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Effects of rotation and input energy flux on convective overshooting

Petri J. Käpylä[1,2], M. J. Korpi[1], M. Stix[2] and I. Tuominen[1]

[1]Observatory, P.O. Box 14, FI-00014 University of Helsinki, Finland

[2]Kiepenheuer-Institut für Sonnenphysik, Schöneckstrasse 6, D-79104 Freiburg, Germany
email: pkapyla@kis.uni-freiburg.de

Abstract. We study convective overshooting by means of local 3D convection calculations. Using a mixing length model of the solar convection zone (CZ) as a guide, we determine the Coriolis number (Co), which is the inverse of the Rossby number, to be of the order of ten or larger at the base of the solar CZ. Therefore we perform convection calculations in the range Co = 0...10 and interpret the value of Co realised in the calculation to represent a depth in the solar CZ. In order to study the dependence on rotation, we compute the mixing length parameters $\alpha_{\rm T}$ and $\alpha_{\rm u}$ relating the temperature and velocity fluctuations, respectively, to the mean thermal stratification. We find that the mixing length parameters for the rapid rotation case, corresponding to the base of the solar CZ, are 3-5 times smaller than in the nonrotating case. Introducing such depth-dependent α into a solar structure model employing a non-local mixing length formalism results in overshooting which is approximately proportional to α at the base of the CZ. Although overshooting is reduced due to the reduced α, a discrepancy with helioseismology remains due to the steep transition to the radiative temperature gradient.

In comparison to the mixing length models the transition at the base of the CZ is much gentler in the 3D models. It was suggested recently (Rempel 2004) that this discrepancy is due to the significantly larger (up to seven orders of magnitude) input energy flux in the 3D models in comparison to the Sun and solar models, and that the 3D calculations should be able to approach the mixing length regime if the input energy flux is decreased by a moderate amount. We present results from local convection calculations which support this conjecture.

Keywords. Convection, hydrodynamics, Sun: interior, Sun: helioseismology, Sun: rotation

1. Introduction

Convection poses a difficult problem for stellar structure modelling. One-dimensional stellar structure models require a parameterization of convection in order to be able to yield the thermal stratification within the CZ. Since convection in the stellar envelopes is in general highly efficient, the stratification is close to adiabatic in much of the CZ and thus a detailed description of convection is not needed there. The most often used way to parameterize convection is to use the mixing length concept in a form introduced by Vitense (1953); see also Böhm–Vitense (1958), which considers convective elements to lose their identity after rising (or descending) the so-called mixing length which is proportional to the local pressure scale height, i.e.

$$l = \alpha H_{\rm p}. \tag{1.1}$$

Using this basic assumption, it is possible to derive equations that relate the velocity and temperature fluctuations to the mean stratification, and thus, to compute the convective energy flux (see, e.g. Chapter 6 of Stix 2002).

Among other conceptual problems, the mixing length formalism neglects the effects of rotation although convection can be significantly influenced by it in regions of the CZ

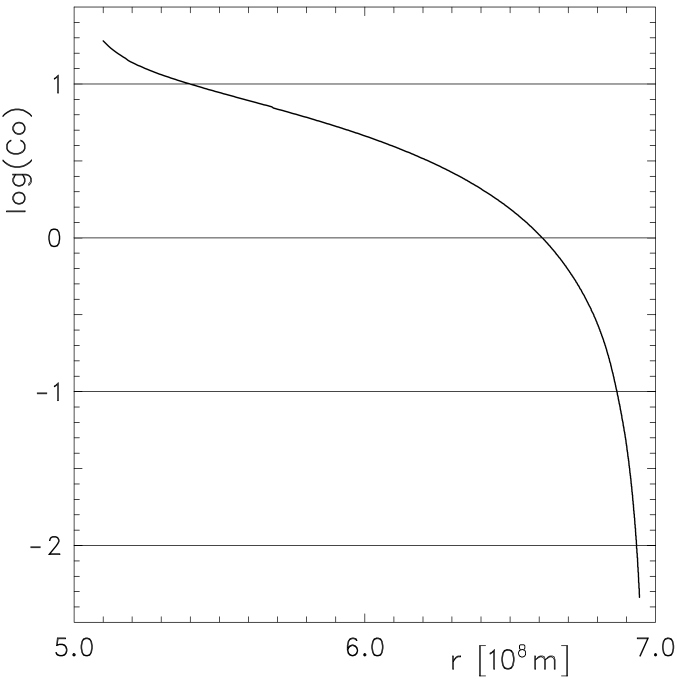

Figure 1. Coriolis number in the solar convection zone according to (1.2).

where the turnover time is longer than the rotation period. In a recent study, Käpylä et al. (2005) estimated the Coriolis number, which is the inverse of the Rossby number, in the solar CZ from

$$\mathrm{Co} = \mathrm{Ro}^{-1} = 2\,\Omega_\odot \tau = 2\,\Omega_\odot \alpha H_\mathrm{p}/u, \tag{1.2}$$

where $\alpha = 1.66$, $\Omega_\odot = 2.6 \cdot 10^{-6}\,\mathrm{s}^{-1}$, and u the convective velocity obtained using a local mixing length model. This relation gives values of the order of 10^{-3} near the solar surface and of the order of ten or larger near the base of the CZ (see figure 1). We compute the mixing length parameters relating the velocity and temperature fluctuations to the mean stratification from local 3D convection calculations in the range $\mathrm{Co} = 0 \ldots 10$, which coincides with the range expected in the Sun. Thus it is possible to probe the influence of rotation on the mixing length relations and take the effect implicitly into account in a solar model employing a non-local formulation of the mixing length concept in order to study overshooting below the CZ.

There is a striking difference between the almost adiabatic overshooting with a very sharp transition to the radiative gradient seen in non-local mixing length models in comparison to the much more subadiabatic and smoother overshooting seen in 3D convection models. In a recent paper Rempel (2004) suggested that this discrepancy arises due to the fact that the two models are simply working in different parameter regimes in the sense that the input energy flux in the 3D models is usually up to 10^7 times larger in comparison to the non-local mixing length models and the Sun. We have performed 3D numerical calculations in which we decrease the input flux by two orders of magnitude in order to study this effect in more detail.

The remainder of the paper is organised as follows: in § 2 a brief description of the model is given and in § 3.1 and § 3.2 the results concerning the effects of rotation and

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input energy flux on convective overshooting are presented. Finally, the main results of the study and remaining problems are summarised in § 4.

2. Numerical model

We use the same model as that described in Käpylä et al. (2005, 2006). The computational domain is a rectangular box situated at a latitude Θ, in which case the rotation vector is represented by $\mathbf{\Omega} = \Omega_0(\cos\Theta, 0, -\sin\Theta)$. In the present study the calculations with rotation are performed at the south pole, i.e. $\Theta = -90^o$. The fluid obeys the ideal gas law and radiation is taken into account only via the diffusion approximation.

In contrast to many earlier studies (e.g. Brummell et al. 2002) we do not use a piecewise polytropic stratification which implies that the thermal conductivity behaves like a step function. Instead, we use a smoothly varying stratification where the logarithmic temperature gradient is computed from

$$\nabla = \nabla_3 + \frac{1}{2}\{\tanh[4(z_{\rm m} - z)] + 1\}\Delta\nabla, \tag{2.1}$$

where $\nabla_3 = 0.15$ is the gradient at the bottom, $\Delta\nabla = \nabla_{\rm CZ} - \nabla_3$ the difference between the gradient in the unstable layer and the applied gradient, and $z_{\rm m}$ the inflection point of the tanh-function, calculated so that $\nabla = \nabla_{\rm ad}$ in the initial state at the base of the convectively unstable region at $z/d = 1$.

In order to regulate the input energy flux we split the heat conduction term, $\partial_t e = \ldots + \Gamma_{\rm cond}$, in the internal energy equation into two parts

$$\Gamma_{\rm cond} = \nabla\cdot[\kappa_{\rm t}\nabla(e - \overline{e}) + \kappa_{\rm h}\nabla\overline{e}], \tag{2.2}$$

where the first term acts only on the fluctuations and the latter only on the mean, i.e. horizontally averaged stratification, and where $e = c_{\rm V}T$. Thus $\kappa_{\rm t}$ and $\kappa_{\rm h}$ can be considered as the turbulent and radiative conductivities, which satisfy $\kappa_{\rm t} \gg \kappa_{\rm h}$ in real stars. We define the conductivies as

$$\kappa_{\rm t} = \gamma\rho\chi_0, \tag{2.3}$$

$$\kappa_{\rm h} = \frac{(\gamma - 1)F_{\rm b}}{g\nabla}, \tag{2.4}$$

where χ_0 is the reference value of the thermal diffusivity, computed from $\mathrm{Pr} = \nu/\chi_0$, where $\mathrm{Pr} = 0.4$ is the Prandtl number and ν the kinematic viscosity. $F_{\rm b}$ is the input energy flux, g the constant gravitational acceleration, and ∇ the mean logarithmic temperature gradient given by (2.1) (for more details, see Käpylä et al. 2006).

3. Results

3.1. *Effects of rotation on mixing length relations*

Using the basic assumption of the mixing length concept, (1.1), it is possible to derive equations that relate the velocity and temperature fluctuations to the mean thermal stratification

$$\overline{u_z'^2} = \frac{\alpha_{\rm u}^2 H_{\rm p} g}{8}(\nabla - \nabla_{\rm ad}), \tag{3.1}$$

$$\overline{T'^2} = \frac{\alpha_{\rm T}}{2}(\nabla - \nabla_{\rm ad})\overline{T}, \tag{3.2}$$

where the bars denote horizontal averaging, primes the fluctuation, and $\nabla_{\rm ad} = (\gamma-1)/\gamma = 0.4$, where $\gamma = c_{\rm P}/c_{\rm V} = 5/3$. Furthermore, adiabatic variation within the convective

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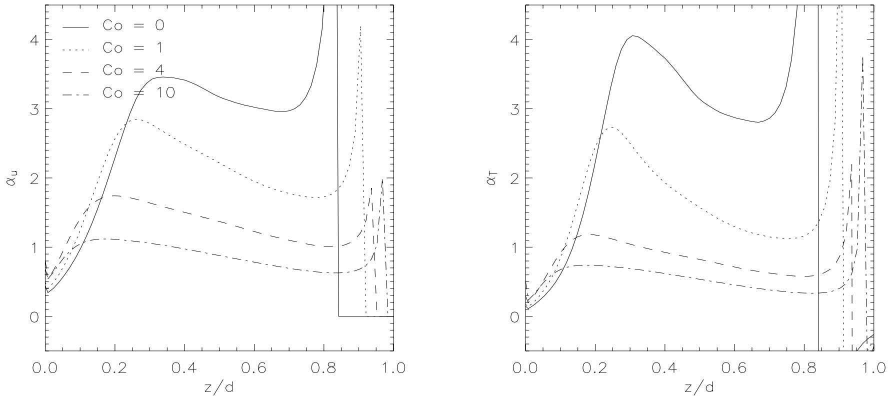

Figure 2. Mixing length parameters according to (3.1) and (3.2).

elements is assumed. Figure 2 shows the results for calculations made at the south pole with approximate Coriolis numbers 0 (no rotation), 1, 4, and 10. It is clear that increasing rotation reduces the convective efficiency and thus the mixing length parameters. If the rotation dependence is intepreted as a depth dependence in the solar CZ, the reduction of the mixing length α can be taken into account in solar models as an implicit way of incorporating the effects of rotation on convection. The main effect of reduced α near the base of the CZ is the reduction of the overshooting when a non-local version of the mixing length model is used (Käpylä *et al.* 2005). The overshooting depth is approximately proportional to the mixing length at the base of the CZ.

3.2. *Effects of input energy flux*

The non-local mixing length models tend to produce a quasi-adiabatic overshoot region with a very sharp transition to the radiative gradient below the CZ (see e.g. figure 9 of Käpylä et al. 2005). Although rotational effects may be able to alleviate the situation, the sharp transition should still show up in the helioseismic inversions of the solar internal structure. On the other hand, numerical 3D calculations always tend to produce overshooting with a much gentler transition to the radiative gradient (e.g. Brummell *et al.* 2002). The main difference between these two approaches is that whereas in the mixing lenght models the input energy flux is the solar flux, i.e. $f = F_{\odot}/\rho c_{\rm s}^3 \approx 10^{-11}$ in the deep layers of the CZ, the 3D models need a much higher flux (up to 10^7 times) in order to bring the thermal relaxation time closer to the dynamical time scale.

Recently, Rempel (2004) suggested that if the input energy flux in 3D calculations was reduced by a moderate amount, the mixing length regime could be approached. Our results (see figure 3) support this conjecture. When the input energy flux is reduced by a factor of 10^2 it is seen that the overshooting depth decreases as the average velocities are reduced, and that the transition to the radiative gradient becomes significantly steeper. If these results are taken at face value it would seem difficult to avoid the quasi-adiabatic overshoot region with steep transition at the base of the solar CZ if extrapolated to the solar regime which is still five orders of magnitude away in terms of the input energy flux. One must, however, bear in mind that in the present models the convectively unstable region spans only little over two pressure scale heights so the effects of compressibility are likely to be weak in comparison to the Sun, affecting the filling factor of downflows. The filling plays a crucial role in the overshoot model of Rempel (2004), with low values

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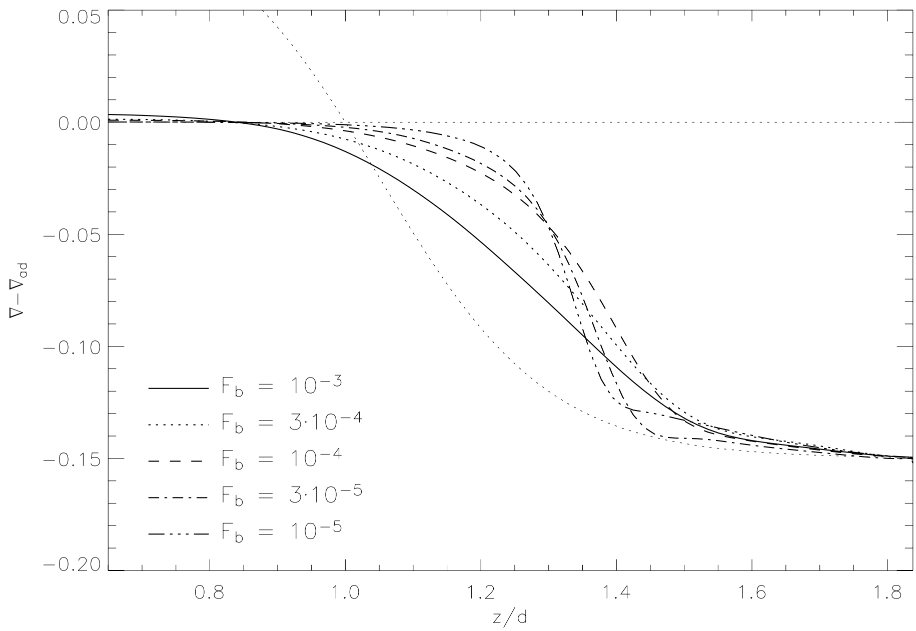

Figure 3. Superadiabatic temperature gradient $\delta = \nabla - \nabla_{\rm ad}$ as a function of the input energy flux $F_{\rm b}$. The input flux is given in units of $\rho_0 (gd)^{3/2}$, see Käpylä *et al.* (2006) for the details. In the present models $\rho c_s^3 \approx \rho_0 (gd)^{3/2}$ at the base of the convectively unstable region leading to dimensionless flux of $f = F_{\rm b}/\rho c_s^3 \approx \mathcal{O}(F_{\rm b})$. The thin dotted curve shows the temperature gradient if the total flux would be transported by radiative diffusion.

($\approx 10^{-5}$) being able to produce overshooting with smooth transition also for the solar energy flux.

4. Conclusions

Three dimensional local convection calculations were used to probe the effects of rotation on mixing length coefficients relating the temperature and velocity fluctuations to the mean thermal stratification. It was found that when the rotational influence on the flow is comparable to that expected in the deep layers of the solar CZ, the mixing length parameters are reduced by a factor of three to five. If a depth-dependent mixing length α is introduced into a solar model, the overshooting at the base of the CZ is reduced approximately in proportion to the reduction of α. Although the depth of the solar CZ can be correctly reproduced in this way, the steep transition to the radiative gradient should still be visible in the helioseismic inversions.

The overshooting in 3D convection calculations is much more subadiabatic with a smooth transition, which is due to the much higher input energy flux used to meet the time step constraints. In the present study we show that decreasing the input flux in 3D calculations leads to more adiabatic overshooting and sharper transition at the base of the CZ. Although this result seems to suggest that the 3D calculations will approach the mixing length regime when the flux is reduced enough, one must bear in mind that in the present models the stratification is rather weak in comparison to the Sun. Thus the effects of compressibility are likely to be underestimated, and lead to too large filling factor for the downflows. Furthermore, the spatial size of the downflow plumes is restricted by the grid size, and it is probable that the filling factor of downflows further decreases when the resolution is increased.

To summarize, we stress the point that rotation should be taken into account in models of convective overshooting since it can exert considerable influence on convection already in slowly rotating stars such as the Sun. Furthermore, high resolution numerical studies of deep convection are needed in order to study whether the convective overshooting is due to very few strong downflows, producing nearly adiabatic overshoot region with a steep transition, or whether downflows of different strengths penetrate into the stable region in a larger area producing smooth overshooting required by helioseismology.

Acknowledgements

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Discussion

J. Christensen-Dalsgaard: Comment: Helioseismology shows that the sound speed gradient is likely smoother in the Sun than in models even without penetration, possibly requiring a subadiabatic gradient in the lower parts of the convection zone.

R.F. Stein: Comment: when you model the base of the convection zone, it is very slightly subadiabatic inside the convection zone.

P.J. Käpylä: This is indeed the case also in our 3D models.

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