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DYNAMICS OF OPEN-ECONOMY BUSINESS-CYCLE MODELS: ROLE OF THE DISCOUNT FACTOR

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This paper examines the dynamic implications of different preference formulations in open-economy business-cycle models with incomplete asset markets. In particular, we study two preference formulations: a time-separable preference formulation with a fixed discount factor, and a time-nonseparable preference structure with an endogenous discount factor. We analyze the moment implications of two versions of an otherwise identical open-economy model—one with a fixed discount factor and the other with an endogenous discount factor—and study impulse responses to productivity and world real-interest-rate shocks. Our results suggest that business-cycle implications of the two models are quite similar under conventional parameter values. We also find that the approximation errors associated with the solutions of these two models are of the same magnitude.

Keywords: Business-Cycle Dynamics, Fixed Discount Factor, Endogenous Discount Factor

1. INTRODUCTION

There have been two popular preference formulations employed in infinite-horizon open-economy real-business-cycle (RBC) models under incomplete asset markets: time-separable preferences with a fixed discount factor, and time-nonseparable preferences with an endogenous discount factor.[1] The former formulation is the "standard" one that is widely used in closed- or open-economy RBC models, under complete and incomplete asset markets. However, it is by now well known that, with this formulation, when the models are solved using the usual linear approximation methods, it is not possible to generate stationary state variables and

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a well-defined wealth distribution in an open-economy setting under incomplete markets.[2] This problem has led several researchers to employ the latter formulation, which generates well-defined steady-state dynamics in these models.

How can we justify the fact that a number of researchers have employed the fixed discount factor formulation despite the well-known problems associated with steady-state dynamics in open-economy RBC models under incomplete asset markets? Three potential reasons emerge for the widespread use of this formulation. First, although the issue of the existence of a well-defined long-run wealth distribution might be important for some economic experiments, there might be no reason to believe that these two preference formulations generate significantly different model dynamics along the dimensions examined by typical RBC studies. To be more specific, it might be the case that the models with these two preference formulations produce very similar moments and impulse responses, which are the main interests of these studies.

Second, considering that most models in this literature are solved using linear approximation methods, potential additional accuracy gains from employing an endogenous discount factor might be insignificant. In other words, although the true solution of the fixed discount factor model, which we call FDM (exact), and the true solution of the endogenous discount factor model, which we call EDM (exact), might exhibit some differences, it is not clear whether the approximate solutions of these two models, which we call FDM (approximate) and EDM (approximate), are different under incomplete markets.

Third, some features of the endogenous discount factor formulation might not be desirable in a representative-agent business-cycle model. For example, the endogenous discount factor formulation implies that agents become more impatient as they become wealthier, and there is a steady-state utility level that relies on a predetermined saving target [cf. Senhadji (1995), Daniel (1997)]. Another undesirable property of the EDM is that it is not possible to examine the state-dependent steady-state level of net foreign assets because the EDM forces consumption to behave in such a way that the net foreign asset position goes back to its initial steady state. Moreover, since a number of macroeconomic time series are nonstationary, it might not be advantageous to have a model generating stationary variables [cf. Correia et al. (1995)].[3]

Despite the wide use of these two preference formulations, there has yet been no rigorous examination of their impact on the cyclical dynamics of open-economy RBC models. The objective of this paper is to provide a comprehensive comparison of the behavior of an open-economy RBC model with incomplete asset markets under the fixed discount factor with that under the endogenous discount factor. In particular, we consider two versions of an otherwise identical small open-economy RBC model, one with a fixed discount factor (FDM), and the other with an endogenous discount factor (EDM), and study the moment implications and impulse responses of these models. We provide empirical support for the three reasons above and argue for the similarity between FDM (exact) and EDM (exact) based on the similarity between FDM (approximate) and EDM (approximate), which we systematically study in this paper.[4]

Section 2 starts with a brief discussion of the problem associated with the steady-state dynamics in open-economy RBC models with incomplete asset markets. Then, we present the two models and their calibration and parameterization. In Section 3, we provide empirical support for the three justifications described above. We analyze the policy functions generated by the two models in Section 3.1. Our discussion in Sections 2 and 3.1 makes transparent the arguments regarding the deterministic and stochastic steady-state dynamics of the two models. We find that although the net foreign asset series in the FDM follows a unit root process, this series exhibits a near unit root behavior in the EDM. More importantly, the coefficients in the policy functions generated by the two models display minimal differences under conventional parameter values. In Section 3.2, we examine the second-moment implications pertaining to business cycles and find that the two models generate almost identical business-cycle moments. Section 3.3 presents the impulse responses to productivity and the world interest-rate shocks. Although some small quantitative differences exist between the impulse responses produced by the two models, qualitative implications are identical. In Section 3.4, we compare the distributions of variables produced by the two models using the Kolmogorov–Smirnov test statistic. The test results suggest that these empirical distributions are not statistically different. In Section 3.5, we evaluate the approximation errors associated with the solutions of these models. We find that, although the solution of the FDM results in slightly larger approximation errors than that of the EDM, the errors are quite small. We provide a brief conclusion in Section 4.

2. MODEL

Before getting into the details of the model economies, we provide a brief discussion of the problem associated with the steady state. Consider the standard time-separable preference formulation in an infinite-horizon open-economy model with incomplete asset markets. In a deterministic setting, the steady state or the long-run wealth depends on the initial conditions of the economy and the steady state is compatible with any level of net foreign assets.[5] In a stochastic environment, since the net foreign asset series follows a unit root process, the model generates nonstationary variables, implying the absence of a well-defined stochastic steady state.[6] In other words, certain model variables do not return to their initial steady-state values when the model is subjected to a temporary shock. Hence, the long-run wealth of the economy changes with the state of nature; that is, the long-run wealth distribution is not well defined.

Researchers have developed several methods that can resolve the steady-state issues associated with the fixed discount factor formulation.[7] As mentioned earlier, one popular alternative is to employ the Uzawa–Epstein-type time-nonseparable preferences with an endogenous discount factor. Unlike the fixed discount factor model, the endogenous discount factor model generates stationary net foreign asset series and, in turn, other state variables become stationary. Moreover, the endogenous discount factor model generates a well-defined stochastic steady state at which, under certain conditions, a unique level of net foreign assets is attained, implying that the long-run wealth distribution of the economy is also well defined.

To evaluate the impact of the discount factor on the dynamics of business cycles, we study a small open-economy RBC model in which agents produce an internationally tradable good using labor and capital.[8] There are two shocks in the model: a productivity shock and a world real-interest-rate shock. Agents have access to world financial markets in which they can buy and sell one-period risk-free bonds at a stochastic world real interest rate. We solve the model using a linear approximation method and choose the parameter values that provide that both versions of the model have the same steady state. We calibrate the model to Canada, an economy that has been extensively studied in the small open-economy RBC literature. In Section 2.1, we present the model with a fixed discount factor. In Section 2.2, we analyze the same model with an endogenous discount factor. In Section 2.3, we explain the derivation of the steady state and the calibration of the model.

2.1. Fixed Discount Factor Model

The optimization problem of the representative agent is the following:

$$\max_{c_t, i_t, n_t, b_t} U(c_t, n_t) = E_0 \sum_{t=0}^{\infty} \beta^t \frac{\left(c_t - \frac{n_t^w}{w}\right)^{1-\sigma} - 1}{1-\sigma}, \qquad \sigma > 1, w > 1,$$

$$\text{subject to } c_t + i_t + b_t = y_t + b_{t-1}(1 + r_{t-1}), \tag{1}$$

$$y_t = z_t k_t^{1-\alpha} n_t^{\alpha}, \tag{2}$$

$$k_{t+1} = (1-\delta)k_t + \phi(i_t/k_t)k_t. \tag{3}$$

In the momentary utility function, c_t is consumption, n_t is labor hours, w is the intertemporal elasticity of substitution in labor supply, β is the fixed discount factor, and σ is the household's coefficient of relative risk aversion. Our momentary utility formulation implies that the elasticity of substitution associated with leisure is zero.[9] In resource constraint (1), i_t is investment, b_t is the net foreign assets at the end of the period t, r_{t-1} is the stochastic world real interest rate from period $t-1$ to period t, and y_t is output. In constraint (2), k_t is the domestic capital stock at the beginning of the period t, z_t is the productivity shock, and α governs the share of income accruing to labor that describes the production function.[10] In constraint (3), δ denotes depreciation rate and $\phi(\cdot)$ represents the standard adjustment cost function with $\phi(\cdot) > 0$, $\phi(\cdot)' > 0$, and $\phi(\cdot)'' < 0$ [cf. Baxter and Crucini (1993)].

Substituting (2) into (1), the first-order conditions of this optimization problem are given as

$$c_t : \left(c_t - \frac{n_t^w}{w}\right)^{-\sigma} = \lambda_t, \tag{4}$$

$$n_t : n_t^w = \alpha y_t, \tag{5}$$

$$i_t : \lambda_t = \mu_t \phi'(i_t/k_t), \tag{6}$$

$$k_{t+1} : \mu_t = \beta E_t\left[\lambda_{t+1}(1-\alpha)\frac{y_{t+1}}{k_{t+1}} + \mu_{t+1} g\left(\frac{i_{t+1}}{k_{t+1}}\right)\right], \tag{7}$$

where

$$g\left(\frac{i_{t+1}}{k_{t+1}}\right) = (1-\delta) + \phi\left(\frac{i_{t+1}}{k_{t+1}}\right) - \phi'\left(\frac{i_{t+1}}{k_{t+1}}\right)\frac{i_{t+1}}{k_{t+1}},$$

$$b_t : \lambda_t = \beta E_t \lambda_{t+1}(1+r_t), \tag{8}$$

where λ_t and μ_t are the Lagrange multipliers associated with the constraints (1) and (3).

Since this problem cannot be solved analytically, we find an approximate solution using the approximation method of King et al. (1988).[11]

2.2. Endogenous Discount Factor Model

This part describes the same model with a time-nonseparable preference formulation in which the discount factor is endogenous.[12] The optimization problem of the representative agent is

$$\max_{c_t, n_t, i_t, b_t} U(c_t, n_t) = E_0 \sum_{t=0}^{\infty} \gamma_t \frac{\left(c_t - \frac{n_t^w}{w}\right)^{1-\sigma} - 1}{1-\sigma}, \qquad \sigma > 1, w > 1,$$

subject to the same constraints (1)–(3), above. The endogenous discount factor, γ_t, is defined as

$$\gamma_t = \exp\left[-\sum_{\tau=0}^{t-1} \theta \ln\left(1 + c_\tau - \frac{n_\tau^w}{w}\right)\right], \quad \theta > 0. \tag{9}$$

The discount factor depends on the level of consumption and labor input in the previous periods; θ denotes the elasticity of the discount factor with respect to utility. The functional form of the endogenous discount factor implies that an increase (decrease) in current consumption (labor input) decreases the weights assigned to all future utility, and, in turn, the agent becomes more impatient.[13]

Using (9), we define an auxiliary variable ψ_t, the time t value of discounted future utility from date $t+1$ onward:

$$\psi_t = E_t \sum_{k=t+1}^{\infty} \frac{\left(c_k - \frac{n_k^w}{w}\right)^{1-\sigma} - 1}{1-\sigma} \frac{\gamma_k}{\gamma_t}. \tag{10}$$

The first-order conditions of the optimization problem above are

$$c_t : \left(c_t - \frac{n_t^w}{w}\right)^{-\sigma} = \lambda_t + \psi_t \theta \left(1 + c_t - \frac{n_t^w}{w}\right)^{-1}, \tag{11}$$

$$n_t : n_t^w = \alpha y_t, \tag{12}$$

$$i_t : \lambda_t = \mu_t \phi'(i_t / k_t), \tag{13}$$

$$k_{t+1} : \mu_t = \left(1 + c_t - \frac{n_t^w}{w}\right)^{-\theta} E_t \left[\lambda_{t+1}(1-\alpha)\frac{y_{t+1}}{k_{t+1}} + \mu_{t+1} g\left(\frac{i_{t+1}}{k_{t+1}}\right)\right], \tag{14}$$

$$b_t : \lambda_t = \left(1 + c_t - \frac{n_t^w}{w}\right)^{-\theta} E_t \lambda_{t+1}(1 + r_t). \tag{15}$$

Using equation (10), we write the following law of motion for the auxiliary variable ψ_t:

$$E_t(\psi_{t+1}) = \left(1 + c_t - \frac{n_t^w}{w}\right)^{\theta} \psi_t - E_t \left[\frac{\left(c_{t+1} - \frac{n_{t+1}^w}{w}\right)^{1-\sigma} - 1}{1-\sigma}\right]. \tag{16}$$

We solve the model using the same approximation method.

2.3. Calibration

We choose the parameter values to ensure that we have the same steady state for both models. The variables without time subscripts refer to the steady-state values of the corresponding variables. The three important steady-state parameters to be discussed here are r, nx/y (net export/output), and θ, since we attain the same steady state by using the same values for r and nx/y and by endogenously determining the value of θ in the EDM.

In the FDM, given r, the steady-state version of equation (8) determines only the discount factor β. Therefore, the number of endogenous variables is less than the number of steady-state equations by one. This implies that any level of foreign assets is compatible with the initial steady state of the model. Following the standard approach in the literature [cf. Baxter and Crucini (1993), Correia et al. (1995)], we draw the value of nx/y (net exports/output) from data to solve this problem because nfa/y (net foreign assets/output) is uniquely determined by nx/y and r.

Unlike the FDM, the EDM does not suffer from the indeterminacy problem because the number of endogenous variables is equal to the number of steady-state equations; that is, the model generates a unique steady-state level of the net foreign assets. However, this hinges on the assumption that the value of θ is known in advance. Using the value of nx/y, which is drawn from the data, we pin down the value of θ endogenously using the steady-state version of equation (15). This guarantees that all the variables including the discount rates in the two models have the same steady-state values. This is also the standard calibration method used in the studies employing the endogenous discounting factor.

We calibrate the structural parameters to correspond to the existing RBC literature and to be consistent with the long-run features of the Canadian economy. Table 1 presents the calibrated values of parameters. We set the quarterly steady-state world real interest rate at 1.21%, which is the average rate calculated using the U.S. 3-month T-Bill rate deflated with the CPI inflation. Following Mendoza (1991), we set the elasticity of substitution, w, to 1.455. The risk aversion parameter, σ, is set to 1.5, which is an intermediate case between the commonly used values of 2 and 1 [cf. Schmitt-Grohé (1998)]. Following Mendoza (1991) and Schmitt-Grohe (1998), we set the share of labor income in the production, α,

TABLE 1. Parameters of the model[a]

Parameter	Description	Value
Preferences		
r	Steady-state real interest rate	1.21%
w	Intertemporal elasticity of substitution in labor supply	1.455
σ	Coefficient of relative risk aversion	1.5
Technology		
α	Share of labor income	0.68
δ	Depreciation rate	2.5%
η	Elasticity of marginal adjustment cost function $\eta = -(\phi'/\phi'')/(i/k)$	10
nx/y	Steady-state net exports to output ratio	0
Shocks		
ρ_z	Persistence of technology shock	0.95
σ_z	Standard deviation of technology shock	0.625%
ρ_r	Persistence of interest-rate shock	0.7
σ_r	Standard deviation of interest-rate shock	0.1%
$\text{Corr}(\varepsilon_z, \varepsilon_r)$	Correlation between technology and interest-rate shocks	0

[a] See Section 2.3 for details.

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to 0.68. The quarterly depreciation rate, δ, is set to 0.025, a widely used value in the RBC literature.

Our benchmark value of nx/y is equal to 0.[14] To examine the sensitivity of our results, we experiment with different values of nx/y, which is equal to the interest payments/output ratio ($-rb/y$), ranging from -0.05 to 0.05 per quarter.[15] Since θ is an increasing function of nx/y at an exponential rate, θ ranges from 0.0074 to 0.0088.[16] The corresponding value of θ is 0.008 when nx/y is equal to 0.

The adjustment cost parameters are chosen so that the steady state of the model is same as the one without adjustment costs. This implies that $\phi(i/k) = i/k$ and $\phi'(i/k) = 1$. The steady-state value of i/k is equal to the depreciation rate, δ. The elasticity of the marginal adjustment cost function, $\eta = -(\phi'/\phi'')(i/k)^{-1}$, is set to 10, to match the volatility of investment in the data [cf. Baxter and Crucini (1993)].

The exogenous shocks, z_t and r_t, follow AR(1) processes with

$$\hat{z}_t = \rho_z \hat{z}_{t-1} + \varepsilon_t^z, \tag{17}$$

$$\hat{r}_t = \rho_r \hat{r}_{t-1} + \varepsilon_t^r \tag{18}$$

where ε^z and ε^r are assumed to follow normal distributions with mean 0 and variance σ_z^2 and σ_r^2. We set the standard deviations of the productivity and interest-rate shocks at 0.615% and 0.1%, respectively, to match the volatility of output in the data. The persistence parameters of shocks, ρ_y and ρ_r, are estimated and they are equal to 0.95 and 0.7.

3. RESULTS

We study the dynamic implications of the FDM and the EDM on five dimensions: first, we compare the policy functions generated by the two models. Second, we study their second-moment implications. Third, we analyze the impulse responses to productivity and interest-rate shocks. Fourth, we compare the distributions of the three variables, which are nonstationary in the FDM, in the two models. We also formally test whether the distributions produced by these two models are statistically different. Finally, we estimate the approximation errors generated by the two models, considering that the FDM generates nonstationary variables that might induce larger approximation errors than those in the EDM.

3.1. Policy Functions

Table 2 presents the coefficients of policy functions for consumption, asset holdings and the net exports of the two models. We concentrate on the coefficients of the endogenous state variables, considering that impulse responses in the next section illustrate the differences in the coefficients associated with shocks. Asset holdings follow a unit root process in the FDM; that is, the coefficient of asset holdings, b_2, is equal to 1. The unit root property implies that a shock in the current period has a permanent impact and the long-run wealth of the economy depends on the shock

TABLE 2. Policy functions

$$\begin{pmatrix} \hat{c}_t \\ \hat{b}_t \\ \hat{n}x_t \\ \hat{y}_t \\ \hat{n}_t \\ \hat{i}_t \\ \hat{k}_{t+1} \end{pmatrix} = \begin{pmatrix} a_1 & b_1 \\ a_2 & b_2 \\ a_3 & b_3 \\ a_4 & b_4 \\ a_5 & b_5 \\ a_6 & b_6 \\ a_7 & b_7 \end{pmatrix} \begin{pmatrix} \hat{k}_t \\ \hat{b}_{t-1} \end{pmatrix} + \text{shocks}_t$$

Parameter[a]	Coefficient	FDM	EDM	Coefficient	FDM	EDM
$nx/y = -0.05$	a_1	0.4616	0.4623	b_1	0.0145	0.0146
$\theta = 0.0074$	a_2	0.4611	0.4606	b_2	1	0.9999
	a_3	0.4611	0.4605	b_3	−0.0121	−0.0122
$nx/y = 0$	a_1	0.4910	0.4998	b_1	0.0154	0.0164
$\theta = 0.008$	a_2	0.4611	0.4542	b_2	1	0.9992
	a_3	0.4611	0.4543	b_3	−0.0121	−0.0129
$nx/y = 0.05$	a_1	0.5245	0.5416	b_1	0.0165	0.0185
$\theta = 0.0088$	a_2	0.4611	0.4485	b_2	1	0.9985
	a_3	0.4611	0.4485	b_3	−0.0121	−0.0136
All cases	a_4	0.6008	0.6008	b_4	0	0
	a_5	0.4129	0.4129	b_5	0	0
	a_6	−1.1384	−1.1384	b_6	0	0
	a_7	0.9465	0.9465	b_7	0	0

[a] nx/y is the initial steady-state value of the net exports/output ratio. θ denotes the elasticity of the discount factor with respect to utility. See Section 3.1 for details.

realizations. This also induces consumption and the net exports to be nonstationary. Although the coefficients of the policy function for consumption (a_1 and b_1) are always larger in the EDM than in the FDM, differences between the coefficients are very small. In other words, consumption is more responsive to the state variables in the EDM compared to the FDM.

As the elasticity of the endogenous discount factor θ decreases, differences in the coefficients of the policy functions disappear. As θ decreases, the discount factor in the EDM responds less to the changes in utility; therefore, the variables in the EDM behave as if the discount factor is almost fixed. For example, when θ is equal to 0.0074 ($nx/y = -5\%$), the policy functions generated by the two models become almost identical and the net foreign asset series in the EDM follows a near unit root process ($b_2 = 0.9999$ in the EDM). In other words, our findings suggest that the moments produced by the EDM converge to those produced by the FDM when the elasticity of the endogenous discount factor, θ, gets arbitrarily small.

The policy functions of output, labor hours, investment, and capital stock generated by the two models are identical and invariant to the value of θ. These variables do not depend on the asset holdings; that is, the coefficients associated with asset holdings are zero in both models. Therefore, even in the FDM, these four variables, unlike consumption and the net exports that depend on the previous period's net foreign asset holdings, become stationary. This results from the momentary preference structure where labor hours depend only on the current output and there is no intertemporal substitution involving labor. This implies that labor hours become stationary, which induces output, investment, and capital stock to be stationary as well. We also examine the sensitivity of policy functions' coefficients to changes in other parameters of the model such as σ and w. We find that our results are robust to those changes.

3.2. Second-Moment Implications

One of the important objectives of the RBC research program is to construct models that are able to replicate certain moments of the data. In this section, we compare the business-cycle moments generated by each model. If the discrepancies between the moments generated by the two models are small, then this suggests that the FDM constitutes a reasonable alternative to the EDM for business-cycle analysis. We simulate the model for 100 periods with our benchmark parameterization and report the average moments over 300 simulations. All results refer to the moments of Hodrick–Prescott (HP 1600) filtered variables [cf. Hodrick and Prescott (1997)].

Panel A in Table 3 reports the second moments generated by productivity shocks. The results suggest that there is no statistically significant difference between the moments produced by the two models. As expected, investment is the most volatile variable, and output is more volatile than consumption. All model variables are procyclical except the net exports and the net foreign assets. Both models predict that correlations of labor hours and consumption with output are equal to 1 as implied by the preference structure.

In Panel B, we present the simulation results with both productivity and the world real-interest-rate shocks. The moments produced by the two models are again quite similar. Adding interest-rate shocks does not change the moments of any of the variables except investment, the net exports, and the net foreign assets. While increasing the volatilities of these three variables, interest-rate shocks dampen the correlations of these variables with output. We also check the sensitivity of our results to the changes in θ and find that these results are quite robust.

We also examine the ability of the models in matching the main characteristics of Canadian business cycles. Our findings suggest that both FDM and EDM are successful in replicating the main features of Canadian business cycles.[17] All results in this section suggest that it is almost impossible to differentiate between business-cycle statistics generated by the FDM and those generated by the EDM.

TABLE 3. Business-cycle moments[a]

	Volatility (%) (σ)		Relative volatility		Correlation with output		Autocorrelation	
	FDM	EDM	FDM	EDM	FDM	EDM	FDM	EDM
A: Productivity shocks								
Output	1.50	1.50	1.00	1.00	1.00	1.00	0.70	0.70
	(0.015)	(0.015)			(0.00)	(0.00)		
Consumption	1.09	1.10	0.73	0.73	1.00	1.00	0.70	0.70
	(0.011)	(0.011)			(0.00)	(0.00)		
Hours	1.03	1.03	0.69	0.69	1.00	1.00	0.70	0.70
	(0.011)	(0.011)			(0.00)	(0.00)		
Investment	4.37	4.37	2.91	2.91	0.97	0.97	0.68	0.68
	(0.042)	(0.042)			(0.001)	(0.001)		
nfa/y	1.30	1.32	0.87	0.88	−0.65	−0.65	0.96	0.96
	(0.022)	(0.022)			(0.004)	(0.004)		
nx/y	0.35	0.36	0.23	0.24	−0.76	−0.78	0.70	0.70
	(0.004)	(0.004)			(0.004)	(0.004)		
B: Productivity and interest rate shocks								
Output	1.50	1.50	1.00	1.00	1.00	1.00	0.70	0.70
	(0.016)	(0.016)			(0.00)	(0.00)		
Consumption	1.09	1.10	0.73	0.73	1.00	1.00	0.70	0.70
	(0.011)	(0.011)			(0.00)	(0.00)		
Hours	1.03	1.03	0.69	0.69	1.00	1.00	0.70	0.70
	(0.011)	(0.011)			(0.00)	(0.00)		
Investment	5.30	5.30	3.53	3.53	0.78	0.78	0.62	0.62
	(0.042)	(0.042)			(0.004)	(0.004)		
nfa/y	2.21	2.10	1.47	1.40	−0.42	−0.44	0.92	0.92
	(0.032)	(0.031)			(0.013)	(0.013)		
nx/y	0.81	0.76	0.54	0.51	−0.30	−0.34	0.54	0.55
	(0.006)	(0.005)			(0.009)	(0.009)		

[a] All moments are averages over 300 simulations, where each simulation consists of 100 periods. All variables, except nfa/y and nx/y, are logged and HP (1600) filtered. nx (net exports) and nfa (net foreign assets) are normalized by output, and HP (1600) filtered. Numbers in parenthesis are standard errors. See Section 3.2 for details.

3.3. Impulse Responses

Constructing models that can generate dynamic responses that are compatible with those in the data is another important objective of the RBC research program. This section compares the impulse responses to productivity and the world real-interest-rate shocks. We consider the benchmark case with $nx/y = 0$ and examine the sensitivity of impulse responses to changes in the persistence of shocks.[18]

Figure 1A presents the impulse responses to a temporary productivity shock, a 1% increase in productivity at the initial period with $\rho_z = 0$. Output, consumption,

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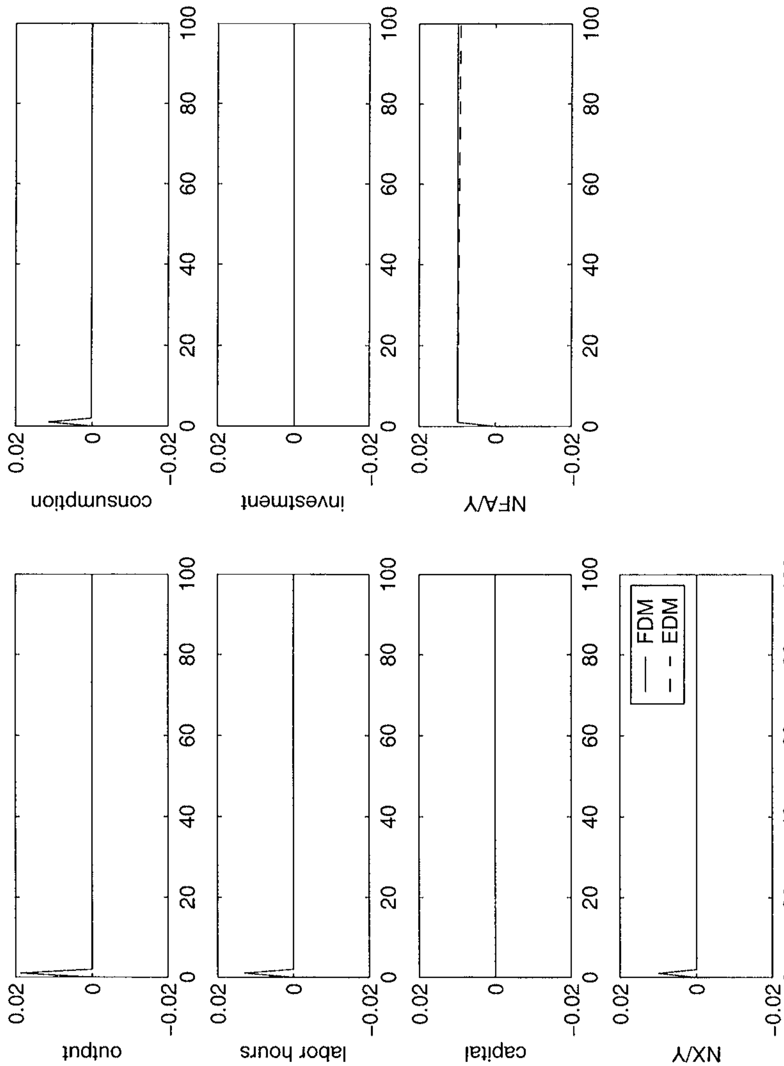

FIGURE 1A. Impulse responses to 1% increase in productivity ($\rho_z = 0$).

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and labor input initially increase; investment and, in turn, capital remain unchanged. The net exports initially rise because consumption increases less than output. The agent is able to maintain a higher consumption level because she receives interest income from its positive foreign asset holdings after the second period.

The impulse responses produced by the two models are quite similar: impulse responses of output, labor hours, capital, and investment are identical, while those of consumption, the net exports/output ratio, and the net foreign assets/output ratio exhibit little difference. In the FDM, as predicted by policy functions, consumption, the net foreign assets, and the net exports exhibit nonstationary behavior and do not return to the initial steady state, whereas the other four variables converge to the initial steady state. In the EDM, all variables slowly converge to the initial steady state.

Figure 1B presents the case with a persistent productivity shock ($\rho_z = 0.95$). We focus only on the three variables that are nonstationary in the FDM because impulse responses of other stationary variables are the same as those in the EDM. To study the sensitivity of our results to the duration of simulations, we examine the impulse responses for 100 periods (25 years) and 400 periods (100 years).[19]

The model economy responds to a positive productivity shock by increasing investment, consumption, labor hours, and output. The net exports decrease at impact because the agent borrows from the rest of the world to increase its capital stock and, in turn, utilizes the increase in productivity. In other words, the pro-borrowing effect initially dominates the pro-saving effect, inducing a fall in the net exports. As the agent starts accumulating foreign assets, the net exports increase, but then decrease in the long run.

The initial increase in consumption is slightly larger in the EDM than in the FDM. This can be explained by the impatience effect in the EDM: as the current consumption rises, the agent discounts future utility more, inducing a further increase in the current consumption. The speed of convergence in the EDM is extremely slow because the net foreign assets follow a near unit root process. In fact, with these parameter values, it takes more than 4,000 periods (1,000 years) for the variables in the EDM to return to their initial steady state. Figures with 100 periods suggest that the dynamic responses of the model variables are almost identical. Impulse responses with 400 periods indicate that there are some minor quantitative differences between the dynamic responses of the two models. Our sensitivity analysis suggests that the impulse responses produced by the two models become similar as θ decreases, as predicted by the policy functions.

None of the variables returns to the initial steady state when the productivity shocks are permanent ($\rho_z = 1$) in Figure 1C. The message of Figure 1C is the same as that of the earlier figures: The qualitative responses of the two models are identical and there are only slight differences in the quantitative results. Unlike in the cases with temporary and persistent shocks, in the case of permanent shocks, the agent, instead of accumulating foreign assets, borrows from abroad by issuing bonds and enjoys a permanent increase in consumption.[20] One interesting observation is that the net foreign assets in the EDM do not reach the new steady

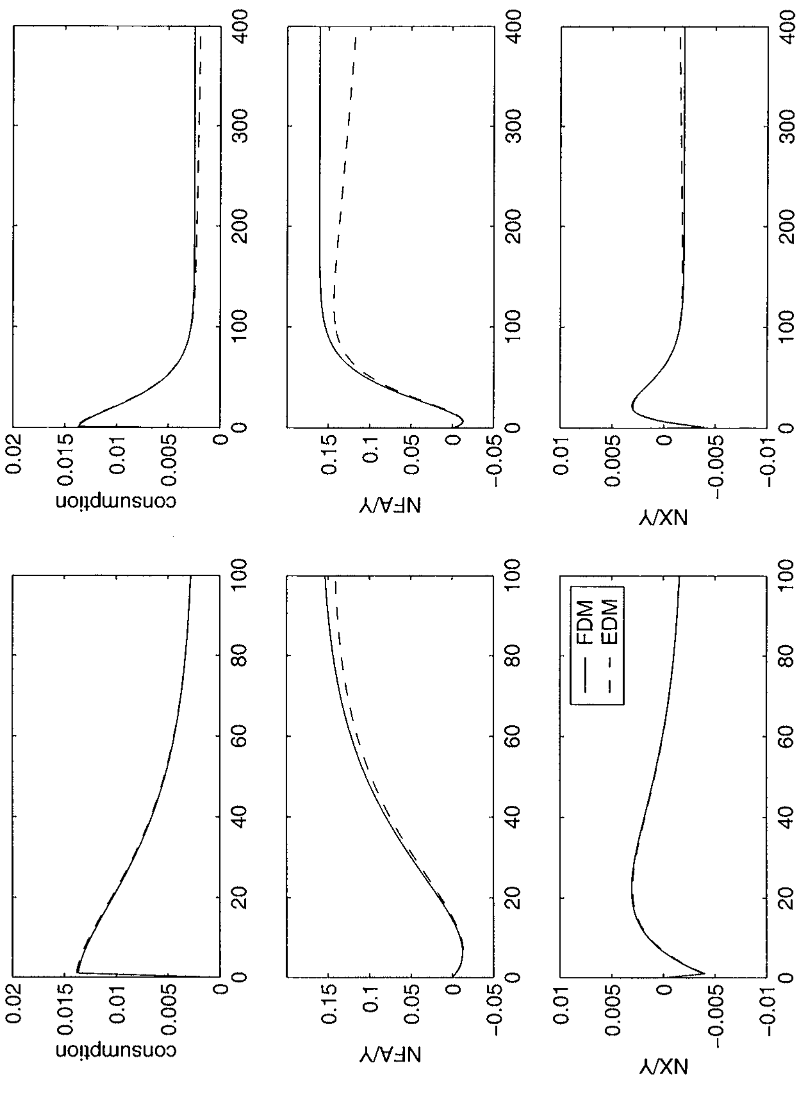

FIGURE 1B. Impulse responses to 1% increase in productivity ($\rho_z = 0.95$).

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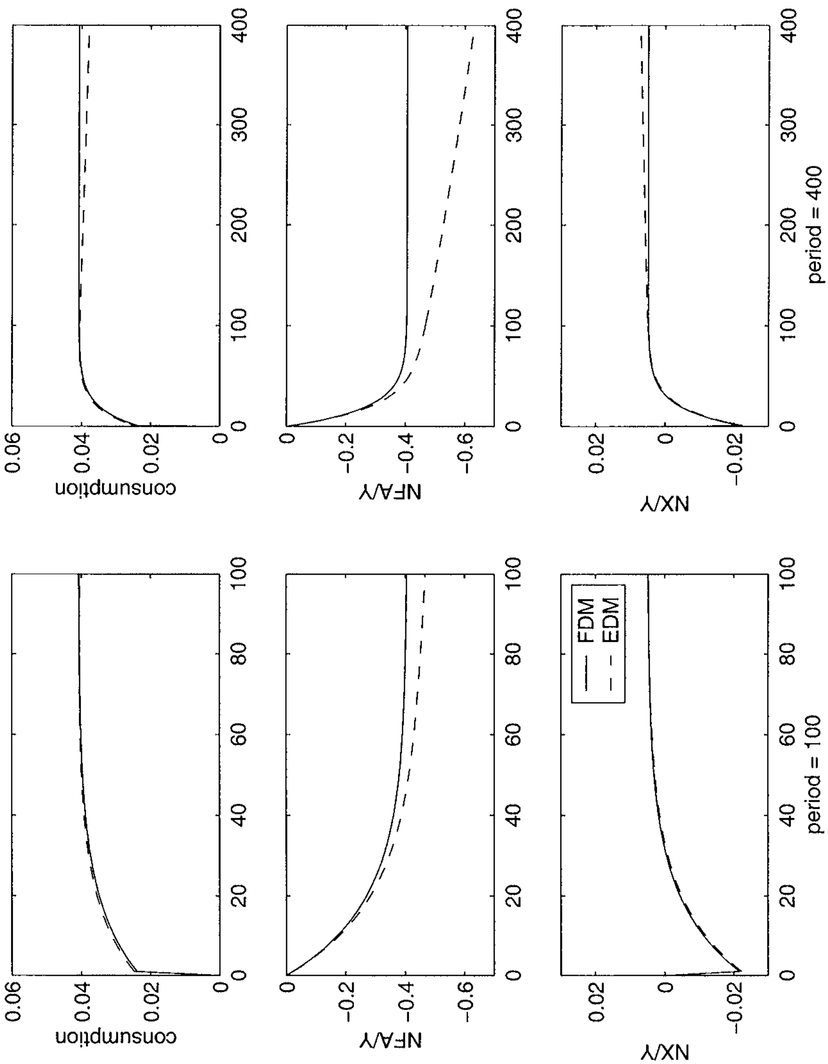

FIGURE 1C. Impulse responses to 1% increase in productivity ($\rho_z = 1$).

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state even after 400 periods and grow over 60% of output in this period. This unrealistic prediction can be interpreted as another unappealing property of the EDM.

We next examine the impulse responses of the model variables to a 0.25% increase in the world real interest rate in Figures 2A–C. Figure 2A reports our results with a temporary interest rate shock ($\rho_r = 0$). Since capital stock is predetermined in the period of impact, labor supply does not respond immediately, and output remains constant. Changes in investment and consumption in the first period trigger changes in output, labor input, and capital stock in the following period. As in the case of productivity shocks, responses of output, labor hours, capital, and investment are identical in the two models, while the behavior of the net foreign assets is slightly different.

In Figures 2B and 2C, we analyze the impulse responses of the variables that are nonstationary in the FDM. When the interest-rate shock is persistent ($\rho_r = 0.7$) in Figure 2B, none of the previous results changes significantly. Figure 2C depicts the impulse responses to a permanent ($\rho_r = 1$) world interest-rate shock. As these graphs show, all variables become nonstationary with permanent interest-rate shocks. Consumption and the net foreign assets in the FDM increase indefinitely because β is always larger than $1/(1+r)$, whereas these variables approach the new steady state in the EDM. In conclusion, although there are some quantitative differences, the two models produce similar qualitative responses except when shocks are permanent.

3.4. Distribution of Model Variables

To further examine the dynamic implications of these models, we analyze the distributions of the model variables. We simulate the two models 5,000 times with both shocks, and record the values of consumption, the net exports/output ratio, and the net foreign assets/output ratio in the 100th and 400th periods in each simulation.[21]

Figure 3 presents histograms of these variables. Although there are some differences in the histograms across the two models, these differences are quite small, especially at the 100th period. Table 4 reports the sample statistics of the three variables in each model: mean, standard deviation, skewness, and kurtosis. These statistics suggest that the two distributions share almost identical statistical properties.

To formally test whether the two distributions are statistically different, we compute the Kolmogorov–Smirnov test statistic that measures the maximum distance between the cumulative density functions of each series generated by the two models.[22] Table 4 reports the test statistics. The statistics show that, except for the distribution of the net foreign asset series in the 400th period, the distributions of variables in the two models are not statistically different at the 1% level.

3.5. Approximation Errors

Since both models are solved using the same linear approximation method, the results in the previous sections may be valid only for the linearized versions of

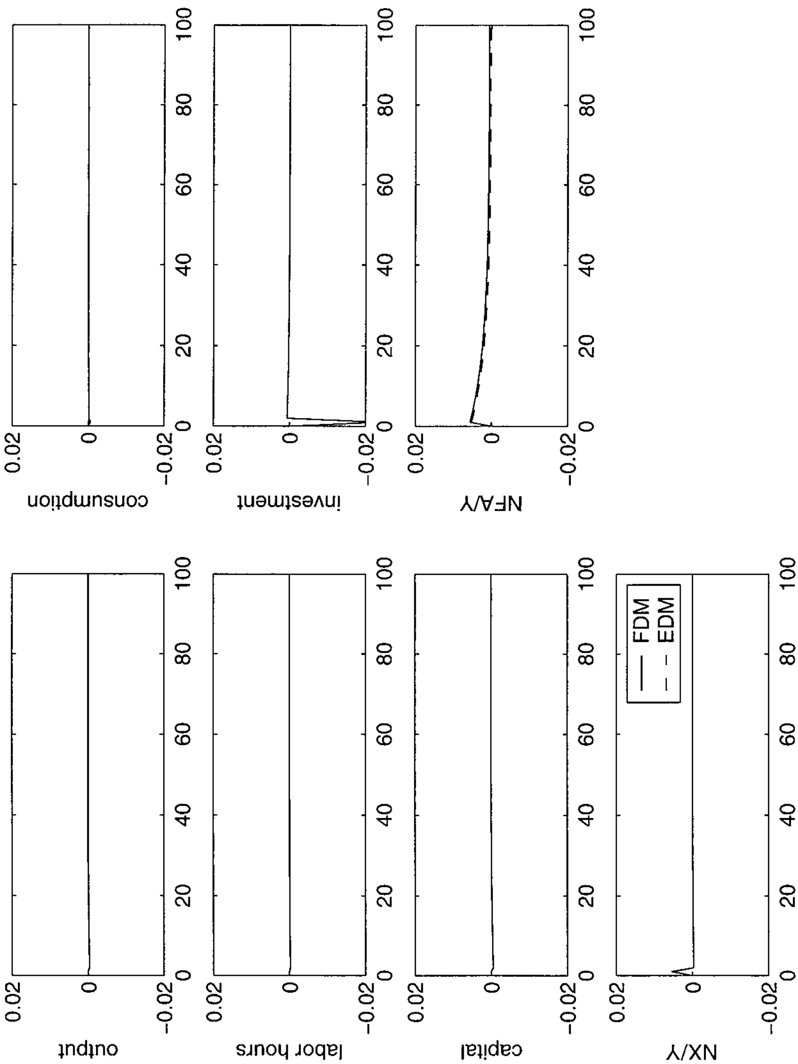

FIGURE 2A. Impulse responses to 0.25% increase in world interest rate ($\rho_r = 0$).

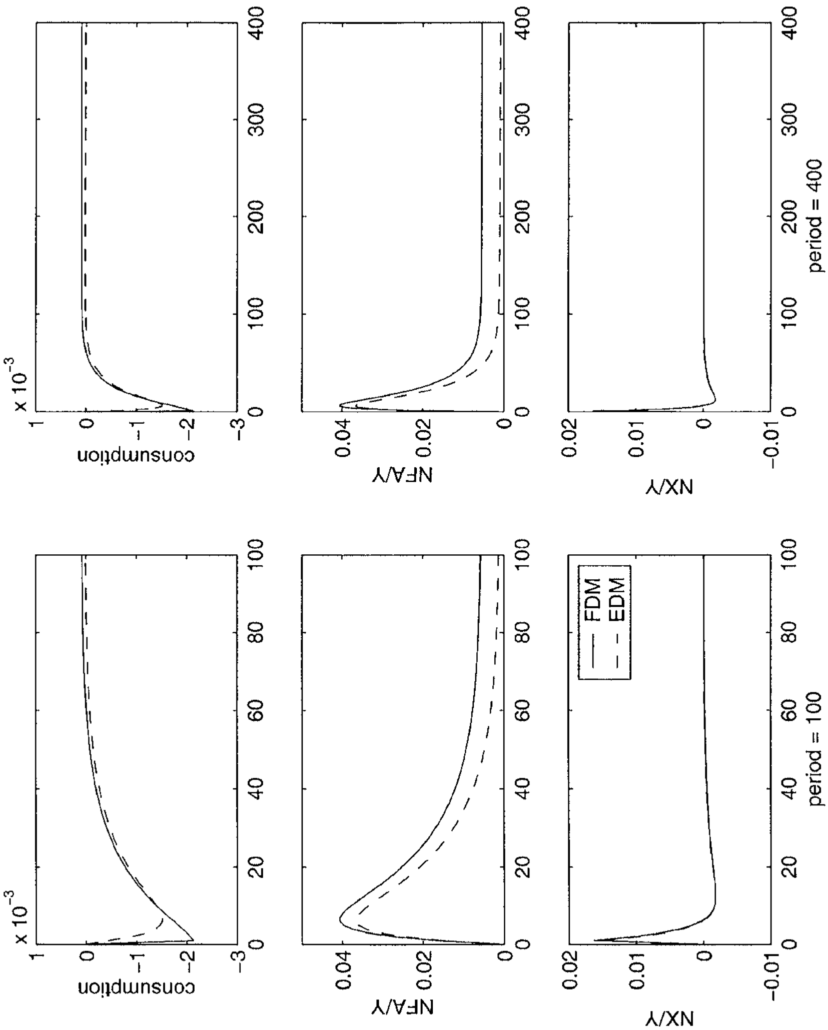

FIGURE 2B. Impulse responses to 0.25% increase in world interest rate ($\rho_r = 0.7$).

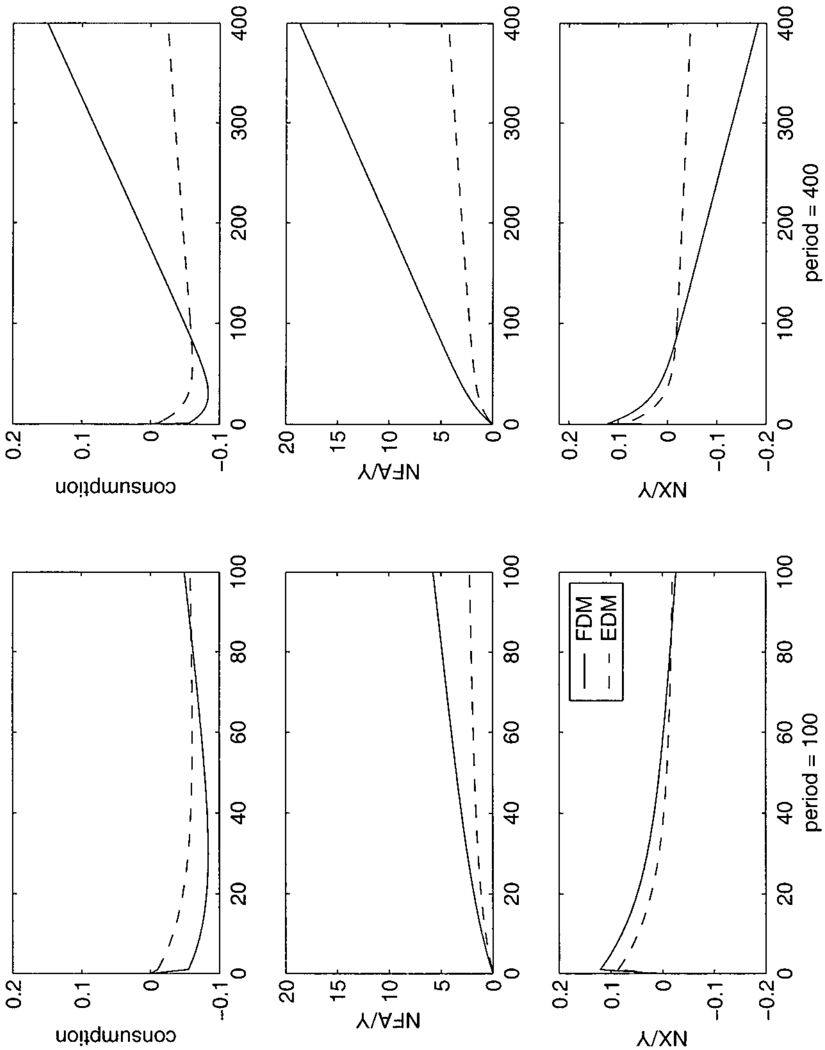

Figure 2C. Impulse responses to 0.25% increase in world interest rate ($\rho_r = 1$).

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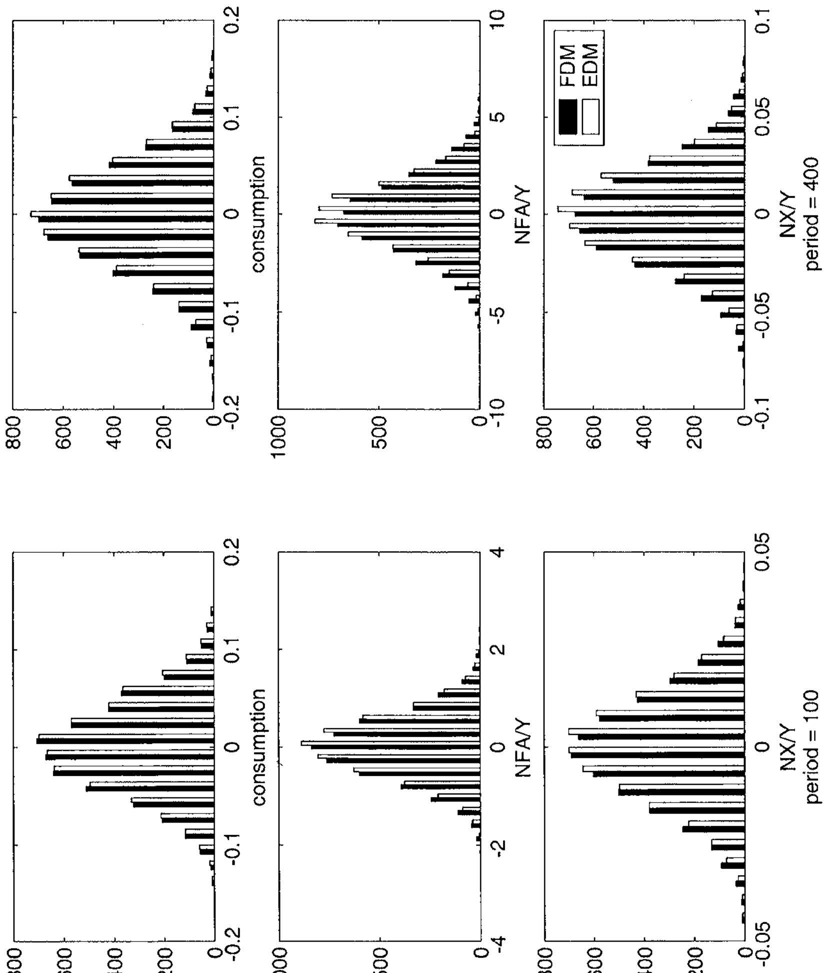

FIGURE 3. Distribution of variables ($\sigma_z = 0.615\%$ and $\sigma_r = 0.1\%$).

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TABLE 4. Sample statistics of empirical distributions[a]

Variable	Statistic	FDM	EDM
100th Period			
Consumption	Mean	0.00029	0.00027
(K-S statistic =	Standard dev.	0.0459	0.0464
0.0062)	Skewness	0.0360	0.0349
	Kurtosis	2.893	2.889
nfa/*y*	Mean	0.0113	0.0103
(K-S statistic =	Standard dev.	0.648	0.603
0.0214)	Skewness	−0.0107	−0.0123
	Kurtosis	3.0649	3.0652
nx/*y*	Mean	−0.00049	−0.00047
(K-S statistic =	Standard dev.	0.0138	0.0132
0.0142)	Skewness	−0.052	−0.050
	Kurtosis	2.955	2.961
400th Period			
Consumption	Mean	−0.001022	−0.001015
(K-S statistic =	Standard dev.	0.052	0.051
0.0116)	Skewness	−0.026	−0.022
	Kurtosis	2.981	2.987
nfa/*y*	Mean	−0.028	−0.025
(K-S statistic =	Standard dev.	1.85	1.55
0.0454)*	Skewness	−0.023	−0.025
	Kurtosis	2.900	2.904
nx/*y*	Mean	0.000184	0.000179
(K-S statistic =	Standard dev.	0.025	0.022
0.0306)	Skewness	0.0259	0.0265
	Kurtosis	2.94	2.95

[a]The distributions are drawn from 5,000 simulations with both productivity and interest-rate shocks, where each simulation consists of 400 periods. K-S statistic denotes the Kolmogorov–Smirnov statistic for testing the difference between two cumulative density functions. If the test statistic is greater than the critical value, then the two distributions are significantly different (noted with *). For all cases, 1% critical value is 0.0326. See Section 3.4 for details.

the FDM and the EDM, FDM (approximate) and EDM (approximate). That is, the exact solutions of these two models, FDM (exact) and EDM (exact), may not produce similar dynamics. This is an important issue, considering that we linearize the model around an initial steady state to which the economy either converges very slowly (in the EDM) or does not converge at all (in the FDM) in response to a shock.[23] This section focuses on the accuracy of the linearized solutions and examines whether the associated approximation errors are large enough to affect our previous results. We also compare the size of approximation errors from the two models. If the approximation errors are small, then we can claim that the results reported in the previous sections are valid for the FDM (exact) and EDM (exact) as well.

Approximation errors are calculated as follows: First, using the benchmark parameterization, we simulate the linearized model and compute the value of each variable at a certain period (every 100th period up to the 400th period). Then, we plug these values into the original budget constraint and calculate the residual—the difference between the RHS and LHS of the budget constraint,

$$c_t + i_t + b_t = y_t + b_{t-1}(1 + r_{t-1}). \qquad \textbf{(19)}$$

Approximation error is defined as the ratio of the absolute value of this residual to output.[24] Approximation errors arise from neglecting higher-order terms in the linearization of the first-order conditions and the budget constraint. There is an additional source of approximation error in the FDM since the net foreign asset series follow a unit root process.

Table 5 reports the mean of approximation errors calculated at every 100th period up to the 400th period over 5,000 simulations.[25] The first panel reports the results with productivity shocks only, while the second panel presents the results with both shocks. The table shows that the FDM produces larger approximation errors than the EDM in all cases, although differences are quite small.[26] This result is compatible with the observation that, in the FDM, the nonstationarity drives the variables away from the initial steady state. This also explains why the approximation errors increase as the length of simulation increases.

TABLE 5. Mean of approximation errors[a] (percentage of output)

	Period			
Point of estimation	100	200	300	400
	Productivity shocks			
FDM ($\times 10^{-14}$)	0.0464	0.0836	0.1114	0.1328
EDM ($\times 10^{-14}$)	0.0399	0.0692	0.0881	0.1002
	Productivity and interest rate shocks			
FDM	0.0006	0.0011	0.0014	0.0017
EDM	0.0005	0.0010	0.0012	0.0014

[a] Approximation errors are calculated as follows: First, using the benchmark parameterization, we simulate the linearized model and compute the value of each variable at certain periods (every 100th period up to the 400th period). Then, we plug these values into the original budget constraint and calculate the residual—the difference between the RHS and LHS of the budget constraint. The approximation error is defined as the ratio of the absolute value of this residual to output. The mean of approximation errors is the average over 5,000 simulations. See Section 3.5 for details.

Another finding is that the absolute magnitude of approximation errors is negligible, less than a $10^{-13}\%$ of output, when the models are simulated with productivity shocks. When both productivity and interest-rate shocks present, the size of approximation errors increases up to 0.17% of output. These findings suggest that, even though the approximation errors in the FDM are larger than those in the EDM, the size of approximation errors is quite small if the models are subjected to productivity shocks only.

We also examine the business-cycle dynamics of fixed and endogenous discount-factor formulations in models with complete markets. In particular, we solve a closed-economy RBC model and a two-country RBC model with complete markets. The results suggest that the models with fixed and endogenous discount factors produce similar business-cycle statistics and impulse responses.[27]

4. CONCLUSION

We examine the dynamic implications of different preference formulations in an open-economy RBC model with incomplete asset markets. In particular, we consider two versions of an otherwise identical small open-economy RBC model—one with a fixed discount factor and the other with an endogenous discount factor.

Our empirical examination reveals five important results. First, while the net foreign asset series in the FDM follows a unit root process, this series exhibits a near unit root behavior in the EDM. Hence, the coefficients in the policy functions generated by the two models exhibit minor differences. Second, the EDM and the FDM generate almost identical business-cycle moments. Third, while there are small quantitative differences between the impulse responses produced by the two models, their qualitative implications are the same. Fourth, the distributions of variables generated by simulations of the two models are not statistically different. Finally, the approximation errors from the solutions of the two models are of the same magnitude and the errors are minimal, especially with productivity shocks.

Understanding the statistical properties of models with nonstationary variables, such as the model with the fixed discount factor presented here, is an important research topic considering that these types of models have been widely used in the dynamic macroeconomics literature. This paper emphasizes the quantitative implications of the fixed discount factor formulation by comparing these implications with those of the endogenous discount factor model. The results suggest that the FDM does not generate any results that are significantly different from those produced by the EDM. More importantly, business-cycle dynamics produced by the two models are almost identical. The endogenous discount factor model exhibits some significantly important undesirable properties, such as the impatience effect and implausible dynamics generated by permanent productivity shocks. In sum, our results rationalize the use of the fixed discount factor in the studies aiming to understand business-cycle dynamics in open economies.

It might be interesting to examine the effects of preference structure on some other aspects such as welfare issues and policy implications. It is possible that the welfare implications or the long-term impact of particular policies may depend on the preference formulation in open economy models with incomplete asset markets.

NOTES

1. Correia et al. (1992, 1995), Baxter (1995), Baxter and Crucini (1995), Rebelo and Vegh (1995), Arvanitis and Mikkola (1996), Kollmann (1996, 1998, 2001), Crucini (1999), Sadka and Yi (1996), van Wincoop (1996), Kouparitsas (1997), van Wincoop and Marrinan (1999), Blankenau et al. (2001), and Kose and Riezman (2001) employ the standard time-separable preferences with a fixed discount factor in their models. Mendoza (1991, 1995), Karayalcin (1995), Uribe (1997), Schmitt-Grohe (1998), and Cook and Devereux (2000) use the Uzawa–Epstein-type time-nonseparable preferences with an endogenous discount factor.

2. As we discuss in detail in Section 2, there are some methods to deal with these problems in open-economy models under incomplete markets.

3. Daniel (1997) notes that it is "intuitively unappealing" to have wealthier agents who are more impatient than less wealthy agents in this utility formulation. Senhadji (1995) criticizes the use of the endogenous discount factor because the representative agent must save enough to reach a saving target in order to attain a fixed utility level determined by the steady state. Correia et al. (1995), after noting that consumption, the net exports, and the net foreign assets are nonstationary in the data, claim that the stationarity of these variables induced by the nonseparable preference formulation is not "necessarily a desirable property" (p. 1100).

4. To be more specific, our ultimate objective is to argue for the similarity between the exact solutions of these two models based on the similarity between their approximate solutions. Our findings suggest that the approximation errors are quite small, implying that the results from these approximate solutions may apply to the exact solutions of these models.

5. We consider the case in which the time preference is equal to the world interest rate. Otherwise, no deterministic steady state exists. If the world interest rate is greater (smaller) than the time-preference rate, agents permanently accumulate (deplete) foreign assets. These issues were first discussed by Helpman and Razin (1982). Mendoza and Tesar (1998) provide a detailed discussion of the steady-state issues in a deterministic model with incomplete asset markets.

6. The stochastic steady state of a variable is defined as its expected mean. The fact that our approximate solution features a unit root in asset holdings does not necessarily imply that the exact solution of the model also has a unit root. The stochastic steady state of the model might be well defined, but it is not possible to capture this steady state by the approximate solution method. It is not possible to provide an exact solution of the incomplete market models with the fixed discount factor formulation unless the state space of asset holdings is bounded.

7. Heathcote and Perri (in press) impose a quadratic adjustment cost on bond holdings to produce stationary asset series. Cardia (1991) uses the uncertain lifetime approach advanced by Blanchard (1985) in her small open-economy model. Harjes (1997) assumes that the world real interest rate depends on the net foreign assets, and Bruno and Portier (1995) assume that net foreign assets negatively affect the households' utility in their small open-economy models. Senhadji (1998) considers a setup with a downward-sloping export demand function. It is also possible to produce stationary equilibria in models with incomplete asset markets by introducing limits on the level of asset holdings [explicit bounds as in Huggett (1993) or implicit bounds as in Levine and Zame (1996, 1999) and Kubler and Schmedders (2000)], or by introducing endogenous solvency constraints [see Alvarez and Jermann (1999)], or enforcement constraints [see Kehoe and Perri (2000)].

8. See Backus et al. (1995) and Baxter (1995) for a survey of open-economy RBC models and their use in studying the sources and transmission of international business cycles.

9. This utility function is introduced by Greenwood et al. (1988) and is widely used in the open-economy RBC models. Correia et al. (1995) and Crucini (1999) compare the dynamic implications of

this utility function with those of the Cobb–Douglas utility function, both with a fixed discount factor. We do not adopt the Cobb–Douglas preference structure because, as shown by Correia et al. (1995), a small open-economy model with the Cobb–Douglas utility function is unable to match the volatility of consumption and the countercyclical behavior of net exports in real data.

10. We do not consider other shocks such as government spending shocks, or net foreign transfers shocks because Correia et al. (1995) convincingly argue that these shocks are not able to generate significant business-cycle dynamics in a small open-economy model.

11. Detailed derivation of the steady state and the linearized first-order conditions of the model are available upon request.

12. This preference formulation was first introduced by Uzawa (1968) and further developed by Epstein (1983, 1987). See Obstfeld (1990) for a theoretical analysis of dynamics of this type of small open-economy model.

13. Epstein (1983) shows that under certain conditions this preference formulation generates "a unique invariant limiting distribution of the state variables." Our parameterization also meets those conditions.

14. The average value of nx/y is near zero (less than 0.3% per quarter) for Canada.

15. This range is wide enough to cover most realistic cases. The observed trade balance rarely exceeds $\pm 5\%$ of output per quarter for the OECD countries.

16. The value of θ used in the literature with the EDM ranges from 0.001 to 0.1, depending on the model specification.

17. Since we set the standard deviations of shocks to match the volatility of output, the models are able to replicate the output volatility. The models slightly understate the volatilities of consumption, labor supply, and the net exports/output ratio. The models exaggerate the correlations of consumption, labor supply, and the net exports with output. Although the persistence of the net exports/output ratio is lower in the models than in the data, the models slightly underpredict the persistence of other variables. However, all of these differences are marginal.

18. We experiment with other values of nx/y and find that impulse responses exhibit similar dynamics.

19. We limit our analysis to 400 periods (100 years) because the time span used in most RBC papers is shorter than 100 years.

20. We also study the endowment economy version of the FDM that shows slightly different impulse response dynamics. With a permanent shock, the FDM behaves like an autarky economy: The net foreign assets do not change and consumption is always equal to output. The net foreign assets in the EDM exhibit the same behavior as in the production economy case. Detailed results of the endowment economy model are available upon request.

21. These simulations are performed with the benchmark parameterization. Since we are interested in levels, we do not filter the series. The distributions of output, labor hours, investment, and capital stock produced by the two models, which are not reported in the figure, are identical as predicted by the policy functions.

22. See Serfling (1980) and Spanos (1986) for theoretical background on this test statistic.

23. Correia et al. (1995) also observe this problem and note that "the accuracy of linear approximations in models with integrated variables, such as ours, is still an open question" (p. 1095). The results we report here also shed light on this issue.

24. Our method is similar to the one of Baxter (1991) who uses the Euler equations to calculate the approximation errors. In particular, she defines the approximation error as the difference between the left- and right-hand sides of the Euler equation, where she uses the approximate solution to calculate the values of the variables.

25. We are unable to use the Den Haan–Marcet (1994) statistic to evaluate the accuracy of approximation since this method works only with stationary variables.

26. Using an open-economy RBC model, Kim and Kim (in press) show that while the linear approximation is quite accurate in measuring the second moments, it produces inaccurate results in the first moments such as welfare.

27. We would like to thank the Associate Editor for suggesting this exercise. Detailed examination of the two models and the results of the exercise are available upon request.

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