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Electron beam acceleration and potential formation induced by the Compton scattering of extraordinary waves

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Abstract. High-energy or relativistic electron beam acceleration along and across a magnetic field, and the generation of an electric field transverse to the magnetic field, both induced by the Compton scattering of almost perpendicularly propagating extraordinary waves, are investigated theoretically based on kinetic wave equations and transport equations. Compton scattering occurs via nonlinear Landau damping of two extraordinary waves interacting nonlinearly with the electron beam, satisfying the resonance condition of $\omega_{\mathbf{k}}-\omega_{\mathbf{k}'}-(k_\perp-k'_\perp)\nu_{\rm d}-(k_\parallel-k'_\parallel)\nu_{\rm b}=m\omega_{\rm ce}$ $(m=0,\pm1)$, where $\nu_{\rm b}$ and $\nu_{\rm d}$ are the parallel and perpendicular velocities of the electron beam, respectively. The transport equations can be derived from the single-particle theory and also from Vlasov–Maxwell equations. The transport equations show that two extraordinary waves accelerate the electron beam in the $\mathbf{k}''$ direction $(\mathbf{k}''=\mathbf{k}-\mathbf{k}')$. Simultaneously, an intense cross-field electric field $\mathbf{E}_0=\mathbf{B}_0\times\mathbf{v}_{\rm d}/c$ is generated via the dynamo effect owing to the perpendicular drift of the electron beam to satisfy the generalized Ohm's law, which means that this cross-field electron drift is identical to the $\mathbf{E}\times\mathbf{B}$ drift. The single-particle theory is very useful for an easy and straightforward understanding of the physical mechanism of the electron beam acceleration and the generation of cross-field electric field, although the rigorously exact transport equations are derived from Vlasov–Maxwell equations.

1. Introduction

Nonlinear wave–particle scattering induced by nonlinear Landau and cyclotron damping of electromagnetic and electrostatic waves in an unmagnetized or magnetized plasma has attracted a great deal of attention in theoretical [1–11] and experimental [12–16] research of plasma physics over recent years. This nonlinear phenomenon is one of the fundamental mechanisms for anomalous resistivity, anomalous transport, plasma heating and acceleration, and plasma current drive in fusion plasmas, as well as space plasmas. Recently, plasma heating due to nonlinear Landau or cyclotron damping of electrostatic waves has been investigated theoretically [3, 4] and experimentally [12–16]. In particular, it has been shown by the present author and co-workers experimentally and theoretically that a strong radial electric field and a radial transport of heated plasma electrons are generated by electrostatic waves excited by nonlinear Landau damping in an

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electron-beam–plasma system, and they can maintain the large and abrupt radial inhomogeneity of electron temperature and density [15–17]. The author has developed a general theory for the transport equations for the nonlinear Landau and cyclotron damping of electromagnetic and electrostatic waves in magnetized plasmas [4, 7]. They are derived from the θ-dependent velocity-space diffusion equation obtained by means of the perturbation theory of Vlasov–Maxwell equations. Based on these works, it has been predicted that high-energy or relativistic electrons can be accelerated via the nonlinear Landau and cyclotron damping of electromagnetic and electrostatic waves in unmagnetized [5, 8, 9] and magnetized [6, 10, 11] plasmas. On the other hand, it was verified theoretically, based on the θ-dependent quasilinear velocity-space diffusion equation, that the plasma acceleration and transport along and across the magnetic field can be induced via the Landau and cyclotron damping of almost perpendicularly propagating electrostatic and electromagnetic waves, and the electric field transverse to the magnetic field can be generated simultaneously by the dynamo effect of the perpendicular particle drift to satisfy the generalized Ohm's law [17–21]. Moreover, the present author has also clarified (theoretically on the basis of the θ-dependent relativistic quasilinear momentum-space diffusion equation) that the relativistic and non-relativistic particle acceleration along and across a magnetic field can be generated by almost perpendicularly propagating electrostatic waves in a relativistic magnetized plasma, and the cross-field electric field is created simultaneously via the perpendicular particle acceleration [19, 20]. This quasilinear process explains the fluctuation-induced anomalous plasma transport and the potential formation in fusion plasmas and an electron-beam–plasma system [15–17] well, as well as explaining the perpendicular ion acceleration and the electric field observed in tokamaks and space plasmas. In particular, the generation of the cross-field electric field is exceptionally important in connection with the stability of the magnetically confined fusion plasma, since it can be considered that this process causes the rapid change of the plasma profile or the collapse of the plasma.

It has been shown that the particle acceleration and transport induced by nonlinear wave–particle scattering or non-resonantly produced quasi-modes in a fusion plasma are potentially important [22–25]. In addition, it has been clarified that the generation of electromagnetic radiation in the space plasma can be explained based on the plasma–maser interaction among the electrostatic lower-hybrid turbulence, accelerated electrons and extraordinary mode radiation, and it has been pointed out that the electrons are strongly accelerated by lower-hybrid turbulence [26–28]. As previously predicted [6], the numerical analysis of the nonlinear wave–particle coupling coefficients has demonstrated that the strong nonlinear scattering and absorption of the extraordinary wave can be caused by nonlinear electron cyclotron damping due to the interaction with high-energy electrons that satisfy the resonance condition of $\omega_{\mathbf{k}} - \omega_{\mathbf{k}'} - (k_\parallel - k'_\parallel)\nu_\parallel = m\omega_{\mathrm{ce}}$ $(m = 1)$ and result in the efficient acceleration of high-energy or relativistic electrons. Here, ω_{ce} is the electron cyclotron frequency. Consequently, the nonlinear scattering of the extraordinary wave may provide an effective means for an acceleration of high-energy or relativistic electrons [29–35].

Moreover, the present author has verified (by numerical analysis of the nonlinear wave–particle coupling coefficients) that a high-energy or relativistic electron beam can be accelerated efficiently along the magnetic field via Compton scattering induced by the nonlinear electron Landau damping of almost perpendicularly

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propagating extraordinary waves [10, 11]. In this nonlinear scattering, two extraordinary waves slightly separated in frequency interact nonlinearly with an electron beam, satisfying the resonance condition of $\omega_{\mathbf{k}} - \omega_{\mathbf{k}'} - (k_\parallel - k'_\parallel)\nu_{\rm b} = 0$ $(m = 0)$, and the electron beam with velocity $\nu_{\rm b}$ can be accelerated or decelerated efficiently to the phase velocity of the beat-wave $\omega_{\mathbf{k}''}/k''_\parallel$ near the speed of light, when $\omega_{\mathbf{k}''}/k''_\parallel$ is slightly larger or smaller than $\nu_{\rm b}$, respectively. Here, $\omega_{\mathbf{k}''} = \omega_{\mathbf{k}} - \omega_{\mathbf{k}'}$ and $k''_\parallel = k_\parallel - k'_\parallel$. For $\omega_{\mathbf{k}}, \omega_{\mathbf{k}'} \lesssim \omega_{\rm h}$ (where $\omega_{\rm h}$ is the upper-hybrid frequency), the efficient acceleration or deceleration occurs via the parallel pondermotive force due to the wave electric field. For $\omega_{\mathbf{k}}, \omega_{\mathbf{k}'} \gtrsim \omega_{\rm R}$ (where $\omega_{\rm R}$ is the right-hand cutoff frequency), the efficient acceleration or deceleration occurs via the nonlinear $\mathbf{v} \times \mathbf{B}$ Lorentz force. For a sufficiently higher frequency $(\omega_{\mathbf{k}}, \omega_{\mathbf{k}'} \gg \omega_{\rm R})$, the rate of acceleration and deceleration decreases enormously and approaches that in a vacuum.

On the other hand, theoretical [36–39] and experimental [40–42] studies of the stimulated Compton scattering of electromagnetic waves in an unmagnetized plasma have been reported by many researchers in recent years, where it was demonstrated that the rapid and strong acceleration and heating of electrons result from this nonlinear scattering. Recently, theoretical and experimental investigations of the multiphoton Compton scattering of an electromagnetic field in which a laser pulse is scattered from ultrarelativistic electron beams (Stanford Linear Accelerator Center (SLAC) beam) have been performed. Hartemann and Kerman [43] have presented the classical theory of multiphoton Compton scattering in which an ultrashort laser pulse in vacuum is scattered from a 50 GeV SLAC beam. Bula *et al.* [44] have reported experimental observations of multiphoton Compton scattering in which terawatt pulses from a Nd:glass laser are scattered from a 46.6 GeV SLAC beam. However, to the best of our knowledge, the only theoretical study concerning the Compton scattering of electromagnetic waves by a high-energy electron beam in a magnetized plasma has been presented by the present author and co-workers [10, 11]. Accordingly, a further detailed theoretical investigation of Compton scattering in a magnetized plasma has been developed.

In the present work, the acceleration and heating of a high-energy or relativistic electron beam due to the Compton scattering induced by the nonlinear electron Landau and cyclotron damping of almost perpendicularly propagating extraordinary waves are investigated theoretically by developing the previous work [10,11]. It was clarified on the basis of single-particle theory [45] that the electron beam can be accelerated not only along the magnetic field but also across the magnetic field, and an electric field transverse to the magnetic field is generated simultaneously. In this nonlinear scattering, two extraordinary waves interact nonlinearly with the electron beam, satisfying the resonance condition of $\omega_{\mathbf{k}} - \omega_{\mathbf{k}'} - (k_\perp - k'_\perp)\nu_{\rm d} - (k_\parallel - k'_\parallel)\nu_{\rm b} = m\omega_{\rm ce}$ $(m = 0, \pm 1)$, and efficiently accelerate or decelerate the electron beam in the $\mathbf{k}''$ direction. Here, $\nu_{\rm b}$ and $\nu_{\rm d}$ are the parallel and perpendicular velocities of the electron beam, respectively, $\mathbf{k} = (k_\perp, 0, k_\parallel)$, $\mathbf{k}' = (k'_\perp, 0, k'_\parallel)$ and $\mathbf{k}'' = \mathbf{k} - \mathbf{k}' = (k''_\perp, 0, k''_\parallel)$. The transport equations can be derived from single-particle theory and also from the velocity-space diffusion equation obtained on the basis of Vlasov–Maxwell equations. They show that the two extraordinary waves can accelerate and decelerate the electron beam in the $\mathbf{k}''$ direction. Simultaneously, an intense cross-field electric field $\mathbf{E}_0 = \mathbf{B}_0 \times \mathbf{v}_{\rm d}/c$ is created via the dynamo effect owing to the cross-field electron drift to satisfy the generalized Ohm's law [17–21,46,47]. This means that the cross-field electron drift is identical to the $\mathbf{E} \times \mathbf{B}$ drift. Single-particle theory is considerably useful for the easy and straightforward understanding of

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the physical mechanisms of particle acceleration and the generation of the cross-field electric field [11,21,22,45–47]. The results obtained are compared with the rigorously exact transport equations derived from Vlasov–Maxwell equations, and they are found to be in approximate and satisfactory agreement. As was previously shown, for the nonlinear scattering of $m=0$ the efficient acceleration or deceleration in the $\mathbf{k}''$ direction can occur when the Doppler-shifted phase velocity of the beat-wave $(\omega_{\mathbf{k}''} - k''_\perp \nu_\mathrm{d})/k''_\parallel$ is slightly larger or smaller than ν_b, respectively. For the nonlinear scattering of $m=\pm 1$, the efficient acceleration in the $\mathbf{k}''$ direction can always occur for $\omega_{\mathbf{k}''}/k''_\parallel$ in the close neighborhood of the value satisfying the resonance condition. For both the scattering processes of $m=0,\pm 1$, the efficient acceleration or deceleration can arise from the pondermotive force due to the wave electric field for $\omega_\mathbf{k}, \omega_{\mathbf{k}'} \lesssim \omega_\mathrm{h}$ and from the nonlinear $\mathbf{v}\times\mathbf{B}$ Lorenz force for $\omega_\mathbf{k}, \omega_{\mathbf{k}'} \gtrsim \omega_\mathrm{R}$, as was also previously shown.

In a magnetized plasma immersed in an electric field, the equation called the generalized Ohm's law, $\mathbf{E}_0 + \mathbf{v}_\mathrm{d} \times \mathbf{B}_0/c = \eta \mathbf{J}$, is satisfied. Here, $\mathbf{J}$ is the electric current density and η is the electrical resistivity. This equation usually shows that the mean ion velocity is determined by $\mathbf{E}_0$, $\mathbf{B}_0$ and $\eta\mathbf{J}$. Conversely, we can explain that the electric field $\mathbf{E}_0$ is enhanced or suppressed when a mechanism of acceleration or deceleration of plasma particles exists. Namely, this equation also expresses the electric field $\mathbf{E}_0$ resulting from the dynamo effect of the cross-field particle drift $\mathbf{v}_\mathrm{d}$. Accordingly, it is clarified that the cross-field electric field $\mathbf{E}_0$ can be generated by the particle acceleration due to Compton scattering of extraordinary waves. In the present work, this nonlinear phenomenon is investigated for a collisionless plasma with $\eta=0$. On the other hand, in single-particle theory the initial value problem should be adopted rigorously, as Dawson has investigated linear Landau damping of longitudinal plasma waves based on such a single-particle method [45]. However, it is extremely difficult to develop the single-particle theory employing the initial value problem in this work, because the kinetic equation is the fourth-order nonlinear equation including two electromagnetic waves in a magnetized plasma. Consequently, in the present work the nonlinear kinetic equation is treated without consideration of the initial value problem, since it is regarded as the most important in clarifying the physical mechanism of the cross-field particle acceleration.

In the next section, single-particle theory for the Compton scattering of extraordinary waves is described. In Sec. 3, the general and exact transport equations derived from Vlasov–Maxwell equations are presented. Finally, a summary of results is given in Sec. 4.

2. Single-particle theory

2.1. *Resonance condition*

In this nonlinear wave–particle scattering, the following well-known resonance condition is satisfied:

$$\omega_\mathbf{k} - \omega_{\mathbf{k}'} - (k_\perp - k'_\perp)\nu_\mathrm{d} - (k_\parallel - k'_\parallel)\nu_\parallel = m\omega_\mathrm{ce}, \tag{1}$$

which means that two extraordinary waves $(\omega_\mathbf{k}, \mathbf{k})$ and $(\omega_{\mathbf{k}'}, \mathbf{k}')$ scatter from the beam electrons whose parallel and perpendicular velocities are $\nu_\parallel$ and ν_d, respectively, where m is an integer. In the present work, the cases of $m=0,\pm 1$ are investigated.

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2.2. *Dispersion relation and polarizations*

The dispersion relation and polarizations of wave electric and magnetic fields for the extraordinary waves are determined by

$$(\boldsymbol{\varepsilon}_{\mathbf{k}} - \mathbf{N}_{\mathbf{k}}) \cdot \mathbf{E}_{\mathbf{k}} = 0, \tag{2}$$

where $\boldsymbol{\varepsilon}_{\mathbf{k}}$ is the dielectric tensor and $N_{\mathbf{k}}^{jl} = (c^2/\omega_{\mathbf{k}}^2)(k^2\delta_{jl} - k_j k_l)$ $(j, l = x, y, z)$. Then the polarizations for the extraordinary wave in a cold plasma are given as follows:

$$\begin{gathered} \frac{E_{\mathbf{k}}^y}{E_{\mathbf{k}}^x} \simeq i\frac{\omega_{\mathbf{k}}\left(\omega_{\mathbf{k}}^2 - \omega_{\mathrm{h}}^2\right)}{\omega_{\mathrm{pe}}^2\omega_{\mathrm{ce}}}, \quad \frac{E_{\mathbf{k}}^z}{E_{\mathbf{k}}^x} \simeq \frac{k_\parallel}{k_\perp}, \\ B_{\mathbf{k}}^x = -\frac{ck_\parallel}{\omega_{\mathbf{k}}}E_{\mathbf{k}}^y, \quad B_{\mathbf{k}}^y \cong 0, \quad B_{\mathbf{k}}^z = \frac{ck_\perp}{\omega_{\mathbf{k}}}E_{\mathbf{k}}^y. \end{gathered} \tag{3}$$

Here, $\omega_{\mathrm{pe}} = (4\pi n_{\mathrm{e}}e^2/m_{\mathrm{e}})^{1/2}$ is the electron plasma frequency, $\omega_{\mathrm{ce}} = eB_0/m_{\mathrm{e}}c$ and $\omega_{\mathrm{h}} = (\omega_{\mathrm{pe}}^2 + \omega_{\mathrm{ce}}^2)^{1/2}$ is the upper-hybrid frequency.

2.3. *Kinetic equation for an electron beam*

In order to derive the transport equations, the single-particle theory for the Compton scattering of extraordinary waves, which is convenient for understanding the detailed physical mechanisms of particle acceleration and the generation of the cross-field electric field, was developed. The previous single-particle theory, in which the perpendicular drift of the electron beam and the cross-field electric field are not taken into account, was described in the Appendix of [11]. The kinetic equation for an electron beam with velocity $\mathbf{v}$, which includes the cross-field electron drift velocity $\mathbf{v}_{\mathrm{d}}$ and the cross-field electric field $\mathbf{E}_0$, is given by

$$m_{\mathrm{e}}\frac{d\mathbf{v}}{dt} = -e(\mathbf{E}_0 + \mathbf{E}_1) - \frac{e}{c}\mathbf{v} \times (\mathbf{B}_0 + \mathbf{B}_1). \tag{4}$$

In addition, it is assumed that the following generalized Ohm's law for a uniform collisionless plasma is satisfied simultaneously:

$$\mathbf{E}_0 + \frac{1}{c}\mathbf{v}_{\mathrm{d}} \times \mathbf{B}_0 = 0. \tag{5}$$

Here,

$$\mathbf{r} = \mathbf{r}_0 + \mathbf{v}_{\mathrm{d}}t + \mathbf{v}_\parallel t + \tfrac{1}{2}\left(\mathbf{r}_{\mathbf{k}} + \mathbf{r}_{\mathbf{k}}^* + \mathbf{r}_{\mathbf{k}'} + \mathbf{r}_{\mathbf{k}'}^* + \mathbf{r}_{\mathbf{k}''}^{(2)} + \mathbf{r}_{\mathbf{k}''}^{(2)*}\right), \tag{6}$$

$$\mathbf{v} = \mathbf{v}_{\mathrm{d}} + \mathbf{v}_\parallel + \tfrac{1}{2}\left(\mathbf{v}_{\mathbf{k}} + \mathbf{v}_{\mathbf{k}}^* + \mathbf{v}_{\mathbf{k}'} + \mathbf{v}_{\mathbf{k}'}^* + \mathbf{v}_{\mathbf{k}''}^{(2)} + \mathbf{v}_{\mathbf{k}''}^{(2)*}\right), \tag{7}$$

$$\mathbf{E}_1 = \tfrac{1}{2}(\mathbf{E}_{\mathbf{k}} + \mathbf{E}_{\mathbf{k}}^* + \mathbf{E}_{\mathbf{k}'} + \mathbf{E}_{\mathbf{k}'}^*), \tag{8}$$

$$\mathbf{B}_1 = \tfrac{1}{2}(\mathbf{B}_{\mathbf{k}} + \mathbf{B}_{\mathbf{k}}^* + \mathbf{B}_{\mathbf{k}'} + \mathbf{B}_{\mathbf{k}'}^*), \tag{9}$$

$$\mathbf{k} \times \mathbf{E}_{\mathbf{k}} = \frac{\omega_{\mathbf{k}}}{c}\mathbf{B}_{\mathbf{k}}, \tag{10}$$

where $\mathbf{v}_{\mathrm{d}} = c\mathbf{E}_0 \times \mathbf{B}_0/B_0^2 = (\nu_{\mathrm{d}}, 0, 0)$, $\mathbf{v}_\parallel = (0, 0, \nu_\parallel)$, and $E_0 = (\nu_{\mathrm{d}}/c)B_0$. The Larmor radius of an electron beam is assumed to be zero; that is, a cold electron beam is considered. The background uniform stationary electric and magnetic fields $\mathbf{E}_0 = (0, E_0, 0)$ and $\mathbf{B}_0 = (0, 0, B_0)$ are in the y and z directions, respectively. $\mathbf{E}_{\mathbf{k}} = \mathbf{E}_{\mathbf{k}}^{(1)}$ and $\mathbf{B}_{\mathbf{k}} = \mathbf{B}_{\mathbf{k}}^{(1)}$ are the first-order electric and magnetic fields of extraordinary waves,

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respectively, and $\mathbf{r}_\mathbf{k} = \mathbf{r}_\mathbf{k}^{(1)} + \mathbf{r}_\mathbf{k}^{(3)}$ and $\mathbf{v}_\mathbf{k} = \mathbf{v}_\mathbf{k}^{(1)} + \mathbf{v}_\mathbf{k}^{(3)}$ are the first- and third-order oscillating terms of an electron beam. They obey $\mathbf{r}_\mathbf{k}, \mathbf{v}_\mathbf{k}, \mathbf{E}_\mathbf{k}, \mathbf{B}_\mathbf{k} \propto \exp[i(\mathbf{k}\cdot\boldsymbol{r} - \omega_\mathbf{k} t)]$ and the beat-wave is assumed to be absent; that is, $\mathbf{E}_{\mathbf{k}''}^{(2)} = \mathbf{B}_{\mathbf{k}''}^{(2)} = 0$. The oscillating terms $\mathbf{r}_\mathbf{k}, \mathbf{v}_\mathbf{k}$ are not assumed to satisfy the initial condition of $\mathbf{r}_\mathbf{k} = \mathbf{v}_\mathbf{k} = 0$ at $t = 0$, so the problem can be solved directly, although we cannot obtain the rigorous temporal evolution of the fourth-order velocity of the electron beam in which the trapping effect is taken into account [45]. Here, * means the complex conjugate. It should be noted that $\mathbf{v}_\mathrm{d}$ is identical to the $\mathbf{E} \times \mathbf{B}$ drift velocity. The other notation are standard.

Equation (5) means that the drift velocity of the electron beam is determined by $\mathbf{E}_0$ and $\mathbf{B}_0$. Simultaneously, it can be also stated that the cross-field electric field $\mathbf{E}_0$ is enhanced or suppressed when the mechanism of acceleration and deceleration of the electron beam exists. Namely, (5) shows $\mathbf{E}_0$ caused by the dynamo effect of $\mathbf{v}_\mathrm{d}$. Accordingly we find that the cross-field electric field $\mathbf{E}_0$ can be generated by the cross-field electron beam acceleration due to the Compton scattering of extraordinary waves [17–21,46,47]. As was interpreted in detail previously [19], the dynamo effect results from the electrical charge separation caused by the Lorentz force $-e\mathbf{v}_\mathrm{d} \times \mathbf{B}_0/c$ so that the electric field created $\mathbf{E}_0 = \mathbf{B}_0 \times \mathbf{v}_\mathrm{d}/c$ is balanced with the Lorentz force.

The kinetic equation of the first order becomes

$$m_\mathrm{e}\frac{d\mathbf{v}_\mathbf{k}^{(1)}}{dt} = -e\mathbf{E}_\mathbf{k} - \frac{e}{c}\mathbf{v}_\mathbf{k}^{(1)} \times \mathbf{B}_0 - \frac{e}{c}(\mathbf{v}_\mathrm{d} + \mathbf{v}_\parallel) \times \mathbf{B}_\mathbf{k}. \tag{11}$$

Next, the second-order kinetic equation is given by

$$m_\mathrm{e}\frac{d\mathbf{v}_{\mathbf{k}''}^{(2)}}{dt} = -\frac{1}{2}e\big(\mathbf{r}_\mathbf{k}^{(1)}\cdot\nabla\big)\mathbf{E}_{\mathbf{k}'}^{*} - \frac{1}{2}e\big(\mathbf{r}_\mathbf{k}^{(1)*}\cdot\nabla\big)\mathbf{E}_\mathbf{k} - \frac{e}{c}\mathbf{v}_{\mathbf{k}''}^{(2)} \times \mathbf{B}_0 - \frac{e}{2c}\mathbf{v}_\mathbf{k}^{(1)} \times \mathbf{B}_{\mathbf{k}'}^{*}$$
$$- \frac{e}{2c}\mathbf{v}_{\mathbf{k}'}^{(1)*} \times \mathbf{B}_\mathbf{k} - \frac{e}{2c}(\mathbf{v}_\mathrm{d} + \mathbf{v}_\parallel) \times \big[\big(\mathbf{r}_\mathbf{k}^{(1)}\cdot\nabla\big)\mathbf{B}_{\mathbf{k}'}^{*} + \big(\mathbf{r}_{\mathbf{k}'}^{(1)*}\cdot\nabla\big)\mathbf{B}_\mathbf{k}\big]. \tag{12}$$

These equations are equivalent to (A6) and (A7) of [11], where $\mathbf{v}_\parallel$ is replaced by $\mathbf{v}_\mathrm{d} + \mathbf{v}_\parallel$. Similarly, the third-order kinetic equation is given by (A8) of [11], where $\mathbf{v}_\parallel$ should also be replaced by $\mathbf{v}_\mathrm{d} + \mathbf{v}_\parallel$. In (A8), only the nonlinear singularity $\omega_{\mathbf{k}''} - k''_\perp \nu_\mathrm{d} - k''_\parallel \nu_\parallel = m\omega_\mathrm{ce}$ $(m = 0, \pm 1)$ should contribute to the acceleration and deceleration of the electron beam. Then we find that only the terms involving $\mathbf{r}_{\mathbf{k}''}^{(2)}$, $\mathbf{v}_{\mathbf{k}''}^{(2)}$ and $\mathbf{v}_\mathbf{k}^{(3)}$ in (A8) should be retained. Consequently, the third-order kinetic equation for Compton scattering is expressed as follows:

$$m_\mathrm{e}\frac{d\mathbf{v}_\mathbf{k}^{(3)}}{dt} = -\frac{1}{2}e\big(\mathbf{r}_{\mathbf{k}''}^{(2)}\cdot\nabla\big)\mathbf{E}_{\mathbf{k}'} - \frac{e}{c}\mathbf{v}_\mathbf{k}^{(3)} \times \mathbf{B}_0 - \frac{e}{2c}\mathbf{v}_{\mathbf{k}''}^{(2)} \times \mathbf{B}_{k'} - \frac{e}{2c}(\mathbf{v}_\mathrm{d} + \mathbf{v}_\parallel) \times \big(\mathbf{r}_{\mathbf{k}''}^{(2)}\cdot\nabla\big)\mathbf{B}_{\mathbf{k}'}. \tag{13}$$

Finally, the fourth-order kinetic equation showing the acceleration and deceleration of the electron beam due to Compton scattering is also given by (A9) of [11], where the replacements

$$\mathbf{v}_\parallel^{(4)} \to \mathbf{v}^{(4)} \quad \text{and} \quad -\frac{e}{c}\mathbf{v}_\mathrm{b}^{(4)} \times \mathbf{B}_0 \to -e\mathbf{E}_0^{(4)} - \frac{e}{c}\mathbf{v}_\mathrm{d}^{(4)} \times \mathbf{B}_0$$

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should be made. Similarly, the terms involving $\mathbf{r}_{\mathbf{k}}^{(3)}$, $\mathbf{v}_{\mathbf{k}}^{(3)}$ and $(\mathbf{r}_{\mathbf{k}''}^{(2)}\cdot\nabla)$ should be retained in (A9), and the fourth-order kinetic equation then becomes

$$\begin{aligned} m_{\mathrm{e}}\frac{d\mathbf{v}^{(4)}}{dt} &= -\frac{1}{4}e\big(\mathbf{r}_{\mathbf{k}}^{(3)}\cdot\nabla\big)\mathbf{E}_{\mathbf{k}}^{*} - \frac{1}{4}e\big(\mathbf{r}_{\mathbf{k}'}^{(3)}\cdot\nabla\big)\mathbf{E}_{\mathbf{k}'}^{*} - \frac{1}{8}e\big(\mathbf{r}_{\mathbf{k}}^{(1)}\cdot\nabla\big)\big(\mathbf{r}_{\mathbf{k}''}^{(2)*}\cdot\nabla\big)\mathbf{E}_{\mathbf{k}'}^{*} \\ &\quad - \frac{1}{8}e\big(\mathbf{r}_{\mathbf{k}'}^{(1)}\cdot\nabla\big)\big(\mathbf{r}_{\mathbf{k}''}^{(2)}\cdot\nabla\big)\mathbf{E}_{\mathbf{k}}^{*} - e\mathbf{E}_0^{(4)} - \frac{e}{c}\mathbf{v}_{\mathrm{d}}^{(4)} \\ &\quad \times \mathbf{B}_0 - \frac{e}{4c}\mathbf{v}_{\mathbf{k}}^{(3)}\times\mathbf{B}_{\mathbf{k}}^{*} - \frac{e}{4c}\mathbf{v}_{\mathbf{k}'}^{(3)}\times\mathbf{B}_{\mathbf{k}'}^{*} \\ &\quad - \frac{e}{8c}\mathbf{v}_{\mathbf{k}''}^{(2)}\times\big[\big(\mathbf{r}_{\mathbf{k}'}^{(1)}\cdot\nabla\big)\mathbf{B}_{\mathbf{k}}^{*} + \big(\mathbf{r}_{\mathbf{k}}^{(1)*}\cdot\nabla\big)\mathbf{B}_{\mathbf{k}'}\big] - \frac{e}{8c}\mathbf{v}_{\mathbf{k}}^{(1)}\times\big(\mathbf{r}_{\mathbf{k}''}^{(2)*}\cdot\nabla\big)\mathbf{B}_{\mathbf{k}'}^{*} \\ &\quad - \frac{e}{8c}\mathbf{v}_{\mathbf{k}'}^{(1)}\times\big(\mathbf{r}_{\mathbf{k}''}^{(2)}\cdot\nabla\big)\mathbf{B}_{\mathbf{k}}^{*}, \end{aligned} \tag{14}$$

where $\mathbf{v}^{(4)}=\mathbf{v}_{\mathrm{d}}^{(4)}+\mathbf{v}_{\parallel}^{(4)}$ and ∇ is only operated to $\mathbf{E}$ and $\mathbf{B}$.

From the above equation, we can obtain the following simple kinetic equations:

$$m_{\mathrm{e}}\frac{d\mathbf{v}}{dt} = e\mathbf{D}, \tag{15}$$

$$\mathbf{E}_0^{(4)} + \frac{1}{c}\mathbf{v}_{\mathrm{d}}^{(4)}\times\mathbf{B}_0 = 0, \tag{16}$$

$$\begin{aligned} \mathbf{D} = &-\frac{1}{4\tilde{\omega}_{\mathbf{k}}}\big(\mathbf{k}\cdot\mathbf{v}_{\mathbf{k}}^{(3)}\big)\mathbf{E}_{\mathbf{k}}^{*} - \frac{1}{4\tilde{\omega}_{\mathbf{k}'}}\big(\mathbf{k}'\cdot\mathbf{v}_{\mathbf{k}'}^{(3)}\big)\mathbf{E}_{\mathbf{k}'}^{*} + \frac{1}{8\tilde{\omega}_{\mathbf{k}}\tilde{\omega}_{\mathbf{k}''}^{*}}\big(\mathbf{k}'\cdot\mathbf{v}_{\mathbf{k}}^{(1)}\big)\big(\mathbf{k}'\cdot\mathbf{v}_{\mathbf{k}''}^{(2)*}\big)\mathbf{E}_{\mathbf{k}'}^{*} \\ &- \frac{1}{8\tilde{\omega}_{\mathbf{k}'}\tilde{\omega}_{\mathbf{k}''}}\big(\mathbf{k}\cdot\mathbf{v}_{\mathbf{k}'}^{(1)}\big)\big(\mathbf{k}\cdot\mathbf{v}_{\mathbf{k}''}^{(2)}\big)\mathbf{E}_{\mathbf{k}}^{*} - \frac{1}{4c}\mathbf{v}_{\mathbf{k}}^{(3)}\times\mathbf{B}_{\mathbf{k}}^{*} - \frac{1}{4c}\mathbf{v}_{\mathbf{k}'}^{(3)}\times\mathbf{B}_{\mathbf{k}'}^{*} \\ &- \frac{1}{8c}\mathbf{v}_{\mathbf{k}''}^{(2)}\times\left[\frac{1}{\tilde{\omega}_{\mathbf{k}'}}\big(\mathbf{k}\cdot\mathbf{v}_{\mathbf{k}'}^{(1)}\big)\mathbf{B}_{\mathbf{k}}^{*} + \frac{1}{\tilde{\omega}_{\mathbf{k}}}\big(\mathbf{k}'\cdot\mathbf{v}_{\mathbf{k}}^{(1)*}\big)\mathbf{B}_{\mathbf{k}'}\right] \\ &+ \frac{1}{8c\tilde{\omega}_{\mathbf{k}''}^{*}}\mathbf{v}_{\mathbf{k}}^{(1)}\times\big(\mathbf{k}'\cdot\mathbf{v}_{\mathbf{k}''}^{(2)*}\big)\mathbf{B}_{\mathbf{k}'}^{*} - \frac{1}{8c\tilde{\omega}_{\mathbf{k}''}}\mathbf{v}_{\mathbf{k}'}^{(1)}\times\big(\mathbf{k}\cdot\mathbf{v}_{\mathbf{k}''}^{(2)}\big)\mathbf{B}_{\mathbf{k}}^{*}, \end{aligned} \tag{17}$$

where $\tilde{\omega}_{\mathbf{k}}=\omega_{\mathbf{k}}-k_{\perp}\nu_{\mathrm{d}}-k_{\parallel}\nu_{\parallel}$, $\tilde{\omega}_{\mathbf{k}'}=\omega_{\mathbf{k}'}-k'_{\perp}\nu_{\mathrm{d}}-k'_{\parallel}\nu_{\parallel}$ and $\tilde{\omega}_{\mathbf{k}''}=\omega_{\mathbf{k}''}-k''_{\perp}\nu_{\mathrm{d}}-k''_{\parallel}\nu_{\parallel}$. It can be easily proved that $D_z/D_x=k''_{\parallel}/k''_{\perp}$, $D_y=0$ because of (3) and $k_y=k'_y=k''_y=0$, thereby the kinetic equation (15) demonstrates that the acceleration or deceleration of the electron beam occurs only in the $\mathbf{k}''$ direction. Equation (16) shows the fourth-order cross-field electric field caused by the change of the cross-field drift velocity ($E_0^{(4)}=(v_{\mathrm{d}}^{(4)}/c)B_0$); that is, $\mathbf{E}_0^{(4)}$ is generated by the dynamo effect of $\mathbf{v}_{\mathrm{d}}^{(4)}$, as has been clarified previously in the similar dynamo effect arising from the quasilinear wave–particle interaction of electrostatic and electromagnetic waves [17–21,46,47]. That is, (5) states that the cross-field electric field $\mathbf{E}_0$ can be generated by the dynamo effect owing to the cross-field acceleration of the electron beam due to the Compton scattering of extraordinary waves. The first four terms on the right-hand side of (17) express the pondermotive force due to the wave electric field ($E_{\mathbf{k}}^{x}, E_{\mathbf{k}}^{z}, E_{\mathbf{k}'}^{x}, E_{\mathbf{k}'}^{z}$) and the remaining five terms result from the nonlinear $\mathbf{v}\times\mathbf{B}$ Lorentz force due to the wave magnetic field ($\mathbf{B}_{\mathbf{k}}, \mathbf{B}_{\mathbf{k}'}$) or the wave electric field ($E_{\mathbf{k}}^{y}, E_{\mathbf{k}'}^{y}$).

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In (6)–(9), (12)–(14) and (17), the numerical factors are corrected because the errors in numerical factors in the higher-order kinetic equations exist in the appendix of [11]. It is noted that $d/dt \approx \gamma_{\rm NL}$ in (15) ($d\nu_{\rm d}^{(4)}/dt \approx \gamma_{\rm NL}\nu_{\rm d}^{(4)} \approx eD/m_{\rm e}$) is comparable to the nonlinear growth rate of Compton scattering given in (57)–(60) in Sec. 3; that is, $\gamma_{\rm NL} \approx A_0 U_{\mathbf{k}}$. Accordingly, when this nonlinear effect is strong, $|d\gamma_{\rm NL}/dt| \approx A_0^2 U_{\mathbf{k}} U_{\mathbf{k}'} \lll \omega_{\rm ce}^2$ ($|m_{\rm e}\, d(c\mathbf{D} \times \mathbf{B}_0/B_0^2)/dt| \lll |e\mathbf{v}_{\rm d}^{(4)} \times \mathbf{B}_0/c|$) is not satisfied necessarily. Then, it is considered reasonable that the particle drift perpendicular to the wave momentum does not appear and the particle drift parallel to the wave momentum appears. When $|d\gamma_{\rm NL}/dt| \lll \omega_{\rm ce}^2$ is satisfied, the particle drift perpendicular to the wave momentum may appear. However the particle drift in this case is negligibly small, since the nonlinear effect is very weak. Accordingly, it is assumed that the condition of (5) is satisfied so that the results consistent with the Vlasov–Maxwell equations given in Sec. 3 are obtained.

2.4. *Electron beam acceleration by Compton scattering of $m = 0$*

First, we consider the Compton scattering of $m=0$. In (15) and (17) only the nonlinear singularity $\omega_{\mathbf{k}''} - k''_\perp \nu_{\rm d} - k''_\parallel \nu_\parallel = 0$ should contribute to the acceleration and deceleration of the electron beam. Thus, we employed the relation

$$\operatorname{Im}\frac{1}{(k''_\parallel \nu_\parallel - k''_\perp \nu_{\rm d} - \omega_{\mathbf{k}''})} = \frac{\pi}{k''_\parallel}\delta(k''_\parallel \nu_\parallel - k''_\perp \nu_{\rm d} - \omega_{\mathbf{k}''})\frac{\partial}{\partial \nu_\parallel}, \tag{18}$$

which comes from $\operatorname{Im}(k''_\parallel \nu_\parallel + k''_\perp \nu_{\rm d} - \omega_{\mathbf{k}''})^{-1} = \pi\delta(k''_\parallel \nu_\parallel + k''_\perp \nu_{\rm d} - \omega_{\mathbf{k}''})$ and the partial integration in velocity-space. Immediately we find the solutions for $\mathbf{v}_{\mathbf{k}''}^{(2)}$:

$$\begin{aligned} v_{x\mathbf{k}''}^{(2)} &= v_{y\mathbf{k}''}^{(2)} = 0, \\ v_{z\mathbf{k}''}^{(2)} &= i\frac{\pi e^2}{2m_{\rm e}^2}\delta(k''_\parallel \nu_\parallel + k''_\perp \nu_{\rm d} - \omega_{\mathbf{k}''})A_{\mathbf{k},\mathbf{k}'}, \end{aligned} \tag{19}$$

where

$$\begin{aligned} A_{\mathbf{k},\mathbf{k}'} = &\left[\frac{k'_\parallel}{\tilde\omega_{\mathbf{k}}^2 - \omega_{\rm ce}^2} - \frac{k_\parallel}{\tilde\omega_{\mathbf{k}'}^2 - \omega_{\rm ce}^2} + \frac{k_\parallel k'^2_\parallel}{k_\perp k'_\perp \tilde\omega_{\mathbf{k}}^2} - \frac{k_\parallel^2 k'_\parallel}{k_\perp k'_\perp \tilde\omega_{\mathbf{k}'}^2}\right] E_{\mathbf{k}}^x E_{\mathbf{k}'}^{*x} \\ &+ i\left[\frac{k'_\parallel}{\tilde\omega_{\mathbf{k}}^2 - \omega_{\rm ce}^2} - \frac{k_\parallel}{\tilde\omega_{\mathbf{k}'}^2 - \omega_{\rm ce}^2}\right]\left[\frac{\omega_{\rm ce}}{\omega_{\mathbf{k}'}} E_{\mathbf{k}}^x E_{\mathbf{k}'}^{*y} - \frac{\omega_{\rm ce}}{\omega_{\mathbf{k}}} E_{\mathbf{k}}^y E_{\mathbf{k}'}^{*x}\right] \\ &+ \frac{1}{\omega_{\mathbf{k}}\omega_{\mathbf{k}'}}\left[\frac{k'_\parallel \tilde\omega_{\mathbf{k}}^2}{\tilde\omega_{\mathbf{k}}^2 - \omega_{\rm ce}^2} - \frac{k_\parallel \tilde\omega_{\mathbf{k}'}^2}{\tilde\omega_{\mathbf{k}'}^2 - \omega_{\rm ce}^2}\right] E_{\mathbf{k}}^y E_{\mathbf{k}'}^{*y}. \end{aligned} \tag{20}$$

Thus, it is now straightforward to find that (15) and (17) become the following simple kinetic equation:

$$m_{\rm e}\frac{d\mathbf{v}^{(4)}}{dt} = -\frac{\pi e^4 \mathbf{k}''}{8m_{\rm e}^3 k''_\parallel}\delta(k''_\parallel \nu_\parallel + k''_\perp \nu_{\rm d} - \omega_{\mathbf{k}''})\frac{\partial}{\partial \nu_\parallel}|A_{\mathbf{k},\mathbf{k}'}|^2. \tag{21}$$

This equation shows the temporal evolution of the fourth-order drift velocity of the electron beam and it states that the acceleration and deceleration of the electron beam occurs in the $\mathbf{k}''$ direction via Compton scattering induced by the nonlinear Landau damping of two extraordinary waves due to nonlinear wave–particle interaction with the electron beam. The first term of $A_{\mathbf{k},\mathbf{k}'}$ originates in the first four

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terms in (17) and expresses the pondermotive force due to the wave electric field. The third term of $A_{\mathbf{k},\mathbf{k}'}$ originates in the last five terms in (17) and expresses the nonlinear $\mathbf{v}\times\mathbf{B}$ Lorentz force. The second term of $A_{\mathbf{k},\mathbf{k}'}$ originates partially in all terms in (17) and expresses both forces.

Assuming that the velocity distribution function of the electron beam is given by

$$g_{\rm b} = \frac{1}{\pi^{1/2}\nu_{\rm tb}}\delta(\nu_x-\nu_{\rm d})\delta(\nu_y)\exp\left[-\frac{(\nu_z-\nu_{\rm b})^2}{\nu_{\rm tb}^2}\right], \tag{22}$$

the transport equation indicating the temporal evolution of the momentum density of the electron beam can be derived by integrating (21) multiplied by $n_{\rm b}g_{\rm b}$ in the velocity-space, and is expressed as

$$\frac{d\mathbf{P}_{\rm b}}{dt} = \frac{e^2\omega_{\rm pb}^2\mathbf{k}''}{16\pi^{1/2}m_{\rm e}^2k_\parallel''|k_\parallel''|\nu_{\rm tb}^2}\varsigma_{0\mathbf{k}''}\exp\left(-\varsigma_{0\mathbf{k}''}^2\right)|A_{\mathbf{k},\mathbf{k}'}|^2_{\nu_\parallel=(\omega_{\mathbf{k}''}-k_\perp''\nu_{\rm d})/k_\parallel''}. \tag{23}$$

Here, $\mathbf{P}_{\rm b} = \int d^3\mathbf{v}n_{\rm b}m_{\rm e}\mathbf{v}g_{\rm b} = (P_{{\rm b}\perp},0,P_{{\rm b}\parallel})$ $(P_{{\rm b}\perp}=n_{\rm b}m_{\rm e}\nu_{\rm d}, P_{{\rm b}\parallel}=n_{\rm b}m_{\rm e}\nu_{\rm b})$ is the momentum density of the electron beam, $\omega_{\rm pb}^2=4\pi n_{\rm b}e^2/m_{\rm e}$, $\varsigma_{0\mathbf{k}''}=(\omega_{\mathbf{k}''}-k_\parallel''\nu_{\rm d}-k_\parallel''\nu_{\rm b})/k_\parallel''\nu_{\rm tb}$, and the term $|g_{\rm b}(\partial/\partial\nu_\parallel)|A_{\mathbf{k},\mathbf{k}'}|^2|$ is neglected in comparison with the term $|A_{\mathbf{k},\mathbf{k}'}|^2|\partial g_{\rm b}/\partial\nu_\parallel|$, because the former term is negligibly smaller than the latter term when $|k_\parallel\nu_{\rm tb}/\omega_{\rm ce}|$, $|k_\parallel'\nu_{\rm tb}/\omega_{\rm ce}|\ll 1$. It is seen from (23) that the acceleration and deceleration of the electron beam occur in the $\mathbf{k}''$ direction and its rate is approximately proportional to $\alpha_0=\varsigma_{0\mathbf{k}''}\exp(-\varsigma_{0\mathbf{k}''}^2)$. Figure 1 shows $|\alpha_0|$ versus $\xi_0=(\omega_{\mathbf{k}''}-k_\parallel''\nu_{\rm d}-k_\parallel''\nu_{\rm b})/\omega_{\rm ce}$ under the parameter of $k_\parallel''\nu_{\rm tb}/\omega_{\rm ce}=0.0005$. The solid curve shows $|\alpha_0|$ with $\alpha_0>0$ and the dotted curve shows $|\alpha_0|$ with $\alpha_0<0$. It is found that the dip appears at the point where the phase velocity of the beat-wave for the Doppler-shifted beat-wave frequency equals the parallel velocity of the electron beam $((\omega_{\mathbf{k}''}-k_\perp''\nu_{\rm d})/k_\parallel''=\nu_{\rm b},\xi_0=0)$, since $\alpha_0=0$ at $\varsigma_{0\mathbf{k}''}=0$. Moreover, we see that when the Doppler-shifted phase velocity of the beat-wave slightly exceeds the parallel velocity of the electron beam $((\omega_{\mathbf{k}''}-k_\perp''\nu_{\rm d})/k_\parallel''>\nu_{\rm b}$ and ξ_0, $\zeta_{0\mathbf{k}''}>0)$, the electron beam can be accelerated efficiently. In contrast, when the Doppler-shifted phase velocity of the beat-wave is slightly smaller than the parallel velocity of the electron beam $((\omega_{\mathbf{k}''}-k_\perp''\nu_{\rm d})/k_\parallel''<\nu_{\rm b}$ and $\xi_0,\varsigma_{0\mathbf{k}''}<0)$, the electron beam can be decelerated efficiently. This behaviour is much the same as the previous result in [11]. It should be noticed that the condition of $\xi_0=0$ means the resonance condition of (1) for $m=0$ and it is not the condition for the beat-wave, because the beat-wave is assumed to be absent in the kinetic equation (4) for the electron beam.

As is found from (3), that the relations of $|E_{\mathbf{k}}^x|\gg|E_{\mathbf{k}}^y|$, $|E_{\mathbf{k}}^z|$ and $|E_{\mathbf{k}'}^x|\gg|E_{\mathbf{k}'}^y|$, $|E_{\mathbf{k}'}^z|$ $(k_\parallel/k_\perp,k_\parallel'/k_\perp'\ll 1)$ are satisfied when $\omega_{\mathbf{k}}\cong\omega_{\mathbf{k}'}\lesssim\omega_{\rm h}$. Then the first term of $A_{\mathbf{k},\mathbf{k}'}$ becomes dominant. On the other hand, the relations of $|E_{\mathbf{k}}^y|\gg|E_{\mathbf{k}}^x|\gg|E_{\mathbf{k}}^z|$ and $|E_{\mathbf{k}'}^y|\gg|E_{\mathbf{k}'}^x|\gg|E_{\mathbf{k}'}^z|$ are satisfied when $\omega_{\mathbf{k}}\cong\omega_{\mathbf{k}'}\gtrsim\omega_{\rm R}$, thereby the third term of $A_{\mathbf{k},\mathbf{k}'}$ becomes dominant, where $\omega_{\rm R}=(\omega_{\rm ce}+(4\omega_{\rm pe}^2+\omega_{\rm ce}^2)^{1/2})/2$ is the right-hand cutoff frequency. Accordingly, the efficient acceleration and deceleration of the electron beam can occur via the pondermotive force due to the $\mathbf{k}$ component of the wave electric field when $\omega_{\mathbf{k}}\cong\omega_{\mathbf{k}'}\lesssim\omega_{\rm h}$ and via the nonlinear $\mathbf{v}\times\mathbf{B}$ Lorentz force due to the $\mathbf{k}$ component of the wave magnetic field or the $\mathbf{k}\times\mathbf{B}_0$ component of the wave electric field when $\omega_{\mathbf{k}}\cong\omega_{\mathbf{k}'}\gtrsim\omega_{\rm R}$, as has been shown previously in detail in [11]. In these two limiting cases, considering that $k_\parallel/k_\perp,k_\parallel'/k_\perp'\ll 1$ and $\omega_{\mathbf{k}}\cong\omega_{\mathbf{k}'}$, it can be

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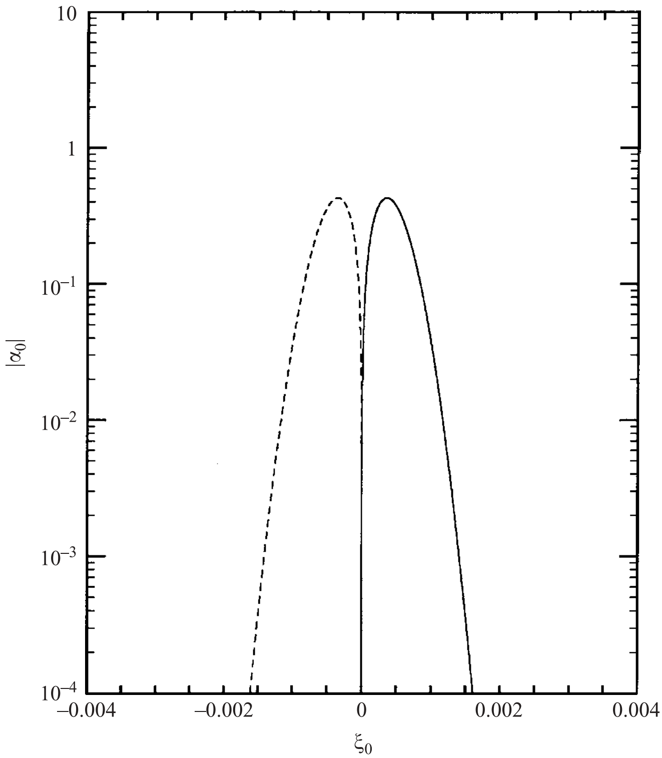

Figure 1. The Compton scattering of $m = 0$: the absolute value of α_0 is shown versus ξ_0 with $k''_{\|}v_{\mathrm{tb}}/\omega_{\mathrm{ce}} = 0.0005$. For the solid curve $\alpha_0 > 0$ and for the dotted curve $\alpha_0 < 0$.

easily confirmed that the parallel component of (23) with $\mathbf{v}_{\mathrm{d}} = 0$ is reduced to (30) and (32) in [11].

2.5. Electron beam acceleration by the Compton scattering of $m = \pm 1$

Next, we consider the Compton scattering of $m = \pm 1$. In (15) and (17) the nonlinear singularity of $\omega_{\mathbf{k}''} - k''_{\perp}v_{\mathrm{d}} - k''_{\|}v_{\|} \mp \omega_{\mathrm{ce}} = 0$ should provide another contribution to the acceleration and deceleration of the electron beam. This nonlinear scattering with $\mathbf{v}_{\mathrm{d}} = 0$ has been investigated in detail by the numerical analysis in [6]. By the use of the relation of

$$\mathrm{Im}\frac{1}{k''_{\|}v_{\|} + k''_{\perp}v_{\mathrm{d}} - \omega_{\mathbf{k}''} \pm \omega_{\mathrm{ce}}} = \pi\delta(k''_{\|}v_{\|} + k''_{\perp}v_{\mathrm{d}} - \omega_{\mathbf{k}''} \pm \omega_{\mathrm{ce}}), \tag{24}$$

the solutions for $\mathbf{v}^{(2)}_{\mathbf{k}''}$ are found as

$$v^{(2)}_{x\mathbf{k}''} = \pm i\frac{\pi e^2}{4m_{\mathrm{e}}^2}\delta(k''_{\|}v_{\|} + k''_{\perp}v_{\mathrm{d}} - \omega_{\mathbf{k}''} \pm \omega_{\mathrm{ce}})F_{\mathbf{k},\mathbf{k}'}, \tag{25a}$$

$$v^{(2)}_{y\mathbf{k}''} = -\frac{\pi e^2}{4m_{\mathrm{e}}^2}\delta(k''_{\|}v_{\|} + k''_{\perp}v_{\mathrm{d}} - \omega_{\mathbf{k}''} \pm \omega_{\mathrm{ce}})F_{\mathbf{k},\mathbf{k}'}, \tag{25b}$$

$$v_{z\mathbf{k}''} = 0, \tag{25c}$$

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where

$$F_{\mathbf{k},\mathbf{k}'} = \left[\frac{k'_\perp}{\tilde{\omega}^2_{\mathbf{k}} - \omega^2_{\rm ce}} - \frac{k_\perp}{\tilde{\omega}^2_{\mathbf{k}'} - \omega^2_{\rm ce}} + \frac{k_\parallel k'_\parallel}{k_\perp \tilde{\omega}^2_{\mathbf{k}}} - \frac{k_\parallel k'_\parallel}{k'_\perp \tilde{\omega}^2_{\mathbf{k}'}}\right] E^x_{\mathbf{k}} E^{*x}_{\mathbf{k}'}$$
$$+ i\frac{\omega_{\rm ce}}{\omega_{\mathbf{k}'}}\left[\frac{k'_\perp}{\tilde{\omega}^2_{\mathbf{k}} - \omega^2_{\rm ce}} + \frac{k_\parallel k'_\parallel}{k_\perp \tilde{\omega}^2_{\mathbf{k}}}\right] E^x_{\mathbf{k}} E^{*y}_{\mathbf{k}'} + i\frac{\omega_{\rm ce}}{\omega_{\mathbf{k}}}\left[\frac{k_\perp}{\tilde{\omega}^2_{\mathbf{k}'} - \omega^2_{\rm ce}} + \frac{k_\parallel k'_\parallel}{k'_\perp \tilde{\omega}^2_{\mathbf{k}'}}\right] E^y_{\mathbf{k}} E^{*x}_{\mathbf{k}'}$$
$$+ i\left[\frac{k'_\perp}{\tilde{\omega}^2_{\mathbf{k}} - \omega^2_{\rm ce}} - \frac{k_\perp}{\tilde{\omega}^2_{\mathbf{k}'} - \omega^2_{\rm ce}}\right]\left[\frac{\omega_{\rm ce}}{\omega_{\mathbf{k}'}} E^x_{\mathbf{k}} E^{*y}_{\mathbf{k}'} - \frac{\omega_{\rm ce}}{\omega_{\mathbf{k}}} E^y_{\mathbf{k}} E^{*x}_{\mathbf{k}'}\right]$$
$$+ \frac{1}{\omega_{\mathbf{k}}\omega_{\mathbf{k}'}}\left[\frac{k'_\perp(\tilde{\omega}^2_{\mathbf{k}} + \omega^2_{\rm ce})}{\tilde{\omega}^2_{\mathbf{k}} - \omega^2_{\rm ce}} - \frac{k_\perp(\tilde{\omega}^2_{\mathbf{k}'} + \omega^2_{\rm ce})}{\tilde{\omega}^2_{\mathbf{k}'} - \omega^2_{\rm ce}}\right] E^y_{\mathbf{k}} E^{*y}_{\mathbf{k}'}. \tag{26}$$

From (15) and (17) we can straightforwardly obtain the following simple kinetic equation showing the temporal evolution of the fourth-order drift velocity of the electron beam:

$$m_{\rm e}\frac{d\mathbf{v}^{(4)}}{dt} = \frac{\pi e^4 \mathbf{k}''}{16 m^3_{\rm e}\omega_{\rm ce}}\delta(k''_\parallel \nu_\parallel + k''_\parallel \nu_{\rm d} - \omega_{\mathbf{k}''} \pm \omega_{\rm ce})|F_{\mathbf{k},\mathbf{k}'}|^2. \tag{27}$$

This equation shows that the acceleration and deceleration of the electron beam arise in the $\mathbf{k}''$ direction from Compton scattering induced by nonlinear cyclotron damping of two extraordinary waves. The first term of $F_{\mathbf{k},\mathbf{k}'}$ originating in the first four terms in (17) expresses the pondermotive force due to the wave electric field. The last term of $F_{\mathbf{k},\mathbf{k}'}$ originating in the last five terms in (17) expresses the nonlinear $\mathbf{v}\times\mathbf{B}$ force. The remaining terms of $F_{\mathbf{k},\mathbf{k}'}$ originating partially in all terms in (17) express both forces.

Assuming that the velocity distribution function of the electron beam is given by (22), the transport equation for the momentum density of the electron beam can be derived similarly from (27) and is represented as

$$\frac{d\mathbf{P}_{\rm b}}{dt} = \frac{e^2\omega^2_{\rm pb}\mathbf{k}''}{64\pi^{1/2}m^2_{\rm e}\omega_{\rm ce}|k''_\parallel|\nu_{\rm tb}}\exp\left(-\varsigma^2_{m\mathbf{k}''}\right)|F_{\mathbf{k},\mathbf{k}'}|^2_{\nu_\parallel=(\omega_{\mathbf{k}''}-k''_\perp\nu_{\rm d}-m\omega_{\rm ce})/k''_\parallel}, \tag{28}$$

where $\varsigma_{m\mathbf{k}''} = (\omega_{\mathbf{k}''} - k''_\perp\nu_{\rm d} - k''_\parallel\nu_{\rm b} - m\omega_{\rm ce})/k''_\parallel\nu_{\rm tb}$ $(m = \pm 1)$. The above equation states that the acceleration of the electron beam always occurs in the $\mathbf{k}''$ direction and its rate is approximately proportional to $\alpha_1 = \exp(-\varsigma^2_{m\mathbf{k}''})$. Figure 2 exhibits α_1 versus $\xi_m = (\omega_{\mathbf{k}''} - k''_\parallel\nu_{\rm d} - k''_\parallel\nu_{\rm b} - m\omega_{\rm ce})/\omega_{\rm ce}$ under the parameter of $k''_\parallel\nu_{\rm tb}/\omega_{\rm ce} = 0.0005$. We see that the electron beam can be accelerated efficiently in the vicinity of $\xi_m = 0$. For $\omega_{\mathbf{k}}$, $\omega_{\mathbf{k}'} \lesssim \omega_{\rm h}$, the first term of $F_{\mathbf{k},\mathbf{k}'}$ owing to $E^x_{\mathbf{k}}$, $E^z_{\mathbf{k}}$, $E^x_{\mathbf{k}'}$ and $E^z_{\mathbf{k}'}$ becomes dominant. For $\omega_{\mathbf{k}}, \omega_{\mathbf{k}'} \gtrsim \omega_{\rm R}$, the last term owing to $E^y_{\mathbf{k}}$ and $E^y_{\mathbf{k}'}$ becomes dominant. Accordingly, it is found in the same way as in the case of $m = 0$ that the efficient acceleration of the electron beam results from the pondermotive force due to the $\mathbf{k}$ and $\mathbf{k}'$ components of $\mathbf{E}_{\mathbf{k}}$ and $\mathbf{E}_{\mathbf{k}'}$ when $\omega_{\mathbf{k}}$, $\omega_{\mathbf{k}'} \lesssim \omega_{\rm h}$, and from the nonlinear $\mathbf{v}\times\mathbf{B}$ force due to $\mathbf{B}_{\mathbf{k}}$ and $\mathbf{B}_{\mathbf{k}'}$ when $\omega_{\mathbf{k}}, \omega_{\mathbf{k}'} \gtrsim \omega_{\rm R}$.

Combining (23) and (28), we get the following transport equations involving both the Compton scatterings of $m = 0$ and ± 1:

$$\frac{d\mathbf{P}_{\rm b}}{dt} = \frac{e^2\omega^2_{\rm pb}\mathbf{k}''}{16\pi^{1/2}m^2_{\rm e}k''_\parallel|k''_\parallel|\nu^2_{\rm tb}}\sum_{m=0,\pm 1}\exp\left(-\varsigma^2_{m\mathbf{k}''}\right)S_m, \tag{29}$$

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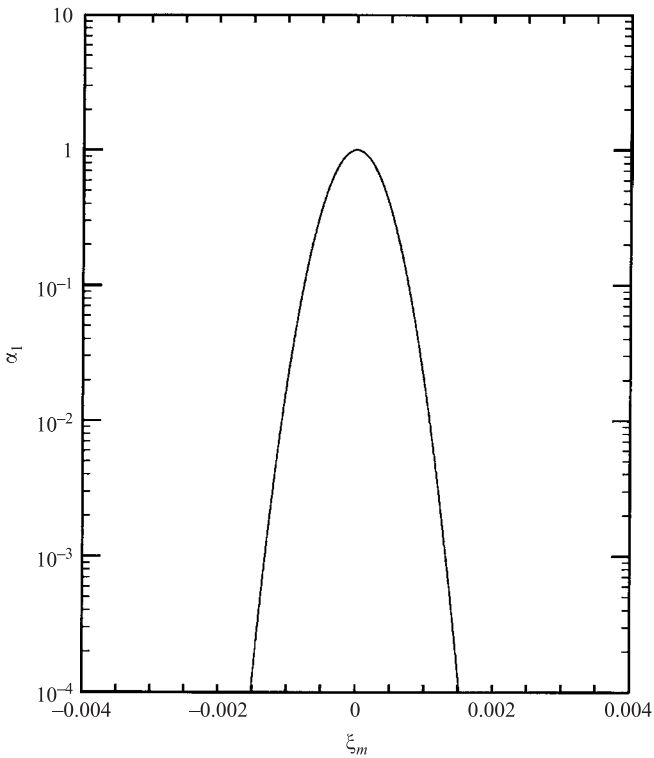

Figure 2. The Compton scattering of $m = \pm 1$, α_1 is shown versus ξ_m with $k''_\parallel \nu_{\rm tb}/\omega_{\rm ce} = 0.0005$.

where

$$S_m = \varsigma_{0\mathbf{k}''} |A_{\mathbf{k},\mathbf{k}'}|^2_{\nu_\parallel = (\omega_{\mathbf{k}''} - k''_\perp \nu_{\rm d})/k''_\parallel}, \quad \text{for } m = 0, \tag{30a}$$

$$S_m = \frac{k''_\parallel \nu_{\rm tb}}{4\omega_{\rm ce}} |F_{\mathbf{k},\mathbf{k}'}|^2_{\nu_\parallel = (\omega_{\mathbf{k}''} - k''_\perp \nu_{\rm d} - m\omega_{\rm ce})/k''_\parallel}, \quad \text{for } m = \pm 1. \tag{30b}$$

It should be noted that the ratio of the acceleration rates for the Compton scattering of $m = \pm 1$ and $m = 0$ is given roughly as follows:

$$\left| \frac{d\mathbf{P}_{\rm b}}{dt} \right|_{m=\pm 1} \Bigg/ \left| \frac{d\mathbf{P}_{\rm b}}{dt} \right|_{m=0} \cong \left| \frac{S_{\pm 1}}{S_0} \right| \sim \left| \frac{k''_\perp}{k''_\parallel} \frac{k''_\perp \nu_{\rm tb}}{\omega_{\rm ce}} \right|. \tag{31}$$

Because $A_{\mathbf{k},\mathbf{k}'}$ and $F_{\mathbf{k},\mathbf{k}'}$ are derived taking into account the polarizations given by (3), the transport equations obtained can only be applied for the Compton scattering of extraordinary waves by a cold electron beam with zero Larmor radius. Accordingly they are not general and rigorously exact. The transport equations for the energy and momentum densities of the electron beam and the conservation laws for the total energy and momentum densities of the waves and the electron beam are derived from Vlasov–Maxwell equations and will be described in the next section. They are general and rigorously exact, and are applicable for the Compton scattering of electromagnetic waves by a warm electron beam with a finite Larmor radius.

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3. Transport equations

In this section the general and exact transport equations for nonlinear Landau damping of electromagnetic waves are derived from Vlasov–Maxwell equations. Because they are mathematically complicated, it is rather difficult to understand the detailed physical mechanism of Compton scattering. First, the kinetic wave equation that is necessary for obtaining the conservation laws for the total energy and momentum densities of waves and particles is given. Second, the θ-dependent velocity-space diffusion equation which leads to the transport equations is presented. Third, the transport equations and the conservation laws are described. Finally, they are applied for the Compton scattering of extraordinary waves.

3.1. Kinetic wave equation

The kinetic wave equation for nonlinear wave–particle scattering due to nonlinear Landau and cyclotron damping of electromagnetic waves in a homogeneous magnetized plasma immersed in the uniform electric field can be derived from the third-order perturbation theory of Vlasov–Maxwell equations in the same manner as in [4, 6, 7] and is given by

$$\frac{\partial U_{\mathbf{k}}}{\partial t} = 2\gamma_{\mathbf{k}} U_{\mathbf{k}} + \sum_{k' \neq 0} \sum_{j,l,l',l''} \frac{\omega_{\mathbf{k}}}{4\pi} A^{jl''l'l}_{\mathbf{k},\mathbf{k}'',\mathbf{k}'} E^{*j}_{\mathbf{k}} E^{l}_{\mathbf{k}} E^{*l'}_{\mathbf{k}'} E^{l''}_{\mathbf{k}'}, \tag{32}$$

where $E^j_{\mathbf{k}}$ refers to the j component of a wave electric field $\mathbf{E}_{\mathbf{k}}$ in the Cartesian coordinates $(j, l, l', l'' = x, y, z)$, $\mathbf{E}_{-\mathbf{k}} = \mathbf{E}^*_{\mathbf{k}}, \omega_{-\mathbf{k}} = -\omega^*_{\mathbf{k}}$, and the wave energy density $U_{\mathbf{k}}$, the linear damping rate $\gamma_{\mathbf{k}} = \mathrm{Im}\omega_{\mathbf{k}}$ and the nonlinear wave–particle coupling coefficient $A^{jl''l'l}_{\mathbf{k},\mathbf{k}'',\mathbf{k}'}$ are represented as follows:

$$U_{\mathbf{k}} = \frac{1}{8\pi} \mathbf{E}^*_{\mathbf{k}} \cdot \left(\frac{\partial}{\partial \omega_{\mathbf{k}}} [(\boldsymbol{\varepsilon}'_{\mathbf{k}} - \mathbf{N}_{\mathbf{k}})\omega_{\mathbf{k}}] \right) \cdot \mathbf{E}_{\mathbf{k}}, \tag{33}$$

$$\gamma_{\mathbf{k}} = -\frac{\omega_{\mathbf{k}}}{8\pi U_{\mathbf{k}}} (\mathbf{E}^*_{\mathbf{k}} \cdot \boldsymbol{\varepsilon}''_{\mathbf{k}} \cdot \mathbf{E}_{\mathbf{k}}), \tag{34}$$

$$A^{jl''l'l}_{\mathbf{k},\mathbf{k}'',\mathbf{k}'} = \sum_{\mathbf{s}} A^{(s)jl''l'l}_{\mathbf{k},\mathbf{k}'',\mathbf{k}'}, \tag{35}$$

$$A^{(s)jl''l'l}_{\mathbf{k},\mathbf{k}'',\mathbf{k}'} = -P_{\mathrm{AH}} \big(C^{(s)jl''l'l}_{\mathbf{k},\mathbf{k}'',\mathbf{k}'} + D^{(s)jl''l'l}_{\mathbf{k},\mathbf{k}'',\mathbf{k}'} \big). \tag{36}$$

Here, $\boldsymbol{\varepsilon}_{\mathbf{k}} = \boldsymbol{\varepsilon}'_{\mathbf{k}} + i\boldsymbol{\varepsilon}''_{\mathbf{k}}$ is the dielectric tensor for electromagnetic waves, $\boldsymbol{\varepsilon}'_{\mathbf{k}}$ and $\boldsymbol{\varepsilon}''_{\mathbf{k}}$ are the Hermitian and anti-Hermitian parts of $\boldsymbol{\varepsilon}_{\mathbf{k}}$, respectively, P_{AH} indicates the anti-Hermitian part of the tensor; that is,

$$P_{\mathrm{AH}} \big[C^{(s)jl''l'l}_{\mathbf{k},\mathbf{k}'',\mathbf{k}'} \big] = \frac{1}{2i} \big(C^{(s)jl''l'l}_{\mathbf{k},\mathbf{k}'',\mathbf{k}'} - C^{*(s)ll'l''j}_{\mathbf{k},\mathbf{k}'',\mathbf{k}'} \big), \tag{37}$$

and the subscript s designates the species of plasma particles. The matrix element $C^{(s)jl''l'l}_{\mathbf{k},\mathbf{k}'',\mathbf{k}'}$ expresses Compton scattering (four-wave scattering or two-wave–particle scattering) and $D^{(s)jl''l'l}_{\mathbf{k},\mathbf{k}'',\mathbf{k}'}$ represents plasma shielding effect (non-resonant wave–wave scattering) associated with beat-waves. These matrix elements are expressed by means of the fairly complex equations and are described in Appendices A and B. They are different from the previously obtained matrix elements in [4, 6, 7] only in the effects due to the cross-field electric field $\mathbf{E}_0$ and the cross-field particle drift velocity $\mathbf{v}_{\mathrm{d}}$. The matrix elements are evaluated by their anti-Hermitian parts

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coming from the nonlinear singularity associated with the resonance condition (1), i.e. $\omega_{\mathbf{k}''} - k''_\perp \nu_{\rm d} - k''_\parallel \nu_\parallel - m\omega_{\rm cs} = 0$, where m is an integer, $\omega_{\rm cs} = e_{\rm s} B_0/m_{\rm s} c$ is the cyclotron frequency for the particles of species s and $\omega_{\rm cs}$ includes the sign of the charge of particles.

The dielectric tensor $\boldsymbol{\varepsilon}_{\mathbf{k}}$ is given by

$$\boldsymbol{\varepsilon}_{\mathbf{k}} = \mathbf{I} + \sum_{\rm s} \boldsymbol{\varepsilon}_{\mathbf{k}}^{(\rm s)}, \tag{38}$$

$$\varepsilon_{\mathbf{k}}^{(\rm s)jl} = -\frac{\omega_{\rm ps}^2}{\omega_{\mathbf{k}}^2} \Bigg\{ \eta_j \eta_l + \sum_{r=-\infty}^{\infty} \int d^3\mathbf{v} \times \left[\frac{[a_{\mathbf{k},r}^{*j} J_r(\mu_{\mathbf{k}})][a_{\mathbf{k},r}^{l} J_r(\mu_{\mathbf{k}})]}{(k_\parallel \nu_\parallel + k_\perp \nu_{\rm d} - \omega_{\mathbf{k}} + r\omega_{\rm cs})^2} + \frac{[b_{\mathbf{k}}^{*j} J_r(\mu_{\mathbf{k}})][b_{\mathbf{k}}^{l} J_r(\mu_{\mathbf{k}})]}{(k_\parallel \nu_\parallel + k_\perp \nu_{\rm d} - \omega_{\mathbf{k}} + r\omega_{\rm cs})^2 - \omega_{\rm cs}^2} \right] g_{\rm s0} \Bigg\}, \tag{39}$$

where $\omega_{\rm ps} = (4\pi n_{\rm s} e^2/m_{\rm s})^{1/2}$ is the plasma frequency for the particles of species s, $\mu_{\mathbf{k}} = k_\perp \nu_\perp/\omega_{\rm cs}$, J_r is the Bessel function of the rth order. The function $g_{\rm s0}(\nu_\perp, \nu_\parallel, t)$ is the background velocity distribution function with $\mathbf{v}_{\rm d} = \mathbf{E}_0 = 0$ and the distribution function $g_{\rm s}(\mathbf{v}, t)$ with $\mathbf{v}_{\rm d}$, $\mathbf{E}_0 \neq 0$ is provided by the solution of the unperturbed Vlasov equation,

$$\left(\mathbf{E}_0 + \frac{1}{c} \mathbf{v} \times \mathbf{B}_0 \right) \cdot \frac{\partial g_{\rm s}}{\partial \mathbf{v}} = 0. \tag{40}$$

The solution of the above equation can be obtained as [**17–21**,**46**,**47**]

$$\begin{aligned} g_{\rm s} &= a_{\rm s} \sum_{l=0}^{\infty} \frac{\nu_x^l}{l!} \left(-\frac{\nu_{\rm d}}{\nu_\perp} \frac{\partial}{\partial \nu_\perp} \right)^l g_{\rm s0}(\nu_\perp, \nu_\parallel, t) \\ &= g_{\rm s0}\big(((\nu_x - \nu_{\rm d})^2 + \nu_y^2)^{1/2}, \nu_\parallel, t \big), \end{aligned} \tag{41}$$

with the normalization constant $a_{\rm s}$, which is determined such that $\int d^3\mathbf{v} g_{\rm s} = \int d^3\mathbf{v} g_{\rm s0} = 1$. The background velocity distribution function $g_{\rm s}(\mathbf{v}, t)$ includes the cross-field particle drift $\nu_{\rm d} = \int d^3\mathbf{v} \nu_x g_{\rm s}$ which is induced by quasilinear wave–particle interaction and the nonlinear Landau and cyclotron damping of electromagnetic waves, and is expressed in the displaced cylindrical coordinates in velocity-space ($\nu_x = \nu_\perp \cos\theta + \nu_{\rm d}, \nu_y = \nu_\perp \sin\theta, \nu_z = \nu_\parallel$). The generalized Ohm's law (5) can be derived by means of the velocity-space integration of (40) multiplied by $\mathbf{v}$. That is, (5) holds for all species of plasma particles and shows that the cross-field electric field $\mathbf{E}_0$ is also produced by the dynamo effect of the cross-field particle drift of the species s. When $\mathbf{v}_{\rm d} = 0$, $g_{\rm s}$ is reduced to $g_{\rm s0}$, being symmetric with respect to the magnetic field. The functions $a_{\mathbf{k},r}^l$ and $b_{\mathbf{k}}^l$ are defined by

$$\begin{gathered} a_{\mathbf{k},r}^x = \frac{k_\parallel}{k_\perp}(r\omega_{\rm cs} + k_\perp \nu_{\rm d}), \quad a_{\mathbf{k},r}^y = i k_\parallel \nu_\perp \frac{\partial}{\partial \mu_{\mathbf{k}}}, \quad a_{\mathbf{k},r}^z = \omega_{\mathbf{k}} - k_\perp \nu_{\rm d} - r\omega_{\rm cs}, \\ b_{\mathbf{k}}^x = \omega_{\mathbf{k}} - k_\parallel \nu_\parallel, \quad b_{\mathbf{k}}^y = i\omega_{\rm cs} \frac{\partial}{\partial \mu_{\mathbf{k}}}, \quad b_{\mathbf{k}}^z = k_\perp \nu_\parallel, \end{gathered} \tag{42}$$

and $I_{jl} = \delta_{jl}$, $\eta_x = \eta_z = 0$, $\eta_y = 1$. The velocity-space integration in (39) is performed in the cylindrical coordinates $\mathbf{v} = (\nu_\perp, \theta, \nu_\parallel)$, which are displaced by $\nu_{\rm d}$ in the x direction. Consequently, $g_{\rm s0}$ appears in the integrand. The term $k_\perp \nu_{\rm d}$ represents the Doppler shift due to the cross-field particle drift. The dielectric tensor (39)

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with $\nu_{\rm d}=0$ is exactly equivalent to (8) in [11]. Further, the two scattering waves satisfy the dispersion relation of (2), and the beat-wave $(\omega_{\mathbf{k}''}, \mathbf{k}'')$ cannot satisfy the dispersion relation, i.e. $|\boldsymbol{\varepsilon}_{\mathbf{k}''} - \mathbf{N}_{\mathbf{k}''}| \neq 0$.

3.2. θ-dependent velocity-space diffusion equation

The θ-dependent velocity-space diffusion equation for the present nonlinear scattering can be derived similarly from Vlasov–Maxwell equations as was described previously in [7]. It is required for obtaining the transport equations and is expressed as

$$\frac{\partial g_{\rm s}}{\partial t} = \sum_{\mathbf{k}\neq 0}\sum_{l,l'} E_{\mathbf{k}}^{*l} E_{\mathbf{k}}^{l'} Q_{\mathbf{k}}^{ll'} g_{\rm s0} + \sum_{\mathbf{k}\neq 0}\sum_{\mathbf{k}'}\sum_{j,l,l',l''} E_{\mathbf{k}}^{*j} E_{\mathbf{k}}^{l} E_{\mathbf{k}'}^{*l'} E_{\mathbf{k}'}^{l''} B_{\mathbf{k},\mathbf{k}'',\mathbf{k}'}^{jl''l'l} g_{\rm s0}, \tag{43}$$

where

$$B_{\mathbf{k},\mathbf{k}'',\mathbf{k}'}^{jl''l'l} = -P_{\rm AH}\big(L_{\mathbf{k},\mathbf{k}'',\mathbf{k}'}^{jl''l'l} + M_{\mathbf{k},\mathbf{k}'',\mathbf{k}'}^{jl''l'l}\big). \tag{44}$$

The θ-dependent velocity-space diffusion coefficients $Q_{\mathbf{k}}^{ll'}$, $L_{\mathbf{k},\mathbf{k}'',\mathbf{k}'}^{jl''l'l}$ and $M_{\mathbf{k},\mathbf{k}'',\mathbf{k}'}^{jl''l'l}$ have fairly complex expressions and are described in Appendix C. Since $Q_{\mathbf{k}}^{ll'}$ and $B_{\mathbf{k},\mathbf{k}'',\mathbf{k}'}^{jl''l'l}$ are expressed in the displaced cylindrical coordinates in velocity-space, $g_{\rm s0}$ appears on the right-hand side of (43). The former coefficient $Q_{\mathbf{k}}^{ll'}$ is the quasilinear velocity-space diffusion coefficient and has been derived previously by the present author [18–21]. The latter coefficient $B_{\mathbf{k},\mathbf{k}'',\mathbf{k}'}^{jl''l'l}$ represents the θ-dependent velocity-space diffusion due to the nonlinear Landau and cyclotron damping of electromagnetic waves. The term $Q_{\mathbf{k}}^{ll'}$ results from the quasilinear wave–particle interaction (quasilinear Landau and cyclotron damping), the term $L_{\mathbf{k},\mathbf{k}'',\mathbf{k}'}^{jl''l'l}$ from the velocity-space diffusion due to Compton scattering, and the term $M_{\mathbf{k},\mathbf{k}'',\mathbf{k}'}^{jl''l'l}$ from the plasma shielding effect [4, 6, 7]. These velocity-space diffusion coefficients have the azimuthal dependence; that is, $\partial Q_{\mathbf{k}}^{ll'}/\partial\theta \neq 0$ and $\partial B_{\mathbf{k},\mathbf{k}'',\mathbf{k}'}^{jl''l'l}/\partial\theta \neq 0$ hold, as described in detail in Appendix C. The azimuthal dependence of $Q_{\mathbf{k}}^{ll'}$ and $B_{\mathbf{k},\mathbf{k}'',\mathbf{k}'}^{jl''l'l}$ expresses the anisotropy of the velocity-space diffusion around the magnetic field and the resulting cross-field particle acceleration or transport in the x direction.

It can be proved easily that there are the simple relations between the θ-dependent velocity-space diffusion coefficients, the dielectric tensor and the nonlinear wave–particle coupling coefficients [4, 6, 7, 19, 21]. Then they are shown as follows:

$$\int d^3\mathbf{v}\, w_{\rm s} Q_{\mathbf{k}}^{ll'} g_{\rm s0} = \frac{\omega_{\mathbf{k}}}{4\pi} P_{\rm AH}\varepsilon_{\mathbf{k}}^{({\rm s})ll'}, \tag{45}$$

$$\int d^3\mathbf{v}\mathbf{p}_{\rm s}\, Q_{\mathbf{k}}^{ll'} g_{\rm s0} = \frac{\mathbf{k}}{4\pi} P_{\rm AH}\varepsilon_{\mathbf{k}}^{({\rm s})ll'}, \tag{46}$$

$$\int d^3\mathbf{v}\, w_{\rm s}\, L_{\mathbf{k},\mathbf{k}'',\mathbf{k}'}^{jl''l'l} g_{\rm s0} = -\frac{\omega_{\mathbf{k}}}{4\pi} C_{\mathbf{k},\mathbf{k}'',\mathbf{k}'}^{({\rm s})jl''l'l}, \tag{47}$$

$$\int d^3\mathbf{v}\mathbf{p}_{\rm s}\, L_{\mathbf{k},\mathbf{k}'',\mathbf{k}'}^{jl''l'l} g_{\rm s0} = -\frac{\mathbf{k}}{4\pi} C_{\mathbf{k},\mathbf{k}'',\mathbf{k}'}^{({\rm s})jl''l'l}, \tag{48}$$

$$\int d^3\mathbf{v}\, w_{\rm s} M_{\mathbf{k},\mathbf{k}'',\mathbf{k}'}^{jl''l'l} g_{\rm s0} = -\frac{\omega_{\mathbf{k}}}{4\pi} D_{\mathbf{k},\mathbf{k}'',\mathbf{k}'}^{({\rm s})jl''l'l}, \tag{49}$$

$$\int d^3\mathbf{v}\mathbf{p}_{\rm s}\, M_{\mathbf{k},\mathbf{k}'',\mathbf{k}'}^{jl''l'l} g_{\rm s0} = -\frac{\mathbf{k}}{4\pi} D_{\mathbf{k},\mathbf{k}'',\mathbf{k}'}^{({\rm s})jl''l'l}, \tag{50}$$

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where $w_s = \frac{1}{2} n_s m_s \nu^2$ and $\mathbf{p}_s = n_s m_s \mathbf{v}$. These relations provide the transport equations that will be demonstrated in the next section.

3.3. Transport equations

The velocity-space integration of the θ-dependent velocity-space diffusion equation multiplied by w_s or $\mathbf{p}_s$ leads to the transport equations because of the simple relations (45)–(50). Thus the transport equations showing the temporal development of the energy and momentum densities of magnetized particles of species s are derived as

$$\frac{\partial U_s}{\partial t} = -\sum_{\mathbf{k}\neq 0} 2\gamma_{\mathbf{k}}^{(s)} U_{\mathbf{k}} - \sum_{\mathbf{k}\neq 0} \sum_{\mathbf{k}'} \sum_{j,l,l',l''} \frac{\omega_{\mathbf{k}}}{4\pi} A_{\mathbf{k},\mathbf{k}'',\mathbf{k}'}^{(s)jl''l'l} E_{\mathbf{k}}^{*j} E_{\mathbf{k}}^{l} E_{\mathbf{k}'}^{*l'} E_{\mathbf{k}'}^{l''}, \tag{51}$$

$$\frac{\partial \mathbf{P}_s}{\partial t} = -\sum_{\mathbf{k}\neq 0} \frac{2\gamma_{\mathbf{k}}^{(s)} \mathbf{k}}{\omega_{\mathbf{k}}} U_{\mathbf{k}} - \sum_{\mathbf{k}\neq 0} \sum_{\mathbf{k}'} \sum_{j,l,l',l''} \frac{\mathbf{k}}{4\pi} A_{\mathbf{k},\mathbf{k}'',\mathbf{k}'}^{(s)jl''l'l} E_{\mathbf{k}}^{*j} E_{\mathbf{k}}^{l} E_{\mathbf{k}'}^{*l'} E_{\mathbf{k}'}^{l''}, \tag{52}$$

where $\mathbf{k}U_{\mathbf{k}}/\omega_{\mathbf{k}}$ is the wave momentum density, $U_s = \int d^3\mathbf{v} w_s g_s$ and $\mathbf{P}_s = \int d^3\mathbf{v} \mathbf{p}_s g_s = (P_{s\perp}, 0, P_{s\parallel})$ ($P_{s\perp} = n_s m_s \nu_d$, $P_{s\parallel} = n_s m_s \nu_{s\parallel}$) are the energy and momentum densities for particles of species s, respectively, $\gamma_{\mathbf{k}}^{(s)} = -(\omega_{\mathbf{k}}/8\pi U_{\mathbf{k}})(\mathbf{E}_{\mathbf{k}}^{*} \cdot \boldsymbol{\varepsilon}_{\mathbf{k}}^{\prime\prime(s)} \cdot \mathbf{E}_{\mathbf{k}})$ is the linear Landau and cyclotron damping rate of the electromagnetic waves ascribed to the particles of species s, $\boldsymbol{\varepsilon}_{\mathbf{k}}^{(s)} = \boldsymbol{\varepsilon}_{\mathbf{k}}^{\prime(s)} + i\boldsymbol{\varepsilon}_{\mathbf{k}}^{\prime\prime(s)}$, $\gamma_{\mathbf{k}} = \sum_s \gamma_{\mathbf{k}}^{(s)}$, and $\boldsymbol{\varepsilon}_{\mathbf{k}}^{\prime(s)}$ and $\boldsymbol{\varepsilon}_{\mathbf{k}}^{\prime\prime(s)}$ are the Hermitian and anti-Hermitian parts of $\boldsymbol{\varepsilon}_{\mathbf{k}}^{(s)}$, respectively. The transport equations (51) and (52) clearly predict that the electromagnetic waves can generate a strong particle acceleration or transport along and across the magnetic field. The first terms show that the particle acceleration in the $\mathbf{k}$ direction and heating result from Landau or cyclotron damping due to the quasilinear wave–particle interaction of the electromagnetic waves. The second terms express the particle acceleration and heating due to nonlinear Landau and cyclotron damping of the electromagnetic waves. It is seen that the particle acceleration owing to one scattering wave of the associated electromagnetic waves occurs in the $\mathbf{k}$ direction, whereas, as described in the next section, the particle acceleration due to the two scattering waves occurs in the $\mathbf{k}''$ direction. When the two scattering waves or one of them are electrostatic, the electrostatic or partially electrostatic matrix elements described in Appendix B should be used for the nonlinear wave–particle coupling coefficients in (51) and (52).

From (32), (51) and (52), as has been previously obtained in [4, 6, 7], the conservation laws for the total energy and momentum densities of waves and particles are given by

$$\frac{\partial}{\partial t}\left(\sum_s U_s + \sum_{\mathbf{k}\neq 0} U_{\mathbf{k}}\right) = 0, \tag{53}$$

$$\frac{\partial}{\partial t}\left(\sum_s \mathbf{P}_s + \sum_{\mathbf{k}\neq 0} \frac{\mathbf{k}}{\omega_{\mathbf{k}}} U_{\mathbf{k}}\right) = 0. \tag{54}$$

Equations (51)–(54) imply that the acceleration and heating of particles can be caused by the quasilinear wave–particle interaction and nonlinear wave–particle scattering of the electromagnetic waves and they are determined by means of the linear damping or growth rate and nonlinear wave–particle coupling coefficients.

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3.4. Two-wave scattering

We now consider the nonlinear scattering induced by only two electromagnetic waves interacting with a high-energy electron beam. The matrix element for nearly perpendicular propagation, which is described in Appendix A, should only be evaluated by taking the contribution from the pole of nonlinear singularity given by (A22), which is associated with the resonance condition (1). As previously proved in [4, 6, 7], the following symmetry relation of matrix elements can be verified to hold:

$$A^{(s)jl''l'l}_{\mathbf{k},\mathbf{k}'',\mathbf{k}'} = -A^{*(s)l''jll'}_{\mathbf{k}',-\mathbf{k}'',\mathbf{k}}. \tag{55}$$

In addition, we assume that

$$A^{jl''l'l}_{\mathbf{k},\mathbf{k}'',\mathbf{k}'} = A^{(b)jl''l'l}_{\mathbf{k},\mathbf{k}'',\mathbf{k}'}, \tag{56a}$$

$$\gamma_{\mathbf{k}} = \gamma^{(b)}_{\mathbf{k}}, \tag{56b}$$

$$A^{(s)jl''l'l}_{\mathbf{k},\mathbf{k}'',\mathbf{k}'} = \gamma^{(s)}_{\mathbf{k}} = 0, \quad \text{for } s \neq b, \tag{56c}$$

where b designates the beam electrons. This assumption means that the linear damping and nonlinear wave–particle scattering of two electromagnetic waves only originate in the high-energy beam electrons. Considering the condition given by (55) and (56), we get the simple expressions for the kinetic wave equations of two electromagnetic waves and the transport equations of the beam electrons, and they are represented as

$$\frac{\partial U_{\mathbf{k}}}{\partial t} = 2\gamma_{\mathbf{k}} U_{\mathbf{k}} - A_0 U_{\mathbf{k}} U_{\mathbf{k}'}, \tag{57}$$

$$\frac{\partial U_{\mathbf{k}'}}{\partial t} = 2\gamma_{\mathbf{k}'} U_{\mathbf{k}'} + \frac{\omega_{\mathbf{k}'}}{\omega_{\mathbf{k}}} A_0 U_{\mathbf{k}} U_{\mathbf{k}'}, \tag{58}$$

$$\frac{\partial U_{\mathrm{b}}}{\partial t} = -2\gamma_{\mathbf{k}} U_{\mathbf{k}} - 2\gamma_{\mathbf{k}'} U_{\mathbf{k}'} + \frac{\omega_{\mathbf{k}''}}{\omega_{\mathbf{k}}} A_0 U_{\mathbf{k}} U_{\mathbf{k}'}, \tag{59}$$

$$\frac{\partial \mathbf{P}_{\mathrm{b}}}{\partial t} = -\frac{2\gamma_{\mathbf{k}}\mathbf{k}}{\omega_{\mathbf{k}}} U_{\mathbf{k}} - \frac{2\gamma_{\mathbf{k}'}\mathbf{k}'}{\omega_{\mathbf{k}'}} U_{\mathbf{k}'} + \frac{\mathbf{k}''}{\omega_{\mathbf{k}}} A_0 U_{\mathbf{k}} U_{\mathbf{k}'}, \tag{60}$$

where

$$U_{\mathbf{k}} = \Gamma_{\mathbf{k}} |E_{\mathbf{k}}|^2,$$

$$A_0 = -\frac{\omega_{\mathbf{k}}}{4\pi \Gamma_{\mathbf{k}} \Gamma_{\mathbf{k}'}} \sum_{j,l,l',l''} A^{jl''l'l}_{\mathbf{k},\mathbf{k}'',\mathbf{k}'} \rho^{*j}_{\mathbf{k}} \rho^{l}_{\mathbf{k}} \rho^{*l'}_{\mathbf{k}'} \rho^{l''}_{\mathbf{k}'},$$

$$\Gamma_{\mathbf{k}} = \frac{1}{8\pi} \sum_{j,l} \left(\frac{\partial}{\partial \omega_{\mathbf{k}}} \left[\left(\varepsilon'^{jl}_{\mathbf{k}} - N^{jl}_{\mathbf{k}} \right) \omega_{\mathbf{k}} \right] \right) \rho^{*j}_{\mathbf{k}} \rho^{l}_{\mathbf{k}}, \tag{61a}$$

$$E^{j}_{\mathbf{k}} = \rho^{j}_{\mathbf{k}} E_{\mathbf{k}}, \quad \sum_{j} \rho^{*j}_{\mathbf{k}} \rho^{j}_{\mathbf{k}} = 1. \tag{61b, c}$$

Here, $\rho^{j}_{\mathbf{k}}$ indicates the polarizations of two waves. Immediately it is found that (57)–(60) lead to the conservation laws for total energy and momentum densities of two electromagnetic waves and a high-energy electron beam,

$$\frac{\partial}{\partial t}(U_{\mathrm{b}} + U_{\mathbf{k}} + U_{\mathbf{k}'}) = 0, \tag{62}$$

$$\frac{\partial}{\partial t}\left(\mathbf{P}_{\mathrm{b}} + \frac{\mathbf{k}}{\omega_{\mathbf{k}}} U_{\mathbf{k}} + \frac{\mathbf{k}'}{\omega_{\mathbf{k}'}} U_{\mathbf{k}'} \right) = 0. \tag{63}$$

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When the linear damping is absent ($\gamma_{\mathbf{k}} = \gamma_{\mathbf{k}'} = 0$), i.e. only nonlinear scattering contributes to acceleration and heating of the electron beam, the following features can be stated under the condition of $\omega_{\mathbf{k}''}, k''_{\|} > 0$. For the positive value of A_0, the usual nonlinear scattering occurs ($\partial U_{\mathbf{k}}/\partial t < 0, \partial U_{\mathbf{k}'}/\partial t > 0$) and the electron beam is accelerated in the $\mathbf{k}''$ direction ($\partial P_{\mathrm{b}\|}/\partial t > 0$). In contrast, for the negative value of A_0, the nonlinear scattering of energy up-conversion in frequency takes place ($\partial U_{\mathbf{k}}/\partial t > 0, \partial U_{\mathbf{k}'}/\partial t < 0$) [26–28] and the electron beam is decelerated in the $\mathbf{k}''$ direction ($\partial P_{\mathrm{b}\|}/\partial t < 0$).

Because U_{b} and $\mathbf{P}_{\mathrm{b}}$ are given by

$$U_{\mathrm{b}} = \tfrac{1}{2} n_{\mathrm{b}} m_{\mathrm{e}} \left(\nu_{\mathrm{b}}^2 + \nu_{\mathrm{d}}^2 + \tfrac{3}{2}\nu_{\mathrm{tb}}^2\right),$$

$$\mathbf{P}_{\mathrm{b}} = (n_{\mathrm{b}} m_{\mathrm{e}} \nu_{\mathrm{d}}, 0, n_{\mathrm{b}} m_{\mathrm{e}} \nu_{\mathrm{b}}),$$

the transport equations (59) and (60) yield the following equations:

$$\frac{\partial}{\partial t}\left[\frac{1}{2} n_{\mathrm{b}} m_{\mathrm{e}} \left(\nu_{\mathrm{b}}^2 + \nu_{\mathrm{d}}^2\right)\right] = -2\gamma_{\mathbf{k}} \frac{k_{\perp}\nu_{\mathrm{d}} + k_{\|}\nu_{\mathrm{b}}}{\omega_{\mathbf{k}}} U_{\mathbf{k}} - 2\gamma_{\mathbf{k}'} \frac{k'_{\perp}\nu_{\mathrm{d}} + k'_{\|}\nu_{\mathrm{b}}}{\omega_{\mathbf{k}'}} U_{\mathbf{k}'} + \frac{k''_{\perp}\nu_{\mathrm{d}} + k''_{\|}\nu_{\mathrm{b}}}{\omega_{\mathbf{k}}} A_0 U_{\mathbf{k}} U_{\mathbf{k}'}, \tag{64}$$

$$\frac{\partial}{\partial t}\left(\frac{3}{4} n_{\mathrm{b}} m_{\mathrm{e}} \nu_{\mathrm{tb}}^2\right) = -2\gamma_{\mathbf{k}} \frac{\omega_{\mathbf{k}} - k_{\perp}\nu_{\mathrm{d}} - k_{\|}\nu_{\mathrm{b}}}{\omega_{\mathbf{k}}} U_{\mathbf{k}} - 2\gamma_{\mathbf{k}'} \frac{\omega_{\mathbf{k}'} - k'_{\perp}\nu_{\mathrm{d}} - k'_{\|}\nu_{\mathrm{b}}}{\omega_{\mathbf{k}'}} U_{\mathbf{k}'} + \frac{\omega_{\mathbf{k}''} - k''_{\perp}\nu_{\mathrm{d}} - k''_{\|}\nu_{\mathrm{b}}}{\omega_{\mathbf{k}}} A_0 U_{\mathbf{k}} U_{\mathbf{k}'}, \tag{65}$$

where $\nu_{\mathrm{tb}} = (2k_{\mathrm{B}}T_{\mathrm{b}}/m_{\mathrm{e}})^{1/2}$. Accordingly, it is seen that (64) shows the acceleration and deceleration of the electron beam and (65) its heating and cooling.

3.5. *Compton scattering*

Finally, we investigate the nonlinear wave–particle coupling coefficient A_0 for the Compton scattering of the almost perpendicularly propagating electromagnetic waves interacting nonlinearly with a high-energy cold electron beam whose Larmor radius is zero. It is assumed that only Compton scattering coming from the first term of (36) contributes to A_0 and the plasma shielding effect resulting from the second term of (36) which is associated with the beat-waves is negligibly small compared with Compton scattering; that is,

$$\left|\sum_{j,l,l',l''} C^{(\mathrm{s})jl''l'l}_{\mathbf{k},\mathbf{k}'',\mathbf{k}'} \rho^{*j}_{\mathbf{k}} \rho^{l}_{\mathbf{k}} \rho^{*l'}_{\mathbf{k}'} \rho^{l''}_{\mathbf{k}'}\right| \gg \left|\sum_{j,l,l',l} D^{(\mathrm{s})jl''l'l}_{\mathbf{k},\mathbf{k}'',\mathbf{k}'} \rho^{*j}_{\mathbf{k}} \rho^{l}_{\mathbf{k}} \rho^{*l'}_{\mathbf{k}'} \rho^{l''}_{\mathbf{k}'}\right|.$$

This means that the beat-waves are absent as is assumed in the previous section. In a previous work [11], it was confirmed that the above condition is satisfied in the numerical analysis. We only treat the cases of $m = 0, \pm 1$, since we need the comparison with the results in Sec. 2 and the electron-beam acceleration due to the Compton scattering of $|m| \geqslant 2$ may be considered to be extremely weak compared with that of $|m| \leqslant 1$ when $k_{\perp}\nu_{\mathrm{tb}}/\omega_{\mathrm{ce}} \ll 1$.

First, we consider the Compton scattering of $m = 0$. It can be easily verified that only the terms $q = m = 0$ and $q' = r = 0$ contribute to $C^{(b)jl''l'l}_{\mathbf{k},\mathbf{k}'',\mathbf{k}'}$, and the matrix elements are given by the modified expression (24) of [6]. Although they

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contain many complicated terms, their roughly approximated expression is simple and shown by

$$A_0 \cong -\frac{1}{4\Gamma_{\mathbf{k}}\Gamma_{\mathbf{k}'}}\frac{\omega_{\mathrm{pb}}^2 e^2 k_\parallel''}{\omega_{\mathbf{k}}\omega_{\mathbf{k}'}^2 m_{\mathrm{e}}^2}\int d^3\mathbf{v}\,\delta(k_\parallel''\nu_\parallel + k_\parallel''\nu_{\mathrm{d}} - \omega_{\mathbf{k}''})\frac{\partial g_{\mathrm{b}}}{\partial \nu_\parallel}. \tag{66}$$

When g_{b} is given by (22), A_0 becomes

$$A_0 \cong \frac{\omega_{\mathrm{pb}}^2 e^2 k_\parallel''}{2\pi^{1/2}\Gamma_{\mathbf{k}}\Gamma_{\mathbf{k}'}\omega_{\mathbf{k}}\omega_{\mathbf{k}'}^2 m_{\mathrm{e}}^2 |k_\parallel''|\nu_{\mathrm{tb}}^2}\zeta_{0\mathbf{k}''}\exp\left(-\varsigma_{0\mathbf{k}''}^2\right). \tag{67}$$

Thus, the transport equation (60) with $\gamma_{\mathbf{k}} = \gamma_{\mathbf{k}'} = 0$ can be shown by

$$\frac{\partial \mathbf{P}_{\mathrm{b}}}{\partial t} = \frac{\omega_{\mathrm{pb}}^2 e^2 k_\parallel''\mathbf{k}''}{2\pi^{1/2}\omega_{\mathbf{k}}^2\omega_{\mathbf{k}'}^2 m_{\mathrm{e}}^2 |k_\parallel''|\nu_{\mathrm{tb}}^2}\varsigma_{0\mathbf{k}''}\exp\left(-\varsigma_{0\mathbf{k}''}^2\right)|E_{\mathbf{k}}||^2|E_{\mathbf{k}'}|^2. \tag{68}$$

Considering that $A_{\mathbf{k},\mathbf{k}'}$ in (23) can be roughly approximated such that $|A_{\mathbf{k},\mathbf{k}'}|^2 \sim (k_\parallel''^2/\omega_{\mathbf{k}}^2\omega_{\mathbf{k}'}^2)|E_{\mathbf{k}}|^2|E_{\mathbf{k}'}|^2$, it can be seen that (68) coincides approximately with (23).

Second, we consider the Compton scattering of $m = \pm 1$. It can be similarly found that only the terms $q = m = \pm 1$ and $q', r = 0, \pm 1$ contribute to $C_{\mathbf{k},\mathbf{k}'',\mathbf{k}'}^{(\mathrm{b})jl''l'l}$. Then the matrix element derived from (24) of [6] can be approximated roughly and is represented as

$$A_0 \cong -\frac{1}{4\Gamma_{\mathbf{k}}\Gamma_{\mathbf{k}'}}\frac{\omega_{\mathrm{pb}}^2 e^2}{\omega_{\mathbf{k}}\omega_{\mathbf{k}'}^2 m_{\mathrm{e}}^2}\int d^3\mathbf{v}\,\delta(k_\parallel''\nu_\parallel + k_\perp''\nu_{\mathrm{d}} - \omega_{\mathbf{k}''} \pm \omega_{\mathrm{ce}})\frac{k_\perp''^2\nu_\perp}{4\omega_{\mathrm{ce}}}\frac{\partial g_{\mathrm{b}}}{\partial \nu_\perp}. \tag{69}$$

For g_{b} given by (22), A_0 becomes

$$A_0 \cong \frac{\omega_{\mathrm{pb}}^2 e^2 k_\perp''^2}{8\pi^{1/2}\Gamma_{\mathbf{k}}\Gamma_{\mathbf{k}'}\omega_{\mathbf{k}}\omega_{\mathbf{k}'}^2 m_{\mathrm{e}}^2\omega_{\mathrm{ce}}|k_\parallel''|\nu_{\mathrm{tb}}}\exp\left(-\varsigma_{m\mathbf{k}''}^2\right). \tag{70}$$

Thus, the transport equation (60) with $\gamma_{\mathbf{k}} = \gamma_{\mathbf{k}'} = 0$ becomes

$$\frac{\partial \mathbf{P}_{\mathrm{b}}}{\partial t} = \frac{\omega_{\mathrm{pb}}^2 e^2 k_\perp''^2\mathbf{k}''}{8\pi^{1/2}\omega_{\mathbf{k}}^2\omega_{\mathbf{k}'}^2 m_{\mathrm{e}}^2\omega_{\mathrm{ce}}|k_\parallel''|\nu_{\mathrm{tb}}}\exp\left(-\varsigma_{m\mathbf{k}''}^2\right)|E_{\mathbf{k}}|^2|E_{\mathbf{k}'}|^2. \tag{71}$$

Because $F_{\mathbf{k},\mathbf{k}'}$ in (28) can be approximated as $|F_{\mathbf{k},\mathbf{k}'}|^2 \sim (k_\perp''^2/\omega_{\mathbf{k}}^2\omega_{\mathbf{k}'}^2)|E_{\mathbf{k}}|^2|E_{\mathbf{k}'}|^2$, we can similarly find that (71) approximately coincides with (28). The ratio of acceleration rates for the Compton scattering of $m = \pm 1$ and $m = 0$ is also found to be given by (31), and it is very small since $|k_\perp''^2\nu_{\mathrm{tb}}/k_\parallel''\omega_{\mathrm{ce}}| \ll 1$. Consequently, the acceleration and deceleration due to the Compton scattering of $m = 0$ are much stronger than those of $m = \pm 1$. Here, the absolute value of ω_{cs} ($e_{\mathrm{s}} = -e$) is used for ω_{ce}.

By using the nonlinear wave–particle coupling coefficient A_0 in which the polarizations and dispersion relations of the extraordinary waves given by (3) are taken into account, the transport equations (68) and (71) can be applied for the Compton scattering of extraordinary waves. The approximate transport equations can be obtained by considering the polarizations of the extraordinary waves in the terms of $|E_{\mathbf{k}}|^2|E_{\mathbf{k}'}|^2$ in (68) and (71). In conclusion, it can be demonstrated that the same results as those from the single-particle theory are obtained from Vlasov–Maxwell equations.

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4. Summary

High-energy or relativistic electron-beam acceleration along and across a magnetic field, and the generation of an electric field transverse to the magnetic field, both induced by the Compton scattering of almost perpendicularly propagating extraordinary waves, have been investigated theoretically based on single-particle theory and Vlasov–Maxwell equations. The transport equations derived independently from both theories show that the electron beam can be accelerated and decelerated in the $\mathbf{k}''$ direction via Compton scattering induced by the nonlinear Landau damping of extraordinary waves. Simultaneously, an intense cross-field electric field $\mathbf{E}_0 = \mathbf{B}_0 \times \mathbf{v}_{\mathrm{d}}/c$ is created via the dynamo effect owing to the perpendicular drift of the electron beam to satisfy the generalized Ohm's law. This implies that the cross-field drift of the electron beam is equivalent to $\mathbf{E} \times \mathbf{B}$ drift. It is easy and straightforward to understand the physical mechanism of the electron beam acceleration and the generation of the cross-field electric field on the basis of single-particle theory, although the transport equations obtained are not general and rigorously exact. In contrast, the general and rigorously exact expressions for the transport equations can be derived from Vlasov–Maxwell equations, although it is rather difficult to understand the physical mechanism. The transport equations derived from both theories agree satisfactorily in the approximated expressions for the Compton scattering of $m = 0, \pm 1$.

It has been clarified that the efficient acceleration or deceleration of the electron beam can occur via the Compton scattering of $m = 0$, corresponding to whether the Doppler-shifted phase velocity of the beat-wave is slightly larger or smaller than the parallel velocity of the electron beam, respectively, as was shown in previous work. On the other hand, the electron beam is always accelerated in the $\mathbf{k}''$ direction via the Compton scattering of $m = \pm 1$ and its acceleration rate is much less than that of $m = 0$. As was previously shown, the consideration of the polarizations of extraordinary waves predicts that the electron-beam acceleration is efficient for wave frequencies lower than the upper-hybrid frequency or exceeding the right-hand cutoff frequency. It can be found easily from the single-particle theory that for frequencies lower than the upper-hybrid frequency, the electron beam can be accelerated by the pondermotive force driven by the $\mathbf{k}$ component of the wave electric field and for frequencies exceeding the right-hand cutoff frequency, the electron beam can be accelerated by the nonlinear $\mathbf{v} \times \mathbf{B}$ Lorentz force driven by the $\mathbf{k}$ component of the wave magnetic field or the $\mathbf{k} \times \mathbf{B}_0$ component of the wave electric field.

In conclusion, the Compton scattering of extraordinary waves can be useful for the efficient acceleration of the high-energy or relativistic electron beam to the speed of light in a magnetically confined plasma. In addition, it is suggested that special consideration should be taken for the generated cross-field electric field, because this electric field seriously affects the profile and stability of the magnetized plasma. A more rigorous theoretical analysis, including the relativistic effect, should be performed in a future study.

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Appendix A. Nonlinear wave–particle coupling coefficients

The nonlinear wave–particle coupling coefficients in (36) are expressed as follows:

$$C^{(\mathrm{s})jl''l'l}_{\mathbf{k},\mathbf{k}'',\mathbf{k}'} = \frac{\omega_{\mathrm{ps}}^2}{\omega_{\mathbf{k}}^2\omega_{\mathbf{k}'}\omega_{-\mathbf{k}'}}\left(\frac{e_{\mathrm{s}}}{m_{\mathrm{s}}}\right)^2 \sum_{q',q,r=-\infty} \int d^3\mathbf{v}\frac{\left[u^{*j}_{\mathbf{k}}(\mathbf{v})J_{q'}(\mu_{\mathbf{k}})\right]W^{l''}_{\mathbf{k}',\mathbf{k}''}(\mathbf{v})}{k_\parallel\nu_\parallel + k_\perp\nu_{\mathrm{d}} - \omega_{\mathbf{k}} + q'\omega_{\mathrm{cs}}}$$
$$\times \frac{1}{k''_\parallel\nu_\parallel + k''_\perp\nu_{\mathrm{d}} - \omega_{\mathbf{k}''} + q\omega_{\mathrm{cs}}}$$
$$\times\left[S^{l}_{\mathbf{k},-\mathbf{k}'}(\mathbf{v})Z^{l'}_{-\mathbf{k}',r}(\mathbf{v})g_{\mathrm{s}0} + S^{l'}_{-\mathbf{k}',\mathbf{k}}(\mathbf{v})Z^{l}_{\mathbf{k},r}(\mathbf{v})g_{\mathrm{s}0}\right], \tag{A 1}$$

$$D^{(\mathrm{s})jl''l'l}_{\mathbf{k},\mathbf{k}'',\mathbf{k}'} = \sum_{j'}\left(\alpha^{(\mathrm{s})jj'l''}_{\mathbf{k}'',\mathbf{k}'} + \alpha^{(\mathrm{s})jl''j'}_{\mathbf{k}',\mathbf{k}''}\right)\left(\beta^{j'll'}_{\mathbf{k},-\mathbf{k}'} + \beta^{j'l'l}_{-\mathbf{k}',\mathbf{k}}\right), \tag{A 2}$$

$$\boldsymbol{\beta}^{ll'}_{\mathbf{k}'',\mathbf{k}'} = (\boldsymbol{\varepsilon}_{\mathbf{k}} - \mathbf{N}_{\mathbf{k}})^{-1}\boldsymbol{\cdot}\boldsymbol{\alpha}^{ll'}_{\mathbf{k}'',\mathbf{k}'}, \tag{A 3}$$

$$\alpha^{jll'}_{\mathbf{k}'',\mathbf{k}'} = \sum_{\mathrm{s}}\alpha^{(\mathrm{s})jll'}_{\mathbf{k}'',\mathbf{k}'}, \tag{A 4}$$

$$\alpha^{(\mathrm{s})jll'}_{\mathbf{k}'',\mathbf{k}'} = \frac{\omega_{\mathrm{ps}}^2}{\omega_{\mathbf{k}}\omega_{\mathbf{k}'}\omega_{\mathbf{k}''}}\frac{e_{\mathrm{s}}}{m_{\mathrm{s}}}\sum_{q,r=-\infty}^{\infty}\int d^3\mathbf{v}\frac{\left[u^{*j}_{\mathbf{k}}(\mathbf{v})J_q(\mu_{\mathbf{k}})\right]S^{l}_{\mathbf{k}'',\mathbf{k}'}(\mathbf{v})}{k_\parallel\nu_\parallel + k_\perp\nu_{\mathrm{d}} - \omega_{\mathbf{k}} + q\omega_{\mathrm{cs}}}Z^{l'}_{\mathbf{k}',r}(\mathbf{v})g_{\mathrm{s}0}, \tag{A 5}$$

and

$$\boldsymbol{\alpha}^{ll'}_{\mathbf{k}'',\mathbf{k}'} = \left(\alpha^{xll'}_{\mathbf{k}'',\mathbf{k}'}, \alpha^{yll'}_{\mathbf{k}'',\mathbf{k}'}, \alpha^{zll'}_{\mathbf{k}'',\mathbf{k}'}\right),$$
$$\boldsymbol{\beta}^{ll'}_{\mathbf{k}'',\mathbf{k}'} = \left(\beta^{xll'}_{\mathbf{k}'',\mathbf{k}'}, \beta^{yll'}_{\mathbf{k}'',\mathbf{k}'}, \beta^{zll'}_{\mathbf{k}'',\mathbf{k}'}\right).$$

The differential operators S, W and Z are defined by

$$S^{l}_{\mathbf{k}'',\mathbf{k}'}(\mathbf{v}) = J_{q-r}(\mu_{\mathbf{k}''})P^{l}_{\mathbf{k}'',\mathbf{k}'}(\mathbf{v}) + J'_{q-r}(\mu_{\mathbf{k}''})Q^{l}_{\mathbf{k}'',\mathbf{k}'}(\mathbf{v}) + J''_{q-r}(\mu_{\mathbf{k}''})R^{l}_{\mathbf{k}'',\mathbf{k}'}(\mathbf{v}), \tag{A 6}$$

$$P^{x}_{\mathbf{k}'',\mathbf{k}'}(\mathbf{v}) = (q-r)\omega_{\mathrm{cs}}\left[\frac{k''_\parallel}{k''_\perp}\frac{\partial}{\partial v_\parallel} + \frac{\omega_{\mathbf{k}''} - k''_\perp\nu_{\mathrm{d}} - k''_\parallel\nu_\parallel}{k''_\perp\nu_\perp}\frac{\partial}{\partial\nu_\perp}\right] + \nu_{\mathrm{d}}U_{q-r}(\mathbf{k}''), \tag{A 7}$$

$$P^{y}_{\mathbf{k}'',\mathbf{k}'}(\mathbf{v}) = -i\left[(q-r)\omega_{\mathrm{cs}}\frac{\omega_{\mathbf{k}''} - k''_\perp\nu_{\mathrm{d}} - k''_\parallel\nu_\parallel}{(k''_\perp\nu_\perp)^2} - 1\right](k_\perp r - k'_\perp q), \tag{A 8}$$

$$P^{z}_{\mathbf{k}'',\mathbf{k}'}(\mathbf{v}) = [\omega_{\mathbf{k}''} - k''_\perp\nu_{\mathrm{d}} - (q-r)\omega_{\mathrm{cs}}]\frac{\partial}{\partial\nu_\parallel} + (q-r)\omega_{\mathrm{cs}}\frac{\nu_\parallel}{\nu_\perp}\frac{\partial}{\partial\nu_\perp}, \tag{A 9}$$

$$Q^{x}_{\mathbf{k}'',\mathbf{k}'}(\mathbf{v}) = -\frac{r}{\nu_\perp}(\omega_{\mathbf{k}''} - k''_\parallel\nu_\parallel), \tag{A 10}$$

$$Q^{y}_{\mathbf{k}'',\mathbf{k}'}(\mathbf{v}) = i\left[k''_\parallel\nu_\perp\frac{\partial}{\partial\nu_\parallel} + (\omega_{\mathbf{k}''} - k''_\perp\nu_{\mathrm{d}} - k''_\parallel\nu_\parallel)\left(\frac{\partial}{\partial\nu_\perp} - \frac{k'_\perp}{k''_\perp\nu_\perp}\right)\right], \tag{A 11}$$

$$Q^{z}_{\mathbf{k}'',\mathbf{k}'}(\mathbf{v}) = -k''_\perp r\frac{\nu_\parallel}{\nu_\perp}, \tag{A 12}$$

$$R^{x}_{\mathbf{k}'',\mathbf{k}'}(\mathbf{v}) = R^{z}_{\mathbf{k}'',\mathbf{k}'}(\mathbf{v}) = 0, \tag{A 13}$$

$$R^{y}_{\mathbf{k}'',\mathbf{k}'}(\mathbf{v}) = -i\frac{k'_\perp}{\omega_{\mathrm{cs}}}(\omega_{\mathbf{k}''} - k''_\perp\nu_{\mathrm{d}} - k''_\parallel\nu_\parallel), \tag{A 14}$$

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$$U_r(\mathbf{k}) = k_\parallel \frac{\partial}{\partial \nu_\parallel} + \frac{r\omega_{\rm cs}}{\nu_\perp}\frac{\partial}{\partial \nu_\perp}, \tag{A 15}$$

$$W^{l''}_{\mathbf{k}',\mathbf{k}''}(\mathbf{v}) = \left[S^{l''}_{\mathbf{k}',\mathbf{k}''}(\mathbf{v})\right]_{\substack{q\to q'\\ r\to q}}, \tag{A 16}$$

$$Z^l_{\mathbf{k},r}(\mathbf{v}) = \frac{\left[w^l_{\mathbf{k}}(\mathbf{v})J_r(\mu_{\mathbf{k}})\right]Y^l_{\mathbf{k},r}(\mathbf{v}) + s_l\nu_{\rm d}J_r(\mu_{\mathbf{k}})U_r(\mathbf{k})}{k_\parallel\nu_\parallel + k_\perp\nu_{\rm d} - \omega_{\mathbf{k}} + r\omega_{\rm cs}}, \tag{A 17}$$

$$Y^x_{\mathbf{k},r}(\mathbf{v}) = Y^y_{\mathbf{k},r}(\mathbf{v}) = k_\parallel\nu_\perp\frac{\partial}{\partial \nu_\parallel} + (\omega_{\mathbf{k}} - k_\perp\nu_{\rm d} - k_\parallel\nu_\parallel)\frac{\partial}{\partial \nu_\perp}, \tag{A 18}$$

$$Y^z_{\mathbf{k},r}(\mathbf{v}) = (\omega_{\mathbf{k}} - k_\perp\nu_{\rm d} - r\omega_{\rm cs})\frac{\partial}{\partial \nu_\parallel} + r\omega_{\rm cs}\frac{\nu_\parallel}{\nu_\perp}\frac{\partial}{\partial \nu_\perp}, \tag{A 19}$$

$$u^x_{\mathbf{k}}(\mathbf{v})J_r(\mu_{\mathbf{k}}) = \left(\frac{r\nu_\perp}{\mu_{\mathbf{k}}} + \nu_{\rm d}\right)J_r(\mu_{\mathbf{k}}), \quad u^y_{\mathbf{k}}(\mathbf{v}) = i\nu_\perp\frac{\partial}{\partial \mu_{\mathbf{k}}}, \quad u^z_{\mathbf{k}}(\mathbf{v}) = \nu_\parallel, \tag{A 20}$$

$$w^x_{\mathbf{k}}(\mathbf{v})J_r(\mu_{\mathbf{k}}) = \frac{r}{\mu_{\mathbf{k}}}J_r(\mu_{\mathbf{k}}), \quad w^y_{\mathbf{k}}(\mathbf{v}) = i\frac{\partial}{\partial \mu_{\mathbf{k}}}, \quad w^z_{\mathbf{k}}(\mathbf{v}) = 1, \tag{A 21}$$

where $s_x = 1$, $s_y = s_z = 0$, and J'_r and J''_r are the first and second derivatives with respect to the argument, respectively. It is easily found that when $\nu_{\rm d} = 0$, (A 1) and (A 2) are reduced to the previous results [4, 6, 7].

The matrix elements for nearly perpendicular propagation can be derived from evaluation of the anti-Hermitian parts of (A 1) and (A 2) by using the following relation,

$$\mathrm{Im}\frac{1}{k''_\parallel\nu_\parallel + k''_\perp\nu_{\rm d} - \omega_{\mathbf{k}''} + m\omega_{\rm cs}} = \pi\delta(k''_\parallel\nu_\parallel + k''_\perp\nu_{\rm d} - \omega_{\mathbf{k}''} + m\omega_{\rm cs}). \tag{A 22}$$

They are given by (24) and (25) in [6] where $\omega_{\mathbf{k}}$ and $\omega_{\mathbf{k}''}$ in $R^{jl''}_{\mathbf{k},\mathbf{k}'}$, $S^{ll'}_{\mathbf{k},\mathbf{k}'}$ and $\delta(k''_\parallel\nu_\parallel - \omega_{\mathbf{k}''} + m\omega_{\rm cs})$ should be replaced by $\omega_{\mathbf{k}} - k_\perp\nu_{\rm d}$ and $\omega_{\mathbf{k}''} - k''_\perp\nu_{\rm d}$, respectively, $g_{\rm s}$ is replaced by $g_{\rm s0}$, $a^l_{\mathbf{k},r}$, $b^l_{\mathbf{k}}$, and $\mathbf{u}_{\mathbf{k}''}(\mathbf{v})$ are provided by (42) and (A 20), and $\psi^l_{\mathbf{k},r}$ should be defined as follows:

$$\psi^x_{\mathbf{k},r} = \frac{k_\parallel}{k_\perp}(r\omega_{\rm cs} + k_\perp\nu_{\rm d})\frac{\partial}{\partial \nu_\parallel} + \frac{r\omega_{\rm cs}}{k_\perp\nu_\perp}(\omega_{\mathbf{k}} - k_\parallel\nu_\parallel)\frac{\partial}{\partial \nu_\perp}, \tag{A 23}$$

$$\psi^y_{\mathbf{k},r} = \omega_{\rm cs}\left(\frac{k_\parallel}{k_\perp}\frac{\partial}{\partial \nu_\parallel} + \frac{\omega_{\mathbf{k}} - k_\perp\nu_{\rm d} - k_\parallel\nu_\parallel}{k_\perp\nu_\perp}\frac{\partial}{\partial \nu_\perp}\right), \tag{A 24}$$

$$\psi^z_{\mathbf{k},r} = (\omega_{\mathbf{k}} - k_\perp\nu_{\rm d} - r\omega_{\rm cs})\frac{\partial}{\partial \nu_\parallel} + r\omega_{\rm cs}\frac{\nu_\parallel}{\nu_\perp}\frac{\partial}{\partial \nu_\perp}, \tag{A 25}$$

where $\mathbf{u}_{\mathbf{k}''}(\mathbf{v}) = (u^x_{\mathbf{k}''}(\mathbf{v}), u^y_{\mathbf{k}''}(\mathbf{v}), u^z_{\mathbf{k}''}(\mathbf{v}))$.

Appendix B. Electrostatic and partially electrostatic nonlinear wave–particle coupling coefficients

When two scattering waves are electrostatic ($\mathbf{E}_{\mathbf{k}} = (\mathbf{k}/k)E_{\mathbf{k}}$, $\mathbf{E}_{\mathbf{k}'} = (\mathbf{k}'/k')E_{\mathbf{k}'}$), the matrix elements become as follows:

$$A_{\mathbf{k},\mathbf{k}'',\mathbf{k}'} = \sum_{\rm s} A^{(\rm s)}_{\mathbf{k},\mathbf{k}'',\mathbf{k}'}, \tag{B 1}$$

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$$A^{(\mathrm{s})}_{\mathbf{k},\mathbf{k}'',\mathbf{k}'} = \sum_{j,l,l',l''} \frac{k_j k'_{l''} k'_{l'} k_l}{k^2 k'^2} A^{(\mathrm{s})jl''l'l}_{\mathbf{k},\mathbf{k}'',\mathbf{k}'}$$

$$= \mathrm{Im}\big(C^{(\mathrm{s})}_{\mathbf{k},\mathbf{k}'',\mathbf{k}'} + D^{(\mathrm{s})}_{\mathbf{k},\mathbf{k}'',\mathbf{k}'}\big), \tag{B 2}$$

$$C^{(\mathrm{s})}_{\mathbf{k},\mathbf{k}'',\mathbf{k}'} = \frac{\omega^2_{\mathrm{ps}}}{k^2 k'^2}\left(\frac{e_\mathrm{s}}{m_\mathrm{s}}\right)^2 \sum_{q',q,r=-\infty} \int d^3\mathbf{v} \frac{J_{q'}(\mu_\mathbf{k}) W_{\mathbf{k}'}(\mathbf{v})}{k_\parallel \nu_\parallel + k_\perp \nu_\mathrm{d} - \omega_\mathbf{k} + q'\omega_\mathrm{cs}}$$

$$\times \frac{1}{k''_\parallel \nu_\parallel + k''_\perp \nu_\mathrm{d} - \omega_{\mathbf{k}''} + q\omega_\mathrm{cs}}$$

$$\times [S_{-\mathbf{k}'}(\mathbf{v}) Z_{\mathbf{k},r}(\mathbf{v}) g_{\mathrm{s}0} + S_\mathbf{k}(\mathbf{v}) Z_{-\mathbf{k}',r}(\mathbf{v}) g_{\mathrm{s}0}], \tag{B 3}$$

$$D^{(\mathrm{s})}_{\mathbf{k},\mathbf{k}'',\mathbf{k}'} = \frac{1}{\varepsilon_{\mathbf{k}''}}\big(\alpha^{(\mathrm{s})}_{\mathbf{k}'',\mathbf{k}'} + \alpha^{(\mathrm{s})}_{\mathbf{k}',\mathbf{k}''}\big)(\alpha_{\mathbf{k},-\mathbf{k}'} + \alpha_{-\mathbf{k}',\mathbf{k}}), \tag{B 4}$$

$$\alpha_{\mathbf{k}'',\mathbf{k}'} = \sum_\mathrm{s} \alpha^{(\mathrm{s})}_{\mathbf{k}'',\mathbf{k}'}, \tag{B 5}$$

$$\alpha^{(\mathrm{s})}_{\mathbf{k}'',\mathbf{k}'} = \frac{\omega^2_{\mathrm{ps}}}{kk'k''}\frac{e_\mathrm{s}}{m_\mathrm{s}} \sum_{q,r=-\infty}^{\infty} \int d^3\mathbf{v} \frac{J_q(\mu_\mathbf{k}) S_{\mathbf{k}''}(\mathbf{v})}{k_\parallel \nu_\parallel + k_\perp \nu_\mathrm{d} - \omega_\mathbf{k} + q\omega_\mathrm{cs}} Z_{\mathbf{k}',r}(\mathbf{v}) g_{\mathrm{s}0}, \tag{B 6}$$

$$\varepsilon_\mathbf{k} = 1 + \sum_\mathrm{s} \varepsilon^{(\mathrm{s})}_\mathbf{k}, \tag{B 7}$$

$$\varepsilon^{(\mathrm{s})}_\mathbf{k} = -\frac{\omega^2_{\mathrm{ps}}}{k^2} \sum_{r=-\infty}^{\infty} \int d^3\mathbf{v} \frac{J^2_r(\mu_\mathbf{k}) U_r(\mathbf{k}) g_{\mathrm{s}0}}{k_\parallel \nu_\parallel + k_\perp \nu_\mathrm{d} - \omega_\mathbf{k} + r\omega_\mathrm{cs}}. \tag{B 8}$$

The differential operators S, W and Z are defined by

$$S_\mathbf{k}(\mathbf{v}) = \frac{1}{\omega_\mathbf{k}} \sum_l k_l S^l_{\mathbf{k},-\mathbf{k}'}(\mathbf{v})$$

$$= J_{q-r}(\mu_\mathbf{k}) U_{q-r}(\mathbf{k}) - J'_{q-r}(\mu_\mathbf{k}) \frac{k_\perp r}{\nu_\perp}, \tag{B 9}$$

$$W_{\mathbf{k}'}(\mathbf{v}) = [S_{\mathbf{k}'}(\mathbf{v})]_{\substack{q\to q' \\ r\to q}}, \tag{B 10}$$

$$Z_{\mathbf{k},r}(\mathbf{v}) = \frac{1}{\omega_\mathbf{k}} \sum_l k_l Z^l_{\mathbf{k},r}(\mathbf{v})$$

$$= \frac{J_r(\mu_\mathbf{k}) U_r(\mathbf{k})}{k_\parallel \nu_\parallel + k_\perp \nu_\mathrm{d} - \omega_\mathbf{k} + r\omega_\mathrm{cs}}. \tag{B 11}$$

In the partially electrostatic cases where one of two scattering waves is electrostatic and the other is electromagnetic, the matrix elements can be obtained by performing the appropriate replacements of $[u^{*j}_\mathbf{k}(\mathbf{v}) J_q(\mu_\mathbf{k})]/\omega_\mathbf{k} \to J_q(\mu_\mathbf{k})/k$, $S^l_{\mathbf{k},-\mathbf{k}'}(\mathbf{v})/\omega_\mathbf{k} \to S_\mathbf{k}(\mathbf{v})/k$, $W^{l'}_{\mathbf{k}',\mathbf{k}''}(\mathbf{v})/\omega_{\mathbf{k}'} \to W_{\mathbf{k}'}(\mathbf{v})/k'$ and $Z^l_{\mathbf{k},r}(\mathbf{v})/\omega_\mathbf{k} \to Z_{\mathbf{k},r}(\mathbf{v})/k$, as described previously [4, 6, 7].

The electrostatic or partially electrostatic matrix elements for nearly perpendicular propagation can be obtained from the matrix elements described in the last paragraph in Appendix A by carrying out the appropriate replacements of

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$a^l_{\mathbf{k},r}/\omega_{\mathbf{k}} \to k_\parallel/k$, $b^l_{\mathbf{k}}/\omega_{\mathbf{k}} \to k_\perp/k$ and $\mu_{\mathbf{k}}\xi^l_{\mathbf{k}}\psi^l_{\mathbf{k},r}/\omega_{\mathbf{k}} \to U_r(\mathbf{k})/k$. Here, $\xi^x_{\mathbf{k}} = \xi^z_{\mathbf{k}} = 1/\mu_{\mathbf{k}}$ and $\xi^y_{\mathbf{k}} = i\partial/\partial\mu_{\mathbf{k}}$, as defined in [6]. In particular, the partially electrostatic elements can be also given by (27)–(30) in [6], where the same replacements as those described in Appendix A should be carried out in order to get the matrix elements from (24) and (25) in [6].

Appendix C. θ-dependent velocity-space diffusion coefficients

The θ-dependent velocity-space diffusion coefficients in (43) and (44) are represented as follows:

$$Q^{ll'}_{\mathbf{k}} = P_{\mathrm{AH}}\left[\frac{1}{\omega^2_{\mathbf{k}}}\left(\frac{e_{\mathrm{s}}}{m_{\mathrm{s}}}\right)^2 \sum_{n,r=-\infty}^{\infty} e^{i(n-r)\theta}\left(H^l_{\mathbf{k},n}(\mathbf{v})\left[w^{*l}_{\mathbf{k}}(\mathbf{v})J_n(\mu_{\mathbf{k}})\right] + K^l_{n,r}(\mathbf{v})\right)Z^{l'}_{\mathbf{k},r}(\mathbf{v})\right], \tag{C 1}$$

$$\begin{aligned} L^{jl''l'l}_{\mathbf{k},\mathbf{k}'',\mathbf{k}'} = {} & \frac{1}{\omega^2_{\mathbf{k}}\omega_{\mathbf{k}'}\omega_{-\mathbf{k}'}}\left(\frac{e_{\mathrm{s}}}{m_{\mathrm{s}}}\right)^4 \sum_{n,q',q,r=-\infty}^{\infty} e^{i(n-q')\theta}\left(H^j_{\mathbf{k},n}(\mathbf{v})\left[w^{*j}_{\mathbf{k}}(\mathbf{v})J_n(\mu_{\mathbf{k}})\right] + K^j_{n,q'}(\mathbf{v})\right) \\ & \times \frac{W^{l''}_{\mathbf{k}',\mathbf{k}''}(\mathbf{v})}{k_\parallel \nu_\parallel + k_\perp \nu_{\mathrm{d}} - \omega_{\mathbf{k}} + q'\omega_{\mathrm{cs}}} \frac{1}{k''_\parallel \nu_\parallel + k''_\perp \nu_{\mathrm{d}} - \omega_{\mathbf{k}''} + q\omega_{\mathrm{cs}}} \\ & \times \left[S^{l'}_{-\mathbf{k}',\mathbf{k}}(\mathbf{v})Z^l_{\mathbf{k},r}(\mathbf{v}) + S^l_{\mathbf{k},-\mathbf{k}'}(\mathbf{v})Z^{l'}_{-\mathbf{k}',r}(\mathbf{v})\right], \end{aligned} \tag{C 2}$$

$$\begin{aligned} M^{jl''l'l}_{\mathbf{k},\mathbf{k}'',\mathbf{k}'} = {} & \frac{1}{\omega_{\mathbf{k}}\omega_{\mathbf{k}'}\omega_{\mathbf{k}''}}\left(\frac{e_{\mathrm{s}}}{m_{\mathrm{s}}}\right)^3 \sum_{j'}\left(\beta^{j'll'}_{\mathbf{k},-\mathbf{k}'} + \beta^{j'l'l}_{-\mathbf{k}',\mathbf{k}}\right) \\ & \times \sum_{n,q,r=-\infty}^{\infty} e^{i(n-q)\theta}\left(H^j_{\mathbf{k},n}(\mathbf{v})\left[w^{*j}_{\mathbf{k}}(\mathbf{v})J_n(\mu_{\mathbf{k}})\right] + K^j_{n,q}(\mathbf{v})\right) \\ & \times \frac{1}{k_\parallel \nu_\parallel + k_\perp \nu_{\mathrm{d}} - \omega_{\mathbf{k}} + q\omega_{\mathrm{cs}}}\left[S^{j'}_{\mathbf{k}'',\mathbf{k}'}(\mathbf{v})Z^{l''}_{\mathbf{k}',r}(\mathbf{v}) + S^{l''}_{\mathbf{k}',\mathbf{k}''}(\mathbf{v})Z^{j'}_{\mathbf{k}'',r}(\mathbf{v})\right]. \end{aligned} \tag{C 3}$$

The differential operators H and K are defined by

$$H^x_{\mathbf{k},n}(\mathbf{v}) = H^y_{\mathbf{k},n}(\mathbf{v}) = k_\parallel \nu_\perp \frac{\partial}{\partial \nu_\parallel} + \frac{1}{\nu_\perp}(\omega_{\mathbf{k}} - k_\perp \nu_{\mathrm{d}} - k_\parallel \nu_\parallel)\frac{\partial}{\partial \nu_\perp}\nu_\perp, \tag{C 4}$$

$$H^z_{\mathbf{k},n}(\mathbf{v}) = Y^z_{\mathbf{k},n}(\mathbf{v}), \tag{C 5}$$

$$K^x_{n,r}(\mathbf{v}) = \nu_{\mathrm{d}} U_n(\mathbf{k})J_n(\mu_{\mathbf{k}}) - \frac{1}{\nu_\perp}(n-r)(\omega_{\mathbf{k}} - k_\parallel \nu_\parallel)J'_n(\mu_{\mathbf{k}}), \tag{C 6}$$

$$K^y_{n,r}(\mathbf{v}) = \frac{i}{\nu_\perp}(n-r)\left[\frac{n}{\mu_{\mathbf{k}}}(\omega_{\mathbf{k}} - k_\perp \nu_{\mathrm{d}} - k_\parallel \nu_\parallel) - k_\perp \nu_\perp\right]J_n(\mu_{\mathbf{k}}), \tag{C 7}$$

$$K^z_{n,r}(\mathbf{v}) = -\frac{k_\perp \nu_\parallel}{\nu_\perp}(n-r)J'_n(\mu_{\mathbf{k}}). \tag{C 8}$$

It can be confirmed that when $\nu_{\mathrm{d}} = 0$, (C 1)–(C 3) are reduced to the previous results [7].

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When two scattering waves are electrostatic, the matrix elements become

$$Q_{\mathbf{k}} = \mathrm{Im}\left[\frac{1}{k^2}\left(\frac{e_{\rm s}}{m_{\rm s}}\right)^2 \sum_{n,r=-\infty}^{\infty} Y_{n,r}(\mathbf{k}) e^{i(n-r)\theta} Z_{k,r}(\mathbf{v})\right], \tag{C 9}$$

$$\begin{aligned} B_{\mathbf{k},\mathbf{k}'',\mathbf{k}'} &= \sum_{j,l,l',l''} \frac{k_j k'_{l''} k'_{l'} k_l}{k^2 k'^2} B^{jl''l'l}_{\mathbf{k},\mathbf{k}'',\mathbf{k}'} \\ &= \mathrm{Im}(L_{\mathbf{k},\mathbf{k}'',\mathbf{k}'} + M_{\mathbf{k},\mathbf{k}'',\mathbf{k}'}), \end{aligned} \tag{C 10}$$

$$\begin{aligned} L_{\mathbf{k},\mathbf{k}'',\mathbf{k}'} &= \frac{1}{k^2 k'^2}\left(\frac{e_{\rm s}}{m_{\rm s}}\right)^4 \sum_{n,q',q,r=-\infty}^{\infty} Y_{n,q'}(\mathbf{k}) \frac{e^{i(n-q')\theta} W_{\mathbf{k}'}(\mathbf{v})}{k_\parallel \nu_\parallel + k_\perp \nu_{\rm d} - \omega_{\mathbf{k}} + q'\omega_{\rm cs}} \\ &\quad \times \frac{1}{k''_\parallel \nu_\parallel + k''_\perp \nu_{\rm d} - \omega_{\mathbf{k}''} + q\omega_{\rm cs}} [S_{-\mathbf{k}'}(\mathbf{v}) Z_{\mathbf{k},r}(\mathbf{v}) + S_{\mathbf{k}}(\mathbf{v}) Z_{-\mathbf{k}',r}(\mathbf{v})], \end{aligned} \tag{C 11}$$

$$\begin{aligned} M_{\mathbf{k},\mathbf{k}'',\mathbf{k}'} &= \frac{1}{kk'k''\varepsilon_{\mathbf{k}''}}\left(\frac{e_{\rm s}}{m_{\rm s}}\right)^3 \sum_{n,q,r=-\infty}^{\infty} Y_{n,q}(\mathbf{v}) \frac{e^{i(n-q)\theta}}{k_\parallel \nu_\parallel + k_\perp \nu_{\rm d} - \omega_{\mathbf{k}} + q\omega_{\rm cs}} \\ &\quad \times [S_{\mathbf{k}''}(\mathbf{v}) Z_{\mathbf{k}',r}(\mathbf{v}) + S_{\mathbf{k}'}(\mathbf{v}) Z_{\mathbf{k}'',r}(\mathbf{v})], \end{aligned} \tag{C 12}$$

where the differential operator Y is defined by

$$Y_{n,q}(\mathbf{k}) = U_n(\mathbf{k}) J_n(\mu_{\mathbf{k}}) + \frac{k_\perp}{\nu_\perp}(q-n) J'_n(\mu_{\mathbf{k}}). \tag{C 13}$$

In the partially electrostatic cases where one of two scattering waves is electrostatic, the matrix elements can be obtained by the appropriate replacements of $(H^l_{\mathbf{k},n}(\mathbf{v})[w^{*l}_{\mathbf{k}}(\mathbf{v}) J_n(\mu_{\mathbf{k}})] + K^l_{n,r}(\mathbf{v}))/\omega_{\mathbf{k}} \to Y_{n,r}(\mathbf{k})/k$, $S^l_{\mathbf{k},-\mathbf{k}'}(\mathbf{v})/\omega_{\mathbf{k}} \to S_{\mathbf{k}}(\mathbf{v})/k$, $W^{l'}_{\mathbf{k}',\mathbf{k}''}(\mathbf{v})/\omega_{\mathbf{k}'} \to W_{\mathbf{k}'}(\mathbf{v})/k'$ and $Z^l_{\mathbf{k},r}(\mathbf{v})/\omega_{\mathbf{k}} \to Z_{\mathbf{k},r}(\mathbf{v})/k$, as mentioned in Appendix B.

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