Robotica (2015) volume 33, pp. 195–207. © Cambridge University Press 2014
doi:10.1017/S0263574714000204

# The role of walking surface in enhancing the stability of the simplest passive dynamic biped

Ali Tehrani Safa and Mahyar Naraghi*

*Mechanical Engineering Department, Amirkabir University of Technology, No. 424, Hafez Avenue, Tehran 15914, Iran*

(Accepted January 28, 2014. First published online: February 27, 2014)

**SUMMARY**
Employing passive dynamics of the simplest point-foot walker, we have shown that the walking surface could have a great role in promoting the gait stability. In this regard, the stabilization of the simplest walking model,[3] between the range of slopes greater than 0.0151 rad. and less than 0.26 rad., has been achieved. The walker like other passive dynamic walking models has no open or closed-loop control system; so, is only actuated by the gravity field. Moreover, no damper or spring is used. Obviously, according to the model's unstable behavior, it is unable to walk on an even flat ramp between the mentioned intervals.[3] Here, instead of restraining the model, we let it explore other smooth surfaces, walking on which, will end in an equally inclined surface. To reach the objective, we employ a parallel series of fixed straight lines (local slopes) passing through contact points of an unstable cycling gait, which is generated by an ordinary ramp. To categorize, we have nicknamed those local slopes that guide the biped to a stable cyclic walking, "Ground Attractors," and the other, leading it to a fall, "Repulsive Directions." Our results reveal that for the slope $<0.26$ rad., a closed interval of ground attractors could be found. Stabilization of those unstable limit cycles by this technique makes obvious the key role of walking surface on bipedal gait. Furthermore, following our previous work,[13] the results confirm that the two thoroughly similar walking trajectories can have different stability. All of these results strongly demonstrate that without considering the effects of a walking surface, we cannot establish any explicit relationship between the walker's speed and its stability.

KEYWORDS: Passive dynamic walking; Stability; Ground attractors; Repulsive directions.

## 1. Introduction

Bipedal Walking is one of the transitional patterns mostly used in the nature. Humans and some animals employ legs to move through complex environments with minimal thought. Actually, they do this as a habit. In this way, their neuromuscular systems help them to be far from falling. In contrast, as an engineering and artificial effort, bipeds imitate legged creatures normally by using complex control systems to conquer predefined obstacles and predicted disturbances. Through the development of bipedal robots, a reduction in design complexity appeared when McGeer showed that walking can be a natural dynamic mode of a two-legged machine.[10,11] He pioneered in introducing the subject of Passive Dynamic Walking (PDW) by using distributed mass models placed on shallow slopes. His models presented stable gaits under the actuation of mere gravity, without any extra energy input. In spite of Human-like walking, high efficiency and control free locomotion, as the most important benefits of PDW, it has been widely demonstrated that stability is a major weakness of PDW,[1,3,4,10,11,13,14] since walking on steep ramps has remained merely a wish for a passive biped.

Throughout the two decades of PDW's life, many researchers have devoted their efforts to amend the stability of such a mechanism by means of various techniques and devices. Adding dampers and springs,[1,4] using upper bodies and foots[5,10,11,18] and distributing the mass,[4,5,10–12] have been some of such endeavors to improve the stability of passive walkers. All of these efforts have been solely

* Corresponding author. E-mail: naraghi@aut.ac.ir

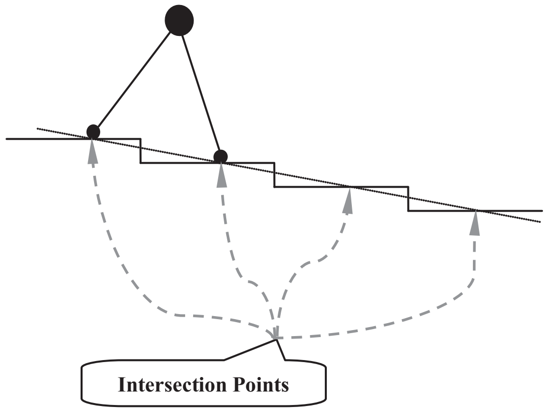


Fig. 1. The simplest walking model: View of the contact points of a period-one limit cycle walking, common between the ramp and stairs.

restricted to biped models, while the complete dynamics of walking appears as an interaction between biped and ground.

Effects of the walking surface are commonly analyzed in the form of random terrains yet the results are usually far from classified and normally close to stochastic approaches.[2,16] This causes the researcher to miss any comparison between different walking surfaces and, probably, extra capacities to produce more stability, efficiency and velocity that a rough terrain can provide a biped with. Moreover, as limit cycle walking does not emerge from uneven terrains, the classical implication of stability vanishes.[2] This is another negative point for a non-systematic approach.

In our previous work,[13] we considered the simplest walking model introduced by Garcia *et al.*[3] placed on stairs instead of ramps. It was shown there, that two different types of walking surface can produce quite similar trajectories, i.e. equal speeds and equal periods, for the same walker. This analogy had been kept until small disturbances were added to the walker. At that point, two different walking surfaces gave us two unlike stability strategies. Stable period-one gait cycles vanished for those stairs that their equivalent ramps had slopes greater than 0.0107 rad., but survived under walking on regular ramps until slope reached 0.0151 rad. The defect of that paper was related to the inability of stairs to produce stable motions for equivalent ramps greater than 0.0107 rad.

Here we compensate the shortage by a more generalized stability analysis of the simplest walking model. The beginning point would be the consideration of the stability related to the walking surface. By scrutinizing perturbed limit cycle walking from a physical point of view instead of a pure mathematical one, we discover and extract excess stability from the ground. Also we define “Ground Attractors” and “Repulsive Directions” to organize our results. The output of the resolved problem reveals several new capacities for the simplest walking model performing stable walking.

## 2. Methods

### *2.1. Problem approaching*

Can the simplest passive walking model reach higher velocities than those proposed by Garcia *et al.*[3]? This is the basic question that we are interested in finding a clear answer to. Here, we look for a starting point to get close to the answer. Referring to our previous experience helps us in finding such starting points. Figure 1 shows the schema of stepping vs. inclined 2D motion quoted from.[13] Regardless of its stability, we suppose the biped exhibiting period-one limit cycle walking; so the step length and the step period are fixed. By adding a small perturbation, the biped encounters two options. One leads to walking on a ramp and another ending in stepping motion. For each one of these choices, if the walker presents a stable gait, then it will return to its initial configuration, i.e. limit cycle walking. As we mentioned before, for slopes less than 0.0107 rad. both the stepping and the inclined motion are stable; then instability appears for stepping motion and beyond the amounts greater than 0.0151 rad. the model doesn’t repeat its earlier figure on inclined surface as well.

https://doi.org/10.1017/S0263574714000204 Published online by Cambridge University Press

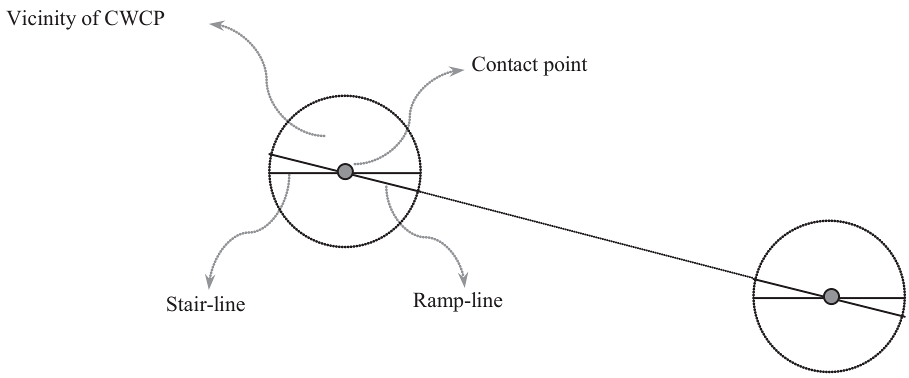


Fig. 2. Schema of the common contact points between a ramp and stairs and their vicinity.

To gain more insight, let us focus on the vicinity of the intersection points, common between a ramp and stairs, and expunge any extra line. Therefore, Fig. 1 will be summarized in Fig. 2, where, two stiff straight lines are passed as walking surfaces through the contact points of the periodic motion. A little focus helps us to find out that there exist a lot of routes terminated to the cycle walking not having been considered before. From the other point of view, up to now, the basin of attraction of the simplest walking model has been confined by impacts supplied by a ramp or a stair-line (Fig. 2). But actually, when a limit cycle walker is deformed under some small disturbances, it can choose any point in the space as the next contact point, being provided by ground collision. In other words, the moment when the perturbed trajectory must be cut, is the most important factor that may guide the biped to fall or not. In real world, this cut is provided by ground and it defines the step period, i.e. the time between two collisions. Researchers normally concentrate on ramps or stairs as flat terrains,[1,3–5,10–14,18] or stochastic walking surfaces as an uneven behavior,[2,16] hence, they have ignored and lost the total capacity of the space around the nominal gait surface. The dashed circle in Fig. 2 encloses the arbitrary impact points of a cyclic walking. Note that the figure is exaggerated.

In this research, we are interested in examining and exploring this area by using systematic approach. To do this, instead of ramps or stairs, we pass an arbitrary series of parallel lines through the Cyclic Walking Contact Points (CWCP). If these lines guide the biped to its initial state at the presence of small disturbances, we term them "Ground Attractors," else, they are called "Repulsive Directions." In the following section, we present the mathematical model of the biped walking on these lines (local slopes).

### 2.2. *Problem modeling*

Classical mechanics describes PDW by a set of differential-algebraic-transitional equations. These equations are derived under some common assumptions including plastic collisions, frictionless joints, no-bounce and no-slip walking, simultaneous heel-strike and toe-off actions, ignorable geometric interference during forward swinging (foot scuffing) and no impulsive reaction between new swing foot and the walking surface. Garcia *et al.* introduced the simplest model of passive dynamic walkers consisting of two rigid mass-less legs hinged at the hip, a point-mass at the hip, and infinitesimal point-masses at the feet.[3] They proved that the model would present stable period – one gait cycles when placed on a shallow regular ramp by a slope smaller than 0.0151 rad. Also, it is able to walk down by other periodic or chaotic behaviors if the slope is kept less than 0.0189 rad. The model's simplicity constantly fascinates researchers to discover walking principles. The governing equations of motion under mentioned conditions are as below:[3]

$$\ddot{\theta} = \sin\theta, \tag{1}$$

$$\ddot{\phi} = \sin\theta + \left(\dot{\theta}^2 - \cos\theta\right)\sin\phi, \tag{2}$$

where the angle between the stance leg and vertical line is $\theta$, and $\phi$ shows the angle between two legs. Note that these equations are described in a dimensionless form; so time is scaled by $\sqrt{g/l}$ ("$g$" is gravity acceleration and "$l$" is the length of the robot's leg). Numerical solution of the above equations

https://doi.org/10.1017/S0263574714000204 Published online by Cambridge University Press

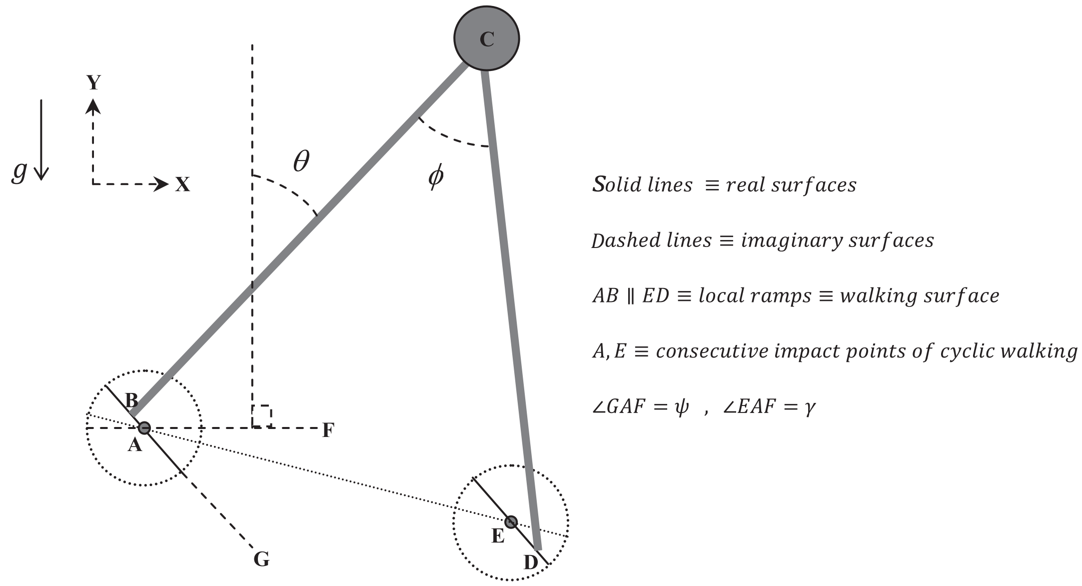


Fig. 3. The biped's configuration at the heel-strike. Solid lines are real and dashed lines are imaginary.

will continue until the heel-strike, which is detected by collision rule, occurs. The time interval between two heel strikes represents the step period, and the collision rule has a vital role in determining it. To avoid complexity and to reduce free parameters, in the present work, we model ground as parallel fixed lines passing through the contact points of a period – one gait cycle generated by an ordinary ramp. These lines would be considered as a general form of "stairs" with non-horizontal surfaces. According to Fig. 3, at the moment of the heel-strike, the following vector relationship can be built:

$$\vec{AB} + \vec{BC} + \vec{CD} + \vec{DE} + \vec{EA} = 0, \tag{3}$$

where, "A" and "E" specify the consecutive CWCP of an arbitrary inclined walking surface; "C" is the hip joint; "B" shows the stance leg contact point and "D" depicts the swing leg impact point. We display the slope angle $E\hat{A}F$ by $\gamma$ and $G\hat{A}F$ by $\psi$. For $\psi = 0$ a common form of stairs appears and for $\psi = \gamma$ the modeling returns to its classic shape, i.e. walking on a ramp. Furthermore, we assume that the friction is sufficient enough to prevent the walker from slippage that happens if $\psi$ is likely to be big. In Fig. 3, both of $\gamma \& \psi$ are defined positive; nevertheless, $\psi$ can choose any amount in the interval of $[-\frac{\pi}{2}, \frac{\pi}{2}]$. We keep $\gamma$ positive, because a passive biped cannot walk uphill. Also by preferring positive initial values, $\theta \& \phi$ get negative values before heel strike. Now we expand Eq. (3). to find the algebraic relation between $\theta, \phi, \gamma, \psi$. According to Cartesian coordinates, we have:

$$\begin{aligned}
\vec{AB} &= \varepsilon_1 \left(-\cos\psi \hat{i} + \sin\psi \hat{j}\right),\\
\vec{DE} &= \varepsilon_2 \left(-\cos\psi \hat{i} + \sin\psi \hat{j}\right),\\
\vec{BC} &= l \left(-\sin\theta \hat{i} + \cos\theta \hat{j}\right),\\
\vec{CD} &= l \left(-\sin(\phi-\theta) \hat{i} - \cos(\phi-\theta) \hat{j}\right),\\
\vec{EA} &= 2l \sin\frac{\phi^*}{2} \left(-\cos\gamma \hat{i} + \sin\gamma \hat{j}\right).
\end{aligned} \tag{4}$$

We show the magnitude of $\vec{AB}$, $\vec{DE}$ by $\varepsilon_1$, $\varepsilon_2$ and if $\vec{AB}$, $\vec{DE}$ choose another direction than which they have been given in Fig. 3, then they will be multiplied by a minus sign in the above equations. This minus sign does not have any role in the results of our calculations and the final equation is independent of both $\varepsilon_1$, $\varepsilon_2$ and the vector's directions. The distance of "EA" is written by supposing that the walker has a cyclic motion through 'A' and 'E'; so $\phi^*$ is the absolute value of $\phi$ when "B"

https://doi.org/10.1017/S0263574714000204 Published online by Cambridge University Press

and "D" coincide with "A" and "E", respectively. How to obtain this value will be explained in the next few lines. Substituting Eqs. (4) into Eq. (3). yields to:

$$-\left(\frac{\varepsilon_1}{l}+\frac{\varepsilon_2}{l}\right)\cos\psi=\sin\theta+\sin(\phi-\theta)+2\sin\frac{\phi^*}{2}\cos\gamma,$$
$$-\left(\frac{\varepsilon_1}{l}+\frac{\varepsilon_2}{l}\right)\sin\psi=\cos\theta-\cos(\phi-\theta)+2\sin\frac{\phi^*}{2}\sin\gamma. \tag{5}$$

Dividing two above equations leads to:

$$\tan\psi=\frac{\cos\theta-\cos(\phi-\theta)+2\sin\frac{\phi^*}{2}\sin\gamma}{\sin\theta+\sin(\phi-\theta)+2\sin\frac{\phi^*}{2}\cos\gamma}. \tag{6}$$

Using the trigonometry laws, it would be written in a better form resulting in:

$$\sin\frac{\phi}{2}\sin\left(\frac{\phi}{2}-\theta-\psi\right)+\sin\frac{\phi^*}{2}\sin(\gamma-\psi)=0,\phi^*>0. \tag{7}$$

This is a more general form of collision rule, not considered before. Note that the merit of the above equation is in covering the whole vicinity of CWCP by the minimum applying of parameters. Now by focusing on the limit case, i.e. when the limit cycle gait occurs, $\Phi-\Phi^*$ and Eq. (7). will be summarized to:

$$\phi=2(\theta+\gamma). \tag{8}$$

This is the foot-strike condition for walking on a ramp with the slope angle of $\gamma$. It clearly verifies that the fixed points of gaits emerging from Eq. (7)., are the same as those obtained from Eq. (8). So to get $\phi^*$ we use Eq. (8). Note that another way to reach Eq. (8). is to put $\psi$ equal to $\gamma$.

After heel-strike detection, conservations of angular momentum about the new stance foot for the whole body, and about the hip for the new swing foot, provide us with the initial angular velocities. Note that we neglect the effect of non-impulsive forces. Furthermore, the new angles of $\theta \& \phi$ are simply defined by observing the fact that the role of swing and stance leg has been interchanged. Therefore, the transitional equations can be written in the form of:[3]

$$\dot\theta^+=(\cos\phi^-)\dot\theta^-,$$
$$\dot\theta^+=\left(1-\cos\phi^-\right)\dot\theta^+,$$
$$\theta^+=\theta^--\phi^-,$$
$$\phi^+=-\phi^-. \tag{9}$$

where superscripts '$-$' and '$+$' show the variables just before and after the heel-strike, respectively. Conditions given by Eq. (9) are not dependent on the shape of the walking surface.

Now our mathematical model is complete by Eqs. (1, 2, 7) and (9). These equations can be summarized by taking the Poincare section at the instant of heel-strike. We have:

$$q_{i+1}^+=S\left(q_i^+\right),q=\left[\theta,\dot\theta,\phi,\dot\phi\right]^T. \tag{10}$$

The variables, just after heel-strike, are shown by $q^+$ and the $i^{th}$ step of the motion is displayed by $i$. McGeer termed this mapping as "Stride Function".[10,11] Gait limit cycles emerge from fixed points of this function. At fixed points we have repetitive gait cycles, so:

$$q^*=S\left(q^*\right). \tag{11}$$

In the next section, we will solve our equations to demonstrate new capacities for the simplest walking model.

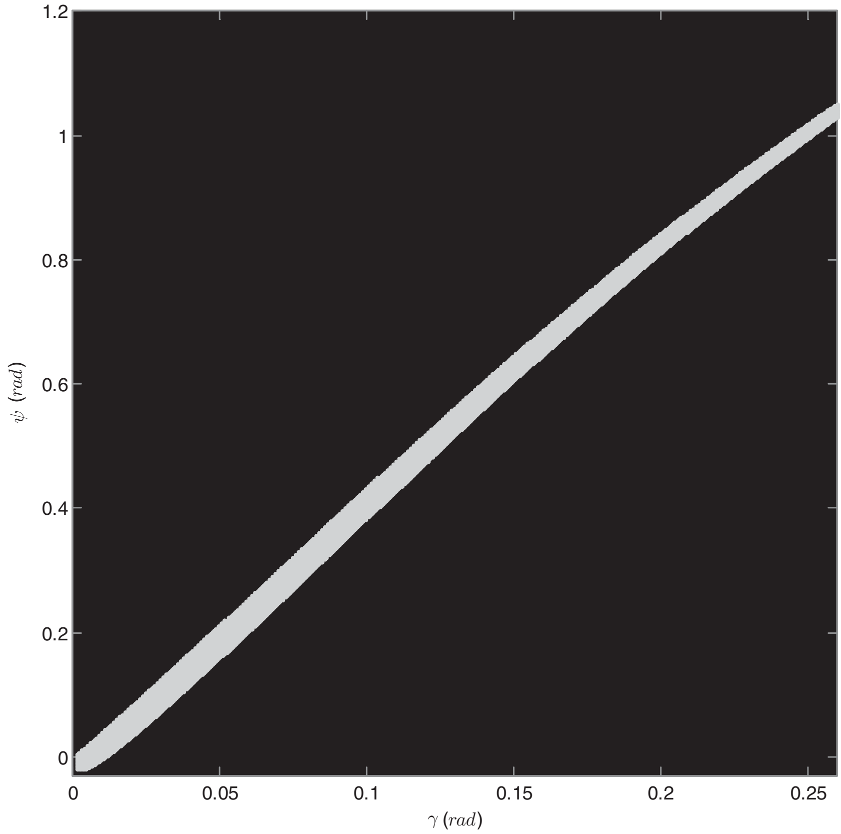


Fig. 4. The Diagram of "Ground Attractors" (light region) and "Repulsive Direction" (dark area) based on Eq. (7). Theoretically, we could find stable limit cycle solutions for $-0.020 \leq \psi \leq 1.05\,\text{rad}\ \&\ \gamma < 0.26\,\text{rad}$ .

## 3. Results

It has been repeatedly demonstrated that PDW poses limit cycle gaits.[1,3–5,10–14,18] Although other kinds of locomotion including chaotic or aperiodic patterns are possible,[3,13,16] from the stability point of view, cyclic walking is more acceptable for a bipedal design. When a legged machine is forced to perform a periodic terrain, detection and elimination of external disturbances are much easier than other cases; since the trajectory of the system is completely known.

Here we are interested in seeking a period-one limit cycle walking in which the stability is passively guaranteed. So we will restrict our attention only to the long-period solutions and discard any other unstable short-period equilibriums.[3] The investigation of the stability of such solutions can be done using several well-known techniques. These methods are divided based on theoretical and experimental, and linear or nonlinear aspects. Floquet multipliers and the calculation of the basin of attraction are two major measures for orbital stability analysis. The basin of attraction is the set of all possible initial conditions that will guide the robot to the existing fixed point.[14] It's the only nonlinear analysis that today we are familiar with. In contrast, Floquet multipliers deal with the Jacobian's eigenvalues of the McGeer's stride function.[10,11] These multipliers indicate how fast a small perturbation vanishes through the step-by-step mapping. Once encountering a stable limit cycle, Floquet multipliers ought to be within the unit circle; the closer to zero, the agiler the limit cycles are to convergence. In the present study, since the computation of the basin of attraction is much expensive and wasteful,[14] we prefer the linear approximated method (Floquet multipliers) to nonlinear numerical analysis (basin of attraction). Detailed procedure of the calculation of Floquet multipliers is discussed in the classical literature of the subject.[3,10] The simulation results of the present study are carried out by MATLAB functions.

Figure 4 depicts the diagram of "Ground Attractors" and "Repulsive Directions" of the simplest walking model ordered by Eq. (7). Once finding period-one limit cycles using Eq. (8). instead of Eq. (7)., the stability adjustments can be done by varying $\psi$ in Eq. (7). The light area shows the

https://doi.org/10.1017/S0263574714000204 Published online by Cambridge University Press

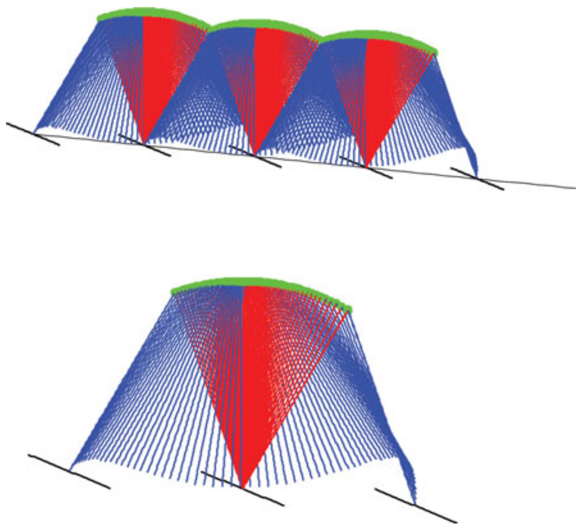

Fig. 5. (Colour online) The stick graph of the simplest walking model with a stable walk on a walking surface defined by $\gamma = 0.100\,\text{rad}.\&\psi = 0.390\,\text{rad}$. The swing leg is shown by blue lines and the red ones express the motion of the stance leg. Note that when the swing and the stance legs get close to each other, the foot-scuffing phenomenon occurs. In the real world, it can be avoided by adding knees or using checkerboard surface.

closed continuous intervals of $\psi$ per $\gamma$, in which the stability is guaranteed; if and only if the model is kept away from large perturbations. This region, as we mentioned, is termed "Ground Attractors". On the other hand, the dark area illustrates those walking surfaces which will guide the biped to fall or to continue other kinds of motion, including period-n limit cycles ($n \neq 1$) or chaotic behaviors, even in the presence of very small disturbances. "Repulsive Directions" generally interpret this zone. Our diagram surprisingly extends the stable behavior of the simplest walking model. Garcia *et al.* demonstrated stable long-period-one limit cycles only for slopes less than 0.0151 rad (0.86 deg);[3] but here, without making any physical change in their model and solely by introducing local smooth surfaces, we improve stability up to the global slope of 0.26 rad(14.9 deg). This means that stability is amended about 1600%. By continuing the simulation beyond the global slope of 0.26 rad., we found out that the vertical contact force will be negative and the stance foot will leave the ground; so the biped begins running. This is the time to stop the simulation.

Primary data shows that although the existence of limit cycles is not related to walking surface (here $\psi$ parameter), their stability is highly surface dependent. However, it is frequently reported that uneven terrains lead to instability;[2,16] or, are sometimes treated as external disturbances.[12] Here, the positive role of walking surface in stabilization of a legged machine is deciphered. Furthermore, in contrast to other parameters influencing dynamic stability of such mechanisms, it is not an overstatement if we claim that the principal role belongs to the walking surface.

Figure 5 indicates the detailed plot of the model's cyclic gait on a walking surface described by $\gamma = 0.100\,\text{rad}.\&\psi = 0.390\,\text{rad}$. The amount of the defined $\psi$ belongs to the model's ground attractors, so if the model starts with appropriate initial conditions, it converges to its nominal long – period limit cycle produced by an ordinary ramp with the slope of $\gamma = 0.100\,\text{rad}$. In the figure, the swing leg is shown by blue lines and the stance leg is displayed by red ones. After heel – strike, the roles of the swing leg and the stance leg interchange. It should be mentioned that during the forward motion of the swing leg, when it approaches the stance leg, the foot – scuffing phenomenon takes place. This undesirable event is avoided by employing knees or constructing a checkerboard surface in the real world.

Let us return to Fig. 4 and consider it once more. First we are interested in declaring that the maximum width of "Ground Attractors" placed in the interval of $0.020 \leq \gamma \leq 0.100\,\text{rad}$, the following is applicable: $(\psi_{\max} - \psi_{\min}) \cong 0.050\,\text{rad}$. This peak only covers 1.6% of all the possible space provided by Eq. (7); since $-\frac{\pi}{2} \leq \psi \leq \frac{\pi}{2}$. So, for the above interval of $\gamma$, 98.4% of the Fig. 4 will be "Repulsive Directions".

https://doi.org/10.1017/S0263574714000204 Published online by Cambridge University Press

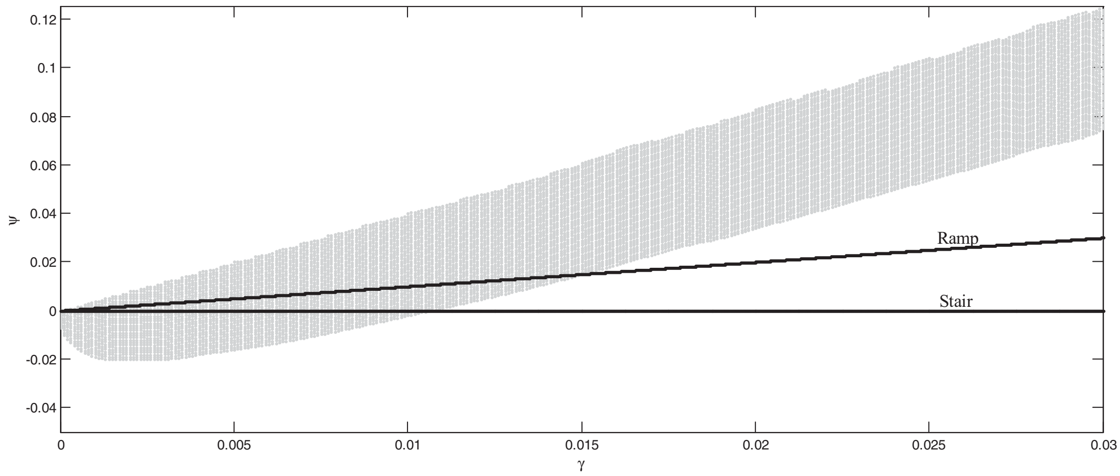


Fig. 6. Comparison of "Ground Attractors" with ramp and stairs is presented. To this end we zoom into Fig. 4 for $\gamma \leq 0.03$ rad.

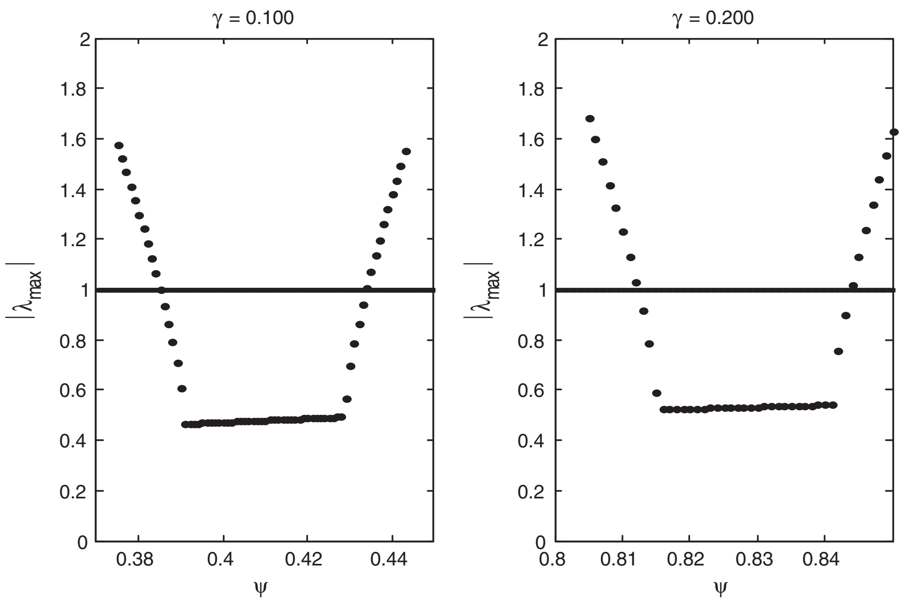


Fig. 7. Plot of maximum Floquet multiplier vs. local slope angle for two values of $\gamma$ is figured.

By increasing or decreasing $\gamma$ beyond this peak interval, the width of ground attractors will diminish. In our diagram, the minimum value of $\psi$ is located around $0.002 \leq \gamma \leq 0.003$ rad, equal to $-0.020$ rad, and the maximum value occurs at $\gamma = 0.2595$ rad when $\psi$ covers the interval of $1.029 \leq \psi \leq 1.050$ rad. In the other limiting case, i.e. when $\gamma \to 0$, although the convergence of both $\psi_{max}$&$\psi_{min}$ to zero is obvious, their variation rates are different. For example, for $\gamma = 10^{-6}$rad, while $\psi_{max} \cong 10^{-6}$rad, the amount of $\psi_{min}$ is about $-0.002$ rad. So in the vicinity of zero, $\psi_{max}$ meets $\gamma$ sooner than $\psi_{min}$.

The next interesting subject rises from the comparison of these results with ramp and stairs. For this objective, we zoom into our diagram for $\gamma \leq 0.03$ rad. and draw the lines $\psi = \gamma$ and $\psi = 0$ to find the intersection points. Figure 6 confirms our expectations, since the ground attractors for ramp and stairs will vanish for $\gamma > 0.015$ & $\gamma > 0.011$ rad. respectively. It is clear from the Fig. 6 that the first-period stable limit cycles for a ramp higher than 0.015 rad, can only be found under the necessary condition of $\psi > \gamma$.

In Fig. 7, the value of $|\lambda_{max}|$ (the maximum Floquet multiplier) vs. $\psi$(local slope angle) is displayed for $\gamma = 0.1$, 0.2rad. The fixed points, meaning the initial conditions just after heel strike, are presented

https://doi.org/10.1017/S0263574714000204 Published online by Cambridge University Press

Table I. The fixed points, i.e. the initial conditions just after heel strike, are presented for some values of $\gamma$. Note that these points are independent of $\psi$.

| $\gamma$ | $\theta^*$ | $\dot{\theta}^*$ | $\phi^*$ | $\dot{\phi}^*$ |
|---|---|---|---|---|
| 0.100 | .3331047466 | −.3480433230 | .8662094938 | −.1226090146 |
| 0.200 | .3338702657 | −.3499287560 | 1.067740538 | −.1812263520 |

in Table I. It can be inferred from the figure that we have a set of minimum points of $|\lambda_{max}|$, meaning that the rate of convergence remains constant and minimum in some intervals of $\psi$. At the end of these intervals, $|\lambda_{max}|$ suddenly grows to exceed one, leading to the disappearance of stability.

It has been shown by a few authors that the correlation between maximum Floquet multiplier and the actual disturbance rejection is not straight forward;[14] so it should be considered that this mapping method is only used to prove stability, and the area of the basin of attraction is independent of the magnitude of the maximum Floquet multiplier. As we observed through our simulation study, the amount of disturbances that the simplest walking model can tolerate varies by altering $\gamma$ or $\psi$. So while the total permitted disturbance that the model can handle for $\gamma = 0.1$, $\psi = 0.41$ rad. is 1.5% of the fixed point's initial condition, this allowable perturbation is 2% for $\gamma = 0.1$, $\psi = 0.40$ rad. and 3% for $\gamma = 0.1$, $\psi = 0.39$ rad.

## 4. Discussion

Today, it is a well-accepted notion that pure, natural dynamics of a bipedal walking is intimately associated with the human walking. In this regard, many valuable researches, based on PDW, have been carried out to reach a better understanding of the human walking.[7–9] Using simple models, which can imitate natural dynamic walking by producing hybrid limit cycles, is a common approach in the literature of PDW. These models possess the generic properties of passive walkers and their simplicity doesn't interfere with the individual characteristics of PDW. Following this trend, the simplest walking model as a well-known walker, has been employed to figure out the unrevealed capacities of PDW as related to the concept of stability. The researches done before, focus on increasing the stability by either adding some mechanical devices such as dampers and springs, or exerting active control strategies. Although these methods extend the robustness of a walker, they are based on altering the nominal trajectories of passive limit cycles; so the walker presents a new behavior, which includes a new step size and a new velocity. In fact, when a stable or unstable hybrid limit cycle is revealed by some simulation efforts, the basin of attraction can be extended or constructed without changing the step size and the step period of the walker. In other words, a physical model revision is not necessary to modify the stability.

Our results prove this hypothesis by adjusting the interaction between the walker and the walking surface in which the stabilization of the unstable limit cycles of the simplest point-foot walker has been guaranteed. This adjustment is done using a series of flat, fixed, parallel local ramps which pass through CWCP. When some disturbances are added to a cyclic walking, each class of the local ramps provides the walker with different subsequent impacts that lead to stability or instability. It is clear from the derived collision condition that the existence of a limit cycle is not dependent on these local ramps. This can be easily checked by observing the limit case, i.e. when a cyclic walking emerges. At that point, as mentioned before, the CWCP are located on an imaginary ramp (global slope) and the parameters participating in collision rule, are reduced to only one parameter, which shows the global slope angle. This global ramp discriminates between the various limit cycles, so by altering the global slope angle, the step size and the walker's speed will change.

The diagram of these modifications is presented in Fig. 8, where the plot of the speed, the consumed energy and the step length in a dimensionless format is shown against global slope angle, i.e. $\gamma$. The values are calculated when the stable cyclic motion is performed by the walker. As it is shown in the figure, while the graphs of the consumed energy and the step length are ascendant on all of the calculated intervals, the diagram of the biped velocity is gradually going to be saturated. This means that the biped will gradually lose efficient walking, since it consumes more energy without obtaining more velocity. In this regard, the maximum speed occurs at $\gamma = 0.26$ rad. with the velocity of $0.2375\sqrt{gl}$. In comparison to regular ramps, our stabilization method is fully successful since the

https://doi.org/10.1017/S0263574714000204 Published online by Cambridge University Press

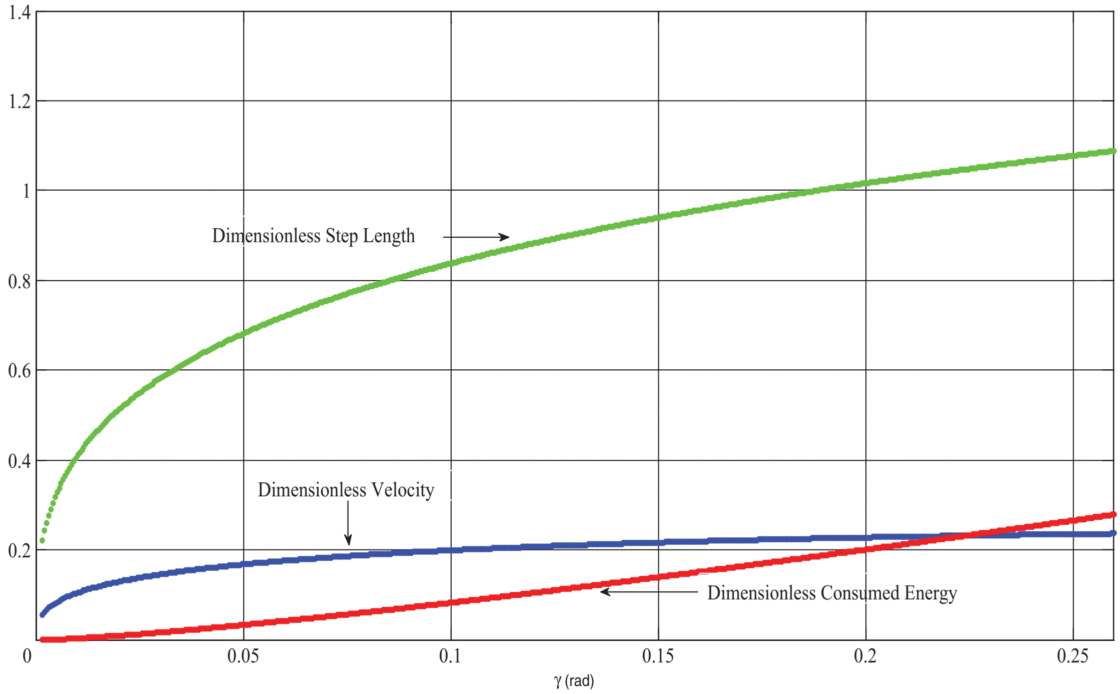


Fig. 8. (Colour online) The normalized speed, consumed energy and step length vs. $\gamma$. It is apparent from the figure that velocity will be gradually saturated while step length continues its growth.

previous maximum stable velocity for the simplest passive walker is about $0.12\sqrt{gl}$, obtained from walking on a slope of 0.015 rad.[3]

It can be inferred from the figure that when the velocity's diagram is going to be saturated, the stability cannot be obtained by local ramps and therefore the ground attractors will vanish. This might guide us to conclude that beyond the maximum velocity, the basin of attraction of any existing limit cycle is completely expunged. However, it should be regarded that we have not considered all of the possible sequences of contact points which might end in a stable cyclic walking. In other words, our focus is only placed on the even terrain produced by flat, fixed, parallel local ramps. Nevertheless, it is not a far-fetched conclusion if we claim that the saturation of the walker's velocity is equal to the diminishing of stability.

By adding up the results of Figs. 6–8, one would be led to a sideway inference about the speed-stability relation. It is an obvious consequence of the overall results (Fig. 4) that for the selected flat, fixed, parallel local ramp, i.e. a constant $\psi$, the walker presents different types of stable walking due to different initial posture conditions. In contrast, when a global slope angle, i.e.$\gamma$, is kept constant, there is an interval of $\psi$ for the walker to present stable walking. The area of the basin of attraction depends on the $\psi$, which is designed for the walker. So the walker can have different stabilities based on different local ramps with final equal velocities and step lengths, since the global slope angles are equal. Now it could be inferred that the two completely similar walkers with equal velocities and equal step lengths (more precisely, equal trajectories) would have different stabilities. We have shown this characteristic in ref. [13]; but here we have generalized it. To describe more specifically, imagine a walker with a repeating motion. Suppose we have passed various flat, fixed, parallel local ramps through the CWCP. Now, if a small disturbance is included in the walking process, based on which a series of local ramps are chosen, the step period will differ and as a result, the stabilities will vary.

It is quite acceptable for a knowledgeable reader to compare the passive strategies presented above with the active control strategy of the swing leg retraction.[6,15,17] However, we insist on avoiding this comparison for the reasons explained as follows. Basically, in the leg retraction theory, the strategy is based on changing the biped's passive trajectories, whereas here, the robot presents its natural dynamics. This difference makes any comparison complicated, since in a passive motion in each step, the amount of the swing leg retraction is different or, even, the swing leg may keep on moving

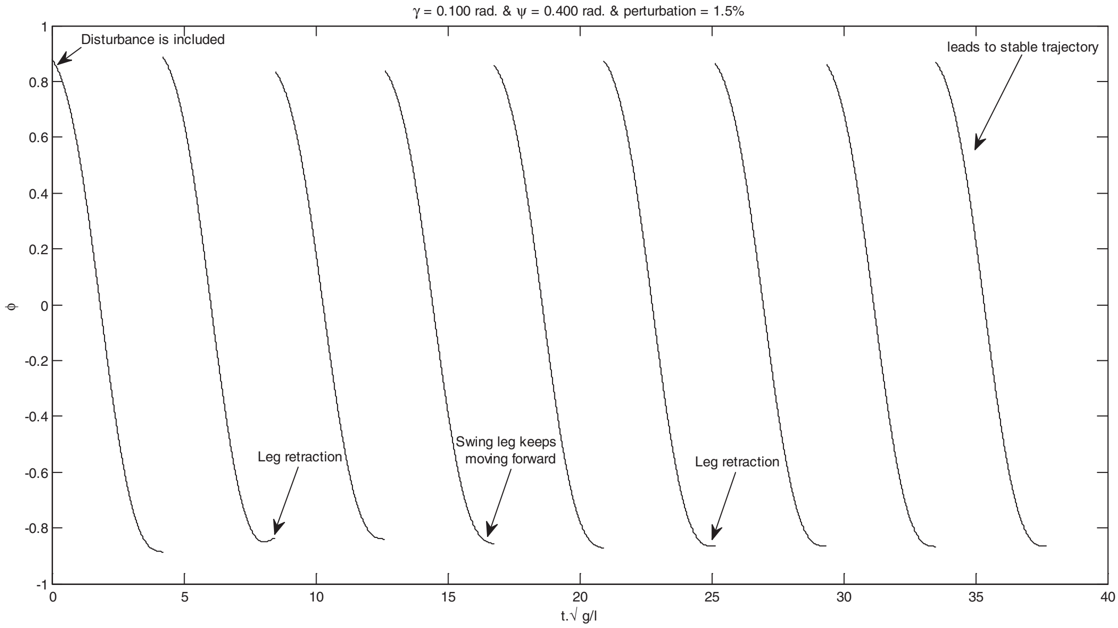


Fig. 9. A disturbed trajectory of a stable cyclic gait containing both negative and positive retraction speeds. The few steps which have positive leg retraction speeds can be eliminated, if the amount of disturbance changes to be about $-1\%$ of the fixed point's initial conditions.

forward and thus we lose any backward phase. On the contrary, in a controlled biped based on swing leg retraction, the amount of the desired retraction is usually fixed and predefined. On the other hand, leg retraction is not a necessary condition for stability;[17] but is a simple rule that eliminates a lot of other possible walking sequences ending in stability. To make the issue more clear, look at the Fig. 9, where a disturbed trajectory of a stable cyclic gait is presented. The walker is placed on the global slope of $\gamma = 0.1$ rad. with the ground attractor of $\psi = 0.4$ rad. The amount of disturbance is supposed to be 1.5% of the fixed point's initial condition. It is understood from the figure that leg retraction occurs only in a few steps (step 2 and step 6), whereas swing leg keeps moving forward in other steps, including periodic motion. If the amount of disturbance is replaced by-1% of the fixed point's initial condition, those few retraction steps will be totally crossed out and walking is revived without any retraction phase!

In addition, as a major difference, according to the active control strategy of the leg retraction theory, walkers with the same step lengths and the same step periods could have various stabilities regarding the dissimilar trajectories; but here it has been shown that the same walking trajectories present unlike stabilities, i.e. stabilities are different under a more restricted condition. That means similar retraction configurations for the same walkers with the same speeds and the same step lengths can lead to unequal stabilities, and vice versa; we can change (specifically increase) the stability of a controlled walking machine by preparing a more appropriate walking surface.

Another interesting subject is the behavior of the model on an uneven terrain. When a walking surface has a ceaseless irregularity, even negligible, the limit cycle walking will vanish.[2] More precisely, the disappearance of periodic walking is related to the variation of the local slope angles during walking. These variations may have a stochastic or a predefined nature. Suppose we have a walking surface with the shape of a semi-circle. When a walker is released from the top of the walking surface, the local slope angle will increase or decrease regarding the convexity or concavity of the surface. In each case, although the walking surface is thoroughly smooth, a passive periodic walking is impossible. So, any uniform or non-uniform variations in the local slope angles raises the chance of falling, because the walker will be far from its equilibrium point and the gait might be near the boundary of the basin of attraction. However, it has been shown by a few researchers that uneven terrains of passive walkers will be feasible if a new strategy for stability is defined.[16] In this regard, the walker will be stable if it never falls. This can be executed by keeping the global average slope (global basin of attraction) constant, in addition to the small variations of the local slope angles.

https://doi.org/10.1017/S0263574714000204 Published online by Cambridge University Press

Since the local slopes have a great role in the construction of the basin of attraction, it is expected that locomotion over an uneven terrain on a steep ramp is possible under some assumptions about the nature of the walking surface. When a global slope angle (average slope angle) is less than 0.0151 rad., according to Fig. 5, the basin of attraction for a limit cycle walking exists for both conditions of the local slope angle ($\psi$) greater and less than the global slope angle ($\gamma$). Otherwise, it will be confined to those circumstances where stability is solely guaranteed for the local slopes greater than the average slope. By using this fact, it is expected that walking over a bumpy surface becomes possible, if the feature of the walking surface provides the walker with enough steep slopes at collision points. The proof of this conjecture is not the concern of this paper and needs more thorough simulations and calculations as the material of a separate research.

At the end, we are interested in arguing the philosophy of the current work. The main question that might have been triggered here is: Would it be acceptable to modify the ground to get more stability? Robots are supposed to get adapted to the environment and thus adapting the environment to them seems rather strange and even far from the major goal of using them. However, we believe it is too soon to judge the theory from the application point of view; besides, no one knows how far the human ideas would go. Even if there is no direct application found for such modification, it could be developed to explore new challenges. In this regard, looking for an answer to each of the following questions might highly affect the route of the research: Is it possible to replace the ground attractors with a simple controller? Can we use the results to study walking on uneven terrains? Can the theory be applied to other kinds of hybrid or impulsive systems?

## 5. Conclusion

The stability of passive walkers has been revisited to reveal the extra capacity of these machines which have not been seen before. In this regard, we employed the simplest walking model[3] and let it examine other smooth surfaces instead of a ramp or stairs. It has been shown that the walking surface can have a vital and fundamental role in the stabilization of a biped during its walking. This role is played by cutting the state trajectories, which is done by the collision rule in the mathematical model. The desired procedure of this cutting is not the concern of the recent paper. In fact, the walking surface (collision rule) cuts walking sequences in which its stability is guaranteed or not; but how it is done and the reason why one leads to a stable gait and another not, is not clearly known. We can only conclude that when the state trajectories are cut, it is likely to be one of the most important factors in a biped stability.

Here, to define categories, we have called that walking surface which provides a stable configuration for a walker, "Ground Attractor", and the other which guides it to its fall, "Repulsive Direction". A "Ground Attractor", which has been uncovered by altering the geometrical collision condition, proves that the stability of a biped might be promoted if its walking surface is revised.

Behind these definitions, it has been numerically shown that the simplest passive walker can reach higher stable velocities which were unachievable by previous efforts. While Garcia *et al.* suggest the maximum stable velocity of $0.12\sqrt{gl}$ by using ordinary ramps, our solution enables their model to experience a velocity somewhere around $0.24\sqrt{gl}$. This means that we have doubled the maximum stable velocity of the simplest walking model only by employing the new walking surface. As a secondary result, it could be concluded that equal velocities will not lead to equal stabilities. This is inferred from the side effects of the $\psi$ parameter through our modeling; i.e. when the walker's speed is adjusted by energy input which is supplied through gravity, the stability will be obtained by varying the angle of the local slope ($\psi$).

The present attempt improves the stability of the simplest passive walker by using local slopes. It is hoped that it could extend the boundaries of the subject and also help the designing of bipedal robots together with the achievement of a better understanding of human and animal walking.

## References

1. E. Borzova and Y. Hurmuzlu, "Passively walking five-link robot," *Automatica* **40**(4), 621–629 (2004).
2. K. Byl and R. Tedrake, "Stability of passive dynamic walking on uneven terrain," *Proceedings of the Dynamic Walking*, Ann Arbor, MI (2006).

3. M. Garcia, A. Chatterjee, A. Ruina and M. Coleman, "The simplest walking model: Stability, complexity, and scaling," *ASME J. Biomech. Eng.* **120**(2), 281–288 (1998).
4. A. Goswami, B. Thuilot and B. Espiau, "A study of the passive gait of a compass-like biped robot: Symmetry and chaos," *Int. J. Robot. Res.* **17**(12), 1282–1301 (1998).
5. J. Hass, J. M. Herrmann and T. Geisel, "Optimal mass distribution for passivity-based bipedal robots," *Int. J. Robot. Res.* **25**(11), 1087–1098 (2006).
6. D. G. E. Hobbelen and M. Wisse, "Swing-leg retraction for limit cycle walkers improves disturbance rejection," *IEEE Trans. Robot.* **24**(2), 377–389 (2008).
7. A. D. Kuo, "A simple model of bipedal walking predicts the preferred speed-step length relation," *ASME J. Biomech. Eng.* **123**(3), 264–269 (2001).
8. A. D. Kuo, "The six determinants of gait and the inverted pendulum analogy: A dynamic walking perspective," *Hum. Mov. Sci.* **26**(4), 617–656 (2007).
9. A. D. Kuo, J. M. Donelan and A. Ruina, "Energetic consequences of walking like an inverted pendulum: Step-to-step transitions," *Exercise Sport Sci. Rev.* **33**(2), 88–97 (2005).
10. T. McGeer, "Passive dynamic walking," *Int. J. Robot. Res.* **9**(2), 62–82 (1990).
11. T. McGeer, "Passive Walking With Knees," *Proceedings IEEE International Robotics & Automation Conference*, IEEE Computer Society, Los Alamitos, CA (1990) pp. 1640–1645.
12. L. Ning, L. Junfeng and W. Tianshu, "The effects of parameter variation on the gaits of passive walking models: Simulations and experiments," *Robotica* **27**(4), 511–528 (2008).
13. A. T. Safa, M. Ghaffari Saadat and M. Naraghi, "Passive dynamic of the simplest walking model: Replacing ramps with stairs," *Mech. Mach. Theory* **42**(10), 1314–1325 (2007).
14. A. L. Schwab and M. Wisse,"Basin of attraction of the simplest walking model," *Proceedings of the ASME Design Engineering Technical Conferences*, Pennsylvania, Paper number DETC2001/VIB-21363 (2001).
15. A. Seyfarth, H. Geyer and H. Herr, "Swing-leg retraction: A simple control model for stable running," *J. Exp. Biol.* **206**(15), 2547–2555 (2003).
16. J. L. Su and J. B. Dingwell, "Dynamic stability of passive dynamic walking on an irregular surface," *ASME J. Biomech. Eng.* **129**(6), 802–810 (2007).
17. M. Wisse, C. G. Atkeson and D. K. Kloimwieder, "Swing Leg Retraction Helps Biped Walking Stability," *Proceedings of the IEEE-RAS International Conferences on Humanoid Robots*, Tsukuba, Japan (2005) pp. 295–300.
18. M. Wisse, A. L. Schwab and F. C. T. Vander Helm, "Passive dynamic walking model with upper body," *Robotica* **22**(6), 681–688 (2004).