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MACROECONOMIC DYNAMICS SURVEY: ENDOGENOUS GROWTH THEORY AND MODELS: THE "FIRST WAVE," 1952–1973

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This paper surveys the first stage in the development of models of endogenous growth over the two decades between 1952 and 1973, including seminal works by Arrow, Frankel, Phelps, Uzawa, and others as faculty and graduate students who interacted in the Stanford–Chicago–MIT–Yale Nexus. The paper also deals with American Economic Association (AEA) and Econometric Society (ES) conference sessions over the period 1964–1968 on growth, technical progress, and innovations and with the contributions in the 1967 Shell volume, in addition to dissertations, working papers, and book chapters by those involved in the "first wave" over the period 1965–1973. Finally, the paper considers the initial efforts at building more sophisticated models based on technical progress, human capital, increasing returns and imperfect competition, and the incompatibility between increasing returns with competitive equilibrium.

Keywords: Endogenous Growth, Collaborative Research Programs

1. INTRODUCTION: SCOPE AND METHOD

In previous papers [Spear and Young (2014, 2015a, 2015b)], we surveyed the origins, evolution, and dissemination of optimal growth, two sector and turnpike, and stochastic growth models. In this paper, we focus on *endogenous* growth theory and models. However, in contrast to our previous findings regarding optimal growth theory and its offshoots, which exhibited fairly direct lines of conceptual development, the endogenous growth story, as will be seen, is multifaceted, with a more complex pattern of intellectual evolution.

Our object here is not to provide a history of endogenous growth per se, as a number of conventional narratives have already appeared in the literature since 1990, including by those involved—indirectly or directly—in its evolution. These

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range from accounts presented in working and published papers to survey articles, Festschrift chapters, and textbooks [Dixit (1990), Stiglitz (1990), Romer (1994, 2014), Aghion and Howitt (1998), Aghion and Howitt (2006), Aghion and Durlauf (2007), among others].

Rather, the approach taken here focuses on individual, institutional, and thematic research programs related to the development of endogenous growth. Individual programs were those in which one or more researchers were involved and were based upon dissertations or collaborative efforts, such as between graduate students and their supervisors, or colleagues. Institutional programs originated in specific settings—seminars, workshops, research centers and sponsored projects, and intra- or inter-university collaboration. Thematic programs focused on specific theoretical elements or their combination such as technology and innovation, learning processes, externalities and returns, the nature of competition, and human capital. We deal here with the evolution and dissemination of watershed published papers, and the institutional settings in which they developed by referring to their *unpublished*, *working*, and *conference* versions accordingly.

This paper surveys the development of what can be considered endogenous models *from 1952 to 1973*, that is to say, the "first wave" of the endogenous approach,[1] including the works of Abramovitz, Haavelmo, Kaldor, Kaldor and Mirrlees, Arrow, Frankel, Phelps, Phelps and Drandakis, and Uzawa. We then focus on the Stanford–Chicago–MIT–Yale (SCMY) nexus of growth theorists and their contributions. The graduate student summer seminars organized by Uzawa in Chicago over the period 1964–1967 are dealt with, including recollections of participants who, besides SCMY students, also included graduate students from Rochester and other institutions.

In addition, American Economic Association (AEA henceforth) and Econometric Society (ES) sessions on growth and capital theory, technical change, technical progress, and theory of innovations over the period 1964–1968 are discussed. The 1967 volume edited by Shell, which included papers emanating from the Chicago groups and the American Economics Association and Econometric Society meetings, by Sheshiski, Norhaus, Cass, and Yaari, and Shell himself, will be discussed. The contributions of Shell, Von-Weizacker, Sheshinski, Inada, Aoki, Nordhaus, Stiglitz, Atkinson and Stiglitz, Sidrauski, Calvo, Razin, and Nelson and Phelps, Uzawa, and others over the period 1965–1973 are also surveyed. These include dissertations, working papers, technical reports, conference presentations, book chapters, and papers published by them.

We then turn to the early development of some of the building blocks of what became the "second wave" of the endogenous approach, that is, endogenous technical progress, human capital, increasing returns to scale (IRTS), and imperfect-competition approaches. Finally, the importance of the realization that a competitive equilibrium was not compatible with increasing returns [Shell (1965, 1973)] is discussed.

2. MODELS IN THE 1950s AND EARLY 1960s

2.1. Central Questions, Inventive Activity, Stylized Facts, Learning, and the "AK" Mark 1 Model

Approaches to economic progress date back to Smith and Ricardo. Capital and population theories, as they affected long-run change—combined with theories of decreasing and increasing returns—provided the basis for studies of growth. Abramowitz (1952, 135–144) set out what he saw as the "central questions in the theory of growth." These were (i) supplies of factors of production; (ii) psychological and qualitative attributes of population; (iii) industrial, commercial, and financial organizations; (iv) legal and political frameworks of economic life; and (v) discovery and exploitation of knowledge. He then set out what he saw as the parallel between the increase in the stock of knowledge and capital stock. As Abramowitz (1952, 146) put it: "Insofar as new applied knowledge results from the deliberate devotion of resources to its discovery and use, the stock of knowledge is increased by a process identical with that which produces an increase in the stock of material equipment".

Despite its importance for the endogenous growth story, his 1952 paper is not widely cited. His 1956 *American Economic Review* (AER henceforth) paper, on the other hand, is widely cited, from Arrow (1962) to Romer (1990), and by many others. In this paper, Abramowitz set out, in a prescient manner, what he saw as "the central problem" facing growth theorists and analysts. As Abramowitz (1952, 13–14) put it: "To identify the causes that explain not only the rate at which are opportunities to raise efficiency increase, but also the pace at which we take advantage of those opportunities will, no doubt, remain the central problem in both of history and theory. . . of economic growth." But, although in his 1956 paper Abramowitz analyzed the data for historical trends in output and the utilization of resources in the US case, he did *not* provide a formal model or method for modeling them. It was left to Solow and others to fill the gap.

Another early approach to endogenizing technical change and knowledge was presented by Haavelmo in his book *A Study in the Theory of Economic Evolution* (1956). Arrow later cited this work in his famous "Learning" paper (1962b). According to Arrow, Haavelmo's model was one of those that influenced the "production assumptions" of his own approach (1962b, 160). Arrow described Haavelmo's model in concise and lucid terms. As he put it, in Haavelmo's approach, output depended on the stocks of capital and knowledge, whereas investment was dependent on output, and both the capital stock and the stock of knowledge. The stock of knowledge, in the simplest form of Haavelmo's model, was a "function of time"; though in its "more sophisticated version," it was the outcome of investment, with "the educational effect of each act of investment decreasing exponentially in time" (1962b,160).

In a 1962 NBER volume, Arrow and Fellner, among others, dealt with what Nelson, who edited the volume, called (1962, 3–4) "crucial conceptual

problems" regarding inventive activity. According to Nelson's introduction (1962, 3), the papers were originally presented at a Spring 1960 University of Minnesota conference jointly sponsored by the NBER and the SSRC. In the introduction to the conference volume, Nelson noted a number of reasons for renewed "interest in inventive activity": (i) technological change; (ii) impact of research and development; (iii) new views on competition and the competitive process; and (iv) shift in focus from "short-term Pareto optimality" to conditions of the "long run." He then made the prescient statement that "in a sense these developments represent a renaissance of Schumpeter" (1962, 3–4), a tendency recalled three decades later by Aghion and Howitt (1992). Fellner had published an abridged version of his conference volume paper with the title "Two propositions in the theory of induced innovations" in *Economic Journal* (*EJ*, 1961). Arrow's oft-cited conference volume paper (1962a) "Economic welfare and the allocation of resources for invention" had earlier appeared under the same title as a RAND paper in December 1959 [Arrow (1959)]. Interesting enough, in the RAND version, Arrow had set out one of reasons for the "possible failure of perfect competition to achieve optimality in resource allocation" as "increasing returns" (1959, 1–2). In the conference volume version, he changed this to read "indivisibilities" (1962a, 609). We will return to this issue later in this paper.

Moving forward, a decade after Abramowitz set down what he saw as the five "central questions," Kaldor set out what he called six "stylized facts of economic growth," which he saw "as a starting point for the construction" of growth models. In his view "none" of the "facts" could "be plausibly explained by the theoretical constructions of neoclassical theory" then extant (1961, 178–179).

Kaldor and Mirrlees (1962, 175, 190) linked "technical progress" and what they called the "technical progress function" to a "purely 'endogenous' growth rate," and "endogenous factors," such as "surplus labor and underemployment," so that "continuous growth" as determined by these factors will "necessarily lead to full employment" (1962, 175). They continued on to stress the need for more scientific education and expenditure on research, and higher quality business management, which would be more effective in enhancing technical improvements (1962, 190).

In his paper on "learning," Arrow (1962b, 155) suggested "an endogenous theory of the changes in knowledge that underlies. . . shifts in production functions." After presenting the model, he set out a research agenda for its further development. In his words (1962b, 172): "It has been assumed here that learning takes place only as a by-product of ordinary production. In fact, society has created institutions, education, and research, whose purpose is to enable learning to take place more rapidly. A fuller model would take account of all these additional variables."

Arrow (1962b) is one of the most widely cited papers in economics. In contrast, Frankel published in the *AER*, in the same year, a "rarely cited paper" [Ljungquist and Sargent (2004, 459)], which, according to Aghion and Howitt (1998, 27), among others, presented *one* of the *earliest recognizable* versions of what later

came to be known as the "AK model" or "AK paradigm" [Aghion and Durlauf (2007, 4)], and it is to this curiosum that we now turn.

One of the difficulties in our effort to make sense of Frankel's contribution is that, on the one hand, it was *ostensibly* "a neglected early contribution to the theory of economic growth" with the "only recent reference. . . in Aghion and Howitt (1998)," according to Cannon (2000, 292, 294, note 4), whereas on the other hand, *even prior to 1998*, Frankel's approach was mentioned by Aghion, Banerjee, and Piketty in CEPR symposium papers (1996, 1997)*, before* the publication of their well-known paper in *Quarterly Journal of Economics* (QJE henceforth) (1999). Another difficulty is that observers have linked Frankel's (1962) approach to those of Romer (1986) and Lucas (1988), and also to that of Rebelo (1991). Romer outlined his approach in his 1983 thesis, Lucas in his 1985 Marshall lectures (published in 1988), and Rebelo, as noted, in 1991. Aghion and Howitt, for their part, maintained that while Frankel's "basic idea," i.e., "his AK model" (1998, 27) can be found in Romer and Lucas, they added the caveat that their respective approaches "actually laid out more than an AK model" (1998, 27, note 2). An additional problem is that while some observers have talked about a linkage between the approaches of Romer and Lucas, others have linked Lucas (1988) to the earlier work of Uzawa (1965); again we will deal with this below. Moreover, still others have maintained the AK story to have started with Harrod (1939) and Domar (1946), linking their approaches to that of Frankel *including Frankel himself, who significantly modified the Harrod–Domar model to essentially develop his own* (1962, 1009). *Finally Frankel's approaches to growth and technological change are not limited to his 1962 AER paper* [Frankel (1955, 1961,1965)].

When queried regarding the development of his approach, Frankel replied that his interest in growth models evolved out of his Ph.D. work. His 1953 Berkeley thesis dealt with Industrial Development Councils in Britain. He wrote "one of the concerns of the Councils, as I recall, was the productivity of British industry" (personal communication, October 8, 2013). In 1955, he published a paper in the *AER* entitled "Obsolescence and technological change in a maturing economy." In this, he attempted to assess the "degree to which. . . innovation is retarded" according to the technological level of industries (1955, 306–309). His paper "Producer goods, consumer goods and acceleration of growth" was published in *EJ* in 1961. In this, he presented "a two-sector growth model. . . ," which integrates and sets forth the relations between the capital stocks and the respective outputs in the investment and consumption sectors (1961, 2). His 1962 *AER* paper and AK model have been extensively dealt with by others and thus will not be discussed here. Albeit limited in its applicability [Aghion et al. (1999)], greatly supplemented by Rebelo's work, and ostensibly supplanted by the approaches of Lucas, Romer, and others, suffice it to cite Aghion, who recently wrote that "Frankel is the true father of the AK model the way we teach it" (personal communication, October 22, 2015).

In 1965, Frankel published a paper in *Kyklos* entitled "Home vs. foreign investment: a case against capital exports." In this, he applied his AK approach to suggest "a framework in which the relation between investment decisions by enterprises, the returns to factors and the indirect development benefits to the economy can be explicitly shown" (1965, 413). As he put it, "the main elements in this approach were presented" in his 1962 *AER* paper (published in 1965, 414, note 6).

Over the period 1964–1966, Phelps and Drandakis, individually and jointly, wrote papers focusing on technical progress, induced invention, and growth. Phelps dealt with the "Golden Rule. . . where technical progress is endogenous" in his Cowles Foundation *Discussion Paper* 176 entitled "Models of Technical Progress and the Golden Rule of Research" (1964, 4, 11) and in its published version under the same title in the *Review of Economic Studies* (RES henceforth) (1966, 134, 137–38). Drandakis and Phelps dealt with a model in which the "absolute time rate of change of the (proportionate) growth rate of capital" is "an endogenous variable." They went on to base their "induced invention hypothesis" on this accordingly in their Cowles Foundation *Discussion Paper* 128 entitled "A model of induced invention, growth and distribution" (1965, 9–11, 28), which later appeared under the same title in the *Economic Journal* (EJ henceforth) (1966, 828–829, 838). At the September 1965 World Congress of the Econometric Society, Drandakis and Phelps presented an amended version of their Cowles Foundation *Discussion Paper* at the session "Investment and technological change" chaired by Goodwin. Interestingly enough, at the same World Congress, Inada presented a paper entitled "Business cycles and endogenous factors for economic growth" at the session "Cycles and growth"; we will return to Inada's contributions below [Program of September 1965 meeting, *Econometrica* (1966, 7, 10)].

In January 1965, Uzawa published a paper in the *International Economic Review* (IER henceforth) entitled "Optimum technical change in an aggregative model of economic growth," which became the basis for what has been called the Uzawa–Lucas model. A number of things regarding this path-breaking paper should be noted here. First, according to the *IER*, the manuscript was "received August 25, 1963, revised April 20, 1964" (1965, 18). Uzawa was first at Stanford and then moved to Chicago in 1964, so he cited both as his institutional affiliation (1965, 31). Moreover, Uzawa acknowledged that he was "greatly indebted for valuable comments and suggestions" to Domar (at MIT), which he had briefly visited. In a footnote, Uzawa set out the object of the model he presented in his 1965 paper. As he put it (1965, 19, note 3): "A number of studies, in particular by. . . Shultz (at Chicago) have been made recently on the implications of the improvement in the quality of labor upon the pattern of economic growth. The model presented below has been intended primarily to serve as a basis for discussions on the economic effects of education." And this, in the context of the notions of "optimum" and "technical change," which reflected Uzawa's interests, is manifest in some of his Stanford Technical Reports between 1960 and 1963.

2.2. Chicago and MIT Seminars, AEA and ES Sessions, Papers and Ph.D. Theses Uzawa's Chicago Summer Seminars, 1964–1967

In contrast to the intellectual dissonance between "saltwater" and "freshwater" economists starting in the mid-1970s, the period from 1964 until this "great divide" was characterized by cross-fertilization and positive interaction between growth theorists and graduate students based at Stanford, Chicago, MIT, and Yale, among other places. This is manifest in the graduate student summer seminars organized by Uzawa at Chicago and sponsored by the NSF (1964-66) and MSSB (1967); in AEA and ES sessions on growth and capital theory, and technical change and innovation over the same period; and in seminars, and publications based upon them, such as the 1967 volume edited by Shell—based on MIT seminars on optimal economic growth, 1965–1966—and the November 1967 Chicago growth seminar led by Uzawa [later published in the *Journal of Political Economy* (JPE henceforth) (1969)]. Output of the seminars included the University of Chicago Technical Reports, presentations by seminar participants over the period 1965–1968 at AEA and ES sessions, dissertation chapters, published papers, and book chapters.

Now, Uzawa's 1965 Chicago graduate student summer seminar has been recounted by Stiglitz in a number of places, asserting "among the topics of discussion was how to incorporate endogenous technological change into growth theoretic models" (1990, 63, note 12). It has also been dealt with by Warsh (2006, 155). In addition, that of 1966 was mentioned by Lucas, who participated (2001). But, the 1964 and 1967 Uzawa Chicago graduate student summer seminars, however, *have been overlooked*, despite their importance.

The 1964 Uzawa Chicago summer seminar included Cass, Ryder, Douglas, Oniki, Goldman, Shell, and Teubal. All were Stanford Ph.D. students, with the exception of Teubal, who was at Chicago (communication from Shell, October 26, 2013) (see Appendix A for a listing of their dissertations). As Stiglitz recalled (1990, 63, note 12), Uzawa's summer 1965 Chicago seminar included Stiglitz, Akerlof, Chaudhuri, Sheshinski, Nordhaus, Lamalfa, Levy, and Krishna. In other words, the group consisted of MIT Ph.D. students, with the exception of Lamalfa, an MIT research fellow, Levy, a Yale Ph.D. student, and Krishna, a Ph.D. student at Chicago. Attending the 1966 Summer seminar was, among others, Lucas, then an assistant professor at Carnegie Tech. Cass, then an Assistant Professor at Yale, also attended, as did Lav, a student from Johns Hopkins; the recollections of Lucas will be presented below [Lucas (2001, 14)]

The 1967 Chicago summer seminar was characterized by the fact that Uzawa did not actively participate at the beginning of the period, as he was obliged to be in Japan that time, and thus the seminar was led, until his return, by his former Chicago Ph.D. student, Sidrauski, who was then an assistant professor at MIT. Graduate students included Calvo, Ethier, Mirman, Narayanan, Nissen, Onitsuka, Razin, and Rhee, and comprised the most diverse group of participants in the seminar's brief history. The group included Ph.D. students from Chicago, Yale,

Berkeley, Rochester, and even the University of Kansas (Cutler, MSSB Projects Summary Report, July 12, 1973).

2.3. AEA and ES Sessions, 1964–1967: Presentations, Thesis Chapters, and Papers

As mentioned above, between 1964 and 1967, papers by student members of Uzawa's summer seminars, and others, were presented at Allied Social Sciences Association (ASSA henceforth) meetings, at sessions on growth and capital theory and technical change, sponsored by the AEA and ES, respectively. For example, at the Chicago December 1964 meetings, in the ES session "Growth models," chaired by Amartya Sen, Douglas, a 1964 Uzawa group participant from Stanford, presented "A two sector growth model with inside money," discussed by Ando, which was based upon his thesis on monetary dynamics (1966). At the December 1964 session "Optimal growth theory," chaired by Phelps, Cass presented a paper entitled "Optimum economic growth in an aggregate model of capital accumulation: the finite horizon case," discussed by Diamond [ASSA (1964) Program, 102,169]. This was a version of his November 1964 Cowles Foundation *Discussion Paper* 178 (1964, 1, note 1), and his thesis chapter III (1965). It was published in *Econometrica* (1966).

At the parallel ES session on "technical change," chaired by Nelson, Shell presented a paper entitled "Invention and technical change," discussed by Hurwicz [ASSA (1964) Program, 169]. According to Shell's recollection, this presentation was partially based on the "endogenous growth" paper he presented at Uzawa's August 1964 Chicago summer seminar entitled "A model of inventive activity and capital formation," circulated as University of Chicago Technical Report 2 (August 1964). This was essentially Chapter III—with the same title—of his Stanford thesis (April 1965), supervised by Arrow and Uzawa (personal communication, December 3, 2015); more details about Shell's Ph.D. thesis are given below.

At the December 1965 New York ASSA meeting, there held a number of AEA and ES sessions relevant to our story. The ES session "Growth theory I" took place on 28 December and the AEA session "Capital theory: technical progress and capital structure" on 29 December, parallel to the ES session "Growth theory II." The importance of these sessions cannot be overstated: Some of the presentations constituted, in our view, fundamental contributions to the ongoing development of the endogenous approach to growth.

The ES session "Growth I" was chaired by Solow. There were three presentations. All of the papers were presented by participants in the 1964 and 1965 Uzawa Chicago summer seminars. Cass and Yaari's paper on growth and accumulation was discussed by McManus, the paper by Akerlof and Stiglitz on investment and income was discussed by Jorgenson,and that by Teubal on monetary policy and growth was discussed by Phelps [ASSA (1965) Program, 67]. An early version of the Cass–Yaari paper had been circulated in November 1965 as Cowles Foundation *Discussion Paper* 198 under the title "Individual saving, aggregate

capital accumulation and efficient growth." This later appeared, in revised form, in Shell's 1967 volume. Their paper presented at the ES session dealt with the effects of technical change based on a Modigliani–Brumberg framework that generated aggregate savings, which determined capital accumulation [*Econometrica* Report (1966a, 118)]. The paper by Akerlof and Stiglitz presented a "new model" with "dynamic properties" differing from earlier Harrod–Domar and neoclassical frameworks, based upon endogenous investment [*Econometrica* Report (1966a, 118)]. Teubal's paper, based upon a chapter from his Chicago Ph.D. thesis "Inflation, unemployment and economic growth," was earlier presented at Uzawa's Chicago Summer seminar in August 1964 (personal communication from Teubal, December 11, 2015).

The second ES session on growth (Growth II), held on December 29, 1965, was chaired by Smyth, and consisted of three presentations. The first was a paper by Uzawa and Sidrauski entitled "Price expectations, inflation and economic growth," discussed by Sato. Other papers were presented by Mortensen on consumption and growth, discussed by Drandakis, and by Britto, entitled "Balanced growth and efficiency with increasing returns to scale," discussed by Cass [ASSA (1965) Program, 90]. We will return to Britto's paper below in the context of a discussion of IRTS; at this point, we focus on that by Uzawa and Sidrauski, which has also not received attention.

With regard to the Uzawa–Sidrauski paper, the following should be noted. First, Sidrauski was Uzawa's Chicago Ph.D. student and later, as a junior faculty member at MIT, led the summer seminar in 1967 when Uzawa was absent from Chicago. Second, the December 1965 ES presentation was later cited by Sidrauski with the title "Inflation, optimum consumption and real cash balances" in his May 1967 *AER* Ph.D. thesis summary (1967, 544). Third, according to the abstract of the December 1965 paper, as published in the *Econometrica* report of the 1965 ES sessions, their analysis of "the effects of monetary fiscal policies on the pattern of growth, and the structure of the monetary and fiscal policies which result in optimum growth" was conducted "in terms of the techniques of optimum economic growth as recently developed by Koopmans, Cass, von Weiszacker, and others" [*Econometrica* Report (1966b, 128)].

However, the December 1965 session that had the most impact on the development of the endogenous approach was the AEA session "Capital theory: technical progress and capital structure" chaired by Solow. It was parallel to the second ES sessions on growth, and held on Wednesday, December 29. *In retrospect, presentations at this session comprised two out of the three major papers in the early endogenous growth triad*, that is to say, those by Shell (1965b, 1966) and Nelson-Phelps(1965a,b, 1966), and Uzawa (1965) being the third. Shell's paper entitled "Inventive activity and capital accumulation" was discussed by Sato and that by Nelson and Phelps, "Investment in humans, technological diffusion, and economic growth," was discussed by Hirshleifer [ASSA Program (1965, 86)].

The Nelson–Phelps paper appeared as a RAND Corporation paper in December 1965, whose abstract read:

> The paper is concerned with the relationship between capital structure and technological progress. The process of education is viewed as an act of investment in people, and educated people are seen as bearers of human capital. According to the models presented, the rate of return to education is greater the more technologically progressive is the economy. This suggests that the progressiveness of the technology has implications for the optimal capital structure in the broad sense. It is concluded that society should build more human capital relative to tangible capital the more dynamic is the technology.

The paper and Hirshleifer's comments were published in the *AER* (1966, 69–75, 81–82). In his comments, Hirshleifer focused on the Nelson–Phelps hypothesis "education promotes growth" (1966, 81), or as they put it in the paper, "education spreads the process of technological diffusion" and thereby economic growth (1966, 70).

> Shell's December 1965 paper "Inventive activity and capital accumulation" (1965b) was based on his August 1964 Uzawa Chicago seminar paper "A model of inventive activity and capital accumulation," which—with the same title—became Chapter III of his thesis (1965a). Shell's December 1965 presentation was discussed by Sato, and the paper and comments by Sato published in the *AER* (1966). According to Shell (personal communication, December 3, 2015), his AEA presentation (1965b) and *AER* (1966) paper "were a specialization of the thesis chapter to the case where the three-sector technology is reduced to one sector, so that research and investment are in the same units as consumption." In his *AER* paper, Shell put it as follows:
>
> > I treat the one-sector model for ease of exposition. This implies that the production possibility frontier is a hyper-plane in the consumption-investment-invention space" (1966, 63, note 2).

In his comments on Shell's paper, Sato wrote (1966, 78):

> Technical progress has been a sort of orphan in capital theory. Even in recent theoretical developments, technical progress has remained largely an exogenous variable. Some have attempted to introduce technical progress functions to rectify this state of affairs. Now Professor Shell attempts to make technical progress an endogenous variable by making the rate of technical progress an increasing function of current expenditure on inventive activity. . . The close analogy between capital accumulation and augmentation of technological knowledge is the most interesting feature of the model.

Regarding the reactions to his AEA presentation, Shell recalled (personal communication November 28, 2015): "Solow served as a neutral chair. Much later (after Romer) he seemed to be very keen on Shell inventive activity. . . I remember warm vibrations (from) Sid Winter, but not the details. Phelps hired me for Penn after this, so he must have been non-negative on this work." A more detailed look at Shell's early work will be presented below.

At this point, let us turn to the December 1966 ASSA meeting in San Francisco, where others from Uzawa's Chicago summer seminars presented papers. At the ES session on 27 December, "Theory of innovations and technical change," chaired by Tang, Nordhaus presented "The theory of invention in the firm," discussed by Cass, and Shell presented "A Schumpeterian model of induced innovation

and capital accumulation," discussed by Phelps. At the 1966 ES session on the theory of economic development, on 28 December, chaired by De Vries, Goldman—a member of the 1964 Uzawa Group—presented "Optimal growth and continual planning revision," based on his Stanford Ph.D. thesis. Later that day, the ES session "Growth theory," chaired by Shell, included a paper by Cass and Yaari entitled "Efficient inter-temporal allocation of consumption in the neoclassical growth model," discussed by Hahn [ASSA (1966) Program, 80, 100, 114].

The next year, at the December 1967 ASSA Washington meeting, the ES session "Optimal growth models," chaired by Archibald, included a paper by Goldman entitled "Sequential and continuous planning," discussed by Koopmans [ASSA Report (1967, 127)]. This paper was published in the *JPE* (1969). Earlier that year, at the June 1967 Econometric Society Tokyo meeting, at the session on "economic growth" chaired by Uzawa, Oniki, another of the members of the 1964 Uzawa group, presented a paper entitled "Comparative dynamics in the theory of optimal growth" [*Econometrica*, Report of Tokyo Meeting (1968, 3)].

2.4. Recollections of the Uzawa Groups, 1966–1967: Papers, Publications, and Ph.D. Theses

> As mentioned above, Lucas attended the 1966 August seminar. In his "Professional Memoir," Lucas (2001) recalled"In 1966 I was invited to a workshop. . . conducted in Chicago by Hirofumi Uzawa. Uzawa was a charismatic figure, an enormous influence on theoretically minded students, who had moved to Chicago from Stanford just after I had left. . . Dave Cass spoke about work he and Yaari were doing, trying to understand the logic of a model of an economy with an infinity of overlapping generations that Samuelson had published in 1958. This paper had not been on reading lists of mine, but I was immediately attracted by its simplicity and the role it had for money. This was the first model I had seen in which paper money had a useful role to perform and had a command over goods in equilibrium. This was exciting, and the following fall I began to use this setup in my graduate courses in macroeconomics. (14–15)

The work Cass "spoke about" at the Uzawa seminar was circulated as Cowles Foundation *Discussion Paper* 215 entitled "A note on the role of money in providing sufficient intermediation" (September 1966). This was noted by the authors themselves, who wrote (1966, 1) "A preliminary version was presented at the symposium on optimum economic growth held at the University of Chicago during August 1966." Lucas (2001, 15) went on to say: "The paper I had written for the Uzawa conference was never finished. I couldn't establish the mathematical result that was to be the paper's centerpiece."

A few months later, Lucas wrote Uzawa (October 28, 1966) and said: "I enclose a paper developed from the first two sections of the paper I gave at your symposium in August. As you had suggested, I developed the treatment of the competitive industry in somewhat more detail. In the new version the competitive industry is also analyzed under rational rather than static expectations" (Lucas Papers, Duke,

Box 1, File 1966). And indeed, after the 1966 Uzawa seminar, Lucas started to work with Prescott on the paper he had presented there, in order to establish the mathematical result needed via Blackwell's dynamic programming approach. The result was eventually their 1971 joint paper entitled "Investment under uncertainty" [Lucas (2001, 16)].

As noted above, the 1967 Uzawa Chicago summer seminar was composed of a heterogeneous group of a dozen or so graduate students. It was, at first, led by Sidrauski, Uzawa's former graduate student, due to Uzawa's absence at the beginning of the workshop. Moreover, it was focused, as was the 1965 summer seminar, in the main, on growth. Among the members of the 1967 group who presented papers based upon their Ph.D. work were Calvo and Razin. Ethier and Mirman attended and were influenced, to one degree or another, by Uzawa. Ethier (personal communication, November 21, 2015) recalled that his "participation was a wonderful experience. . . that was due to Uzawa himself, who had a great ability to stimulate students." Mirman, for his part (personal communication, November 20, 2015), recalled Uzawa and Cass, who visited the seminar, somewhat discouraging his decision to work on uncertainty. Mirman also specifically recalled Calvo as being "the star of the group." Calvo recalled (personal communication, 22 November 2015) presenting a paper entitled "A vintage model of optimum economic growth" at the Chicago seminar. This paper became part of his later Yale Ph.D. thesis entitled "Efficiency and optimality in a heterogeneous capital model-putty clay" (1974), supervised by Koopmans, Chapter IV of which was eventually published in *Econometrica* (1976).

Razin's contributions and recollections are most relevant here. He was a Chicago Ph.D. student of Uzawa's, and a member of the 1967 Chicago summer seminar. In a review essay, he recalled (2008, 132):

> Theories that explain endogenous technological change were known as early as (the) 1960s. Arrow's (1962) model of learning by doing, Uzawa's (1965) model of human capital driven productivity improvements, and Shell's (1967) model of inventive technological activity. These models are part of the "First Wave" of the literature on (endogenous) economic growth. . . I contributed to developing the theory of the effect of accumulation human capital, in addition to the accumulation of physical capital, on growth, in my doctoral thesis from 1969 [chapters from which were published in *RES* (1972a), and *Metroeconomica* (1972b)]. . . . I noticed that in Uzawa's model private and social returns coincide and applied the Uzawa model to the competitive economy. Years later this theory was extended by Lucas (1988).

Interestingly enough, Razin's *RES* paper entitled "Optimum investment in human capital" (1972a) was first presented in September 1970 at the Second World Econometric Society Congress under the title "The optimum rate of investment in human capital" at the session entitled "Education," chaired by Phelps [Program, *Econometrica* (1971, 23)]. Moreover, as Razin recalled (personal communication, November 19, 2015) regarding his 1967 paper and 1969 dissertation: "Actually the dissertation was split into a 'normative' version, later published in *RES* (1972a)

and the market-based version, later published in *Metroeconomica* (1972b), then a theory journal."

In another recent recollection, Razin first wrote (personal communication, November 20, 2015) that: "Uzawa is well known. . . for his seminal contribution to endogenous growth. In his article, in the 1965 *IER*, productivity permanently increases as the result of permanent accumulation of human capital. Uzawa was thus a first mover in the new growth theory." He continued: "My Ph.D. dissertation ("Investment in human capital and economic growth: a theoretical study," University of Chicago, June 1969) has been an (and the first) application to the market economy of Hiro's (Uzawa's)human capital optimal growth theory." Because of its importance for our narrative, we present Razin's further recollections in detail below (personal communication, November 20, 2015):

> I started to develop the paper at the 1967 summer camp. This summer camp, the 1967 MSSB-NSF sponsored Chicago seminar, was led by Uzawa. The student group was: Burton, Calvo, Ethier, Marvy, Mirman, Narayanan, Nissen, Onitsuka, Rhee, Smith, Tsushima, and me. Miguel Sidrauski was asked by Uzawa to be in charge. Uzawa was absent most of the time since he began his move from Chicago to Tokyo. There I interacted with several bright young students that were working on cutting edge research projects: Lenny Mirman (Rochester), Bill Ethier (Rochester), Guillermo Calvo (Yale). Hiro got stranded in Japan. He asked Miguel Sidrauski (his former student, and then the young star from MIT, who died from cancer less than two years afterwards) to lead the "Summer Camp." We, the students, and Miguel, our leader, were highly interactive. It was an exciting first experience of an interaction within a research group. I started to develop a paper on human capital and economic growth, following an influential paper by Hiro Uzawa, which demonstrates how a centrally planned economy can efficiently affect growth through the allocation of the economy's resources between a sector that produces know how and other productive sectors.
>
> In my paper which turned out to be my Ph.D. Dissertation (1969), I applied these ideas to decentralized, market-based, equilibrium model of an economy which accumulates human capital and physical capital. I noted that "economists have long noticed that people play an important role in the process of production, and in return, they are rewarded by an amount which constitutes the largest fraction of income." I guess I heard it from T. W. Schultz. In my model, the accumulation of human capital is the driving force behind a permanent rate of increase in income per capita. This feature was dubbed in the 1980s as endogenous growth. I presented an unfinished paper on human capital accumulation and economic growth at a 1967 conference [the November 1967 Uzawa Chicago Conference on Growth, to be discussed below; see *JPE* (1969, 574)]. The human capital-physical capital growth paradigm turned out to be important, because one stream of the Endogenous Growth Theory, decade and a half later, in the mid-1980s, grew out of another Chicago "school" that my endogenous growth model was almost identical to the Lucas' well cited paper on endogenous growth.
>
> Perhaps my contribution that appeared in print [*RES* (1972)], when the profession got a little "tired" of what seems to have been over-occupation in the theory of economic growth after over a decade of significant developments in this area (has

been overlooked). I was extremely successful in terms of visibility and citations with another early endogenous growth paper of mine (in which the endogenous population growth cum education is the driving force for a permanent rise in income per capita because parents trade off "quality for quantity" of their children in the growth model, *AER* 1975).

2.5. MIT Seminar on Optimal Growth and Shell Volume, 1965–1967

Over the period 1965–1966, Shell led a seminar on the theory of optimal economic growth at MIT, papers from which were published in the 1967 volume he edited. The papers were written by members of Uzawa's 1964 and 1965 groups, such as Shell, Cass, and Ryder (1964 group); Sheshinski, Nordhaus, Duta-Chaudhuri, Akerlof (1965 group); and others then at MIT and other institutions, such as Bardhan, Bruno, Chase, and Samuelson (MIT), Marglin (Harvard), and Carter (IBRD).

Some of the papers were based upon chapters from Ph.D. dissertations, such as those of Sheshinski and Ryder; others were based upon working papers, such as that of Cass and Yaari, whose volume paper entitled "Individual saving, aggregate capital accumulation and efficient growth" appeared earlier as Cowles Foundation *Discussion Paper* 198 (November 1965). There were also papers specifically prepared for the Shell seminar, such as that of Bardhan. Finally, Shell contributed two chapters, but more about this is mentioned below.

Bardhan provided recollections of Shell's MIT seminar; again, because of their importance to our narrative, we report his recollections at length here (personal communications, December 21 and 23, 2015). He wrote (December 21, 2015):

> I finished my Ph.D. dissertation in Cambridge University in UK on Economic Growth and International Trade in 1965 under the supervision of James Meade and Frank Hahn. Many of the chapters used optimum growth models, with the use of calculus of variations and Pontryagin's Maximum Principle. A somewhat revised version of my dissertation was published as a book with the title, *Economic Growth, Development, and Foreign Trade: A Study in Pure Theory*, John Wiley Interscience, 1970. I joined MIT in 1966, where I carried out the revisions on the dissertation for publication. Some of the revised chapters were published in *Econometrica*, *RES*, *QJE* and *IER*.
>
> Karl Shell was then my colleague at MIT. He was running a seminar on optimum growth where he asked me to present a paper, which later became a chapter in his edited book. This paper was written for Karl, it was not a part of my dissertation.
>
> My main inspiration in this period was Robert Solow (whom as well as Ken Arrow I met as a graduate student in Cambridge, UK, when they spent a sabbatical year there). Even before I finished my dissertation, on the basis of reading some of my chapters Bob invited me to join MIT after the completion of my Ph.D. Bob used to read all my papers and give the most valuable comments. The other person who had helped me throughout in this period was Jim Mirrlees, who was on the faculty in Cambridge, UK (he was also an examiner of my Ph.D. dissertation), and then visited MIT for a semester.

> Optimal Growth, and Growth Theory in general was in the air those days in both Cambridge, UK and Cambridge, Mass. I interacted intensively with the younger growth thory crowd at MIT, particularly Joe Stiglitz, George Akerlof, Eytan Sheshinsky, Miguel Sidrauski and others. Coming as I did from India, my friends and mentors Amartya Sen, Sukhamay Chakrabarty, and T.N. Srinivasan were also a big influence on my work.

In answer to the question as to whether the recollection by Joe Stiglitz that "endogenous growth" was a subject of discussion at Shell's seminar and at MIT at the time, Bardhan wrote (December 23, 2015):

> Joe is absolutely right. Endogenous Growth was very much a part of our discussion, particularly with Joe (Stiglitz), George (Akerlof) and Karl (Shell). Arrow's 1962 Learning by Doing paper was a major inspiration for all of us (I myself published a few papers on that topic), and Learning by Doing is central to endogenous growth. Karl was, however, particularly interested in R&D as a source of endogenous growth, and that was more often discussed in his seminar.
>
> What none of us used much was the link between endogenous growth and imperfect competition, which Paul Romer developed more than two decades later. But American endogenous growth theorists do not remember that the same issue of *RES* 1962 that carried Arrow's Learning by Doing article also had an article by Kaldor and Mirrlees which developed a theoretical model of endogenous growth with dynamic economies of scale and imperfect competition. (We, graduate students at Cambridge UK, studied that paper intensively.) Many years later when Paul Romer was my colleague at Berkeley, I told him about the 1962 Kaldor–Mirrlees paper, which he had never heard of.)

With regard to chapters in Shell's 1967 volume based on the dissertations of Ryder and Sheshinsky, the following may be said. Ryder's paper (1967b) was based upon work he had presented at the 1964 Uzawa Chicago workshop [Ryder (1965)] and his Stanford thesis entitled "Optimal accumulation and trade in an open economy" (1967a). Sheshinski's chapter in Shell's volume entitled "Optimal accumulation with learning by doing" is based, in part, on Chapter V of his 1966 MIT thesis entitled "Optimal growth with disembodied learning by doing." His chapter in the Shell volume, however, includes an extension from "closed" and "two sector models" to a two-sector "open model" (1967, 44–48).

Before proceeding to survey (below) Shell's contributions over the period 1964–1969 (his work from 1969 to 1973 will be dealt with subsequently), we turn to the November 1967 Chicago seminar on "growth theory" led by Uzawa, papers from which appeared in the *JPE* in 1969. Among these were papers based upon dissertations of two Stanford students, Goldman and Ryder, who attended Uzawa's 1964 Chicago seminar group, along with Shell. Goldman's November 1967 presentation was titled "Sequential planning and continuous planning revision," discussed by McFadden and Radner. It was then presented at the December 1967 ASSA ES session under the title "Sequential and continuous planning," and discussed by Koopmans, as noted above. Ryder's contribution "Optimal accumulation in a two sector neoclassical economy with non-shiftable capital" was based upon the "second model" presented in his Stanford Ph.D. thesis (1967a).

Finally, Teubal, a member of Uzawa's 1964 group, presented a paper to the November 1967 Chicago seminar on "agricultural and industry development," whereas Razin, a member of the1967 Chicago group, presented a paper entitled "Investment in human capital and economic growth," albeit neither paper was included in the 1969 *JPE* published version of the November 1967 seminar due to "editorial considerations," according to Uzawa ("Symposium on theory of economic growth", *JPE* 1969, 574). Razin's paper was part of his Ph.D. thesis and eventually published in *Metroeconomica* (1972) under the same title, also as noted above.

2.6. Shell's Contributions and Recollections, 1964–1969

As recounted above, Shell attended Uzawa's 1964 Chicago summer seminar for graduate students. The paper he presented there was based upon the first draft of his thesis Chapter III entitled "A model of inventive activity and capital accumulation." This paper was circulated under this title as University of Chicago Technical Report Number 2, dated August 1964. According to Shell's recollections (personal communication, November 10, 2015), "it was also my job market paper delivered at MIT in Fall 1964." We have already discussed Shell's ASSA presentations. What then is the relationship between his thesis, 1966 *AER* paper, and the two chapters he published in the 1967 volume he edited?

First, his paper that comprises Chapter I of his edited 1967 volume entitled "Optimal programs of capital accumulation for an economy in which there is exogenous technical change" was based on Chapters I and II of his 1965 Stanford Ph.D. thesis, which initially dealt with "the aggregative model of Solow and Swan" (thesis Chapter I) and then with the "two sector model" of growth of Meade and Uzawa (thesis Chapter II). Second, Shell recalled (personal communication, February 8, 2014) Chapter III of his thesis entitled "A model of inventive activity and capital accumulation":

> is *2-sector* inventive activity and capital accumulation; it is the basis for the 1967 book chapter (volume Chapter IV of the same title). Chapter III (of the thesis) also has the 1966 (*AER*) *1-sector* phase diagram as a special two sector case. So the *AER* (1966) phase diagram is in Chapter III (of the thesis), but only the *AER* (1966 paper) deals directly and simply with the *1-sector* model. Chapter IV (of the thesis) is some comparative static supporting chapter III.

In May 1967, Shell attended "The ad hoc meeting of experts on the role of advanced skills and technologies in industrial development," held at the United Nations in New York, and presented a paper entitled "Technological knowledge and economic growth" [Shell (1969)]. He recalled (personal communication, January 16, 2016):

> Re-reading the UN conference paper was pleasant for me. It is what I had intended: a provocative think piece with ready references. The main theme is modeling endogenous growth from inventive activity. The sundry theme was "teaching" dynamic optimization in this context. I very much doubt that an untenured 20-something could have published something like this in a major journal. In the conference, Amartya

and I formed a coalition of sorts. Amartya was analyzing the development (i.e., poor country) growth model of Arthur Lewis in which the real wage rate is set much higher than market clearing. Profits are saved. Wages are consumed. Then the composition of output (c, z) depends on the technology k. High k means low saving. Low k means low consumption. The "opposition" accused of trying to dump our used cars on poor people. Amartya said that the choice of technique is driven by the social demand price of investment. I showed where that price comes from.

2.7. Building Blocks: Technical Progress and Innovation, Human Capital, Growth, Increasing Returns and Imperfect Competition, 1952–1973

A decade after Abramowitz set out the "central questions" in growth theory, as noted above, Kaldor and Mirrlees (1962) describe the relationship between technical progress and endogenous growth, and Arrow (1962) set out his "endogenous theory" of knowledge-based growth. In the years that followed, the focus was, in the main, on the roles of technical progress and innovation in the growth process, as noted above. Some, however, also attempted to bring in other factors such as the relationship between human capital and growth, and that between increasing returns in a market economy and growth, in *both* static and dynamic frameworks. The human capital approach was ostensibly introduced by Uzawa (1965), and extended by Razin in his Ph.D. thesis—supervised by Uzawa—and published paper, as noted above.

The implications of IRTS for the market economy are evident in the works of Aoki over the period 1967–1968. Aoki's approach developed out of his 1967 Minnesota Ph.D. thesis entitled "Increasing returns to scale and the market mechanism," supervised by Chipman (1967a). He was then recruited to Stanford by Arrow. At Stanford, he circulated *Technical Reports* on IRTS, sponsored by the NSF and Stanford IMSSS, and emanating from his thesis (1967b, 1968). As Aoki recalled (personal communication, October 15, 2013): "My approach was not explicitly dynamic as in the work of Romer. However, it is important to be analytically clear even in (the) static framework about how increasing returns could be consistent with the market economy—either through tax and subsidy schemes (in the case of external economies) or the emergence of the firm (in the case of internal economies) this is what my original contributions did."

Interestingly enough, another Ph.D. student *directly introduced* IRTS into a von Neumann type of balanced growth model at about the same time. Britto "assumed increasing returns to scale" in Chapter IV of his Brown Ph.D. dissertation entitled "Efficient allocation of resources in a growing economy" (1966). Moreover, he actually presented his approach in the December 1965 ASSA ES session entitled "Economic Growth II," chaired by Smyth. At that session, as noted above, Uzawa and Sidrauski also presented a paper, as did Mortensen. Britto's paper was entitled "Balanced growth and efficiency with increasing returns to scale," and was discussed by Cass.

According to the abstract of his presentation, Britto presented a model "of a two good economy with increasing returns to scale." In the first section, he

defined the "the von Neumann path of balanced growth," and demonstrated "its inter-temporal efficiency." In the second section, he showed that the von Neumann path "exists. . . is unique, and. . . is the only efficient path intertemporally" [*Econometrica* Report (1966, 129)].

Returning to the Kaldor–Mirrlees (1962) "new model," *it should be recalled here that their approach "assumed" imperfect competition*. Indeed, as they wrote (1962, 175, 180): "Each entrepreneur, operating in imperfectly competitive markets, aims at the maximum attainable growth of his own business. . . the share of profits resulting from a model must be higher than a certain minimum (the so-called 'degree of monopoly' or 'degree of imperfect competition')." We will return to imperfect competition below. At this point, let us survey contributions regarding endogenous growth made by two *additional* important international conferences, the First World Congress of the Econometric Society (ES) (1965) and the International Economic Association (IEA henceforth) Conference on growth (1970), and also contributions made in the period in between them.

At the First World Congress of the ES held in Rome in 1965, there were a number of sessions dealing with growth, technological change, and technological progress. Of the session on "Cycles and growth," chaired by the Ichimura, a paper entitled "Business cycles and endogenous factors for economic growth" was presented by Inada, with Jorgenson and Hansen as discussants [Program First World Congress, *Econometrica* (1966, 7)]. Inada continued his interest in the subject as manifest in his subsequent paper entitled "Endogenous technological progress and steady growth," which first appeared as a Stanford Technical Report in 1968 [Inada (1968)] and was subsequently published in the *RES* (1969).

The session entitled "Investment and technological change," chaired by Goodwin, included the paper by Drandakis and Phelps entitled "A model of induced invention, growth and distribution," previously circulated in July 1965 as Cowles Foundation *Discussion Paper* 198 and later published in the *EJ* (1966), as noted above (Program First World Congress, *Econometrica*, 1966, 10). The session entitled "Causes and consequences of technical progress," chaired by Mansfield, included Levhari's paper under the title "Extensions of Arrow's learning by doing" and that by von Weizsäcker entitled "Tentative notes on a two sector model with induced technical progress" [Program First World Congress, *Econometrica* (1966, 11–12)]. Both of these papers later appeared in the *RES* [Levhari (1966a), von Weizsäcker (1966)]. Levhari also expanded on this in his *RES* paper entitled "Further implications of learning by doing" [Levhari (1966b)].

In their 1969 *EJ* piece, entitled "A new view of technological change," Atkinson and Stiglitz questioned the approaches of Kennedy (1964) and Samuelson (1965) regarding the impact of "induced invention and innovation on economic growth and distribution" (1969, 577–578). In his paper entitled "An economic theory of technological change," Nordhaus used what he termed a "microtheoretic approach," to provide "a reasonable model of the inventive process." This paper was *originally* presented at the December 1968 Chicago ES session entitled "Theory of innovation," chaired by Phelps, under the title "The sources of productivity

change," and was discussed by Domar and Stiglitz. It appeared as Cowles Foundation *Discussion Paper* 265 in February 1969 and was published in the *AER* (1969).

The 1970 IEA Conference entitled "Models of economic growth," held in Jerusalem, was attended by leading growth theorists such as Atkinson, Bliss, Bruno, Diamond, Dixit, Hahn, Levhari, Mirrlees, Shell, Sheshinski, Stern, Stiglitz, Teubal, Uzawa, von Weizsäcker, and Yaari, among others. The conference proceedings were published in 1973. In the introduction to the conference volume, edited by Mirrlees and Stern, Mirrlees outlined "some of the uses to which growth models have been put" (1973, xii–xxi). For our purpose here, the most important paper in the volume was by Shell, entitled "Inventive activity, industrial organization and economic growth" (1973 [1970]). Shell actually based Section II of his paper, which he called "the pure monopoly model," on his paper entitled "The Schumpeterian model of induced innovation and capital accumulation," which he had earlier presented at the ES meeting in December 1965 (1973, 80, note 3), as noted above. Moreover, he again stressed what he saw as the limitations of "constant returns to scale" in "contemporary growth theory," which he had been emphasizing from 1965 Ph.D thesis onward. As he put it (1973, 80):

> Contemporary growth theory relies heavily on the assumption of constant returns to scale. If technical knowledge is an argument of the production function, then constant returns in *all* factors is not an attractive hypothesis. If the firm doubles its conventional factors, capital and labor, output should be at least doubled since mere replication is always a possibility. Therefore, if the firm doubles its conventional factors and doubles its stock of knowledge (as measured say, in patents held), then the firm's output must be more than doubled. If the firm does indeed face these increasing returns to scale, then it is glaringly obvious that specification of industrial organization will not be straightforward. For example, the competitive model with free entry costs or costless adjustment of inputs will not work. By Euler's theorem, if factors were awarded the marginal products, then payments to conventional factors would exhaust output, leaving no room for inventive activity.

The importance of Shell's assertion here cannot be overstated. In a note to this, Shell continued (1973, 80, note 2): "This paragraph on increasing returns to scale at the firm level is to be taken as argument by *reductio ad absurdum.* I wish to show the incompatibility of competition and frequently encountered technological assumptions. I do not mean to argue that decreasing returns to scale (especially at the economy level) are impossible." Shell's underlying message is clear: *competitive equilibrium is not viable with IRTS*.

It is somewhat ironic that Shell felt the need to highlight this result. The observation that the price-taking behavior in perfectly competitive markets would generally not be compatible with situations in which there were a small numbers of agents goes back to the beginnings of marginalist economic theory and the work of Cournot, Edgeworth, and Walras, and is codified in Marshall's *Principles*. The applicability of Euler's theorem on homogeneous functions came along somewhat later but was well established in Samuelson's *Foundations of Economic Analysis*.

The impetus of Shell's observation, and the reason he included explicit taxation in support of research in his own model of external increasing returns to knowledge, was the simple fact that if external returns to scale occur without economic costs, models that incorporate these effects are observationally equivalent to the Solow model with exogenous productivity growth.

That Shell's admonition to the profession was necessary is readily apparent in the way models that build on Marshall's idea of economies of scale external to the actual operation of firms treat the economics of these externalities. Indeed, we can trace a lineage from Allyn Young's 1928 paper through Marvin Frankel's original deployment of the first AK models to Paul Romer's thesis and Robert Lucas's work on the mechanics of development.[2] It is only with Romer's papers on increasing returns and technological innovation, wherein he introduces the Dixit–Stiglitz model of monopolistic competition, that growth theory finally internalizes Shell's critique.

Once *formalized* by Dixit and Stiglitz (1974, 1977) and Ethier (1982), it is not surprising that "imperfect competition"—*along with IRTS*—became a building block of the *first stage* in the "Second Wave" of the endogenous growth approach; more about this will be presented in our next paper.

NOTES

1. We do not discuss the origins of the application of the notion of increasing returns here, as the idea goes back to Smith (1776) and Marshall (1890). For one of the earliest *modern* views, see Young (1928). For an assessment of the idea seven decades after Young, see the papers in Arrow et al. (1998).

2. Young's treatment of increasing returns is quite nuanced, but he still characterizes the interplay between internal and external returns to scale by noting that returns to specialization lead "the representative firm. . . to lose its identity. . . its internal economies dissolve into the internal and external economies of the more highly specialized undertaking that are its success and are supplemented by new economies." (1928, 538)

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APPENDIX

Participants in the Uzawa summer seminars, selected years: Names, institutions, dissertations

Date	Name	University	Dissertation (Date Submitted)
1964	Cass	Stanford	Studies in the theory of optimum economic growth (1965)
	Douglas	Stanford	Studies in monetary dynamics (1966)
	Goldman	Stanford	Economic growth and international trade: a study in the theory of economic development
	Oniki	Stanford	A theoretical study on the demand for education (1968)
	Ryder	Stanford	Optimal accumulation and trade in an open economy (1967)
	Shell	Stanford	Patterns of technical change and capital accumulation (1965)
	Teubal	Chicago	Inflation, unemployment and economic growth (1967)
1965	Akerlof	MIT	Wages and capital (1966)
	Chaudhuri	MIT	Planning in a multi-regional economy: a case study of India (1966)
	Krishna	Chicago	Production relations in manufacturing plants: an exploratory study (1967)
	Levy	Yale	Racial imbalance act of Massachusetts (1969)
	Nordhaus	MIT	A theory of endogenous technical change (1967)
	Sheshinski	MIT	Essays on the theory of production and technical progress (1966)
	Stiglitz	MIT	Studies in the theory of economic growth and income distribution (1966)

1967	Burton	Northwestern	The development process in a dual economy (1971)
	Calvo	Yale	Efficiency and optimality in a heterogeneous capital model – putty-clay (1974)
	Ethier	Rochester	Models of international trade and investment (1970)
	Mirman	Rochester	Two essays on uncertainty and economics (1970)
	Narayanan	Yale	The two-sector model of growth (1973)
	Nissen	Berkeley	Consistent and inconsistent intertemporal behavior (1968)
	Onitsuka	Chicago	International capital movements and economics growth (1970)
	Razin	Chicago	Investment in human capital and economic growth: a theoretical study (1969)
	Rhee	Kansas	Foreign debt and economic growth (1972)
	Sidrauski	Chicago	Rational choice and patterns of growth in a monetary economy (1967)
	Smith	Northwestern	On risk and optimal rates of resource utilization in a theory of the firm

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