J. Fluid Mech. (2002), *vol.* 468, *pp.* 107–119. © 2002 Cambridge University Press
DOI: 10.1017/S0022112002001489 Printed in the United Kingdom

Elastohydrodynamic rebound of spheres from coated surfaces

By ROBERT H. DAVIS†, DEAN A. RAGER AND BRIAN T. GOOD

Department of Chemical Engineering, University of Colorado, Boulder, CO 80309–0424, USA

(Received 24 July 2001 and in revised form 12 April 2002)

Experiments were performed to measure the rebound velocities of small plastic and metal spheres dropped from various heights onto a smooth quartz surface coated with a thin layer of viscous fluid. The spheres stick without rebounding for low impact velocities, due to viscous dissipation in the thin fluid layer. Above a critical impact velocity, however, the lubrication forces in the thin layer cause elastic deformation and rebound of the spheres. The apparent coefficient of restitution increases with the ratio of the Stokes number to its critical value for rebound, where the Stokes number is a dimensionless ratio of the inertia of the sphere to viscous forces in the fluid. The critical Stokes number required for rebound decreases weakly with increasing values of a dimensionless elasticity parameter which is a ratio of the viscous forces which cause deformation to the elastic forces which resist deformation. The experimental results show good agreement with an approximate model based on lubrication theory for undeformed spheres and scaling relations for elastic deformation.

1. Introduction

Collisions of small particles with other particles or surfaces play key roles in industrial and natural processes such as filtration, agglomeration, granular flow, sand blasting, pollen capture, and clean-room applications. The surfaces are wet in many cases, which can cause the particles to stick or have reduced kinetic energy due to viscous losses. The classical Hertzian analysis (cf. Love 1927) for dry surfaces assumes perfectly elastic collisions, so that the coefficient of restitution (defined as the ratio of the magnitudes of the rebound and impact velocities) is unity. In practice, the dry coefficient of restitution, e_{dry}, is reduced by plastic deformation (Johnson 1985), elastic waves (Hunter 1957), vibrations (Hunter 1957; Reed 1985; Sondergaard, Chaney & Brennen 1990), viscoelasticity of the solids (Falcon *et al.* 1998; Ramirez *et al.* 1999), and adhesive forces (Dahneke 1971).

When the surfaces are wet, the coefficient of restitution is further reduced by viscous losses in the thin fluid layer between the colliding surfaces. Davis, Serayssol & Hinch (1986) first analysed this problem, which they called an *elastohydrodynamic collision*, by numerically solving the coupled lubrication equation for the fluid flow and pressure and solid-elasticity equation for the Hertzian deformation of the surfaces. They showed that the collision and rebound process is governed by two dimensionless parameters:
Stokes number

$$St = mv_o/(6\pi\mu a^2), \tag{1}$$

† Author to whom correspondence should be addressed: robert.davis@colorado.edu

elasticity parameter

$$\epsilon = 4\theta\mu v_o a^{3/2}/x_o^{5/2}, \tag{2}$$

where $m = 4\pi a^3\rho_s/3$ is the mass of the ball, a is its radius, ρ_s is its density, μ is the fluid viscosity, v_o is the impact velocity when starting at a separation x_o between the surfaces, $\theta = (1-v_1^2)/(\pi E_1) + (1-v_2^2)/(\pi E_2)$, and v_i and E_i are Poisson's ratio and the Young's modulus of elasticity for the ball ($i = 1$) and plane ($i = 2$). The same analysis applies for the collision of two spheres, with a and m then equal to the reduced radius and mass, respectively (Davis *et al.* 1986).

Davis *et al.* (1986) showed that no rebound occurred when the Stokes number is less than a critical value ($St < St_c$), due to viscous dissipation of the initial kinetic energy of the sphere. The value of the critical Stokes number for rebound is predicted to depend weakly on the elasticity parameter, increasing from $St_c \approx 1$ at $\epsilon = 10^{-2}$ to $St_c \approx 8$ at $\epsilon = 10^{-8}$. Rebound is predicted for $St > St_c$, with $e_{wet} = v_r/v_o$ increasing with increasing St, where v_r is the maximum velocity achieved during rebound. An approximate analytical model which provides a closed-form solution in good agreement with the numerical solution has been presented by Lian, Adams & Thorten (1996).

Elastohydrodynamic rebound was verified by Barnocky & Davis (1988), who dropped small steel and acrylic balls onto a quartz surface covered with a thin viscous layer. They found that the critical drop height for rebound increased with increasing fluid thickness and viscosity, and with decreasing ball size and density. The experimental results for the critical Stokes number are in good agreement with the theory of Davis *et al.* (1986), but the coefficient of restitution for $St > St_c$ was not determined. Lundberg & Shen (1992) subsequently examined the influence of a drop of viscous oil on the collision between a large roller and a large ball, finding that the coefficient of restitution generally decreased with increasing viscosity of the oil. More recently, several groups have investigated elastohydrodynamic collisions and rebound for total immersion of solid spheres in a viscous liquid over broad ranges of the Stokes number (Zhang *et al.* 1999; Zenit & Hunt 1999; Gondret *et al.* 1999; Gondret, Lance & Petit 2002; Joseph *et al.* 2001). No rebound of spheres from surfaces was observed for Stokes numbers below a critical value of $St_c \approx 10$. Above this critical value, the coefficient of restitution was observed to increase rapidly and then gradually with increasing Stokes number, becoming close to the value for dry collisions at $St \approx 500$. Both Gondret *et al.* (2002) and Joseph *et al.* (2001) fit all their data on a single curve of e_{wet}/e_{dry} versus St, independent of elastic properties, with moderate scatter on a semi-log plot.

In the current paper, we describe experiments on elastohydrodynamic collisions and rebound for small metal and plastic balls impacting a hard surface coated with a thin layer of viscous fluid. The purpose is to determine the apparent coefficient of restitution and how it depends on fluid and solid properties. Thus, the earlier work of Barnocky & Davis (1988) is extended to provide quantitative data for non-zero rebound velocities when $St > St_c$. A simple theory accounting for particle inertia, viscous lubrication, elastic deformation, and losses within the solids is used to analyse the data.

2. Materials and methods

Small solid balls (Small Parts, Inc.) of Nylon 66 ($\rho_s = 1.14\,\mathrm{g\,cm^{-3}}$, $E_1 = 2.84 \times 10^{10}\,\mathrm{g\,cm^{-1}\,s^{-2}}$, $v_1 = 0.35$) or stainless steel 302 ($\rho_s = 7.96\,\mathrm{g\,cm^{-3}}$, $E_1 = 2.00 \times$

10^{11} g cm^{-1} s^{-2}, $\nu_1 = 0.28$) were dropped one at a time from various heights onto a quartz disk ($E_2 = 7.26 \times 10^{11}$ g cm^{-1} s^{-2}, $\nu_2 = 0.17$) which was dry or covered with a thin layer of a Newtonian oil. The balls have radii of $a = 0.32$, 0.48 and 0.64 cm. Scanning electron micrographs (SEMs) revealed small (1–5 μm) widely separated pits on the surfaces of the steel balls (Barnocky & Davis 1988). The SEMs of the nylon balls revealed bumps and filaments typically 2–20 μm high on their surfaces (Zeng, Kerns & Davis 1996; Joseph *et al.* 2001). The quartz disk used for most experiments is 0.64 cm thick and has a diameter of 5.08 cm; it is optically smooth. Measurements using the method of Smart & Leighton (1989) gave hydrodynamic roughnesses of approximately 4 μm for the steel balls and 20 μm for the nylon balls. In this method, the balls of 0.32 cm diameter were allowed to settle in a viscous fluid onto the quartz disk, which was subsequently inverted, and the times required for the ball to fall one and two radii from the disk were recorded to determine the initial separation (assumed equal to the hydrodynamic roughness). The hydrodynamic roughnesses are larger than the root-mean-square roughnesses determined by profilometry or microscopy for similar materials (e.g. Joseph *et al.* 2001). This result is expected because the balls contact the plane, prior to its inversion, on the largest roughness elements present in sufficient density to support the balls. Also, calculation of the suction lubrication pressure as the balls fall away from the inverted plane indicates that cavitation may occur for separations less than 0.1 μm for the 0.32 cm nylon ball and less than 3.5 μm for the 0.32 cm steel ball. Thus, the measured hydrodynamic roughness of 4 μm for the steel ball may have been due to cavitation, with the actual roughness being smaller than this value.

The two fluids used are silicon-based oils (Brookfield Engineering Laboratories, Inc.) with viscosities of $\mu = 9.9$ and 125 g cm^{-1} s^{-1} and densities of $\rho = 0.972$ and 0.973 g cm^{-3}, respectively, at 23°C. The oil was applied to the top surface of the quartz disk with a small brush and allowed to stand to obtain a constant thickness which was calculated using the difference in mass between the wet and dry disk. For most experiments, an oil thickness of δ = 80, 150, or 250 μm was used. The experiments were performed at room temperature (23 ± 2 °C), and temperature-dependent viscosity corrections were applied when calculating the Stokes number and elasticity parameter.

The balls were dropped by first placing them in a hole with diameter slightly larger than the ball drilled in an aluminium plate of 0.64 cm thickness. The balls were held in place manually with a finger and then released by rapidly moving the finger straight down. The height h_o from which the ball was dropped onto the quartz disk was varied between 4 and 90 cm. An impact velocity of $v_o = \sqrt{2gh_o}$ is obtained by neglecting air resistance, where $g = 980$ cm s^{-2} is the gravitational acceleration. Each experiment was photographed using an Olympus OM–1 35 mm camera and a General Radio Company Type 1531-A stroboscope (typically operated at about 100 flashes per second). Measured impact and rebound velocities were determined from the distance between successive ball images on each picture, with a correction applied for gravitational acceleration/deceleration. Most of the measured impact velocities are within ±5 cm s^{-1} of the theoretical value of $v_o = \sqrt{2gh_o}$. For sufficiently small rebound velocities, the maximum rebound height, h_r, is evident on the photographs, and the measured rebound velocities from the successive ball images in these cases are generally within ±5 cm s^{-1} of the expected value $v_r = \sqrt{2gh_r}$. The uncertainty in measuring ball velocities from the photographs is also estimated to be ±5 cm s^{-1}. The quartz disk was mounted in an aluminium holder and inclined 3–4° from horizontal so that the rebound images could be easily distinguished from the impact

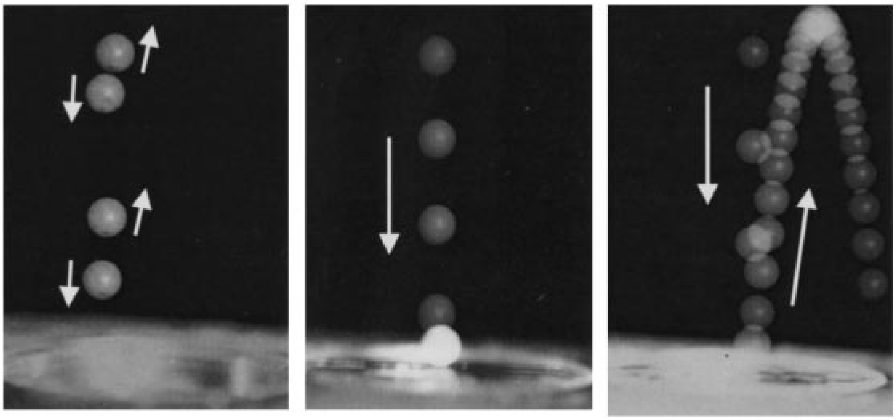


FIGURE 1. Strobed photographs of a nylon sphere of radius 0.32 cm dropped onto (*a*) a dry quartz surface from a height of 20 cm (4000 flashes per minute), (*b*) a quartz surface overlaid with a thin layer of fluid with 9.9 g cm^{-1} s^{-1} viscosity and 80 μm thickness from a height of 20 cm (8000 flashes per minute), and (*c*) the quartz surface with the same fluid layer from a height of 30 cm (8000 flashes per minute). The target disk is visible (but not fully in focus) at the bottom of each panel, and the light area on the surface is a reflection of the strobe light.

images. Additional experiments showed that the critical impact velocity for rebound is insensitive to the angle of inclination of the quartz disk up to 20° from horizontal, although systematic measurements of rebound velocities for different angles have not yet been made.

3. Experimental results

Figure 1 shows typical strobotic photographs for the nylon ball with $a = 0.32$ cm dropped onto the quartz surface with and without a thin layer of the 9.9 P fluid with $\delta = 80$ μm. For the dry surface, the sphere rebounds with a velocity only slightly lower than the impact velocity. In contrast, no rebound is observed when the ball is dropped from $h_o = 20$ cm onto the wetted surface. For $h_o = 30$ cm, rebound from the wet surface is observed, but the rebound velocity is much less than the impact velocity, due to viscous losses in the oil layer. Small ink dots were placed on the ball surface for some experiments, and these show a small rotation of approximately 2° between successive images on the photographs as the ball dropped, and a larger rotation of typically 5–10° between successive images during rebound.

Figure 2 shows that the rebound velocity is proportional to the impact velocity for dry collisions, over the range studied. The slopes of the lines give dry coefficients of restitution at the 90% confidence level of $e_{dry} = 0.88 \pm 0.04$, 0.86 ± 0.02 and 0.84 ± 0.02 for nylon spheres of radius 0.32, 0.48 and 0.64 cm, respectively, and $e_{dry} = 0.77 \pm 0.02$, 0.57 ± 0.05 and 0.35 ± 0.05, respectively, for steel spheres of the same sizes. The small losses for nylon are probably due to a combination of elastic waves, viscoelastic behaviour, and vibrations of the thin quartz disk, whereas vibrations dominate the larger losses with steel balls. We repeated the dry experiments with a quartz disk of 1.27 cm thickness and achieved higher values of $e_{dry} = 0.89 \pm 0.04$, 0.94 ± 0.03 and 0.90 ± 0.02, respectively, for nylon spheres of radius 0.32, 0.44 and 0.64 cm, and $e_{dry} = 0.93 \pm 0.02$, 0.79 ± 0.05 and 0.67 ± 0.07, respectively, for steel spheres of radius 0.32, 0.48 and 0.64 cm. The increased dry coefficients of restitution indicate

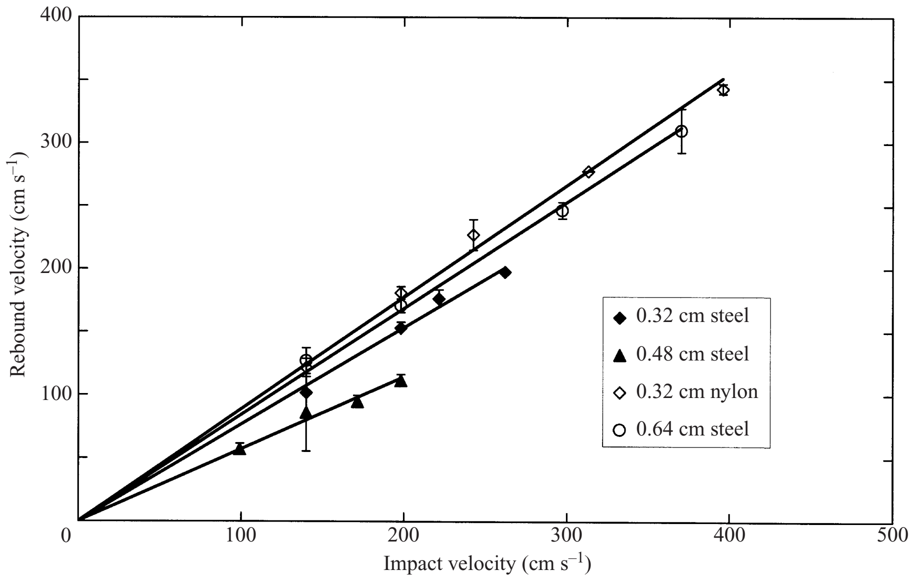
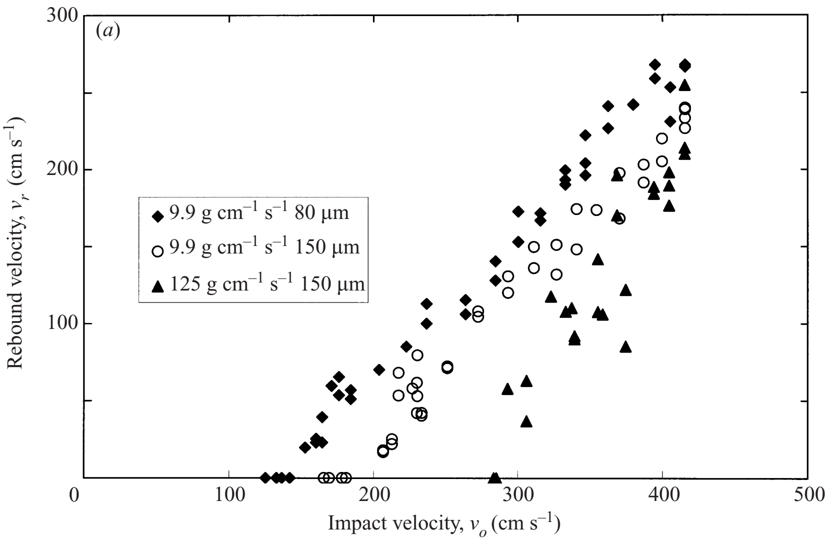


FIGURE 2. Linear increase of rebound velocity with impact velocity for collisions of nylon and steel spheres with a dry quartz disk of 0.64 cm thickness. The error bars represent plus and minus one standard deviation for typically 3–5 repeats for selected conditions.

less vibrational losses, consistent with previous findings that the dry coefficient of restitution increases with plate thickness until the plate thickness is approximately four times the ball diameter (Sondergaard *et al.* 1990). No statistically significant variation in the dry coefficients of restitution with impact location (e.g. centre versus edge of target) was observed. Previously, Sondergaard *et al.* (1990) showed that the apparent coefficient of restitution decreased with impact distance from the charged edge of a plate, due to increased vibrational losses, but the difference is greater than our experimental uncertainty only for steel balls greater than 0.64 cm in diameter.

Figure 3 for wet collisions shows a different behaviour in that no rebound is observed until a critical impact velocity is reached, and then the rebound velocity increases rapidly at first and then almost linearly with impact velocity above the critical value. The critical impact velocity is higher for greater fluid thickness or viscosity (figure 3*a*), due to increased viscous dissipation. On the other hand, the critical impact velocity is lower for a larger or more dense ball (figure 3*b*), due to greater inertia which allows the ball to more easily penetrate the viscous layer and achieve elastic deformation.

The experimental results are plotted in dimensionless form as the coefficient of restitution versus Stokes number for the nylon balls of radii 0.32, 0.48 and 0.64 cm, respectively, in figures 4(*a*), 4(*b*) and 4(*c*). As observed previously (Gondret *et al.* 2002; Joseph *et al.* 2001) for fully immersed collisions, the coefficient of restitution is zero below a critical Stokes number and then increases rapidly with increasing Stokes number above the critical number before levelling out at large Stokes numbers. However, the critical Stokes number is much less than the value of $St_c \approx 10$ observed for fully immersed collisions (Gondret *et al.* 2002; Joseph *et al.* 2001), and it is observed to increase with increased fluid-layer thickness and decrease with increased fluid viscosity. These findings are in at least qualitative agreement with the theory of Davis *et al.* (1986), as the dimensionless elasticity parameter defined by (2) increases



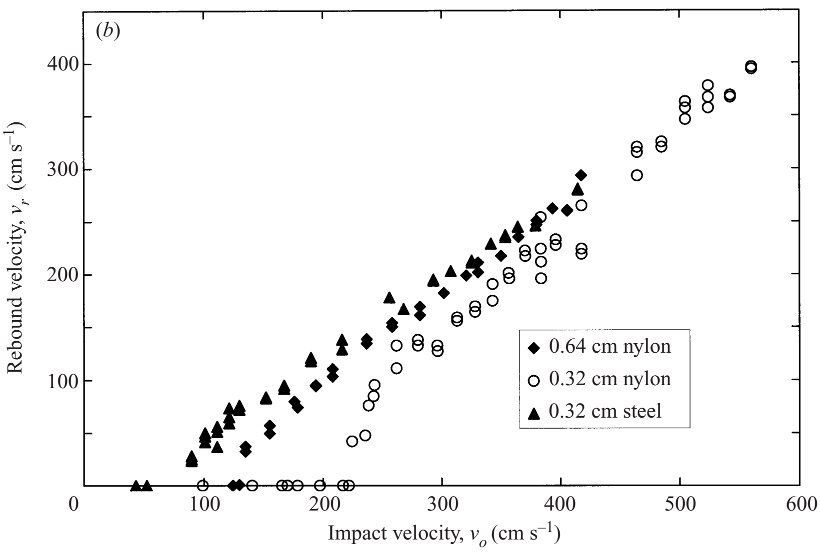


FIGURE 3. Increase in rebound velocity with impact velocity above a critical value for spheres dropped on a quartz disk overlaid with a thin viscous layer: (*a*) nylon spheres of 0.48 cm radius; (*b*) fluid of 9.9 g cm^{-1} s^{-1} viscosity and 80 µm thickness.

with increasing fluid viscosity and decreases with increasing layer thickness. A thicker layer causes more viscous dissipation prior to significant elastic deformation, whereas a more viscous fluid exerts a greater lubrication force and causes more elastic deformation. On the other hand, the critical Stokes number appears to be independent of or increase weakly with the ball size, whereas the elasticity parameter increases with ball size and so a larger ball might be expected to bounce more easily (lower St_c).

The coefficient of restitution versus the Stokes number for the steel balls of radii 0.32, 0.48 and 0.64 cm is shown in figures 5(*a*), 5(*b*) and 5(*c*), respectively. The greater density of steel compared to nylon yields larger Stokes numbers for the same ball sizes, drop heights, and fluid viscosities. Moreover, the critical Stokes numbers for

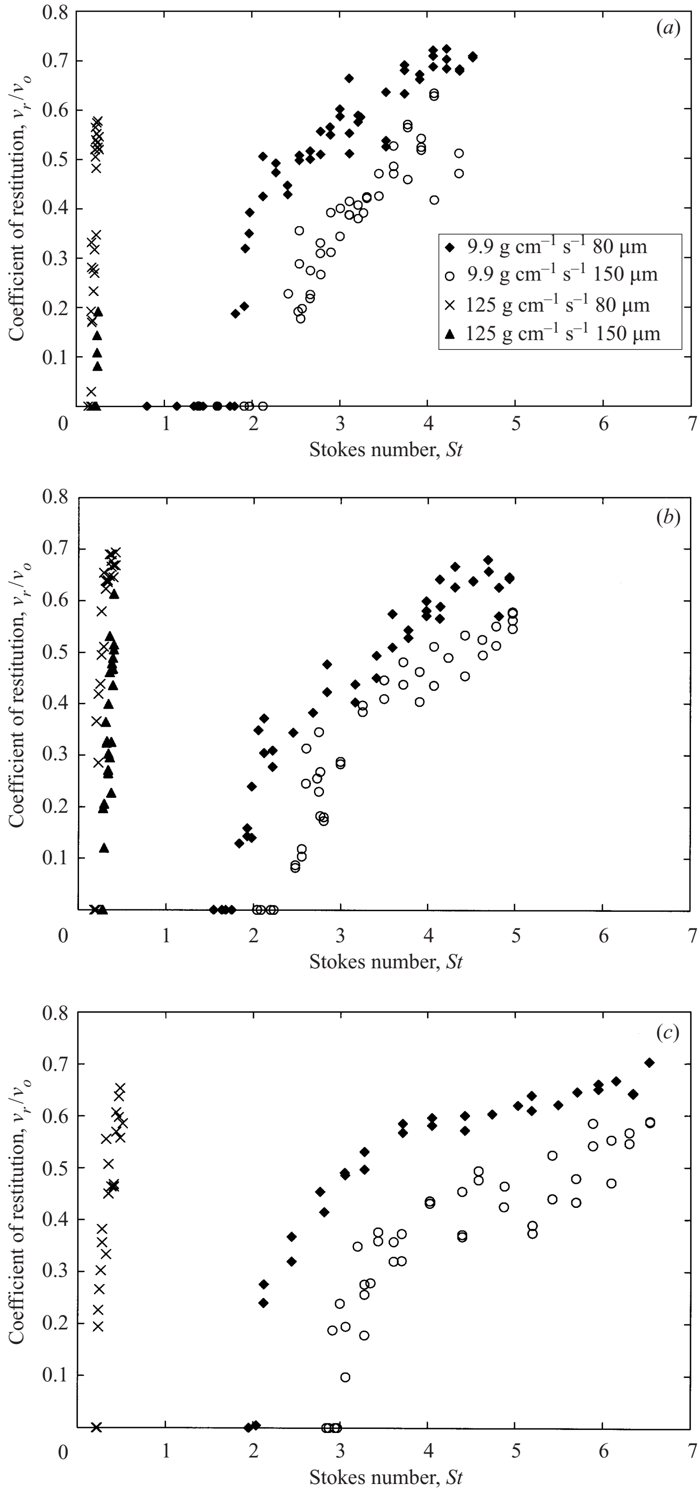


FIGURE 4. Coefficient of restitution versus Stokes number for nylon balls of radius (*a*) 0.32 cm, (*b*) 0.48 cm, and (*c*) 0.64 cm.

rebound are higher for the heavier steel balls. The dimensionless elasticity parameter is much smaller for a steel ball ($\epsilon \approx 10^{-6}$–10^{-4}) than for a nylon ball ($\epsilon \approx 10^{-3}$–10^{-2}), because of the larger Young's modulus for steel and because the greater density of steel requires a smaller impact velocity at fixed Stokes number. The smaller elasticity parameter implies a larger critical Stokes number (Davis *et al.* 1986), and the stiffer and heavier steel balls penetrate further into the oil layer before elastic deformation is significant, causing greater viscous losses and reducing the apparent coefficient of restitution at fixed Stokes numbers. Unlike for the nylon spheres, the coefficient of restitution at large Stokes numbers decreases significantly with increasing ball size; this finding is consistent with the dry coefficient of restitution, which decreases with increasing mass of the steel balls due to greater vibrations of the thin quartz disk (Sondergaard *et al.* 1990) Indeed, figure 5(a) shows that using the quartz disk of thickness 1.27 cm with the steel sphere of radius 0.32 cm results in wet coefficients of restitution which are 20–25% higher than those for the thinner disk, which is about the same as the percent increase observed for the dry coefficient of restitution. There is also considerable scatter in the data, which may be due to local variations in coating thickness and surface roughness, as well as the uncertainty in the velocity measurements.

4. Comparison with theory

In this section, an approximate model based on lubrication theory for undeformed spheres and scaling arguments for the elastic deformation and rebound is presented and compared with experimental results. Following the asymptotic solution of Davis *et al.* (1986) for small deformations, the viscous lubrication force resisting the near-contact motion of a sphere toward a plane (or another sphere) is

$$F_L = 6\pi\mu a^2 v/x, \tag{3}$$

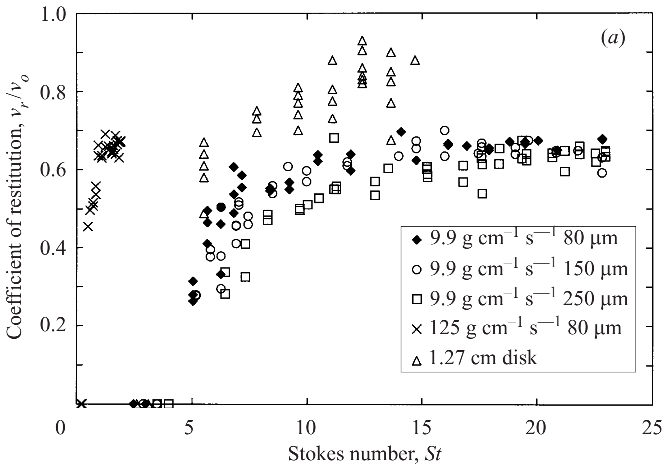
where v is the instantaneous relative velocity of the sphere toward the plane and x is the instantaneous distance between the nose of the undeformed sphere and plane. Equation (3) requires that $x \ll a$ and $Re_L = \rho v x/\mu \ll 1$; both of these conditions are easily met in the experiments of this paper. It also requires that the fluid motion is quasi-steady (so that the transient term in the fluid momentum equation can be neglected); this constraint implies $\rho x/(\rho_s a) \ll 1$, which is also met in the experiments. Finally, the effects of changes in fluid viscosity and density due to the lubrication pressure (Barnocky & Davis 1989) are neglected. Then, combining the kinetic equations $\mathrm{d}x/\mathrm{d}t = -v$ and $m\,\mathrm{d}v/\mathrm{d}t = -F_L$, and integrating subject to the initial condition $v = v_o$ when $x = x_o$, yields (Davis *et al.* 1986)

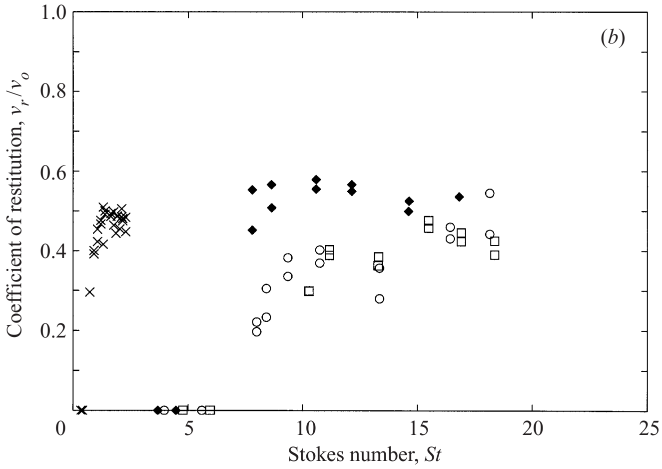
$$v/v_o = 1 - \ln(x_o/x)/St, \tag{4}$$

showing that the sphere slows down due to viscous forces as it approaches the plane. The initial separation is chosen as $x_o = 2\delta/3$, as recommended by Barnocky & Davis (1986) to allow for the sphere to penetrate a sufficient distance into the viscous layer for lubrication forces to become significant.

In this approximate model, the sphere is assumed to stop and then rebound when it reaches a separation $x = x_r$ (to be determined). To account for energy dissipation in the solid, it is assumed that the magnitude of the rebound velocity is reduced by a factor e_{dry} (Joseph *et al.* 2001) and therefore equal to

$$v_r/v_o \equiv e_{wet} = e_{dry}(1 - St_c/St), \quad St > St_c, \tag{5}$$





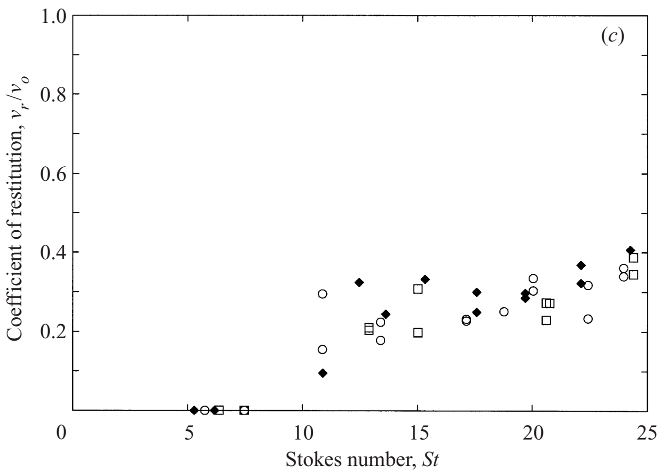


FIGURE 5. Coefficient of restitution versus Stokes number for steel balls of radius (*a*) 0.32 cm, (*b*) 0.48 cm, and (*c*) 0.64 cm. The triangles in (*a*) for the quartz target of thickness 1.27 cm, with $\delta = 80\,\mu m$ at $\mu = 9.9$ g cm^{-1} s^{-1}; all other experiments are for the quartz target of thickness 0.64 cm.

where

$$St_c = \ln(x_o/x_r) \tag{6}$$

is the critical Stokes number required for rebound.

In principle, the rebound velocity of the sphere is further reduced by viscous lubrication forces as it returns to the initial separation x_o. However, these tensile forces during rebound require a suction pressure to draw the fluid back into the gap between the receding surfaces, and the magnitude of the surface pressure is several hundred atmospheres, or larger, for the conditions of our experiments. Since the fluid will cavitate at these high tensile stresses, the resistance to motion during rebound is small relative to the resistance during approach (Barnocky & Davis 1988), and so it is neglected.

When the colliding surfaces have sufficiently larger microscopic roughness elements, then physical contact will occur when the distance between the nominal surfaces decreases to the height of the roughness elements. In this case, $x_r = x_b$, where x_b is the effective height of the roughness bumps, as described previously (Davis 1987; Barnocky & Davis 1988; Joseph *et al.* 2001). For relatively smooth spheres, on the other hand, we set x_r equal to an elasticity length scale, at which substantial elastic deformation of the sphere and plane occur due to the applied load from the lubrication force. From the theory of linear elasticity (e.g. Timoshenko & Goodier 1970), the deformation of the sphere and plane at the axis of symmetry due to the distributed lubrication force F_L scales as

$$\delta_h \approx \frac{\theta F_L}{r_h}, \tag{7}$$

where r_h is a characteristic distance between the axis of symmetry and the location of the applied load. Since the surface of the sphere near the axis of symmetry may be approximated by $h = x + r^2/2a$, where r is the radial coordinate, a characteristic radial distance over which the lubrication force is distributed is $r_h = \sqrt{2ax}$. Substituting this result and (3) into (7) yields

$$\delta_h \approx \frac{\theta}{(2ax)^{1/2}} \frac{6\pi\mu a^2 v}{x}. \tag{8}$$

The deformation at the axis of symmetry is considered to be substantial when it is comparable to the distance separating the undeformed surfaces, or $\delta_h = x = x_r$. The relative velocity in (8) could be provided by (4), but we instead make the simpler choice of $v = v_o/2$ in (8), so that the sphere maintains a significant fraction of its velocity by the time deformation becomes important, yielding

$$x_r = (3\pi\theta\mu a^{3/2} v_o/\sqrt{2})^{2/5}. \tag{9}$$

This length scale is almost the same as the deformation length scale $x_1 = (4\theta\mu a^{3/2} v_o)^{2/5}$ identified by Davis *et al.* (1986), with $x_r = 1.23\, x_1$. Then, combining (6) and (9) yields an expression for the critical Stokes number:

$$St_c = \frac{2}{5} \ln\left(\frac{\sqrt{2} x_o^{5/2}}{3\pi\theta\mu v_o a^{3/2}}\right) = \frac{2}{5} \ln\left(\frac{4\sqrt{2}}{3\pi\epsilon}\right) = 0.40 \ln(1/\epsilon) - 0.20, \tag{10}$$

where ϵ is the dimensionless elasticity parameter given by (2).

Equations (5) and (10) combined give the predicted dependence of the wet coefficient of restitution on the dry coefficient of restitution, the Stokes number, and the elasticity

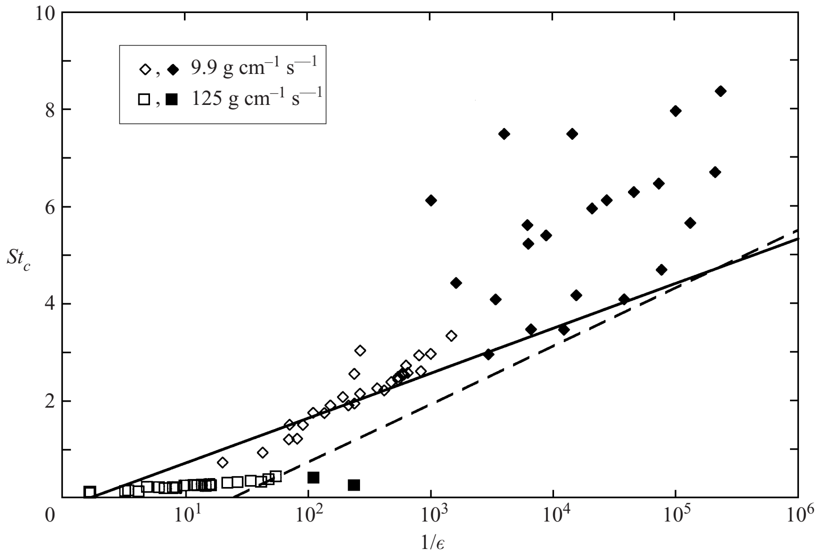


FIGURE 6. Critical Stokes number as a function of the elasticity parameter for nylon (open symbols) and steel (closed symbols) balls. The solid line is from (10) and the dashed line is from (11).

parameter. The effects of capillary forces as the ball enters the thin fluid layer are neglected in the theory, as justified previously (Barnocky & Davis 1988). It may be possible to apply (10) to the fully immersed experiments of Joseph *et al.* (2001) and Gondret *et al.* (1999, 2002), using the terminal velocity for v_o and choosing x_o to be sufficiently small that lubrication theory applies but not so small that the sphere has significantly slowed or deformed.

In figure 6, the critical Stokes number required for rebound is plotted versus $(1/\epsilon)$. As expected, the critical Stokes number increases as ϵ decreases due to the reduction in energy conversion to elastic deformation. However, the experimental data fall below the prediction of (10) for the more viscous fluid, as was observed previously (Barnocky & Davis 1988). This case corresponds to $St_c < 1$, and the large viscous forces cause the spheres to deform and bounce without penetrating very far into the fluid layer, and without meeting the condition $\epsilon \ll 1$ for the elastohydrodynamic theory of Davis *et al.* (1985) to apply. In contrast, the experimental values of St_c exceed the theoretical prediction for large values of $(1/\epsilon)$; as discussed previously (Barnocky & Davis 1988), this finding is probably a result of additional viscous dissipation which occurs during penetration and flattening of the nose of the sphere for collisions with large inertia. As an alternative to (10), the dashed line in figure 6 is the fit provided by Lian *et al.* (1996) to the full numerical solution by Davis *et al.* (1986) for the elastohydrodynamic rebound process:

$$St_c = 0.52 \ln(1/\epsilon) - 1.67. \tag{11}$$

However, (11) does not generally fit the data better than does (10).

Equation (5) represents a 'master curve' for e_{wet}/e_{dry} versus St/St_c. In figure 7, we compare the experimental data for the fluid of $9.9\,\mathrm{g\,cm^{-1}\,s}$ viscosity with both nylon and steel balls to this prediction, using (10) to determine the critical Stokes number for each experiment. Although there is considerable scatter, the data appear to collapse reasonably well on the single master curve, with no adjustable parameters. A better fit of (5) to the data would be achieved if St_c was chosen as an adjustable

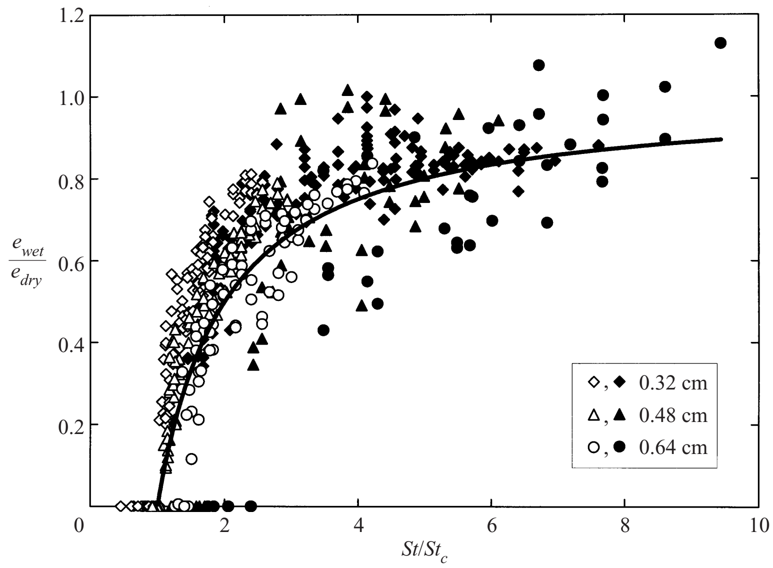


FIGURE 7. Coefficient of restitution for wet collisions, normalized by that for dry collisions, versus the ratio of the Stokes number to its critical value (determined from (10)) for nylon (open symbols) and steel (closed symbols) balls impacting a quartz surface overlaid with an 80–250 μm layer of fluid with 9.9 g cm^{-1} s^{-1} viscosity. The theoretical curve is from (5).

parameter for each set of experiments. Also, the typical elasticity length scale from (9) is about 3–9 μm for the steel balls and 10–30 μm for the nylon balls. Since these values are comparable to the measured hydrodynamic roughnesses, the latter may have contributed to the scatter in the data. The few values of $e_{wet}/e_{dry} > 1$ for the steel balls with radii of 0.48 and 0.64 cm are probably due to the large uncertainties and relatively small values of e_{dry} for these balls.

5. Concluding remarks

The coefficient of restitution (e_{wet}), defined as the rebound velocity divided by the impact velocity, was measured for steel and nylon balls dropped onto a nearly horizontal quartz surface overlaid with a thin layer of viscous fluid. Below a critical value of the Stokes number, which provides a measure of the inertia of the ball relative to viscous forces, both theory and experiment show that no bouncing occurs, because the kinetic energy of the ball is lost to viscous dissipation without enough elastic deformation for the ball to bounce. Above the critical Stokes number, the large lubrication pressure required to squeeze the fluid out of the narrow gap results in enough elastic deformation to cause rebound after the sphere is brought to rest. The coefficient of restitution then increases with increasing Stokes number (St) and asymptotes at large Stokes numbers to the value for dry collisions (e_{dry}). The critical Stokes number (St_c) is shown to increase weakly with decreasing values of an elasticity parameter (ϵ), which is a dimensionless ratio of viscous forces to elastic forces, so that less rebound occurs when the solids are stiff or the fluid layer is thick.

Although there is considerable scatter, the data collapse reasonably well on a single curve, $e_{wet}/e_{dry} = 1 - St_c/St$ for $St > St_c$, developed from lubrication theory together with scaling arguments to estimate the critical Stokes number as a function of the elasticity parameter. This simple relationship may find use in predicting particle capture probabilities in filtration and agglomeration processes and in predicting the coefficient of restitution as input for dynamic simulations of wet granular flow.

This work was supported by NASA grant NCC3-796 and by NSF grant CTS-9712604.

REFERENCES

Barnocky, G. & Davis, R. H. 1988 Elastohydrodynamic collision and rebound of spheres: Experimental verification. *Phys. Fluids* **31**, 1324–1329.

Barnocky, G. & Davis, R. H. 1989 The influence of pressure-dependent density and viscosity on the elastohydrodynamic collision and rebound of two spheres. *J. Fluid Mech.* **209**, 501–519.

Dahneke, B. 1971 The capture of aerosol particles by surfaces. *J. Colloid Interface Sci.* **37**, 342.

Davis, R. H. 1987 Elastohydrodynamic collisions of particles. *Physico-Chem. Hydrodyn.* **9**, 41–52.

Davis, R. H., Serayssol, J.-M. & Hinch, E. J. 1986 The elastohydrodynamic collision of two spheres. *J. Fluid Mech.* **163**, 479–497.

Falcon, E., Laroche, C., Fauve, S. & Coste, C. 1998 Behavior of one inelastic ball bouncing repeatedly off the ground. *Eur. Phys. J.* B **3**, 45–47.

Gondret, P., Hallouin, E., Lance, M. & Petit, L. 1999 Experiments on the motion of a solid sphere toward a wall: From viscous dissipation to elastohydrodynamic bouncing. *Phys. Fluids* **11**, 2803–2805.

Gondret, P., Lance, M. & Petit, L. 2002 Bouncing motion of spherical particles in fluids. *Phys. Fluids* **14**, 643–652.

Hunter, S. C. 1957 Energy absorbed by elastic waves during impact. *J. Mech. Phys. Solids* **5**, 162–171.

Johnson, K. L. 1985 *Contact Mechanics*. Cambridge University Press.

Joseph, G. G., Zenit, R., Hunt, M. L. & Rosenwinkel, A. M. 2001 Particle–wall collisions in a viscous fluid. *J. Fluid Mech.* **433**, 329–346.

Lian, G., Adams, M. J. & Thorten, C. 1996 Elastohydrodynamic collisions of solid spheres. *J. Fluid Mech.* **311**, 141–152.

Love, A. E. H. 1927 *A Treatise on the Mathematical Theory of Elasticity*, 4th edn. Dover.

Lundberg, J. & Shen, H. H. 1992 Collisional restitution dependence on viscosity. *J. Engng Mech.* **118**, 979–989.

Ramirez, R., Pöschel, T., Brillantov, N. V. & Schwager, T. 1999 Coefficient of restitution of colliding viscoelastic spheres. *Phys. Rev.* E **60**, 4465–4472.

Reed, J. 1985 Energy losses due to elastic waves propagation during an elastic impact. *J. Phys.* D **18**, 2329–2337.

Smart, J. R. & Leighton, D. T. 1989 Measurement of the hydrodynamic surface–roughness of noncolloidal spheres. *Phys. Fluids* A **1**, 52–60.

Sondergaard, R., Chaney, K. & Brennen, C. E. 1990 Measurements of solid spheres bouncing off flat plates. *Trans. ASME: J. Appl. Mech.* **57**, 694–699.

Timoshenko, S. P. & Goodier, T. N. 1970 *Theory of Elasticity*, 4th edn. McGraw-Hill.

Zeng, S., Kerns, E. T. & Davis, R. H. 1996 The nature of particle contacts in sedimentation. *Phys. Fluids* **8**, 1389–1396.

Zenit R. & Hunt, M. L. 1999 Mechanics of immersed particle collisions. *Trans. ASME: J. Fluids Engng* **121**, 179–184.

Zhang, J., Fan, C. Zhu, R. Pfeffer R. & Qi, D. 1999 Dynamic behavior of collision of elastic spheres in viscous fluids. *Powder Technol.* **106**, 98–109.