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GROWTH, PENSIONS, AND THE AGING JONESES

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We incorporate keeping-up-with-the-Joneses (KUJ) preferences into the Blanchard–Yaari framework and develop a model of balanced growth. In this context we investigate status preference, demographic shocks, and pension policy. We find that a higher degree of KUJ lowers economic growth, whereas, in contrast, a decrease in the fertility and mortality rates increase it. In the second part of the paper we extend the model by incorporating a pay-as-you-go (PAYG) pension system with a statutory retirement date. The latter implies that the growth rate is higher under PAYG. We also consider the implications of pension reform under both defined benefit and defined contribution schemes.

Keywords: Relative Consumption, Overlapping Generations, Endogenous Growth, Pension Reform

1. INTRODUCTION

Since the 1980s, one of the main streams of research investigating the conditions for sustained economic growth has viewed the accumulation of capital, broadly measured, as one of its key sources. This approach, often termed the "*AK*" framework, has been propagated, among others, by authors such as Romer (1986), Barro (1990), and Rebelo (1991). In its canonical, single-factor form, the *AK* model yields a constant balanced growth rate with no transitional dynamics. In contrast to the standard Solow model, tax and government infrastructure policy [see Barro (1990)] affects the growth rate by changing the rate of return on capital.

The studies cited above all employ the representative agent (RA) framework. In this paper we seek to extend the insights of the *AK* setting by adopting the

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overlapping generations (OLG) approach to consumer behavior. Specifically, we use the Blanchard (1985)–Yaari (1965) continuous-time framework to model the decisions of finite-lived consumers. A central characteristic of the Blanchard–Yaari (BY) model is the demographic turnover from old to young population cohorts. Because the asset-poor young replace the asset-rich old, demographic turnover influences the economy's saving and, thus, its accumulation of capital, and, given *AK* production technology, its balanced growth rate. A key advantage, then, of the BY framework is that it enables us to consider how demographic parameters influence economic growth in a model in which capital accumulation plays the decisive role. Because population dynamics obviously depends on factors such as birth and mortality rates, we can employ our model to investigate how demographic shocks affect the balanced growth rate.[1] Due to its well-defined population dynamics, we can also use the BY model to investigate the effects of public policies, such as pay-as-you-go (PAYG) pensions, that have a significant intergenerational component.

Another important property of the BY framework, as recently shown by Fisher and Heijdra (2009) in an exogenous growth setting, is that the importance of demographic turnover also depends on agents' preferences, and specifically on their attitude to status. This allows us to ask whether or not status competition is an engine of economic growth. Recent authors who investigate this issue using an endogenous growth, representative agent (RA) framework with consumption as the reference good include Alvarez-Cuadrado et al. (2004), Liu and Turnovsky (2005), and Turnovsky and Monteiro (2007). The specification used by these authors is often termed the "keeping up with the Joneses" (KUJ) model of status preferences.[2] This literature does not, however, deliver an unambiguous answer regarding whether or not status competition is growth-promoting. Indeed, the relationship between status and growth in this context is highly sensitive to the specification of preferences and technology.[3]

The RA model is, moreover, a restrictive one to analyze the implications of status preferences, because all agents end up with the same consumption and asset holdings in the symmetric equilibrium, a situation implying that no one wins the "rat race." In contrast, agents of different ages, or "vintages," in the OLG framework possess distinct stocks of wealth and enjoy distinct levels of consumption. An economy-wide shift in KUJ then has age-dependent effects in the OLG context. A framework in which differences among individuals persist over time is, we believe, a promising avenue for exploring the macroeconomic implications of status competition. Another important task in this paper, then, is to extend the findings of Fisher and Heijdra (2009) to the endogenous growth context.[4]

Among our results, we show that the balanced growth rate, due to intergenerational turnover in financial wealth, is lower in the BY framework than on its RA counterpart. Furthermore, we show that an increase in KUJ lowers economic growth, because generational turnover, which tends to reduce growth, becomes more important as the degree of status preference rises. As an additional test of the model, we consider demographic disturbances characteristic of advanced societies: a reduction in fertility and a rise in longevity. We find that although a decline in fertility and an increase in longevity both raise the growth rate, they have opposite implications for the consumption–capital ratio: the latter rises in response to a "baby bust" and falls subsequent to a jump in life expectancy.

In the second part of the paper, we turn to the main policy application of our model, an investigation of the growth implications of a PAYG pension system. Because pension policies have important cross-cohort effects, an OLG framework such as this is an appropriate one to employ. In addition, researchers since at least Feldstein (1974) have been concerned with how public pensions influence the economy's long-run growth path. A key characteristic of our model is that agents are fully productive until retirement; i.e., there is no diminution of ability as agents age.[5] Moreover, because the PAYG system under consideration features a statutory retirement date, agents cease to be productive after that time.[6] We find that economic growth is higher—given our design of the scheme—under PAYG. This is because PAYG under statutory retirement introduces a life-cycle into human wealth earnings that alters the effects of demographic turnover in the BY setting. Specifically, PAYG pensions introduce a "wedge" between the human wealth of newborns and the *average* stock of human wealth. Because newborns possess more human wealth than older agents, who face "mandatory" retirement under PAYG, the intergenerational turnover in human wealth provides a countervailing positive element to the turnover in financial wealth, which, as indicated, lowers economic growth in the BY framework.

The paper is organized as follows: Section 2 outlines the firm and household sectors of the economy, which are aggregated in Section 3 to determine the macroeconomic equilibrium. Section 4 derives the balanced rate of endogenous growth, whereas Section 5 investigates how the latter is influenced by demographic and status preference shocks. Section 6 introduces the PAYG system into our OLG framework, which we analyze in Section 7. There we consider, first, the effect of public pensions on the rate of growth and, second, how an increase in the statutory retirement age influences the growth rate in defined benefit and defined contribution schemes. We close in Section 8 with brief concluding remarks and include Appendices containing supporting mathematical results.

2. THE MACROECONOMY

2.1. Firms

We begin by first analyzing the economy's firm sector. This permits us to describe the engine of endogenous growth, which relies, in the spirit of Romer (1989) and Saint-Paul (1992), on an interfirm externality. The latter also leads to a constant (real) interest rate, a result that simplifies the derivation of the macroeconomic equilibrium. The firm sector is made up of a large number of perfectly competitive firms producing a homogeneous good. At the individual firm level, output technology is Cobb–Douglas:

$$Y_i(t) = F[K_i(t), L_i(t)] \equiv Z(t) \cdot K_i(t)^{\varepsilon} L_i(t)^{1-\varepsilon}, \qquad 0 < \varepsilon < 1, \tag{1}$$

where $Y_i(t)$ represents the net output[7] of firm i, $K_i(t)$ is the capital stock, $L_i(t)$ is the labor supply (coinciding here with the population), and $Z(t)$ is the total factor productivity common to all firms. For simplicity, we assume that capital accumulation does not incur adjustment costs. As usual under profit maximization, the rental values of capital and labor correspond to their marginal physical products,

$$\begin{aligned} w(t) &= \frac{\partial Y_i(t)}{\partial L_i(t)} = (1-\varepsilon)Z(t) \cdot k_i(t)^{\varepsilon}, \\ r(t) &= \frac{\partial Y_i(t)}{\partial K_i(t)} = \varepsilon Z(t) \cdot k_i(t)^{\varepsilon-1}, \end{aligned} \tag{2}$$

where $k_i(t) \equiv K_i(t)/L_i(t)$ is the capital–labor ratio. Moreover, because each firm faces the same wage rate and rental rate of capital, each has the same capital–labor ratio, $k_i(t) = k(t)$, which implies that firm output corresponds to $Y_i(t) = Z(t)L_i(t)k^{\varepsilon}(t)$. To obtain economy-wide relationships, we define $Y(t) \equiv \sum_i Y_i(t)$, $K(t) \equiv \sum_i K_i(t)$, and $L(t) \equiv \sum_i L_i(t)$. In turn, the interfirm externality is given by

$$Z(t) = Z_0 \cdot k(t)^{1-\varepsilon}, \qquad Z_0 > 0, \tag{3}$$

which implies that *individual* firms benefit from a rise in the *average* capital intensity. Aggregating firm output and substituting for $Z(t) = Z_0 \cdot k(t)^{1-\varepsilon}$, we obtain the constant–returns to scale, economy-wide production function $Y(t) = Z_0 K(t)$. Substituting $Z(t) = Z_0 \cdot k(t)^{1-\varepsilon}$ into the marginal productivity conditions (3), we calculate expressions for the wage and the interest rate:

$$w(t) = (1-\varepsilon)y(t) = (1-\varepsilon)Z_0 k(t), \qquad r(t) = r = \varepsilon Z_0 > 0. \tag{4}$$

Clearly, the interest rate is a positive constant, a result that is the source of ongoing growth. In contrast, agents can look forward to ongoing wage growth.

2.2. Households

We assume that the economy consists of agents with different birth dates, or "vintages," who compare their own consumption $\bar{c}(v, \tau)$ to the average level of consumption $c(\tau)$. Following Fisher and Heijdra (2009), for a consumer born at time v ($v \leq t$), lifetime utility at t equals

$$\Lambda(v, t) = \int_t^{\infty} U[\bar{c}(v, \tau), c(\tau)] e^{(\rho+\beta)(t-\tau)} \, d\tau = \int_t^{\infty} \ln \bar{x}(v, \tau) e^{(\rho+\beta)(t-\tau)} \, d\tau, \tag{5}$$

where ρ is the rate of time preference, β is the given instantaneous death probability (independent of age), and $\bar{x}(v, \tau)$ is the instantaneous subfelicity function, defined as

$$\bar{x}(v,\tau) \equiv \frac{\bar{c}(v,\tau) - \alpha c(\tau)}{1-\alpha}, \qquad \alpha < 1, \tag{6}$$

where $\bar{c}(v,\tau)$ is individual consumption, $c(\tau)$ is the economy's average level of consumption, and the parameter α determines the agent's attitude to status competition. If $0 < \alpha < 1$, agents exhibit jealousy of the consumption of others. On the other hand, if $\alpha < 0$, then agents admire the consumption of others. The preferences in (6) satisfy the conditions for "keeping up with the Joneses" (KUJ).[8]

The budget identity of an agent born at time v equals

$$\dot{\bar{a}}(v,\tau) = (r+\beta)\bar{a}(v,\tau) + w(\tau) - \bar{c}(v,\tau), \tag{7}$$

where $\bar{a}(v,\tau)$ represents assets, r is the fixed interest rate, and $w(\tau)$ is the cohort-independent wage rate earned by agents who supply one unit of work effort. Assets yield an annuity income of $(r+\beta)\bar{a}(v,\tau)$, which consists of interest payments $r\bar{a}(v,\tau)$ and annuity receipts $\beta\bar{a}(v,\tau)$. Employing standard methods, we calculate the following time profile for $\bar{x}(v,\tau)$:

$$\frac{\dot{\bar{x}}(v,\tau)}{\bar{x}(v,\tau)} = r - \rho, \qquad r > \rho. \tag{8}$$

The necessary condition $r > \rho$ implies that we focus on a rising profile of $\bar{x}(v,\tau)$. In (8) we also obtain the usual BY result that the probability of death β cancels out along individual time profiles, because the (higher) annuity rate of return $r+\beta$ is offset by the (greater) effective rate of time preference $\rho+\beta$.[9] In fact, at the aggregate level, the crucial demographic parameter [see (16) below] is the fertility rate, denoted by η.

The next step is to calculate the intertemporal budget constraint of the individual. Integrating (7) subject to the transversality condition $\lim_{\tau\to\infty} \bar{a}(v,\tau)e^{-(r+\beta)(\tau-t)} = 0$ yields

$$\int_t^\infty [(1-\alpha)\bar{x}\,(v,\tau) + \alpha c(\tau)]\, e^{(r+\beta)(t-\tau)} d\tau = \bar{a}(v,t) + h(t), \tag{9}$$

where $h(t) = \int_t^\infty w(\tau)e^{(r+\beta)(t-\tau)}d\tau$ is age-independent human wealth.[10] Equation (9) states that the present discounted value of a weighted average of individual subfelicity and average consumption—where the weights depend on the parameter α—corresponds to the aggregate of the agent's financial and human wealth. Integrating (8) to obtain $\bar{x}(v,\tau) = \bar{x}(v,t)e^{(r+\beta)(\tau-t)}$, $\tau \geq t$, we can show that (9) reduces to

$$(1-\alpha)\frac{\bar{x}(v,t)}{\rho+\beta} = \bar{a}(v,t) + h(t) - \alpha\Gamma(t), \tag{10}$$

where $\Gamma(t) \equiv \int_t^\infty c(\tau)e^{(r+\beta)(t-\tau)}d\tau$ is the weighted average of future consumption. Substitution of $\bar{x}(v,t)$ from (6) into (10) yields an expression for individual consumption as a function of average consumption and wealth:

$$\bar{c}(v,t) = (\rho+\beta)\,[\bar{a}(v,t) + h(t)] + \alpha\,[c(t) - (\rho+\beta)\Gamma(t)]\,. \tag{11}$$

Equation (11) directly illustrates how status preferences determine individual consumption dynamics.[11] The term $[c(t) - (\rho+\beta)\Gamma(t)]$ represents a measure of the evolution of average consumption. If this term is negative, average consumption is expected to rise. Under the jealousy motive, $0 < \alpha < 1$, agents then suppress current consumption in order not to fall behind in future consumption. If, on the other hand, the term is positive, then average consumption is expected to fall, which, if agents are jealous, causes them to raise their consumption now.

3. AGGREGATION AND THE MACROECONOMIC EQUILIBRIUM

In this section of the paper we first specify the economy's demography. This is necessary to aggregate the individual relationships and, thus, to describe the OLG macroeconomy. Letting η represent the birth rate, the (constant) population growth rate is $n \equiv \eta - \beta$, with β, as indicated, the mortality probability. Through time, individual population cohorts $L(v,t)$ shrink as their members die off. The proportion of each cohort v in $L(t)$ at time t equals

$$l(v,t) \equiv \frac{L(v,t)}{L(t)} = \eta e^{\eta(v-t)}, \qquad t \geq v, \tag{12}$$

which enables us to define the per capita average values of consumption and financial assets,

$$c(t) \equiv \int_{-\infty}^{t} l(v,t)\bar{c}(v,t)\,dv, \qquad a(t) \equiv \int_{-\infty}^{t} l(v,t)\bar{a}(v,t)\,dv, \tag{13}$$

where $c(t)$ represents, furthermore, the consumption externality from the individual's point of view. Aggregating individual consumption (11), we obtain

$$c(t) = (\rho+\beta)\,[a(t) + h(t)] - \alpha\,[c(t) - (\rho+\beta)\Gamma(t)]. \tag{14}$$

Subtracting (14) from (11), we find that

$$\bar{c}(v,t) - c(t) = (\rho+\beta)\,[\bar{a}(v,t) - a(t)], \tag{15}$$

where the difference between individual and average consumption depends on the difference between individual and average financial wealth, a fact we use below to draw distinctions between the BY and RA frameworks.

The key step in deriving the growth equilibrium is obtaining the differential equations for average consumption and financial assets, $\dot{c}(t)$ and $\dot{a}(t)$. The details of this exercise are given in Appendix A. Using the expressions for $\dot{c}(t)$ and $\dot{a}(t)$, the rate of return and aggregate relationships of the production sector, and the fact

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that only physical capital is used for savings, $k(t) \equiv a(t)$, we derive the following macroeconomic equilibrium:

$$\frac{\dot{c}(t)}{c(t)} = r - \rho - \frac{\eta(\rho+\beta)}{1-\alpha} \cdot \frac{k(t)}{c(t)}, \quad \mathbf{(16)}$$

$$\dot{k}(t) = [r-n]k(t) + w(t) - c(t), \quad \mathbf{(17)}$$

$$w(t) = (1-\varepsilon)y(t), \qquad r = \varepsilon Z_0, \quad \mathbf{(18)}$$

$$y(t) = Z_0 k(t). \quad \mathbf{(19)}$$

The dynamics of consumption is described by (16), whereas (17) governs the accumulation of physical capital, where $n \equiv \eta - \beta$. Equation (18) reiterates the expressions for the wage and the interest rate, whereas (19) is the per capita version of the production function. In contrast to equations (17)–(19), which emerge in the usual Ramsey framework, equation (16) for consumption dynamics merits additional comment. The third term on the right-hand side of (16) is typical in the BY setting and represents the effect that demographic turnover has on consumption dynamics and, as we show below, on economic growth. To see this, we evaluate (15) at $v = t$ and impose $k(t) \equiv a(t)$. This yields $[c(t) - \bar{c}(t,t)] = (\rho+\beta)k(t)$, which, if substituted into (16), results in the following alternative representation of $\dot{c}(t)/c(t)$:

$$\frac{\dot{c}(t)}{c(t)} = r - \rho - \frac{\eta}{1-\alpha} \cdot \frac{c(t) - \bar{c}(t,t)}{c(t)}. \quad \mathbf{(20)}$$

The term $[c(t) - \bar{c}(t,t)]$, corresponding to the difference between average and newborn consumption, measures the effect of intergenerational turnover. In the BY framework, older generations are replaced by newborns. Because, however, agents are born with no financial wealth, their consumption *levels* fall short of those of their older counterparts. Consequently, the replacement of asset-rich by asset-poor population cohorts reduces the growth rate of *average* consumption. This is the case even though the growth rate of *individual* consumption, $\dot{\bar{c}}(v,\tau)/\bar{c}(v,\tau)$, is the same for each generation facing the given interest rate r. Observe further that in (20) the importance of intergenerational turnover depends not only on the birth rate η but also on the KUJ parameter α.[12] Specifically, higher values of α increase the importance of demographic transition and lower the growth rate of average compared to individual consumption. In Section 6 we analyze effect of status preference on the OLG balanced growth rate.

4. STEADY-STATE GROWTH

In this model the single accumulable factor of production, broad capital, has the constant–returns to scale property. Consequently, the long-run equilibrium is characterized by a sustainable, balanced growth rate, denoted by $\hat{\gamma}$. Furthermore, the economy exhibits no transitional dynamics. To see why this is the case, let

$\chi(t) \equiv c(t)/k(t)$ represent the consumption–capital ratio and employ (16)–(19) to derive $\dot{\chi}(t)/\chi(t)$:

$$\frac{\dot{\chi}(t)}{\chi(t)} = [r - \rho + Z_0 - n] + \chi(t) - \frac{\eta(\rho+\beta)}{1-\alpha} \cdot \frac{1}{\chi(t)}. \qquad (\mathbf{21})$$

It is straightforward to show that (21) is an unstable differential equation. Consequently, a stable equilibrium is achieved only if the consumption–capital ratio attains a constant value, $\chi(t) \equiv \hat{\chi}$, $\forall t \geq 0$, which, in turn, implies that the economy grows at the rate $\hat{\gamma}$ through time. The resulting steady-state growth profiles of capital, wages, and consumption are $\hat{k}(t) = \hat{k}_0 e^{\hat{\gamma}t}$, $\hat{w}(0) = \hat{w}_0 e^{\hat{\gamma}t}$, and $\hat{c}(t) = \hat{c}_0 e^{\hat{\gamma}t}$, where $\hat{k}(0) = \hat{k}_0$, $\hat{w}(0) = \hat{w}_0$, and $\hat{c}(0) = \hat{c}_0$ denote their respective initial values.

To determine the solution for the balanced growth rate, we evaluate (16) and (17) along the steady-state profile:

$$\hat{\gamma} = r - \rho - \frac{\eta(\rho+\beta)}{1-\alpha}\frac{1}{\hat{\chi}}, \qquad \hat{\gamma} = r + (1-\varepsilon)Z_0 - n - \hat{\chi}, \qquad (\mathbf{22})$$

where $\hat{\chi} \equiv \hat{c}/\hat{k}$ is the consumption–capital ratio along the balanced growth path. To further simplify the problem, we define the *growth-adjusted* interest rate as $\hat{r}_g \equiv r - \hat{\gamma}$ and reexpress (22) as

$$(\hat{r}_g - \rho)\chi = \frac{\eta(\rho+\beta)}{1-\alpha}, \qquad \hat{\chi} = \hat{r}_g + (1-\varepsilon)Z_0 - n. \qquad (\mathbf{23})$$

Combining the relationships in (23), we form the polynomial

$$\Phi(s) \equiv (s-\rho)\cdot[s + (1-\varepsilon)\,Z_0 - n] - \frac{\eta(\rho+\beta)}{1-\alpha}, \qquad (\mathbf{24})$$

where $\Phi(\hat{r}_g) = 0$ solves for the growth-adjusted interest rate $\hat{r}_g$. There is only one feasible solution with $\hat{\chi} > 0$. This is satisfied with $\hat{r}_g \equiv r - \hat{\gamma} > \rho$ and $\hat{r}_g > n - (1-\varepsilon)Z_0$.[13] The Euler and market-clearing relationships can also be combined to determine the polynomial $\Gamma(s)$ that solves for the consumption–capital ratio, i.e., $\Gamma(\hat{\chi}) = 0$:

$$\Gamma(s) \equiv s^2 - [\rho + (1-\varepsilon)\,Z_0 - n]\,s - \frac{\eta(\rho+\beta)}{1-\alpha}. \qquad (\mathbf{25})$$

Using (22), we illustrate in Figure 1 the OLG balanced growth equilibrium. The positively sloped locus EE_{BY} represents the Euler equation, whereas the downward-sloping line CA depicts market clearing. The relationships (both solid) have the following slopes:

$$\left.\frac{d\hat{\gamma}}{d\hat{\chi}}\right|_{\text{EE}_{\text{BY}}} = -\frac{\eta(\rho+\beta)}{(1-\alpha)\chi^2} > 0, \qquad \left.\frac{d\hat{\gamma}}{d\hat{\chi}}\right|_{\text{CA}} = -1.$$

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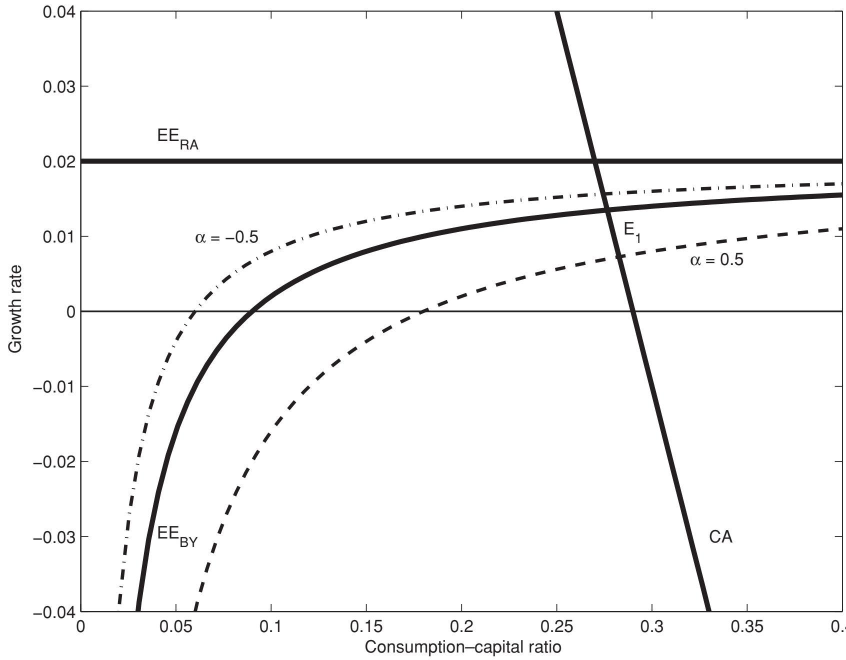

FIGURE 1. Growth and the KUJ effect.

The intersection of EE_{BY} and CA at point E_1 illustrates the OLG solution $(\hat{\chi}_1, \hat{\gamma}_1)$ determined by (24) and (25). The positively sloped EE_{BY} locus reflects the fact that the higher the consumption–capital ratio $\hat{\chi}$, the weaker is the intergenerational turnover effect, which implies a high growth rate. Along the negatively sloped CA line, higher values of $\hat{\chi}$ translate directly into lower rates of growth $\hat{\gamma}$. In addition, we depict in Figure 1 the solid horizontal line EE_{RA}, representing the Euler equation for the RA case. Observe that it lies uniformly above its EE_{BY} counterpart. The growth rate in the RA economy is simply the difference between the fixed interest rate and the given rate of time preference, $r - \rho$ (so that $\hat{r}_g = \rho$). The relationships in Figure 1 are generated using a numerical solution of the model that assumes the structural parameters take the following values:

$$\varepsilon = 0.20, \quad r = 0.06, \quad \rho = 0.04, \quad \alpha = 0, \quad \beta = 0.02, \quad \eta = 0.03. \qquad \textbf{(26)}$$

Clearly, the balanced growth rate $\hat{\gamma}$ is lower in the BY case than for its RA counterpart, whereas the consumption–capital ratio $\hat{\chi}$ is higher. Indeed, whereas the balanced growth rate in the RA economy depends only on technology and the pure of rate of time preference, in the BY setting, according to (16), it is also a function of agents' attitude to status, parameterized by α, as well as demographic parameters, η and β, that reflect intergenerational turnover. We next employ our

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OLG equilibrium to investigate the effect of changes in agents' attitude to consumption externalities and one-time demographic shocks.

5. COMPARATIVE STATIC EFFECTS

To determine the effects of demographic and status preference shocks on the OLG growth rate, we evaluate (24) and (25) at the solution values $(\hat{\chi}, \hat{r}_g)$:

$$\Phi(\hat{r}_g, \eta, \beta, \alpha) \equiv (\hat{r}_g - \rho) \cdot [\hat{r}_g + (1-\varepsilon)Z_0 - (\eta - \beta)] - \frac{\eta(\rho+\beta)}{1-\alpha} \equiv 0,$$
$$\Gamma(\hat{\chi}, \eta, \beta, \alpha) = \hat{\chi}^2 - [\rho + (1-\varepsilon)Z_0 - (\eta - \beta)]\,\hat{\chi} - \frac{\eta(\rho+\beta)}{1-\alpha} \equiv 0. \tag{27}$$

In all instances, the economy jumps immediately its new steady-state growth path. We consider first the consequences of an increase in the parameter α scaling status preference. Using (27), it is straightforward to show that

$$\frac{\partial \hat{r}_g}{\partial \alpha} = -\frac{\partial \hat{\gamma}}{\partial \alpha} = -\frac{\partial \Phi(\hat{r}_g, \eta, \beta, \alpha)/\partial \alpha}{\partial \Phi(\hat{r}_g, \eta, \beta, \alpha)/\partial \hat{r}_g} > 0,$$
$$\frac{d\hat{\chi}}{d\alpha} = -\frac{\partial \Gamma(\hat{\chi}, \eta, \beta, \alpha)/\partial \alpha}{\partial \Gamma(\hat{\chi}, \eta, \beta, \alpha)/\partial \hat{\chi}} > 0, \tag{28}$$

where the signs in (28) imply that a rise in α causes a decline in the growth rate and an increase in the consumption–capital ratio. The larger the status externality, the more important is intergenerational turnover, which implies that average consumption rises at the expense of saving, leading to a permanent fall in $\hat{\gamma}$ and rise in $\hat{\chi}$. In terms of Figure 1, the increase in α from 0 to 0.5 causes the EE_{BY} to shift down to its dashed counterpart (CA is unaffected), leading to a new equilibrium featuring a lower value of $\hat{\gamma}$ and an increase in $\hat{\chi}$. Intuitively, this occurs because new population cohorts, born with no financial wealth, enter the economy placing a greater weight on status preference than did prior generations on entering the economy. The rate of saving falls immediately, because adjustment to the new, lower balanced growth path is instantaneous, as *all* agents seek to attain a higher benchmark level of consumption. This is the endogenous growth analogue of the result of Fisher and Heijdra (2009), showing that a rise in status preference leads in steady state to a decline in the stock of capital and a rise in consumption. One distinction is that here adjustment takes place instantly, whereas the Fisher and Heijdra (2009) findings feature an initial *increase* in consumption, followed by a continuous decline in its level, accompanied by a reduction in the capital stock. Another distinction is that in Fisher and Heijdra (2009) the real interest *rises* due to the steady-state decline in the physical capital stock, which leads to a "steepening" of the consumption time profile of preshock generations. In contrast, the real interest is fixed in this model and these cohort effects do not take

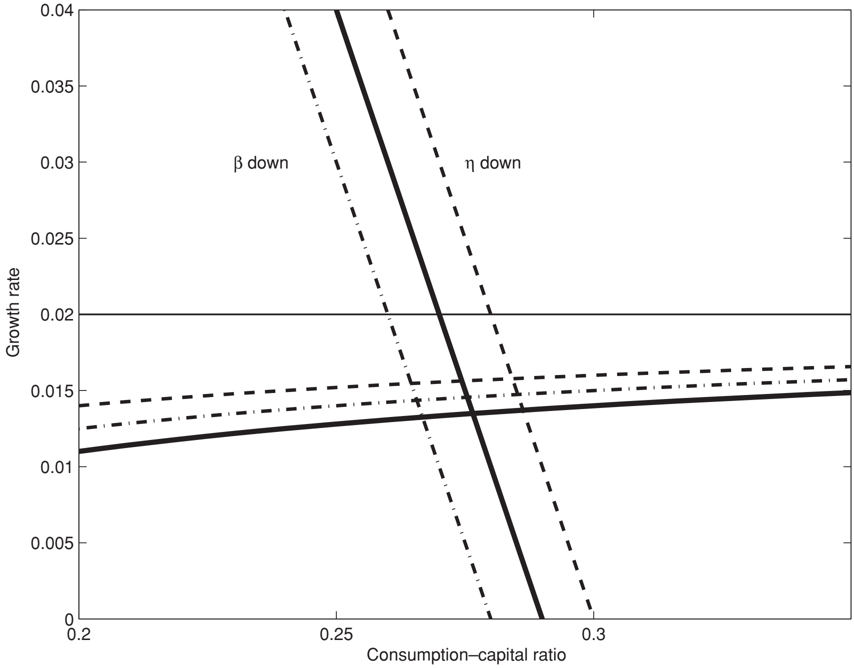

FIGURE 2. Growth and demographic shocks.

place. Finally, for comparison, observe that we also depict in Figure 1 the case of consumption admiration, i.e., a *fall* in α from 0 to -0.5, which causes EE_{BY} to shift up from its original position and results in a rise in $\hat{\gamma}$ and a fall in $\hat{\chi}$.

Considering next the case of a baby bust, differentiation of (27) with respect to η yields

$$\begin{aligned} \frac{\partial \hat{r}_g}{\partial \eta} &= -\frac{\partial \hat{\gamma}}{\partial \eta} = -\frac{\partial \Phi(\hat{r}_g, \eta, \beta, \alpha)/\partial \eta}{\partial \Phi(\hat{r}_g, \eta, \beta, \alpha)/\partial \hat{r}_g} > 0, \\ \frac{d\hat{x}}{d\eta} &= -\frac{\partial \Gamma(\hat{\chi}, \eta, \beta, \alpha)/\partial \eta}{\partial \Gamma(\hat{\chi}, \eta, \beta, \alpha)/\partial \hat{\chi}} < 0, \end{aligned} \qquad \mathbf{(29)}$$

where the signs in (29) follow from $\hat{r}_g > \rho$ and $\hat{r}_g > n - (1 - \varepsilon)Z_0$. According to (29), a decline in fertility, because it reduces the importance of intergenerational turnover, leads to an increase in economic growth and a rise in the consumption–capital ratio. Because new population cohorts are born without financial wealth, a fall in the rate at which they join the economy implies that the population becomes, on average, wealthier, which, in turn, raises the *average* rate of capital accumulation and growth. Graphically, this shock is illustrated in Figure 2, where η falls from 0.03 to 0.02. The new Euler and the market-clearing relationships (dashed) result in a new equilibrium with higher values of $\hat{\gamma}$ and $\hat{\chi}$. Observe, moreover,

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that jealousy and lower birth rates have opposite implications for balanced growth. This implies that generational turnover is still important in ageing societies that have a high degree of status consciousness, e.g., with α close to unity.

Finally, turning to the case of a longevity boom, we find[14]

$$\frac{\partial \hat{r}_g}{\partial \beta} = -\frac{\partial \hat{\gamma}}{\partial \beta} = -\frac{\partial \Phi(\hat{r}_g, \eta, \beta, \alpha)/\partial \eta}{\partial \Phi(\hat{r}_g, \eta, \beta, \alpha)/\partial \hat{r}_g} < 0,$$
$$\frac{d\hat{\chi}}{d\beta} = -\frac{\partial \Gamma(\hat{\chi}, \eta, \beta, \alpha)/\partial \beta}{\partial \Gamma(\hat{\chi}, \eta, \beta, \alpha)/\partial \hat{\chi}} > 0; \tag{30}$$

this leads to a higher growth rate and—in contrast to a baby bust—a *lower* consumption–capital ratio. Because agents live longer, they have the incentive to accelerate the accumulation of capital, directly raising the balanced growth rate $\hat{\gamma}$. However, because this gain is spread out over a longer lifetime, consumption falls *relative* to the stock of capital. We also illustrate this in Figure 2, which depicts the shift up in EE_{BY} and the shift down in CA (dash-dotted), which lead to an increase in $\hat{\gamma}$ and a fall in $\hat{\chi}$ in the case where β falls from 0.02 to 0.01.[15]

6. INTRODUCING A PAYG PENSION SYSTEM

We now extend the basic growth model to incorporate a PAYG pension system. Letting $\bar{p}(v, \tau)$ denote taxes (and transfers if negative), contributions are paid and benefits are received according to the following scheme:

$$\bar{p}(v, \tau) = \begin{cases} \theta \cdot w(\tau) & \text{for } \tau - v \leq u_R \\ -\pi \cdot w(\tau) & \text{for } \tau - v > u_R, \end{cases} \tag{31}$$

where θ is the contribution rate, π is the benefit rate [both indexed to the wage $w(\tau)$], and u_R is the statutory retirement age. For realism, we assume that workers earn more than pensioners, so that $(1-\theta) > \pi$. As in the benchmark specification, labor supply is exogenous, although modified to reflect the PAYG system:

$$\bar{n}(v, \tau) = \begin{cases} 1 & \text{for } \tau - v \leq u_R \\ 0 & \text{for } \tau - v > u_R. \end{cases} \tag{32}$$

According to (32), agents supply a full unit of labor during their whole working lives. There is no diminution in productivity until retirement, after which agents cease to work. The mandatory nature of retirement in this scheme forces worker productivity to fall to zero after u_R. This permits us to define the macroeconomic participation rate as

$$\frac{N(t)}{L(t)} \equiv \int_{-\infty}^{t} \bar{n}(v, t) l(v, t)\, dv = \int_{t-u_R}^{t} l(v, t)\, dv = 1 - e^{-\eta u_R}, \tag{33}$$

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where $N(t)$ is the work force. Clearly, the participation rate rises with u_R, whereas a baby bust (a decline in η) reduces it. This formulation allows us to define the *dependency ratio* as

$$\text{dep} \equiv \text{dep}\,(u_R, \eta) = \frac{e^{-\eta u_R}}{1 - e^{-\eta u_R}}. \tag{34}$$

Not only does the PAYG system place a wedge between the workforce and the population, it also implies that an agent's human wealth is age-dependent. Letting $\bar{h}(v, \tau)$ represent individual human wealth, its weighted average equals

$$h(t) \equiv \int_{-\infty}^{t} l(v, t)\bar{h}(v, t)\, dv. \tag{35}$$

Next, we impose sustainability of the PAYG system by assuming that contributions always equal payouts at all points in time:

$$\int_{t-u_R}^{t} \theta w(t) L(v, t)\, dv = \int_{-\infty}^{t-u_R} \pi w(t) L(v, t)\, dv. \tag{36}$$

Substituting $L(v, t) = L(t) \cdot l(v, \tau)$, using (12), the closure rule (36) reduces to

$$\theta \cdot [1 - e^{-\eta u_R}] = \pi \cdot e^{-\eta u_R}, \tag{37}$$

where one of θ, u_R, and π must be used to balance the PAYG budget. Observe that under a defined benefit (DB) scheme, π and u_R are held constant while θ balances the budget. In contrast, θ and u_R are held constant while π balances the budget under a defined contribution (DC) scheme. It is straightforward to show the following relationships hold:

$$\theta + \pi = \theta e^{\eta u_R}, \qquad 1 - \theta - \pi = 1 - \theta e^{\eta u_R} = 1 - \frac{\pi}{1 - e^{-\eta u_R}}, \tag{38}$$

which allow us to state the *replacement ratio* as

$$rr\,(\pi, \eta, u_R) = \frac{\pi}{1 - \theta} = \frac{\pi}{1 - \text{dep}\,(u_R, \eta) \cdot \pi}. \tag{39}$$

Consequently, for a DB scheme,

$$\frac{\partial(1 - \theta - \pi)}{\partial \pi} < 0, \qquad \frac{\partial(1 - \theta - \pi)}{\partial u_R} < 0, \tag{40}$$

whereas for a DC scenario,

$$\frac{\partial(1 - \theta - \pi)}{\partial \theta} < 0, \qquad \frac{\partial(1 - \theta - \pi)}{\partial u_R} > 0. \tag{41}$$

We use (40) and (41) below to investigate how a change in the statutory retirement date affects the balanced growth rate.

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To solve the modified model, we follow the same procedure outlined above. The firm's problem is solved as in Section 2.1, with $L(t)$ replaced by $N(t)$. Nevertheless, because labor supply, according to (33), depends on the retirement date u_R, so does the wage rate,

$$w(t) = (1-\varepsilon)\frac{Y(t)}{N(t)} = (1-\varepsilon)\frac{Z_0 k(t)}{1-e^{-\eta u_R}}, \quad \textbf{(42)}$$

where we substitute $Y(t) = Z_0 K(t)$, $N(t) \equiv [1-e^{-\eta u_R}]L(t)$ and use $k(t) \equiv K(t)/L(t)$ to obtain (42). Observe that a later retirement date lowers the wage in partial equilibrium.

Regarding the household's problem, we proceed along the same lines as in Section 2.2, with the exception that the agent's choices are made subject to (31). Consequently, we replace (7) with

$$\dot{\bar{a}}(v,\tau) = [r(\tau)+\beta]\bar{a}(v,\tau) + w(\tau) - \bar{c}(v,\tau) - \bar{p}(v,\tau). \quad \textbf{(43)}$$

Similarly, we replace $\bar{h}(v,t)$ with $h(t)$ in the expression (11) for individual consumption. In turn, an active agent possesses the following stock of lifetime human wealth:

$$\begin{aligned}\bar{h}(v,t) &\equiv \int_t^{v+u_R} w(\tau)e^{(r+\beta)(t-\tau)}d\tau - \int_t^{\infty} \bar{p}(v,\tau)e^{(r+\beta)(t-\tau)}d\tau \\ &= \int_t^{v+u_R}(1-\theta)w(\tau)e^{(r+\beta)(t-\tau)}d\tau + \int_{v+u_R}^{\infty}\pi w(\tau)e^{(r+\beta)(t-\tau)}d\tau, \quad \textbf{(44)}\end{aligned}$$

where we use (31) to obtain the second equality of (44). Substituting the path of wages in (44), $w(\tau) = w(t)\cdot e^{\hat{\gamma}(\tau-t)}$, $\tau \geq t$ (with $\hat{\gamma}$ determined in equilibrium), a worker's human wealth simplifies to[16]

$$\begin{aligned}\bar{h}(v,t) = \frac{w(t)}{r_g+\beta}\cdot\big[(1-\theta)\cdot\big[1-e^{(r_g+\beta)(t-v-u_R)}\big] \\ +\pi\cdot e^{(r_g+\beta)(t-v-u_R)}\big], \quad t-v \leq u_R. \quad \textbf{(45)}\end{aligned}$$

Correspondingly, a retired person's lifetime human wealth is given by

$$\bar{h}(v,t) = \pi\int_t^{\infty} w(\tau)e^{(r+\beta)(t-\tau)}d\tau = \frac{\pi w(t)}{r_g+\beta}, \qquad t-v > u_R. \quad \textbf{(46)}$$

To determine the economy's Euler equation, we use the method described in Appendix A for the standard formulation. It is straightforward to show that (16) becomes

$$\frac{\dot{c}(t)}{c(t)} = r(t) - \rho - \frac{\eta(\rho+\beta)}{1-\alpha}\cdot\frac{k(t)+h(t)-\bar{h}(t,t)}{c(t)}, \quad \textbf{(47)}$$

where consumption dynamics now depends on the intergenerational turnover in human wealth, corresponding to $[h(t)-\bar{h}(t,t)]$. We show in Appendix C that

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$[h(t) - \bar{h}(t,t)]$ equals

$$h(t) - \bar{h}(t,t) = -w(t)e^{-\beta u_R}(1-\theta-\pi)\cdot\frac{e^{-nu_R} - e^{-r_g u_R}}{r_g - n} < 0, \qquad (48)$$

where $1-\theta > \pi$.[17] Clearly, agents are born with a greater stock of human wealth than average, because they can look forward to the relatively longest period of high earnings. Moreover, the PAYG system affects the macroeconomy through the turnover in human wealth. Indeed, because newborn agents possess more human wealth than their older counterparts, this *mitigates* the fact newborns are asset poor financially compared to older agents. This lessens the effects of intergenerational turnover in (47) and increases the growth in *average* consumption compared to an economy without a PAYG scheme, a result we prove subsequently. To sum- up, PAYG macroeconomic equilibrium consists of (47), where $[h(t) - \bar{h}(t,t)]$ is given by (48). The expressions for the interest rate are the same as stated in (18) and (19), whereas we replace the expression for the wage with (42). Finally, regarding market clearing, we replace (17) with[18]

$$\dot{k}(t) = [r-n]k(t) + w(t)(1-e^{-\eta u_R}) - c(t). \qquad (49)$$

7. PENSION POLICY AND ECONOMIC GROWTH

To investigate the implications of pension policy for economic growth, we first derive the modified economic dynamics. For the Euler relationship we substitute the equation for the wage $w(t)$ from (42) in that of $[h(t) - \bar{h}(t,t)]$ from (48) and use $y(t) = Z_0 k(t)$. We then substitute the resulting expression in (47) to calculate

$$\frac{\dot{c}(t)}{c(t)} = r - \rho - \sigma\Omega(\hat{\gamma})\cdot\frac{k(t)}{c(t)}, \qquad (50)$$

where

$$\sigma \equiv \frac{\eta(\rho+\beta)}{1-\alpha} > 0,$$

$$\Omega(\hat{\gamma}) \equiv 1 - \frac{(1-\varepsilon)re^{-\beta u_R}}{\varepsilon(1-e^{-\eta u_R})}(1-\theta-\pi)\cdot\frac{e^{-nu_R} - e^{-\hat{r}_g u_R}}{\hat{r}_g - n} > 0.$$

Observe that the difference between (50) and the Euler equation (16) of the basic model is that $\dot{c}(t)/c(t)$ now depends on $\Omega(\hat{\gamma})$, which is a function of $\hat{\gamma}$ (through $\hat{r}_g$) and incorporates features of the pension system. Similarly, combining (49) with (42) and $y(t) = Z_0 k(t)$, the market-clearing condition simplifies to

$$\dot{k}(t) = [r + (1-\varepsilon)Z_0 - n]\,k(t) - c(t). \qquad (51)$$

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Evaluating (50) and (51) along the steady-state growth path $\hat{\gamma}$ and, as before, letting $\hat{\chi} \equiv \hat{c}/\hat{k}$, we obtain

$$\hat{\gamma} = r - \rho - \sigma\Omega(\hat{\gamma}) \cdot \frac{1}{\hat{\chi}}, \qquad \hat{\gamma} = r + (1-\varepsilon)Z_0 - \hat{\chi} - n. \tag{52}$$

Observe that the expression for market clearing is identical to that in the basic model, implying that PAYG pensions affect growth only through the Euler relationship. To distinguish the framework with public pensions from that of the basic framework, we let $\hat{r}_g^P (\equiv r - \hat{\gamma}^P)$ and $\hat{\chi}^P$ represent, respectively, the growth-adjusted interest rate and the consumption–capital ratio under the PAYG plan. The system (52) becomes

$$\left(\hat{r}_g^P - \rho\right)\hat{\chi}^P = \sigma\Omega(\hat{\gamma}), \qquad \hat{\chi}^P = \hat{r}_g^P + (1-\varepsilon)Z_0 - n. \tag{53}$$

Combining the expressions in (53), we obtain the polynomial determining $\hat{r}_g^P$,

$$\Phi(\hat{r}_g, \pi, \theta, u_R) \equiv (\hat{r}_g - \rho) \cdot [\hat{r}_g + (1-\varepsilon)Z_0 - n] - \sigma\Omega(\hat{\gamma}) \equiv 0, \tag{54}$$

where we indicate that the solution depends on the parameters of the PAYG system. Equally, the polynomial solving for $\hat{\chi}^P$ corresponds to

$$\Gamma(\hat{\chi}, \pi, \theta, u_R) = \hat{\chi}^2 - [\rho + (1-\varepsilon)Z_0 - n]\,\hat{\chi} - \sigma\Omega(\hat{\gamma}) \equiv 0. \tag{55}$$

We next show that the economy with PAYG pensions has a higher growth rate and a lower consumption–capital ratio than the economy lacking them. To do so, we linearize the polynomial $\Phi(s)$ given in (24) from the basic model of the PAYG equilibrium determined in (54). This yields

$$\begin{aligned}&\left(\hat{r}_g^P - \rho\right) \cdot \left[\hat{r}_g^P + (1-\varepsilon)Z_0 - n\right] \\ &\quad - \sigma + \left[2\hat{r}_g^P - (\rho+n) + (1-\varepsilon)Z_0\right]\left(\hat{r}_g - \hat{r}_g^P\right) = 0.\end{aligned} \tag{56}$$

Evaluating the first term in (56) at the PAYG equilibrium using (54), we solve for $(\hat{r}_g - \hat{r}_g^P) = (\hat{\gamma}^P - \hat{\gamma})$:

$$\hat{r}_g - \hat{r}_g^P = \hat{\gamma}^P - \hat{\gamma} = \frac{\sigma[1 - \Omega(\hat{\gamma}^P)]}{2\hat{r}_g^P - (\rho+n) + (1-\varepsilon)Z_0} > 0, \tag{57}$$

which implies that the balanced rate of growth is *higher* if agents receive PAYG pensions, assuming the Aaron condition holds.[19] The reason for our finding is that the introduction of the PAYG system imposes a life cycle on human wealth earnings that does not otherwise obtain in the standard BY framework. In our specification, PAYG pensions put part of the population in a lower income stream, because $(1-\theta) > \pi$. This strengthens the effect of the turnover in human wealth, $[h(t) - \bar{h}(t,t)] < 0$, because agents are now born with relatively more human wealth than their older counterparts, who face reduced nonasset retirement income. This, in turn, weakens the negative implications that demographic turnover has,

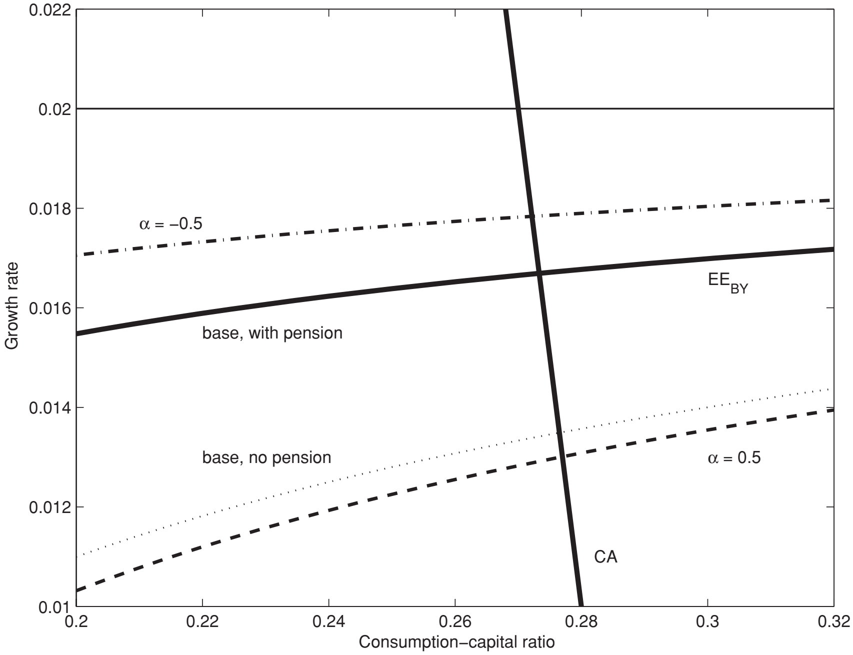

FIGURE 3. Growth, pensions, and the KUJ effect.

in general, for economic growth. Under consumption smoothing, agents respond to the fall in old age income by increasing saving during their working lives, which raises the rate of capital accumulation and implies that $\hat{\gamma}^P - \hat{\gamma} > 0$. In turn, the expression for $\hat{\chi} - \hat{\chi}^P$, obtained by linearizing $\Gamma(s)$ from (25) about the equilibrium solved for in (55), equals

$$\hat{\chi}^P - \hat{\chi} = -\frac{1 - \Omega(\hat{\gamma}^P)}{2\hat{r}_g^P - (\rho + n) + (1 - \varepsilon)Z_0} < 0. \quad \textbf{(58)}$$

We depict in Figure 3 the influence of PAYG, where the baseline pension system parameters equal

$$\text{dep} = 0.20, \quad u_R = 59.73 \text{ years}, \quad \theta = 0.1, \quad \pi = 0.5, \quad \text{rr} = 0.45. \quad \textbf{(59)}$$

The growth equilibrium with pensions is illustrated by the intersection of the solid EE_{BY} and CA relationships: because the solid EE_{BY} locus lies entirely above its counterpart in Figure 1 with no pensions, the solution $(\hat{\chi}, \hat{\gamma})$ under PAYG involves a higher growth rate and a lower consumption–capital ratio. Figure 3 furthermore illustrates how KUJ modifies the role of the pension system. Admiration ($\alpha = -0.5$) augments the effect of PAYG, raising the growth rate even more, whereas jealousy ($\alpha = 0.5$) reverses it. Indeed, under our parameterization, jealousy is more powerful than PAYG, because $\hat{\gamma}^P_{\alpha=0.5} < \hat{\gamma}^P_{\alpha=0}$, a result that can be seen in Figure 3 by comparing the dotted and dashed relationships.

To determine the effects of an increase in the statutory retirement date on the growth equilibrium, we evaluate (54) and (55) at the solution values $(\hat{\chi}, \hat{r}_g)$:

$$\Phi\left(\hat{r}_g^P, \pi, \theta, u_R\right) \equiv \left(\hat{r}_g^P - \rho\right) \cdot \left[\hat{r}_g^P + (1-\varepsilon)Z_0 - n\right] - \sigma\Omega(\hat{\gamma}^P) \equiv 0, \tag{60}$$

$$\Gamma(\hat{\chi}^P, \pi, \theta, u_R) = (\hat{\chi}^P)^2 - [\rho + (1-\varepsilon)Z_0 - n]\,\hat{\chi}^P - \sigma\Omega(\hat{\gamma}^P) \equiv 0. \tag{61}$$

Specifically, we analyze the implications of an increase in the statutory retirement age u_R from 59.73 to 65 years under both DB and DC schemes. Under a DB system the contribution rate θ adjusts to maintain budget closure. Differentiation of (60) and (61) with respect to u_R in this case yields

$$\begin{aligned} \left.\frac{\partial \hat{r}_g^P}{\partial u_R}\right|_{\mathrm{DB}} &= -\left.\frac{\partial \hat{\gamma}^P}{\partial u_R}\right|_{\mathrm{DB}} = -\frac{\left[\partial\Phi\left(\hat{r}_g^P, \pi, \theta, u_R\right)/\partial u_R\right]\big|_{\mathrm{DB}}}{\partial\Phi\left(\hat{r}_g^P, \pi, \theta, u_R\right)/\partial \hat{r}_g^P}, \\ \left.\frac{\partial \hat{\chi}^P}{\partial u_R}\right|_{\mathrm{DB}} &= -\frac{[\partial\Gamma(\hat{\chi}^P, \pi, \theta, u_R)/\partial u_R]|_{\mathrm{DB}}}{\partial\Gamma(\hat{\chi}^P, \pi, \theta, u_R)/\partial\hat{\chi}^P}. \end{aligned} \tag{62}$$

We can show, using (40), that a higher retirement age has an ambiguous effect on growth and the consumption–capital ratio. Nevertheless, we can identify the distinct positive and negative implications of a rise in u_R for growth. An increase in u_R raises labor supply, lowering the wage $w(t)$, which, in turn, shrinks the turnover in human wealth, $[h(t)-\bar{h}(t,t)]$, and lowers $\hat{\gamma}^P$. On the other hand, a later statutory retirement age means that a larger fraction of the population participates in the labor force. This increases generational turnover in human wealth and raises $\hat{\gamma}^P$. The latter effect is augmented by the fact that the *contribution rate* θ paid by the active part of the population *falls* to maintain the PAYG budget balance under DB.

For a statutory rise in u_R under DC, the comparative statics expressions correspond to

$$\begin{aligned} \left.\frac{\partial \hat{r}_g^P}{\partial u_R}\right|_{\mathrm{DC}} &= -\left.\frac{\partial \hat{\gamma}^P}{\partial u_R}\right|_{\mathrm{DC}} = -\frac{\left[\partial\Phi\left(\hat{r}_g^P, \pi, \theta, u_R\right)/\partial u_R\right]\big|_{\mathrm{DC}}}{\partial\Phi\left(\hat{r}_g^P, \pi, \theta, u_R\right)/\partial \hat{r}_g^P}, \\ \left.\frac{\partial \hat{\chi}^P}{\partial u_R}\right|_{\mathrm{DC}} &= -\frac{[\partial\Gamma(\hat{\chi}^P, \pi, \theta, u_R)/\partial u_R]|_{\mathrm{DC}}}{\partial\Gamma(\hat{\chi}^P, \pi, \theta, u_R)/\partial\hat{\chi}^P}, \end{aligned} \tag{63}$$

and are ambiguous in sign, where (41) is used to obtain this result. The general implications of a rise in u_R under DC are similar to those under DB, with the important exception that in DC case the *payout rate* π received by retired workers *increases* under the closure rule. This mitigates against the effect of a longer working life and tends to lower generational turnover $[h(t) - \bar{h}(t,t)]$ in human wealth. For our parameterization in (26) and (59), the negative effect of lower wages dominates, implying that $\mathrm{EE_{BY}}$ shifts down in Figure 4 and growth falls. This is the case whether the PAYG system is DB or DC. Observe in Figure 4,

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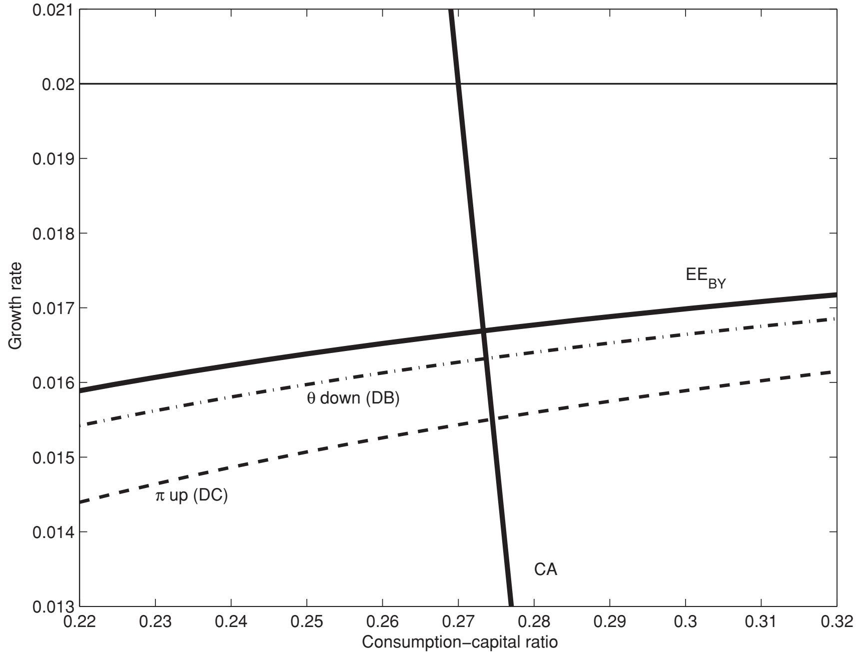

FIGURE 4. Growth and later retirement.

however, that the fall in $\hat{\gamma}^P$ is significantly greater in the DC case—due to the rise in the payout rate π to the retired—than it is in the DB scenario. Our results suggest, then, that pension reform is more effective at maintaining growth under a DB system.

8. CONCLUSIONS

A key limitation of the RA model compared to its OLG counterpart is its assumption of agent homogeneity. The effects of demographic evolution cannot, of course, be addressed in the RA setting. In contrast, the BY version of the OLG framework is particularly suited, due to its convenient dynamic structure, to studying the macroeconomic implications of demographic change. In this paper we seek to investigate the implications of intergenerational turnover for endogenous growth by extending our BY framework—featuring well-defined demography and consumption externalities—to the standard AK model. We show that the BY model extends naturally to the AK setting and provides a new avenue by which to study the relationship between demography and economic growth. Among our findings, we determine that the balanced growth rate, due to intergenerational turnover, is lower in the BY framework than on the basic AK model. Regarding demographic shocks, we show that a fall in fertility and a decline in mortality—both characteristic of modern, industrialized societies—lead to a rise in the balanced growth rate. A greater

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degree of status preference, in contrast, leads to a decline in economic growth, because the effects of intergenerational turnover become more pronounced.

In the second part of the paper we introduce a policy intervention, PAYG pensions with a mandatory retirement date. These directly affect the life-cycle return to human wealth. In particular, we find that a PAYG system increases the balanced growth rate compared to that in an economy that lacks one. This is because PAYG pensions impose an income stream on the retired that is lower than that enjoyed by the active part of the population. This creates a intergenerational turnover in the stock of human wealth—with newborns possessing *more* human wealth than average—that acts as a countervailing influence to the fact that newborns, compared to older population cohorts, have no financial wealth. With respect to changes in the parameters of the pension system, we show, using a numerical parameterization of the model, that an increase in the statutory retirement date lowers balanced growth under both DB and DC schemes, although the decline in growth is much less under DB.

NOTES

1. In non–endogenous growth contexts, recent authors who employ the BY framework to consider the effects of demographic shocks include Bettendorf and Heijdra (2006) and Heijdra and Ligthart (2006). The former authors employ a small open economy framework with nontraded goods and also model the implications of pension shocks. In this research, demographic shocks are time-dependent, though cohort-independent, an approach we follow.

2. Strictly speaking, Alvarez-Cuadrado et al. (2004) and Turnovsky and Monteiro (2007) employ a "catching up with the Joneses" approach in which reference consumption depends on past consumption and evolves over time.

3. Alvarez-Cuadrado et al. (2004) find that the role of reference consumption in determining the response to macroeconomic shocks depends on whether AK or the more flexible Cobb–Douglas technology is assumed. In Liu and Turnovsky (2005), the effect of KUJ on balanced growth is a function of the intertemporal elasticity of substititution. Turnovsky and Monteiro (2007) show that consumption externalities affect the long-run equilibrium if and only if work effort is endogenous.

4. See also the recent paper of Wendner (2008b), who incorporates production as well as consumption externalities into his OLG framework. His work features a careful comparison along the balanced growth paths of the decentralized and socially optimal equilibria.

5. Our specification can be considered a special case of Blanchard's (1985), in which labor productivity declines with age, a concept also adopted by Bettendorf and Heijdra (2006) and Wendner (2008a), who also incorporates consumption externalities.

6. See Boucekkine et al. (2002) for a demographic model of growth that has an increasing disutility of work effort and retirement choice. These modeling features, along with realistic demography, are also incorporated by Heijdra and Romp (2009).

7. That is, net output incorporates capital stock depreciation.

8. KUJ is satisfied with $U[-] \equiv \ln x(v, \tau)$, because $\partial^2 U[\cdot]/\partial \bar{c} \partial c = c/(c - \alpha \bar{c})^2 > 0$. See Dupor and Liu (2003) and Liu and Turnovsky (2005) for detailed characterizations of relative consumption preferences.

9. Neverthless, the *level* of an agent's consumption does depend on β.

10. Observe that in a growth context in which wages follow the path $w(\tau) = w(v)e^{\hat{\gamma}(\tau - v)}$ (where the growth rate $\hat{\gamma}$ is determined in Section 4), human wealth does depend on the date of birth v.

11. If $\alpha = 0$ an agent consumes, as in the standard setting, out of his or her wealth according to $\rho + \beta$, the marginal propensity to consume.

12. In the BY framework it is important to emphasize that the key mechanism is the continuous succession of new disconnected generations, rather than finite horizons. Indeed, from (20) it is clear that our results hold even in the special case infinite horizons ($\beta = 0$) and positive birth rates ($\eta > 0$). We thank an anonymous referee for pointing this out.

13. In Appendix B we derive the conditions for a sensible solution of the steady-state growth profile. In particular, we show that $\hat{r}_g > \rho$ is necessary for $\bar{k}(0, t) > 0$. We also determine the necessary conditions for $\bar{c}(0, t) > 0$, along with the upper and lower bounds on the status parameter α.

14. The sign of $[\partial \hat{r}_g/\partial \beta]$ follows from that of $[\partial \Phi/\partial \beta]$, which equals $\hat{r}_g - \rho - \eta/(1-\alpha) < 0$. Because we can show that $\hat{r}_g < \rho + \eta$ (see Appendix B), the latter holds whether or not $0 < \alpha < 1$ or $\alpha < 0$.

15. In the endogenous fertility approaches of Zhang et al. (2001) and Boucekkine et al. (2002), growth is driven by human capital accumulation. In this research improvements in life expectancy affect fertility and, in turn, the incentive to acquire skills. In contrast to (30), Boucekkine et al. (2002) derive a "hump-shaped" relationship between longevity and growth as changes in the former alter the balance between students, workers, and the retired. For example, Boucekkine et al. (2002) show that if the economy already consists of a high proportion of retirees, then further improvements in life expectancy reduce growth, because then an even greater proportion of the population is made up agents with obsolete skills.

16. In the absence of a pension system, $\theta = \pi = u_R = 0$, individual human wealth reduces to

$$h(v, \tau) = \frac{w(t)}{r_g + \beta}\left[1 - e^{(r_g+\beta)(t-v)}\right].$$

17. The sign of (48) is guaranteed as long as the *Aaron condition*, $r_g > n$, which implies (see Appendix D) that the PAYG reduces human wealth at birth, holds. Clearly, as the population ages through reductions in fertility and mortality, this burden of the PAYG system increases. We thank an anonymous referee for emphasizing this to us.

18. See Appendix C for the derivation of (49).

19. The sign of (57) is positive because

$$1 - \Omega(\hat{\gamma}) = \frac{(1-\varepsilon)re^{-\beta u_R}}{\varepsilon(1-e^{-\eta u_R})}(1-\theta-\pi)\cdot\frac{e^{-nu_R} - e^{-r_g u_R}}{r_g - n} > 0.$$

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APPENDIX A: DERIVATION OF EQUATIONS (16) AND (17)

To calculate the expression for $\dot{c}(t)/c(t)$ in (16), we use Leibnitz's Rule to differentiate $c(t)$, stated in (13), with respect to t:

$$\begin{aligned}\dot{c}(t) &\equiv l(t,t)\bar{c}(t,t) + \int_{-\infty}^{t} l(v,t)\dot{\bar{c}}(v,t)\,dv + \int_{-\infty}^{t} \dot{l}(v,t)\bar{c}(v,t)\,dv \\ &= \eta\bar{c}(t,t) + \int_{-\infty}^{t} l(v,t)\dot{\bar{c}}(v,t)\,dv - \eta\int_{-\infty}^{t} l(v,t)\bar{c}(v,t)\,dv \\ &= \int_{-\infty}^{t} l(v,t)\dot{\bar{c}}(v,t)\,dv - \eta\,[c(t)-\bar{c}(t,t)], \end{aligned} \quad \textbf{(A.1)}$$

where we use (12) and (13) to obtain the second and third equalities of (A.1). The next step, using the definition of $\bar{x}(v,\tau)$ in (6), is to substitute for $\dot{\bar{c}}(v,t)$ in the first term of (A.1). Because $\dot{\bar{c}}(v,t) \equiv (1-\alpha)\dot{\bar{x}}(v,t) + \alpha\dot{c}(t)$, this yields

$$\begin{aligned}\dot{c}(t) &= (1-\alpha)\int_{-\infty}^{t} l(v,t)\dot{\bar{x}}(v,t)\,dv + \alpha\int_{-\infty}^{t} l(v,t)\dot{c}(t)\,dv - \eta\,[c(t)-\bar{c}(t,t)] \\ &= (1-\alpha)\,[r(t)-\rho]\int_{-\infty}^{t} l(v,t)\bar{x}(v,t)\,dv + \alpha\dot{c}(t)\int_{-\infty}^{t} l(v,t)\,dv - \eta\,[c(t)-\bar{c}(t,t)], \end{aligned} \quad \textbf{(A.2)}$$

where we substitute for $\dot{\bar{x}}(v,t) = (r-\rho)\bar{x}(v,t)$ using (8) to obtain the second equality of (A.2). Analogously to the definition of average consumption, $x(t) \equiv \int_{-\infty}^{t} l(v,t)\bar{x}(v,t)\,dv$. Furthermore, because cohort weights sum to unity, $\int_{-\infty}^{t} l(v,t)dv \equiv 1$, and $x(t) = c(t)$

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holds in equilibrium, we can rewrite (A.2) to obtain the aggregate Euler equation,

$$\dot{c}(t) = [r(t) - \rho]\, c(t) - \frac{\eta}{1-\alpha} \cdot [c(t) - \bar{c}(t, t)], \tag{A.3}$$

where $[c(t) - \bar{c}(t, t)]$ corresponds to intergenerational turnover in consumption. To convert (A.3) into the expression (16) in the main text, we evaluate (11) at $v = t$ to find

$$\bar{c}(t, t) = (\rho + \beta)h(t) - \alpha[c(t) - (\rho + \beta)\Gamma(t)], \tag{A.4}$$

where $\bar{a}(t, t) = 0$, because newborns possess only human wealth. In turn, aggregating (11) over cohort weights implies that

$$c(t) = (\rho + \beta)\,[a(t) + h(t)] - \alpha[c(t) - (\rho + \beta)\Gamma(t)]. \tag{A.5}$$

Taking the difference between (A.5) and (A.4), we obtain $[c(t) - \bar{c}(t, t)] = (\rho + \beta)a(t)$, so that (A.3) reduces to (16):

$$\frac{\dot{c}(t)}{c(t)} = r - \rho - \frac{\eta\,(\rho + \beta)}{1-\alpha}\frac{k(t)}{c(t)}.$$

To derive (17), we differentiate $a(t)$ stated in (13) with respect to t to find

$$\dot{a}(t) = -\eta \int_{-\infty}^{t} l(v, t)\bar{a}(v, t)dv + \int_{-\infty}^{t} l(v, t)\dot{\bar{a}}(v, \tau)dv, \tag{A.6}$$

where we again use the fact that $\bar{a}(t, t) = 0$ and substitute for $\dot{l}(v, t) = -\eta l(v, t)$. Substituting for $\dot{\bar{a}}(v, \tau)$ from the budget identity (7) in (A.6), we obtain

$$\begin{aligned} \dot{a}(t) &= -\eta \int_{-\infty}^{t} l(v, t)\bar{a}(v, t)dv \\ &\quad + \int_{-\infty}^{t} l(v, t)\,[(r + \beta)\bar{a}(v, \tau) + w(\tau) - \bar{c}(v, \tau)]\,dv \\ &= (r - n)a(t) + w(t) - c(t), \end{aligned} \tag{A.7}$$

where $n \equiv \eta - \beta$. Finally, the fact that physical capital is the only form of savings, i.e., $a = k$, means that (A.7) is equivalent to the market-clearing relationship (17)

$$\dot{k}(t) = [r - n]k(t) + w(t) - c(t). \tag{A.8}$$

APPENDIX B: CONDITIONS ON STEADY-STATE PROFILES

B.1. CONDITION FOR $\bar{k}(0, t) > 0$

We begin by evaluating (7) at $a \equiv k$ and substituting for $\hat{w}(t) = \hat{w}_0 e^{\hat{\gamma} t}$. Together with (15), this yields

$$\dot{\bar{k}}(v, t) = (r + \beta)\bar{k}(v, t) + \hat{w}_0 e^{\hat{\gamma} t} - \bar{c}(v, t), \tag{B.1}$$

$$\bar{c}(v, t) - \hat{c}(t) = (\rho + \beta)[\bar{k}(v, t) - \hat{k}(t)]. \tag{B.2}$$

Combining these expressions, we obtain the following differential equation in $\dot{\bar{k}}(v, t)$:

$$\dot{\bar{k}}(v, t) = (r - \rho)\,\bar{k}(v, t) + (\eta + \rho + \hat{\gamma} - r)\,\hat{k}_0 e^{\hat{\gamma} t}. \tag{B.3}$$

Solving (B.3) for $v = 0$, subject to $\bar{k}(0, 0) = 0$, gives

$$\bar{k}(0, t) = \frac{\rho + \eta + \hat{\gamma} - r}{r - \hat{\gamma} - \rho} \cdot \hat{k}(t) \cdot \left[e^{(r-\hat{\gamma}-\rho)t} - 1\right], \tag{B.4}$$

which yields the following conditions for $\bar{k}(0, t) > 0$:

$$\text{(i) } r - \hat{\gamma} \equiv \hat{r}_g > \rho; \qquad \text{(ii) } r - \hat{\gamma} \equiv \hat{r}_g < \rho + \eta. \tag{B.5}$$

B.2. CONDITION FOR $\bar{c}(0, t) > 0$

Using the solutions $\hat{k}(t) = \hat{k}_0 e^{\hat{\gamma} t}$, $\hat{w}(t) = \hat{w}_0 e^{\hat{\gamma} t}$, and $\hat{c}(t) = \hat{c}_0 e^{\hat{\gamma} t}$ along the growth path, we can rewrite the Euler equation (16) and the market-clearing condition (17) as

$$(1 - \alpha)\,(r - \rho - \hat{\gamma})\,\hat{c}_0 = \eta(\rho + \beta)\hat{k}_0, \tag{B.6}$$

$$[r - \hat{\gamma} - (\eta - \beta)]\,\hat{k}_0 = \hat{c}_0 - \hat{w}_0.$$

Evaluating (B.2) at $v = 0$ and substituting for $\hat{c}(t)$ employing (B.6), newborn consumption equals

$$\bar{c}(0, t) = \frac{\eta(\rho + \beta)\hat{k}(t)}{(1 - \alpha)\,(r - \rho - \hat{\gamma})} + (\rho + \beta)[\bar{k}(0, t) - \hat{k}(t)]. \tag{B.7}$$

Employing the solution (B.4) for $\bar{k}(0, t)$, we substitute $[\bar{k}(0, t) - \hat{k}(t)]$ into (B.7) and, after simplifying, obtain the expression for $\bar{c}(0, t)$ in terms of $\hat{k}(t)$:

$$\bar{c}(0, t) = \frac{(\rho + \beta)\hat{k}(t)}{r - \rho - \hat{\gamma}} \left[(\eta + \rho + \hat{\gamma} - r)\,e^{(r-\hat{\gamma}-\rho)t} + \frac{\alpha\eta}{1 - \alpha}\right]. \tag{B.8}$$

A sensible solution for newborn consumption, i.e., $\bar{c}(0, t) > 0$, requires that

$$(\eta + \rho + \hat{\gamma} - r) + \frac{\alpha\eta}{1 - \alpha} > 0, \tag{B.9}$$

a condition automatically satisfied for $0 \leq \alpha < 1$. If, instead, $\alpha < 0$, then we must determine a lower bound on the status parameter, so that

$$r - \hat{\gamma} \equiv \hat{r}_g < \eta + \rho + \frac{\alpha\eta}{1 - \alpha}, \tag{B.10}$$

a task we perform now.

B.3. UPPER AND LOWER BOUNDS ON α

We first derive the upper bound on the status parameter α. To do so, we use the polynomial $\Phi(s)$ stated in (24) to prove $\hat{r}_g < \rho + \eta \equiv r_{g1}$, stated in (B.5). This holds if $\Phi(r_{g1}) > 0$.

Evaluating $\Phi(r_{g1})$, we find

$$\Phi(r_{g1}) = \eta \cdot \left[(1-\varepsilon)Z_0 - (\rho+\beta)\,\frac{\alpha}{1-\alpha}\right]. \tag{B.11}$$

For $\alpha < 0$, $\Phi(r_{g1}) > 0$ is automatically satisfied. For $\alpha > 0$, the following upper bound obtains:

$$\frac{\alpha}{1-\alpha} < \frac{(1-\varepsilon)Z_0}{(\rho+\beta)}. \tag{B.12}$$

To determine the lower bound on α, we prove $\hat{r}_g < \rho + \eta + \alpha\eta/(1-\alpha) \equiv r_{g2}$, given in (B.10) for $\alpha < 0$. Evaluating $\Phi(s)$ at $s = r_{g2}$, we show that

$$\Phi(r_{g2}) = \frac{\eta}{1-\alpha} \cdot \left[\frac{\alpha\eta}{1-\alpha} + (1-\varepsilon)Z_0\right]. \tag{B.13}$$

The result $\Phi(r_{g2}) > 0$ holds as long as

$$\frac{\alpha}{1-\alpha} > -\frac{(1-\varepsilon)Z_0}{\eta}. \tag{B.14}$$

Combining (B.12) and (B.14), we state the required range for α:

$$-\frac{(1-\varepsilon)Z_0}{\eta} < \frac{\alpha}{1-\alpha} < \frac{(1-\varepsilon)Z_0}{(\rho+\beta)}. \tag{B.15}$$

APPENDIX C: DERIVATION OF (48) AND (49)

To find the expression for (48), we first solve for $h(t)$ by substituting (45) and (46), which describe, respectively, the stocks of human wealth for the working and retired parts of the population, into (35). This yields

$$h(t) = \frac{w(t)}{r_g+\beta} \cdot \left[(1-\theta)[1-e^{-\eta u_R}] + \eta[\pi - (1-\theta)]e^{-\eta u_R} \cdot \frac{1-e^{-(r_g-n)u_R}}{r_g-n} + \pi e^{-\eta u_R}\right]. \tag{C.1}$$

Employing the PAYG balanced budget rule, we can simplify (C.1) and obtain

$$h(t) = \frac{w(t)}{r_g+\beta} \cdot \left[1 - e^{-\eta u_R} - \eta e^{-\beta u_R}(1-\theta-\pi) \cdot \frac{e^{-n u_R} - e^{-r_g u_R}}{r_g-n}\right]. \tag{C.2}$$

The stock of human wealth represents the present discounted value of wages, adjusted by the features of the PAYG system and demographic parameters. Evaluating (45) at $v = t$, we can show that newborn agents begin life with the following stock of human wealth:

$$\bar{h}(t,t) = \frac{w(t)}{r_g+\beta} \cdot [1 - e^{-\eta u_R} + (1-\theta-\pi)e^{-\beta u_R} \cdot [e^{-n u_R} - e^{-r_g u_R}]]. \tag{C.3}$$

Combining (C.2) and (C.3), the turnover in human wealth then equals

$$h(t) - \bar{h}(t,t) = -w(t)e^{-\beta u_R}(1-\theta-\pi) \cdot \frac{e^{-n u_R} - e^{-r_g u_R}}{r_g-n} < 0. \tag{C.4}$$

To derive (49) for the PAYG case, we modify (A.6) to reflect the working and retired phases of life:

$$\dot{a}(t) = -\eta \int_{-\infty}^{t} l(v,t)\bar{a}(v,t)dv + \int_{t-u_R}^{t} l(v,t)\dot{\bar{a}}(v,\tau)dv + \int_{-\infty}^{t-u_R} l(v,t)\dot{\bar{a}}(v,\tau)dv. \tag{C.5}$$

Using (43) and (31) to substitute for $\dot{a}(t)$ in (C.5), we obtain

$$\dot{a}(t)=(r-n)a(t) + (1-\theta)w(t)\int_{t-u_R}^{t} l(v,t)dv + \pi w(t)\int_{-\infty}^{t-u_R} l(v,t)dv - c(t), \tag{C.6}$$

where we use the definitions of average consumption and asset holdings in (13). Substituting for the cohort weights $l(v,t) = \eta e^{\eta t}$, $t \geq v$, we evaluate (C.6) as

$$\dot{a}(t) = (r-n)a(t) + (1-\theta)w(t)(1-e^{-\eta u_R}) + \pi w(t)e^{-\eta u_R} - c(t). \tag{C.7}$$

To simplify (C.7) and solve for the market-clearing condition, we impose the PAYG budget constraint $\theta \cdot [1 - e^{-\eta u_R}] = \pi \cdot e^{-\eta u_R}$ and $a(t) \equiv k(t)$ and obtain

$$\dot{k}(t) = [r-n]k(t) + w(t)(1-e^{-\eta u_R}) - c(t). \tag{C.8}$$

APPENDIX D: PAY-AS-YOU-GO AND THE WELFARE OF NEWBORNS

Using (45), we can ask whether the PAYG system imposes a net burden on newborns. The scenario we consider is the following: assume that newborns, whether or not they are participants in the PAYG scheme, retire exogenously at age u_R. If the newborn agent pays no contributions and receives no future benefits ($\theta = \pi = 0$), his or her human wealth is

$$[\bar{h}(t,t)]_N = \frac{w(t)}{r_g + \beta}\left[1 - e^{-(r_g+\beta)u_R}\right]. \tag{D.1}$$

In contrast, if the agent is within the pension system, his or her human wealth corresponds to

$$[\bar{h}(t,t)]_P = [\bar{h}(t,t)]_N - \frac{w(t)}{r_g + \beta}\left\{\theta\left[1 - e^{-(r_g+\beta)u_R}\right] - \pi e^{-(r_g+\beta)u_R}\right\}. \tag{D.2}$$

Substituting for the balanced-budget rule $0 = -\theta[1 - e^{-\eta u_R}] + \pi e^{-\eta u_R}$, (D.1) simplifies to

$$[\bar{h}(t,t)]_P - [\bar{h}(t,t)]_N = -\frac{w(t)}{r_g + \beta} \cdot (\theta + \pi)e^{-\beta u_R}[e^{-n u_R} - e^{-r_g u_R}]. \tag{D.3}$$

The question of whether PAYG pensions reduce or augment $\bar{h}(t,t)$ depends on the sign of the term $[e^{-n u_R} - e^{-r_g u_R}]$, which, in turn, depends on r_g relative to n. If the well-known *Aaron condition* holds, i.e., $r_g > n$, then the sign of (D.3) is negative and the PAYG system reduces human wealth at birth.