t) \cdot \sin \omega_i t. \qquad (61)$$

After a certain amount of time, this signal is removed from the reference $y_r(t)$ by choosing the amplitude of excitation signal as

$$a_i(t) = \frac{a_i}{1 + \exp(\gamma(t - t_C))}, \qquad (62)$$

where γ is a large number and t_C is the time of excitation signal removal.

Remark 5. A modification on adaptation law in Eq. (34) seems to be necessary in presence of uncertainty due to modelling errors, wheels slip and surface friction, sensors noise and localization errors, kinematic and dynamic disturbances, quantization and discretization errors and other sources of uncertainty. The robust adaptive law may be proposed by the following modification:

$$\dot{\hat{\theta}} = \Gamma W^T E_1 + f_\theta(t), \qquad (63)$$

where $f_\theta(t)$ is a signal for robustness and it can be designed with robust adaptive techniques such as σ-modification (e.g. see the textbook of Ioannou[23]). This modification is not the subject of this paper. Various applications of these techniques can be found in the literature.[20–27]

4. Simulation Results

In this section, some computer simulations are performed to evaluate the performance of the proposed controller. In these simulations, parameters are chosen to match with a real world mobile robot, and Gaussian white noise is also added to the states to simulate a localization system such as odometry. Real physical parameters of the WMR and control parameters are summarized in Table II. In order to show the performance of the adaptive tracking controller, a non-adaptive controller is also designed based on feedback linearization technique

Table II. Simulation parameters values for WMR.

Simulation 2	Simulation 1	Parameter	Simulation 2	Simulation 1	Parameter
$0.006\ \mathrm{Kg \cdot m^2}$	$0.0025\ \mathrm{Kg \cdot m^2}$	I_m	0.05 m	0.15 m	r
$3\ \mathrm{Kg \cdot m^2}$	$15.625\ \mathrm{Kg \cdot m^2}$	I_C	0.3 m	0. 75 m	b
0.02 s	0.02 s	dt	0.15 m	0.3 m	d
2 m	7.5 m	R	0.2 m	0.1 m	L
(2.5 m, 5.5 m)	(10 m, 25 m)	(x_g, y_g)	0.2 Kg	1 Kg	m_w
0.2615	7.2	K_1	10 Kg	36 Kg	m_C
0.2668	2.592	K_2	$0.002\ \mathrm{Kg \cdot m^2}$	$0.005\ \mathrm{Kg \cdot m^2}$	I_w

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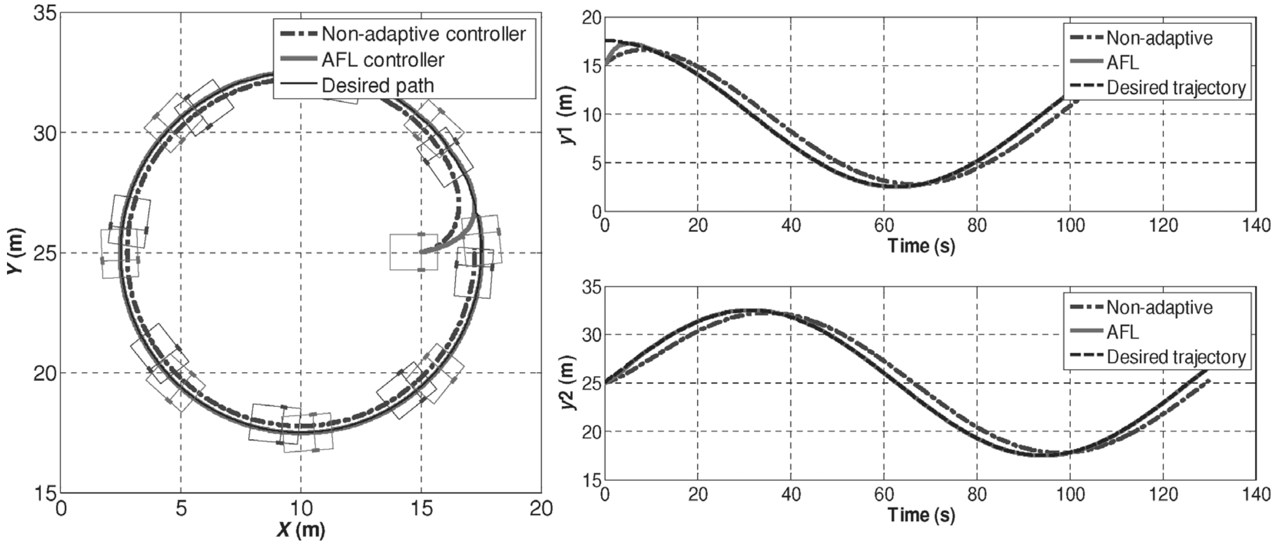

Fig. 4. WMR trajectory for both controllers: Non-adaptive controller with unknown parameters (dashed-dotted line) and adaptive feedback linearizing (AFL) controller in presence of unknown parameters (bold solid line).

in presence of unknown parameters. In the first simulation, a smooth desired trajectory is chosen as following:

$$\left.\begin{aligned} x_r(t) &= x_g + R\cos(\omega_r t), \\ y_r(t) &= y_g + R\sin(\omega_r t), \end{aligned}\right\} \tag{64}$$

where (x_g, y_g) and R are the centre and radius of circular trajectory, respectively and $\omega_r = 0.05$. The initial values of the estimated parameters of WMR are selected as $\hat{\theta}(0) = [1, 1, 1, 1, 1, 1]^T$. According to Table II, the real values of parameters vector are chosen to be $\theta = [6.06, 0.284, 0.54, 6.48, 1.26, 1.8]^T$. All simulations are carried out using Euler method with the time step of 0.02 s. In order to provide a smooth navigation, a critically damped system is chosen by setting $\beta_{1j} = 2\sqrt{\beta_{2j}}$ with $\beta_{2j} = 1$ in error Eq. (46) which together with $\alpha_j = \beta_{2j}/\beta_{1j}$ provides the presented SPR conditions for the transfer function (48). Figure 4 shows the desired and WMR trajectories for both adaptive and non-adaptive tracking controllers. Figure 5 shows the estimated parameters by adaptation law in Eq. (34).

Another simulation was performed for the following desired trajectory with different set of parameters which are summarized in Table II:

$$\left.\begin{aligned} x_r(t) &= x_g + R\sin(2\omega_r t), \\ y_r(t) &= y_g + R\sin(\omega_r t), \end{aligned}\right\} \tag{65}$$

where $\omega_r = 0.05$ and this time, the initial values of estimated parameters are selected to be $\hat{\theta}(0) =$

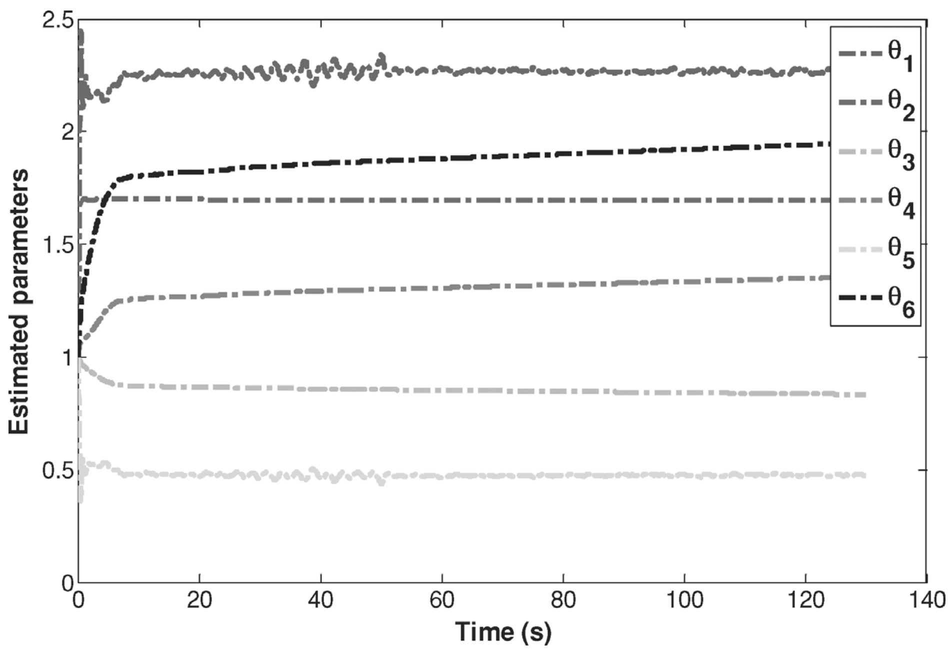

Fig. 5. Estimated WMR parameters $\hat{\theta}_i$, $i = 1, \ldots, 5$.

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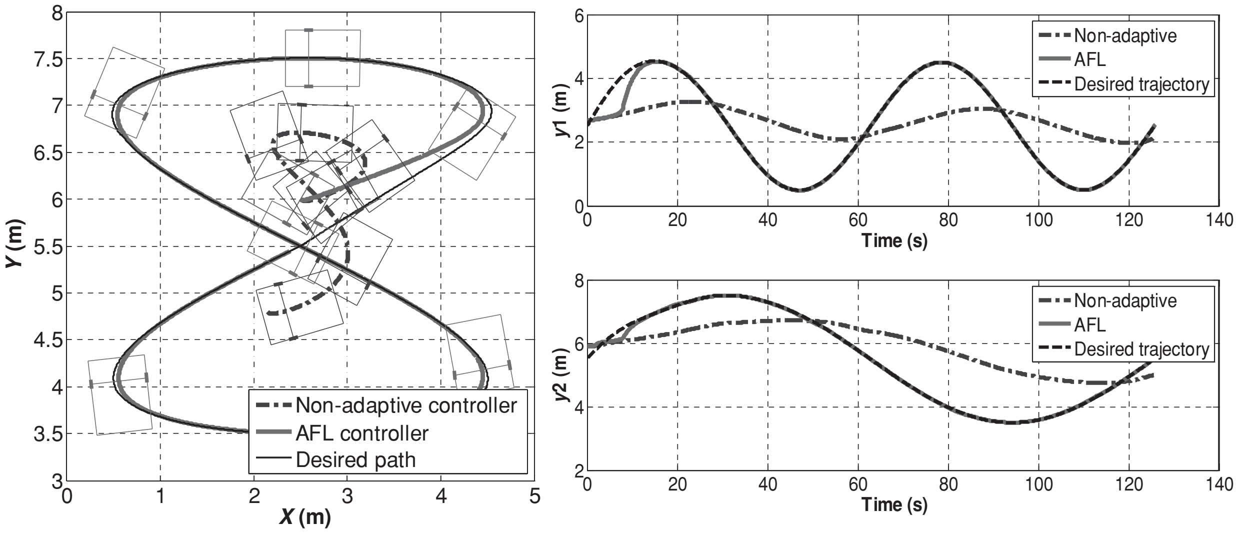

Fig. 6. WMR trajectory for both controllers: Non-adaptive controller with unknown parameters (dashed-dotted line) and adaptive feedback linearizing controller in presence of unknown parameters (bold solid line).

$[1, 1, 1, 1, 1, 1]^T$. The real values of parameters vector are selected to be $\theta = [20.52, 0.144, 0.46, 5.87, 0.5, 0.48]^T$. Figure 6 shows the performance of trajectory tracking controllers. Estimated parameters are also demonstrated by Fig. 7.

Simulation results show that the non-adaptive controller fails to track desired trajectory because of uncertain WMR parameters while the adaptive controller demonstrates a successful tracking. In addition, noisy odometry data has undesirable effects on parameters estimates which may be easily removed by robust adaptive techniques (e.g. see the work of Martins[27]). Note that ω_r must be chosen small because the controller performance degrades when ω_r is far from zero.

5. Experimental Results

This section presents an experimental evaluation of the proposed adaptive controller on robuLAB 10 WMR from Robosoft Inc. which is shown in Fig. 8. The robuLAB 10 is a differentially-driven WMR which is equipped with sonar, a laser range finder, a wireless LAN for the communication, 12V batteries, two DC motors to drive wheels that each wheel is equipped with an incremental encoder for the localization system which updates the relative pose of the WMR every 200 ms. Two passive castor wheels are placed in the rear and front of the WMR to preserve its equilibrium. The WMR does not accept the motors voltage as the input. It is only commanded by linear and angular velocities which are denoted by $v_{ref,1}(t)$ and $v_{ref,2}(t)$, respectively.

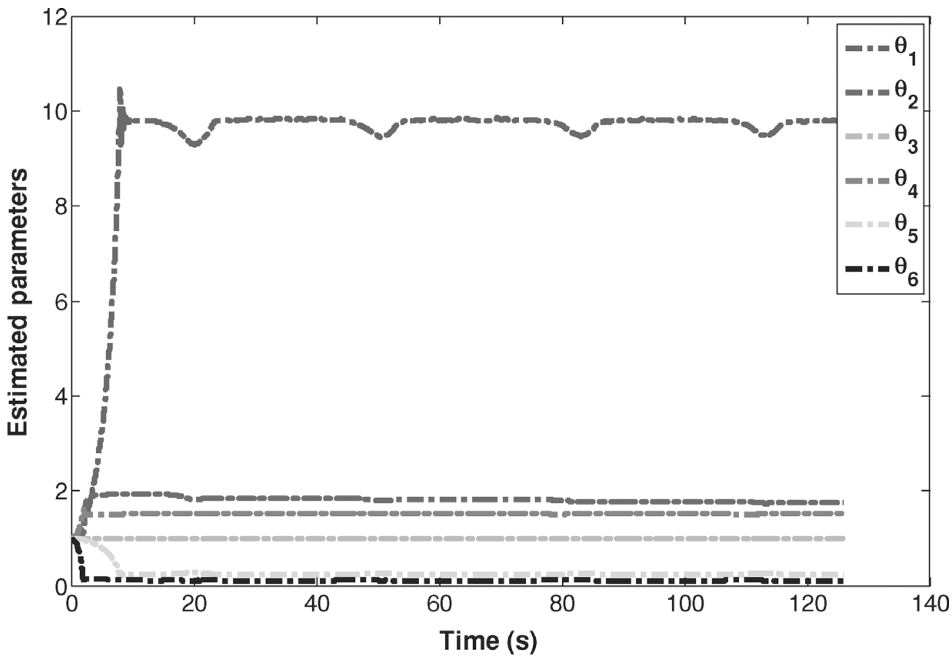

Fig. 7. Estimated WMR parameters $\hat{\theta}_i$, $i = 1, \ldots, 6$.

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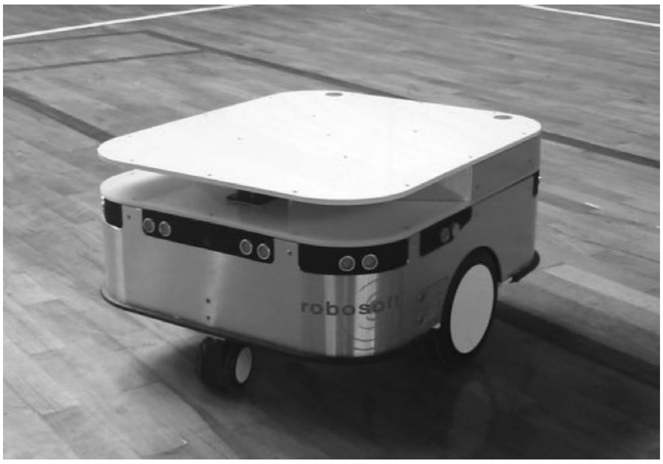

Fig. 8. RobuLAB 10 wheeled mobile platform.

Therefore, in order to test the proposed control law (34) experimentally, it is assumed that the velocities of DC motors are controlled by proportional derivative (PD) controllers. In addition, it is assumed that DC motors are identical and their inductances are negligible. By incorporating PD controllers into WMR dynamics, one may achieve the following model whose inputs are linear and angular velocities. This model is proposed by De La Cruz *et al.* for the first time:[31]

$$\dot{x} = f(x) + g_1(x,\theta)\,v_{ref,1} + g_2(x,\theta)\,v_{ref,2}, \tag{66}$$

$$y = h(x) = [x + L\cos\varphi,\; y + L\sin\varphi]^T, \tag{67}$$

where

$$\left.\begin{aligned} f(x) &= \big[v_1\cos\varphi \quad v_1\sin\varphi \quad v_2 \quad \theta_1 v_2^2 - \theta_2 v_1 \\ &\qquad - \theta_3 v_1 v_2 - \theta_4 v_2\big]^T, \\ g_1(x,\theta) &= [0 \quad 0 \quad 0 \quad \theta_5 \quad 0]^T, \\ g_2(x,\theta) &= [0 \quad 0 \quad 0 \quad 0 \quad \theta_6]^T, \end{aligned}\right\} \tag{68}$$

and $x = [x, y, \varphi, v_1, v_2]^T$ denotes the state vector, $\theta_i, i = 1, \ldots, 6$ are uncertain parameters which are functions of physical parameters of the robot. Note that the non-parametric uncertainties are ignored in model (66).

The interested reader is referred to the work of Martins *et al.*[27] and the work of De La Cruz *et al.*[31] for more details about the above presented model. In this experiment, Microsoft Robotics Studio (MSRS) is used to implement the proposed controller. The MSRS executes the controller program code and generates the linear and angular velocities to be commanded to RobuLAB 10 through a wireless LAN. It is assumed that there is little knowledge about the parameters of the WMR. The controller parameters for the experiment are chosen similar to simulation parameters. A circle-shaped trajectory is considered as the reference input which is specified by $y_{1r}(t) = x_g + R\cos(\omega_r t)$, $y_{2r}(t) = y_g + R\sin(\omega_r t)$ where $(x_g, y_g) = (0.5\,\text{m}, 0.5\,\text{m})$ and $R = 4\,\text{m}$ represent the centre and radius of the circle, respectively. The initial values of WMR motion are set to $x(0) = 2\,\text{m}$; $y(0) = 0.25\,\text{m}$; $\varphi(0) = 80^0$; $v_1(0) = 0$; $v_2(0) = 0$. In order to show the tracking performance and robustness of the proposed controller to parametric uncertainties, a feedback linearizing control law (without adaptation) is also tested on the WMR. Figure 9 illustrates one of experimental results which shows the desired trajectory (dashed line) and trajectories of robuLAB 10 which are the result of a feedback linearzing controller (dashed-dotted line) and our proposed controller (bold solid line). As shown by Figs. 9 and 10, experimental results also verify that our proposed controller is effective in presence of uncertain parameters. One may achieve better results by well tuning of the controller parameters. In spite of the robustness of the proposed controller to parametric uncertainties, the following problems may have undesirable effects in our experiments: (1) non-idealities of the mechanical system such as backlash, (2) wheels slippage, (3) actuators saturation, (4) quantization errors, (5) communication delays, (6) PD approximation error of the WMR model in Eq. (66) (see the work of De La Cruz[31] for more details) and (7) low frequency of the odometry system, which may induce some non-linear effects in the closed loop system. Improvement of the presented results demands more investigations which determine the direction of our future works.

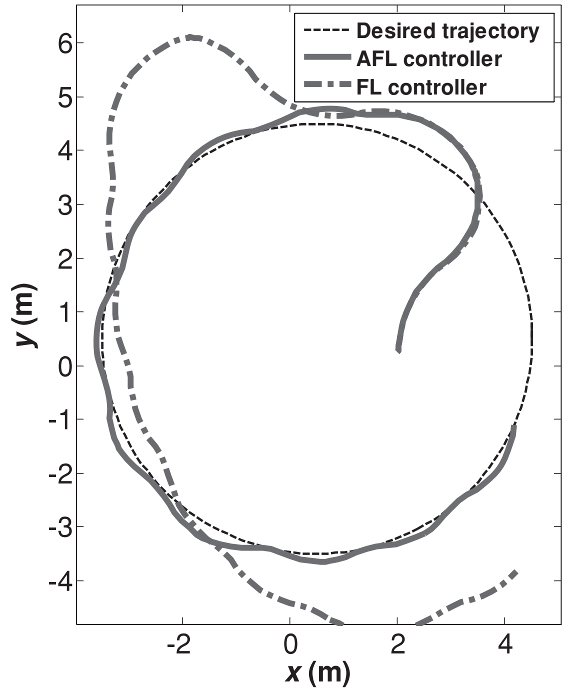

Fig. 9. Desired trajectory (dashed line), WMR trajectory by the feedback linearizing (FL) controller (FL, dashed-dotted line), the adaptive feedback linearizing (AFL) controller (AFL, bold solid line) in *x*–*y* plane.

6. Conclusion and Future Works

An adaptive feedback linearization trajectory tracking controller is proposed for a non-holonomic WMR in presence of uncertainty in the parameters of the kinematic and dynamic models and actuators dynamics. The adaptation law is derived by SPR-Lyapunov design, and it shows a satisfactory performance when well tuning of control parameters is provided. Computer simulations show that the proposed controller is robust to parametric uncertainty. The experimental results on a commercial WMR show that the proposed controller is effective. The presented work

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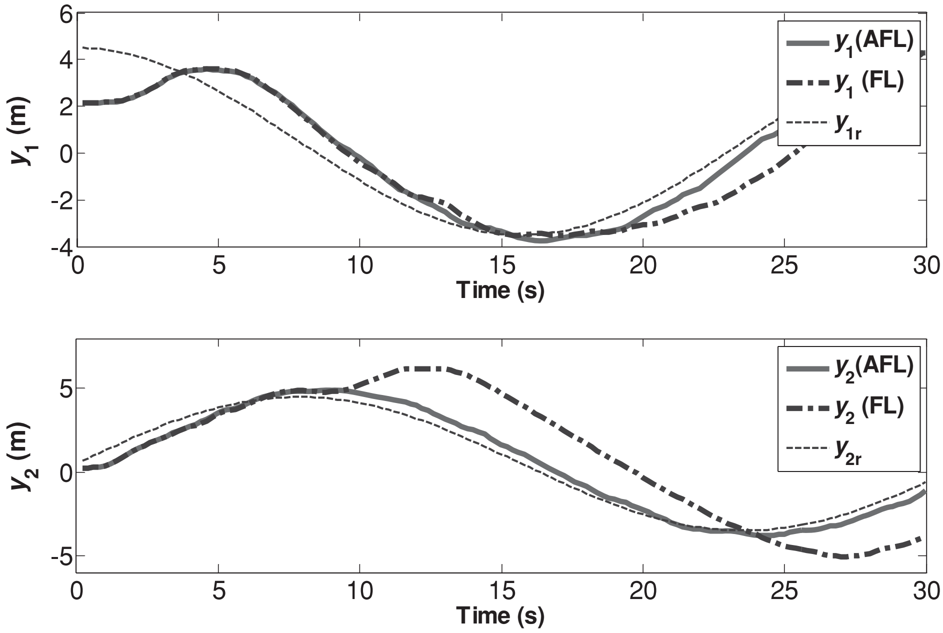

Fig. 10. Desired trajectory (dashed line), the WMR trajectory by the feedback linearizing controller (dashed-dotted line), the proposed controller (bold solid line).

in this paper has provided a good chance to continue our future researches. Referring to Remark 5, our next research will be on the robustness of the adaptive trajectory tracking controller in presence of different types of uncertainties in the WMR models.

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