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Endurance improvement by battery dumping strategy considering Peukert effect for electric-powered disposable UAVs

X. Feng and Y. Sun
School of Aeronautics
Northwestern Polytechnical University
Xi'an
China

M. Chang and J. Bai
changmin@nwpu.edu.cn
Unmanned System Research Institute
Northwestern Polytechnical University
Xi'an
China

ABSTRACT

Electric-powered disposable unmanned aerial vehicles (UAVs) have wide applications due to their advantages in terms of long time flight and load capacity. Thus, improving their endurance has become an important task to enhance the performance of these UAVs. To achieve this, we investigated a battery dumping strategy which splits the battery into several packs that are used and dumped in sequence to reduce the dead weight. The Peukert effect is also considered. In this paper, the sensitivity analysis method was employed to analyse the endurance benefits for different battery weight ratios, Peukert constants and capacities, quantitatively. The results show that the endurance benefits are significantly affected by all three parameters. For ideal batteries, the endurance can be improved by 20% and 28% respectively when employing a double-pack or triple-pack battery strategy (for a battery weight ratio of 0.4), but these benefits will fall rapidly if the Peukert constant exceeds 1.0 or the battery weight declines. Besides, the endurance will be 10% longer if the lift coefficient rather than the velocity remains constant after the battery packs are dumped at a Peukert constant of 1.2.

Keywords: Disposable UAV; battery dumping strategy; Peukert effect; endurance

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NOMENCLATURE

A_W	aspect ratio
C	battery capacity
C_{D0}	zero-lift drag coefficient
C_L	lift coefficient
D	drag coefficient
e	Oswald efficiency number
$(f_p)_{\text{battery}}$	battery weight ratio
k	induced drag constant
κ_{battery}	battery specific energy
L	lift coefficient
n	Peukert constant
n_{critical}	critical n value
P_{battery}	output power
P_{cruise}	power required for level flight
q	dynamic pressure
R_t	battery hour rating
t	endurance
$t_1/\ t_2$	endurance for each phase
t_{total}	total endurance
V	flying velocity
W	weight of the UAV
$(W_P)_{\text{battery}}$	energy contained in the battery
η_{bp}	combined efficiency of battery power system

Subscripts

battery	battery condition
bp	battery power system
critical	critical value
cruise	cruise condition
1,2,3...	number of current phase

1.0 Introduction

There are numerous applications where disposable UAVs may be needed to carry out civil or military missions. For example, the UAV may be used to monitor the state of the ocean or gather environmental information. It would be abandoned when exhausting the onboard supply. It can also be used by the army to carry out tasks such as accurate strike or self-destruction during the mission(1) (e.g. loitering munition(2,3)). The rapid development of Micro Electro-Mechanical System (MEMS) has increased the importance of such disposable UAVs because they can be equipped with lightweight low-cost MEMS for flight stability, navigation and autonomy.(4,5) Powerful evidence of this is provided by the fact that some disposable UAVs or even flying sensor modules have been developed and studied broadly due to their potential to be produced and discarded in large quantities.(1,5–8) However, the tasks they can perform are accordingly ordinary and onefold. There is another kind of disposable UAV that is more complicated and suitable for diverse tasks, being able to patrol for a long time over a designated area, with a variety of mission capabilities such as area observation, information collection, target detection, air communication relay or precision strike. This paper focusses on this kind of disposable UAV.

Endurance is an important performance indicator of a disposable UAV, as it may need to patrol for a long time to observe objects. Disposable UAVs are currently powered by electric energy or fossil-fuel energy, with the latter usually coming with high cost and complex design.(9,10) Hence, electric energy is used more widely, as this power system is cheaper, quieter and easier to maintain. Nevertheless, lithium batteries have a much lower specific energy density (approximately 0.150kWh/kg) than aircraft fuels (approximately 12 kWh/kg) or animal fats (approximately 11kWh/kg). Electric-powered UAVs thus have much shorter endurance than large, fuel-powered vehicles and animals, even considering the different energy conversion efficiencies,(11,12) thus requiring urgent improvement. Many contributions such as references (13) and (14) have presented studies of conceptual parameters of electric-powered UAVs and their influence on endurance. From these, it can be found that traditional ways to improve the endurance are: increasing the lift-to-drag ratio, adding battery weight, optimizing flight and battery discharge strategies or resorting to solar energy.(15) However, if the conceptual parameters of the UAV are basically determined, it is difficult to further improve its endurance performance by using traditional design methods.

Some researchers have considered how batteries are installed in order to address this impasse. Generally, batteries in disposable UAVs are fixed to the vehicle. The battery pack remains a part of the vehicle during the whole flight, which means that the exhausted battery cannot be discarded and increases the dead weight during flight. Many researchers have tried to find solutions to decrease the dead weight in order to improve the endurance. Karan et al.(16) proposed to increase the endurance of a multi-rotor vehicle via mid-air docking and in-flight battery switching. However, this method is aimed at rotorcraft flying in a finite aera and it may be difficult for fixed-wing disposable UAVs to dock in the air due to their rapid movement. Improving the endurance using a battery dumping strategy was reported by Tan Chang et al.(17) That research focused on the influence of vehicle weight parameters (such as the Battery Mounting and Dumping Device (BMDD) weight ratio) and battery dumping strategy. This strategy represents a desirable approach to increase the endurance of fixed-wing disposable UAVs. However, considering the effect of temperature on the Peukert effect, that research used an ideal battery model, thus the decrease of the available capacity during discharge was not taken into account. The influence of the Peukert effect on battery dumping strategies has not been studied.

The Peukert effect has a significant effect on the available battery capacity. Wilhelm Peukert(18) performed constant-current discharge experiments with lead accumulators, confirming that the current and time during discharge satisfied the equation $\Delta t \times I^n = \text{const}$. The Peukert constant n represents the current drain, implying a loss in the available capacity at large discharging currents. Doerffel and Abu-Sharkh(19) confirmed that the Peukert effect is also applicable to other kinds of batteries such as lithium-ion batteries. Yu-Hua Sun et al.(20) proposed multilevel Peukert equations to predict the remaining capacity of lead-acid batteries under different discharge currents. Traub(21–23) proposed corrected equations for predicting the discharge time at large currents considering the Peukert effect and later described the influence of the battery weight ratio on flight range and endurance. The results showed that the optimal endurance will be obtained when the battery weight ratio is 2/3 independent of their type, as discussed below. Cheng Feng et al.(24) evaluated flight endurance with consideration of the Peukert effect and the influence of temperature on the batteries. All these contributions and others remind us that the influence of the Peukert effect on the battery discharge performance cannot be ignored. Although, in some works, some high-performance lithium-ion batteries show very low Peukert constants (~1.0) in ideal conditions, this may also be impacted by different ambient temperatures, manufacturers and discharging cycles, thus the Peukert constant may be much higher in practical applications.(23) Hence, it is still worth researching battery dumping strategies.

Considering the engineering practicability of battery dumping strategies, the extended endurance benefits of double- and triple-pack batteries are studied in this paper. The influence of the battery weight ratio and the Peukert constant n on the endurance are investigated by sensitivity analysis. Finally, a case of the conceptual design of a disposable UAV is presented to investigate the endurance improvement resulting from a battery dumping strategy for different battery capacities and flight velocities.

2.0 BATTERY WEIGHT AND ENDURANCE GOVERNING EQUATIONS

For a typical small electric-powered disposable UAV, it is usually assumed that the total weight remains unchanged throughout its flight. As this kind of UAV will maintain level flight for most of its flying time,(25) it is reasonable to consider that the UAV maintains level flying at sea level during the whole flight to simplify the calculation. Considering the efficiency of the electric power system, the required power and output power satisfy

$$P_{battery} * \eta_{bp} = P_{cruise} \qquad \ldots (1)$$

If the battery is ideal, meaning that no current drain occurs during the discharge, the endurance t can be expressed as(14)

$$t = \frac{(W_P)_{battery}\eta_{bp}}{P_{cruise}} = \frac{(m_P)_{battery}\kappa_{battery}\eta_{bp}}{P_{cruise}} \qquad \ldots (2)$$

where $(W_P)_{battery}$ is the energy contained in the UAV's battery, P_{cruise} is the power required for level flight, η_{bp} is the combined efficiency of electric power system, $(m_P)_{battery}$ is the mass of the battery and $\kappa_{battery}$ is the specific energy of the battery considering the impact of deep discharge.

However, the lithium-ion batteries generally used on disposable UAVs are not ideal and exhibit current drain. The battery's discharge rate impacts its available capacity. The higher the current drawn, the lower the available capacity that can be expected from the battery. This phenomenon is known as the Peukert effect.(22,23) For example, a 4000mAh lithium battery can supply power for exactly an hour at a current of 4A. However, if the required current is

2A, the battery will be capable of supplying the output for more than 2h. Meanwhile, for an 8A current, the discharge time may be less than half an hour.

The discharge time of lithium-ion batteries can be expressed as C/I^n, where C represents the battery capacity and the parameter n is the Peukert constant, which depends on the type and development level of the battery. Unfortunately, this equation only applies when the discharge current is 1 A, but when applied in a disposable UAV, the battery is often required to supply a large current to provide sufficient power. Under this condition, this equation loses accuracy and must be corrected. A correction method proposed in reference (22) is shown below:

$$t = \frac{R_t}{I^n}\left(\frac{C}{R_t}\right)^n, \qquad \ldots(3)$$

where R_t refers to the battery hour rating (in hours), typically being 1 h for small rechargeable batteries; The Peukert constant n of a lead-acid battery is 1.3, while it is 1.0 for an ideal battery. As battery technology has advanced, the value of n for lithium-ion batters has dropped to about 1.2(26) or even lower (~1.0). However, the Peukert constant may change as described above. Thus, in this article, four values of n (1.0, 1.1, 1.2 and 1.3) are considered to study the influence of the Peukert effect on the battery dumping strategy. In addition, research(26) has also been carried out using other methods to correct the equation under high-rate discharging, such as $C = I^n t - It$. However, equation (3) is used to correct it herein. Therefore, the output power $P_{battery}$ of the battery pack can be expressed as

$$p_{battery} = UI = U\frac{C}{R_t}\left(\frac{R_t}{t}\right)^{\frac{1}{n}} \qquad \ldots(4)$$

For a disposable UAV maintaining level flight at sea level, power is consumed to maintain the thrust by overcoming the drag in the direction opposite to the velocity, which can be expressed as

$$P_{cruise} = D * V \qquad \ldots(5)$$

The drag of a small electric-powered disposable UAV during level flight can be expressed as

$$D = qS\left(C_{D0} + kC_L^2\right) = \frac{1}{2}\rho V^2 S\left(C_{D0} + \frac{C_L^2}{\pi A_W e}\right), \qquad \ldots(6)$$

where q is the dynamic pressure, C_{D0} is the zero-lift drag coefficient of the UAV, k is the induced drag constant that can be expressed as $k = 1/\pi e A_W$, C_L is the lift coefficient, A_W is the aspect ratio and e is the Oswald efficiency number. Thus, the total drag on the disposable UAV can be expressed as the sum of the zero-lift drag and induced drag, while the required power can be expressed as

$$P_{cruise} = DV = \frac{1}{2}\rho V^3 S C_{D0} + \frac{2W^2 k}{\rho V^2 S} \qquad \ldots(7)$$

Then, applying equations (4) and (7) to equation (1), the endurance t can be derived as

$$V\frac{C}{R_t}\left(\frac{R_t}{t}\right)^{\frac{1}{n}}\eta_{bp} = \frac{1}{2}\rho V^3 S C_{D0} + \frac{2W^2 k}{\rho V S}$$

$$t = {R_t}^{1-n}\left[\frac{\eta_{tot} UC}{1/2\rho V^3 S C_{D0} + 2W^2 k/\rho V S}\right]^n \qquad \ldots(8)$$

This equation is valid for any flight velocities inside the envelope. According to the aircraft performance estimation equations above, when the UAV reaches its maximum endurance, the following relationship should be satisfied:

$$C_{D0} = \frac{1}{3}kC_L^2$$

$$V = \sqrt{\frac{2W}{\rho S}\sqrt{\frac{k}{3C_{D0}}}} \qquad \ldots(9)$$

The maximum endurance can thus be expressed as

$$t_{\max} = {R_t}^{1-n}\left(\frac{\eta_{bp} UC}{\left(2/\sqrt{\rho S}\right) C_{D0}^{1/4}\left(2W\sqrt{k/3}\right)^{3/2}}\right)^n \qquad \ldots(10)$$

3.0 ANALYSIS OF BATTERY WEIGHT DISTRIBUTION STRATEGY

The battery weight ratio is one of the key factors affecting the endurance performance of an electric-powered disposable UAV. A higher battery weight ratio means there are more batteries inside the disposable UAV. This, however, does not always lead to longer endurance, because the greater weight of the batteries will increase the total weight, meaning that much more power will be required, which causes greater energy consumption. Generally, the endurance of a typical electric-powered disposable UAV is unimodal with increasing $(f_p)_{battery}$. The maximum endurance occurs when $(f_p)_{battery}$ reaches 2/3.(17,21,22) It is worth mentioning that this conclusion is applicable not only to disposable UAVs but also to other electric-powered UAVs, such as multi-rotor aircrafts.(16) However, when the total weight of the UAV is constant, the battery weight will be limited by the structure, airborne equipment and task load, which means it will be hard to reach a high level. Currently, the battery weight ratio $(f_p)_{battery}$ of an electric-powered disposable UAV may lie in the range of [0.2, 0.4], hence we take three values (0.2, 0.3 and 0.4) to study the effect of the battery dumping strategy on extending the endurance of a disposable UAV.

3.1. Double-pack battery strategy

In the previous section, an expression for the endurance t of a single-pack battery UAV was introduced. It is now assumed that the battery is divided into two packs having the same voltage. This means that the difference between the two battery packs lies in their capacity and

Table 1
Maximum endurance for double-pack battery strategy

n \ (f_p)battery	**0.2**	**0.3**	**0.4**
$n = 1.0$	1.09	1.14	1.20
$n = 1.1$	1.02	1.08	1.14
$n = 1.2$	–	1.02	1.09
$n = 1.3$	–	–	1.03

mass, which are assumed to be linearly related. The two battery packs adopt the relay discharge method. Pack 1 is used first, and when its energy is exhausted, power will be supplied from pack 2. We assume that the weight ratio of pack 1 to the total mass of the battery is x and that the entire flight phase can be divided into two parts. After the first phase of the flight, pack 1 is dumped and the flying attitude is reset based on the updated flight weight to ensure the maximum endurance. Since the weight ratio of pack 1 to the total battery is x, the maximum endurance of the two stages t_1 and t_2 and the total endurance t_{total} can be expressed as

$$t_1 = {R_t}^{1-n}\left(\frac{\eta_{bp}UxC}{\left(2/\sqrt{\rho S}\right)C_{D0}^{1/4}\left(2W\sqrt{k/3}\right)^{3/2}}\right)^n$$

$$t_2 = {R_t}^{1-n}\left(\frac{\eta_{bp}U\left(1-x\right)C}{\left(2/\sqrt{\rho S}\right)C_{D0}^{1/4}\left(2\left(1-(f_p)_{battery}x\right)W\sqrt{k/3}\right)^{3/2}}\right)^n$$

$$t_{total} = t_1 + t_2 = A\left(x^n + \left(\frac{1-x}{\left(1-(f_p)_{battery}x\right)^{3/2}}\right)^n\right) \qquad \ldots(11)$$

where here and below $A = {R_t}^{1-n}\left[\frac{\eta_{bp}UC}{(2/\sqrt{\rho S})C_{D0}^{1/4}(2W\sqrt{k/3})^{3/2}}\right]^n$.

According to equation (11), the factor A is a fixed value, so t_{total} is only affected by the variables in the second bracket, i.e. x, $(f_p)_{battery}$ and n. For the $(f_p)_{battery}$ values of 0.2, 0.3 and 0.4 and n values of 1.0, 1.1, 1.2 and 1.3, we calculate the endurance for x varying in the range of [0, 1], and solve for the maximum endurance. The maximum endurance is shown in Table 1, while the ratios of the two battery packs at the maximum endurance are shown in Table 2. The table only lists the parts where endurance benefits occur, expressed (here and below) as a multiple of the endurance with a single-pack battery. If the endurance provided by the double-pack battery is less than that of a single-pack battery at any pack ratio, the space is filled with "–". The endurance for $(f_p)_{battery}$ of 0.2 and 0.4 is shown in Fig. 1.

From these tables and the figure, it can be concluded that all three main variables have a significant impact on the endurance extension. The main variations observed are as follows:

(1) When the Peukert constant is fixed, the endurance extension becomes more obvious with larger $(f_p)_{battery}$, while if $(f_p)_{battery}$ is fixed, the endurance extension gets worse as n

Table 2
The ratio of the two battery packs at the maximum endurance

n, pack number	***(f_p)battery***	**0.2**	**0.3**	**0.4**
$n = 1.0$	**Pack 1**	54%	56%	58%
	Pack 2	46%	44%	42%
$n = 1.1$	**Pack 1**	53%	55%	58%
	Pack 2	47%	45%	42%
$n = 1.2$	**Pack 1**	–	53%	57%
	Pack 2	–	47%	43%
$n = 1.3$	**Pack 1**	–	–	55%
	Pack 2	–	–	45%

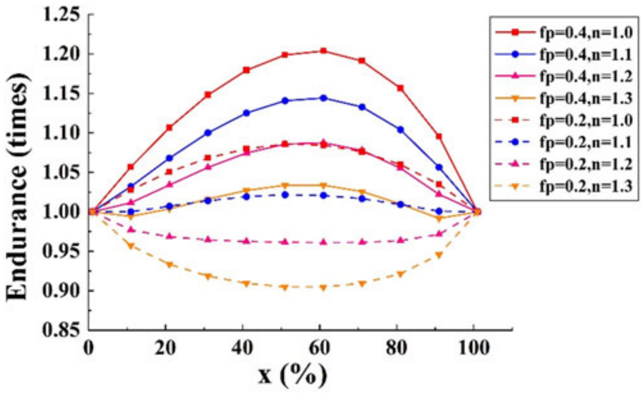

Figure 1. Endurance for different values of the battery weight ratio $(f_p)_{battery}$ and Peukert constant n for the double-pack battery strategy. x is the weight ratio of pack 1, and the endurance is expressed as a multiple of that obtained for the conventional single-pack battery UAV.

becomes larger. As the total capacity is fixed, the battery weight can also be considered to be fixed. Thus, larger $(f_p)_{battery}$ means a reduction in the total UAV weight, resulting in lower velocity, required power and also current drain. However, if $(f_p)_{battery}$ is fixed, a larger n leads to greater loss in the available capacity, that is, a greater loss in endurance;

(2) Define the maximum value of n at which the double-pack battery strategy yields an endurance benefit as the critical n value $n_{critical}$. As $(f_p)_{battery}$ declines, so does $n_{critical}$. Considering unit changes of 0.1 in the n value, the $n_{critical}$ value at $(f_p)_{battery}$ of 0.3 is 1.2, while the $n_{critical}$ value at $(f_p)_{battery}$ of 0.2 is only 1.1. This phenomenon can also be explained by the loss in available capacity mentioned in (1);

(3) When an endurance extension occurs, it generally shows a rising trend first but then declines with further increase of the pack 1 ratio x, or a slight drop appears at both ends. This probably occurs because, if the capacity of one pack is too small, it will suffer from serious current drain during its discharge, as the required current changes little. The x

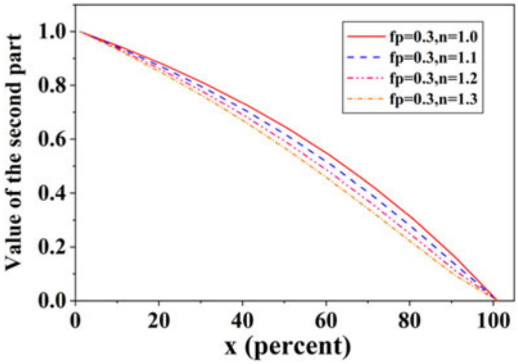

Figure 2. The value of the second term in equation (12) vs x for different values of n. As it can be drawn as a convex curve, this term becomes more linear as n increases, meaning that its value will be smaller for given x.

ratios that achieve the maximum endurance lie between 53% and 58%, very close to an equal split, and indeed some research has suggested that the equipartition law should be employed to split batteries in order to reduce the weight and design difficulties of BMDDs(17);

(4) Ignoring the effects of temperature and fully considering the current state of battery technology development ($n = 1.1$, 1.2), it is estimated that the endurance extension obtained using the double-pack battery strategy lies between 2% and 14%. This result is not as attactive as the use of an ideal battery (9–20%) because of the Peukert effect.

According to the conclusions above, n has a significant effect on the endurance extension of the electric-power disposable UAVs. Using equation (11), it can be found that n mainly affects the endurance via the second multiplication term, which can be written using the function

$$f(x) = x^n + \left(\frac{1 - x}{\left(1 - (f_p)_{battery} x\right)^{3/2}} \right)^n \qquad \ldots (12)$$

From a mathematical point of view, when $n = 1.0$, the first term in equation (12) corresponds to a straight line while the second term plots as a convex function curve. The sum of the two terms is always greater than 1 when x lies in the range of [0, 1], so whatever the value of $(f_p)_{battery}$ within the range considered here, a positive endurance extension is achieved. Meanwhile, when n is greater than 1.0, the first term becomes a concave curve, and its value gets smaller and smaller as n grows. Setting the battery weight ratio to 0.3, the second term in equation (12) is shown in Fig. 2 as a function of x at different n values. As the second term can be drawn as a convex curve and its value remains under 1.0, this term will show more linear growth with n, meaning that its value will become smaller for given x. Besides, the value of the second term decreases as $(f_p)_{battery}$ declines because of its position in the function. Therefore, when n is sufficiently large and $(f_p)_{battery}$ is sufficiently small, the value of this function will be below 1.0, indicating a decrease in endurance.

From a physical point of view, when n is greater than 1.0, the Peukert effect needs to be considered, and its influence increases with the growth of n. As the battery is divided into two packs, the capacity of each pack (in ampere-hours) will be lower than that of a single-pack battery. Thus, in the first phase, the capacity loss will be more serious, as the required current does not change with respect to the UAV with a single-pack battery. In the second phase, there will be a balance because the required current declines as the total weight decreases. When n is large enough, this loss will offset the benefits of the battery dumping strategy, and the endurance will decline.

In summary, the double-pack battery strategy will only result in a satisfactory endurance improvement when the battery weight ratio is large enough and the battery discharge performance is excellent (n is small).

3.2. Triple-pack battery strategy

Similarly, we now assume that the total battery is split into three packs, each having the same voltage and only differing in their capacities and masses. The packs discharge in order, with switching occurring when the previous one is exhausted. The variables a and b represent the proportions of the total weight represented by pack 1 and pack 2, respectively. Dividing the flight into three phases, the power supply will be switched to the next battery pack and the exhausted one will be dumped at the end of the first two phase. The flight velocity will then be re-matched for optimal endurance based on the new flight weight.

As the energy per unit weight of battery is assumed to be constant in this work, the energy proportions are the same as the weight proportions for all the battery packs. Therefore, the endurance for each phase, t_1, t_2, t_3 and the summary t_{total} can be expressed as

$$t_1 = {R_t}^{1-n}\left(\frac{\eta_{bp}UaC}{\left(2/\sqrt{\rho S}\right)C_{D0}^{1/4}\left(2W\sqrt{k/3}\right)^{3/2}}\right)^n$$

$$t_2 = {R_t}^{1-n}\left(\frac{\eta_{bp}UbC}{\left(2/\sqrt{\rho S}\right)C_{D0}^{1/4}\left(2\left(1-(f_p)_{battery}a\right)W\sqrt{k/3}\right)^{3/2}}\right)^n$$

$$t_3 = {R_t}^{1-n}\left(\frac{\eta_{bp}U(1-a-b)C}{\left(2/\sqrt{\rho S}\right)C_{D0}^{1/4}\left(2\left(1-(f_p)_{battery}(a+b)\right)W\sqrt{k/3}\right)^{3/2}}\right)^n$$

$$t_{total} = t_1+t_2+t_3 = A\left(a^n+\left(\frac{b}{\left(1-(f_p)_{battery}a\right)^{3/2}}\right)^n+\left(\frac{1-a-b}{\left(1-(f_p)_{battery}(a+b)\right)^{3/2}}\right)^n\right) \quad \ldots(13)$$

According to these equations, the total endurance t_{total} varies with a, b, $(f_p)_{battery}$ and n. Setting $(f_p)_{battery}$ to 0.2, 0.3 and 0.4 and n to 1.0, 1.1, 1.2 and 1.3, a relationship between the endurance and a, b can be derived, and the results are listed in Table 3 and 4. These tables only list the benefits of endurance while "–" indicates no benefit in endurance or even a loss. From these data, it can be concluded that the triple-battery strategy is more advantageous when n equals 1.0, while both strategies offer similar benefits in terms of endurance enhancement when n is above 1.0. Besides, the endurance of the triple-battery strategy decreases more rapidly with increasing n.

Figure 3 shows the variation of the endurance with n when the battery weight ratio is set at 0.3. When $n = 1.0$, indicating an ideal battery model, the proportion of each pack for the optimal endurance is 38%, 33% and 39% respectively, and the benefit reaches 1.19. When $n = 1.2$,

Table 3
Maximum endurance with triple-pack battery strategy

n \ (f_p)battery	0.2	0.3	0.4
$n = 1.0$	1.12	1.19	1.28
$n = 1.1$	1.02	1.09	1.18
$n = 1.2$	–	1.02	1.09
$n = 1.3$	–	–	1.03

Table 4
Ratios of three battery packs at maximum endurance

n, pack number \ (f_p)battery		0.2	0.3	0.4
$n = 1.0$	**Pack 1**	37%	38%	41%
	Pack 2	33%	33%	33%
	Pack 3	30%	29%	26%
$n = 1.1$	**Pack 1**	53%	37%	40%
	Pack 2	0%	33%	33%
	Pack 3	47%	30%	27%
$n = 1.2$	**Pack 1**	–	53%	57%
	Pack 2	–	0%	0%
	Pack 3	–	47%	43%
$n = 1.3$	**Pack 1**	–	–	55%
	Pack 2	–	–	0%
	Pack 3	–	–	45%

the proportions at which the maximum endurance is achieved when using the triple-pack strategy are similar to those obtained when using the double-pack strategy, as the proportion of one of the packs is zero. In this case, the endurance benefit decreases to 1.02. If the value of n rises further, the benefit of the triple-pack battery becomes lower than that obtained using the double-pack battery. We thus define $n_{critical}$ (the critical value of n) as the value at which the triple-pack strategy degenerates to the double-pack strategy. If n exceeds $n_{critical}$, few arrangements of the triple-pack battery strategy will result in an endurance benefit. Moreover, the endurance performance of the triple-pack strategy will be even worse than that with a single-pack battery, in accordance with the results obtained for the double-pack strategy.

Figure 4 shows the variation of the endurance with the proportions of pack 1 and pack 2 for two values of n and three values of $(f_p)_{battery}$. As seen in each row of the figure, the battery dumping strategy offers remarkable endurance benefits when using high battery weight ratios, as analysed in subsection 3.1. In particular, when $n = 1.2$, this strategy causes a loss of endurance when $(f_p)_{battery}$ is low (below 0.3). This means a higher battery weight ratio will help to increase the endurance benefit of the battery dumping strategy.

In terms of $n_{critical}$, decreasing the battery weight ratio also leads to a reduction of $n_{critical}$. When $(f_p)_{battery}$ is 0.2, benefits can only be obtained when n is less than 1.1, which demands high-performance batteries. The reason for this is probably that the total weight of the UAV

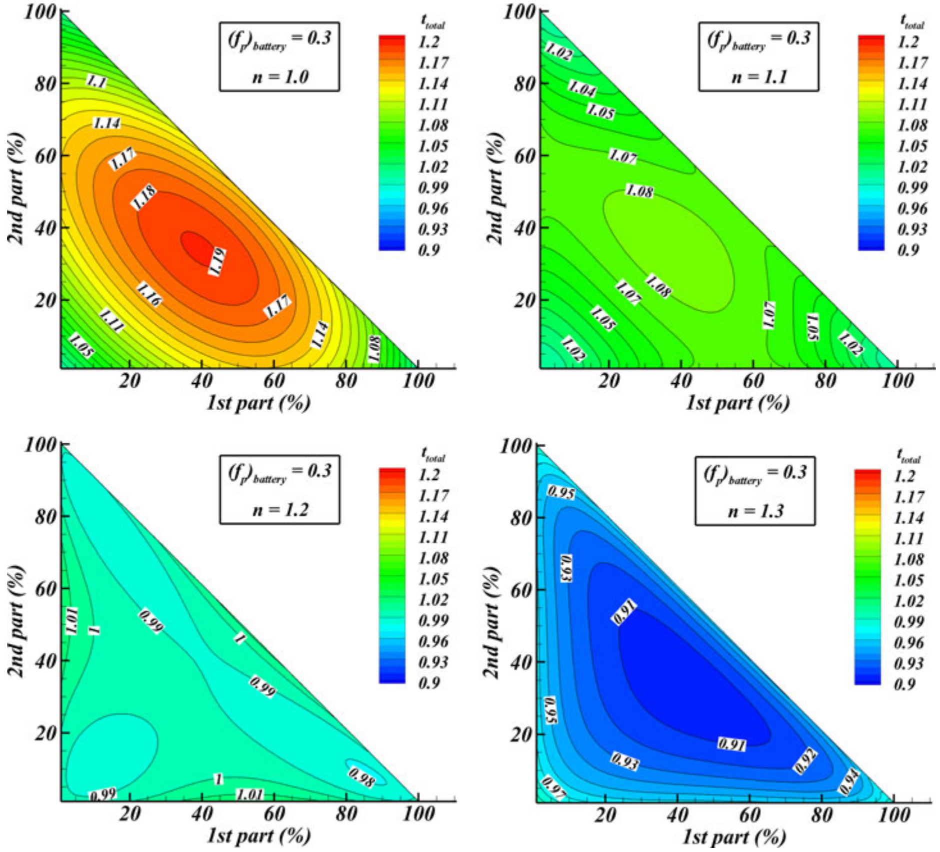

Figure 3. The relationship between the endurance and the pack proportions for each n value when using the triple-pack battery strategy ($(f_p)_{battery} = 0.3$).

becomes too large and great power will be needed to keep it flying, leading to serious current drain. Therefore, similar to the case of the double-pack strategy, the battery dumping strategy will offer remarkable endurance extension benefits only when the battery weight ratio is large enough and n is very small. Besides, the triple-pack strategy appears to be more sensitive to n compared with the double-pack strategy. One possible reason for this is that the battery has been split into more packs, thus the battery rated capacity is much smaller, resulting in a more severe loss of the available capacity. Considering the variation in the ambient temperature, manufacturer, and age of different batteries described above, n may change by more than 0.1, indicating that the benefits obtained using the triple-pack strategy are less certain than those resulting from the double-pack strategy.

4.0 CASE STUDY OF WEIGHT DECOMPOSITION FOR A DISPOSABLE UAV

Equation (10) describes the relationship between the endurance and flight velocity V when using a single-pack battery. After introducing the battery dumping strategy, equation (10) must be adapted to describe the sum of the various flight phases. The endurance in each phase

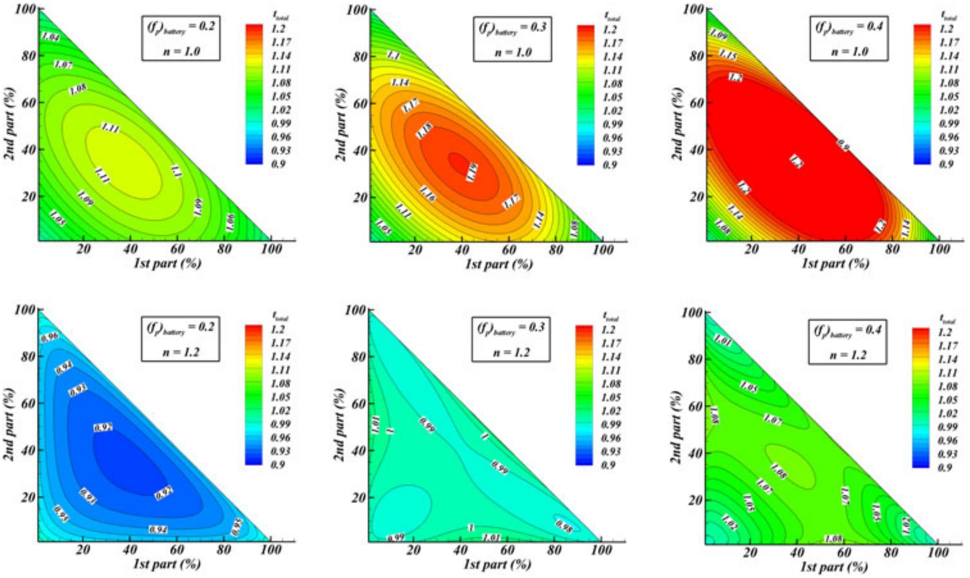

Figure 4. The relationship between the endurance in the triple-pack battery strategy and the pack ratios ($n = 1.0$, 1.2).

should be calculated using its own capacity, flight velocity and weight. If it is assumed that the same lift coefficient can be used for the different flight phases, the endurance with the double-pack battery can be expressed as

$$t_1 = {R_t}^{1-n}\left(\frac{\eta_{tot}UxC}{\frac{1}{2}\rho V^3 SC_{D0} + 2W^2k/\rho VS}\right)^n$$

$$t_2 = {R_t}^{1-n}\left(\frac{\eta_{tot}U\,(1-x)\,C}{\frac{1}{2}\rho V_2^3 SC_{D0} + 2W_2^2k/\rho V_2 S}\right)^n$$

$$t_{total} = t_1 + t_2 \qquad \ldots (14)$$

where $W_2 = W\left(1 - x\left(f_p\right)_{battery}\right)$, $V_2 = V\sqrt{1 - x\left(f_p\right)_{battery}}$.

Similarly, the endurance when using the triple-pack battery can be expressed as

$$t_1 = {R_t}^{1-n}\left(\frac{\eta_{tot}UaC}{\frac{1}{2}\rho V^3 SC_{D0} + 2W^2k/\rho VS}\right)^n$$

$$t_2 = {R_t}^{1-n}\left(\frac{\eta_{tot}UbC}{\frac{1}{2}\rho V_{22}^3 SC_{D0} + 2W_{22}^2k/\rho V_{22} S}\right)^n$$

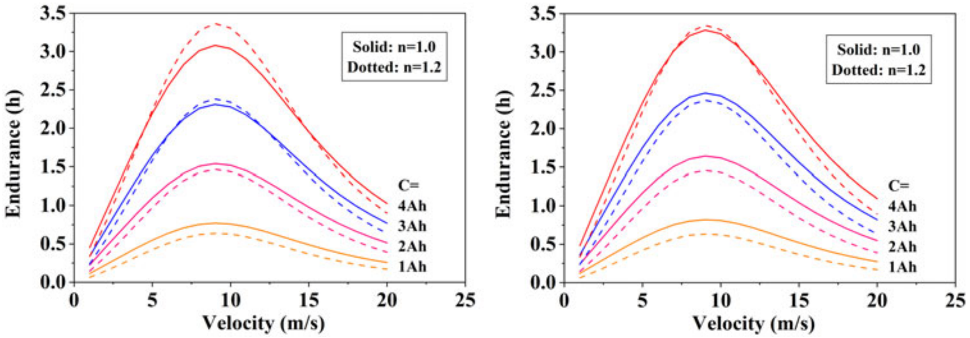

Figure 5. Available endurance at different velocities (left: two packs; right: three packs).

$$t_3 = {R_t}^{1-n}\left(\frac{\eta_{tot}U(1-a-b)C}{\frac{1}{2}\rho V_3^3 SC_{D0} + 2W_3^2 k/\rho V_3 S}\right)^n$$

$$t_{total} = t_1 + t_2 + t_3 \qquad \ldots (15)$$

where

$$W_{22} = W\left(1 - a\left(f_p\right)_{battery}\right),\ V_{22} = V\sqrt{1 - a\left(f_p\right)_{battery}};$$

$$W_3 = W\left(1 - (a+b)\left(f_p\right)_{battery}\right),\ V_3 = V\sqrt{1 - (a+b)\left(f_p\right)_{battery}}$$

Setting $(f_p)_{battery}$ to 0.3, a case analysis of the battery dumping strategy for a disposable UAV can now be carried out. The proportions of the packs for which the UAV can reach its maximum endurance are applied in equations (14) and (15), respectively. Taking the velocity V as an independent variable, the effects of the velocity (and thus the required power) and the Peukert effect on the endurance were studied for different battery capacities (1, 2, 3, and 4Ah) and n values (1.0 and 1.2). The parameters of the disposable UAV are taken from reference (18). The take-off gross weight of the disposable UAV is $W = 9.34$N, its wing area is $S = 0.32\text{m}^2$, the ambient air density is $\rho = 1.2\text{kg/m}^3$, the zero-lift drag coefficient is $C_{D0} = 0.015$, the induced-drag constant is $k = 0.13$, the electric power system efficiency is $\eta_{bp} = 0.5$, the operating voltage is $U = 11.1$V and the battery hour rating is $R_t = 1$h. The resulting endurance is shown as Fig. 5:

The basic variation of the characteristics is similar to that mentioned for a single-pack battery in reference (18), as follows:

(1) With increasing battery capacity, the endurance of the disposable UAVs increases. This is understandable because the battery weight is fixed, so more capacity means more supplementary energy. When the Peukert effect is considered at a capacity of 4Ah, the use of the double-pack battery strategy can reach a maximum endurance of 3.4h.

(2) With increasing velocity, the endurance first rises then decreases. At low velocity, the disposable UAV must fly at a high angle-of-attack, at which the lift-to-drag ratio is relatively low. Meanwhile, the thrust is also used directly to balance part of the weight. Therefore, high power is required to supply the thrust. As the velocity gradually increases, a situation

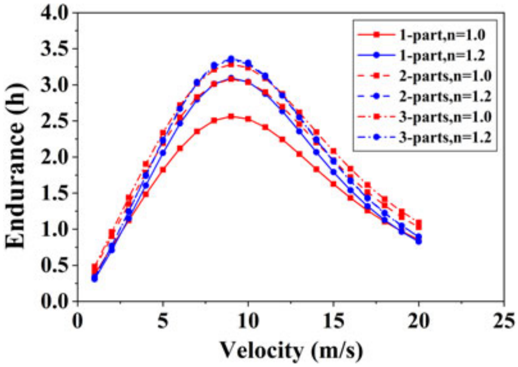

Figure 6. Comparison of endurance achievable when using the double- versus single-pack battery strategy at different velocities ($C = 4$Ah).

with high lift-to-drag ratio is reached. The required power decreases, while the endurance increases. However, as the velocity increases further, the drag increases in a super-linear fashion, thus the required power increases again.

(3) The Peukert effect obviously affects the endurance of the disposable UAV. The required power is fixed, as is the battery weight, thus the current required is consistent for different capacities. Larger capacity means smaller current drain. For the double-pack battery, when the battery capacity is 1 or 2Ah, as the capacity is significantly lower than the current, the Peukert effect appears to reduce the endurance. However, when the capacity is increased to 3 or 4Ah, the capacity can exceed the current value under the condition of low required power (e.g. $V = 10$m/s). Therefore, considering the influence of the Peukert effect, the endurance will be longer than that with an ideal battery. The benefit of the Peukert effect is about 10% at 4Ah, where the endurance will be more than twice that at 2Ah. However, with rising or declining velocity, the current will gradually increase and the Peukert effect will once again weaken the performance characteristics.

(4) The Peukert effect on the triple-pack battery is basically similar to that on the double-pack battery. However, as the number of battery packs is greater, the ampere-hour rating of each pack will be lower than for the double-pack battery. Therefore, it is harder to gain benefits via the Peukert effect. The benefit can only be achieved at $C = 4$Ah, and it is quite small.

To observe the endurance extension in comparison with the single-pack battery at different velocities, we compare the endurance of the single-, double- and triple-pack batteries at the same operating condition in Fig. 6. The battery capacity is set to 4Ah, and the Peukert constant to 1.0 or 1.2. The results reveal that the battery dumping strategy has a positive effect on the endurance, while the benefit when using ideal batteries is also significantly higher than that for lithium-ion batteries with $n = 1.2$ (the maximum difference between the three red lines is greater than that of the blue lines). When $n = 1.0$ (red line), the endurance when using the battery dumping strategy shows obvious differences at each velocity, in the following order: triple-pack battery > double-pack battery > single-pack battery; while when $n = 1.2$ (blue line), the two battery splitting strategies show almost no difference and the benefits (10%) are less obvious than for the single battery (29%).(22)

In the derivation of equation (14), we assume the same lift coefficient for the several flight phases. This means that, if the flight weight changes by B times, the flight velocity will change

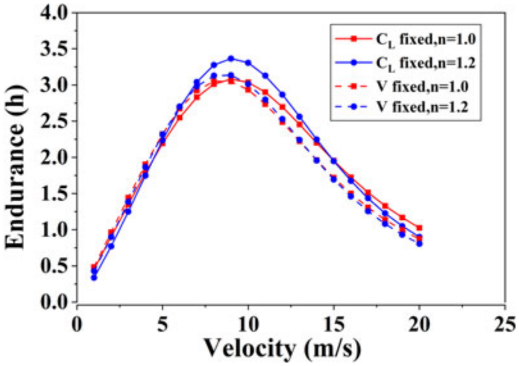

Figure 7. Comparison between the endurance in flight with constant velocity versus constant C_L with the double-pack battery strategy ($C = 4$Ah, $(fp)_{battery} = 0.3$).

correspondingly by $\sqrt{B}$ times. However, there is also another possible option, which is to maintain the flight velocity constant throughout all phases. The flight attitude of the aircraft, such as the angle-of-attack, must then be adjusted to maintain level flight. Taking the double-pack battery as an example, the expression for the endurance then becomes

$$t_1 = R_t^{1-n}\left(\frac{\eta_{tot}UxC}{\frac{1}{2}\rho V^3 SC_{D0} + 2W^2k/\rho VS}\right)^n$$

$$t_2 = R_t^{1-n}\left(\frac{\eta_{tot}U(1-x)C}{\frac{1}{2}\rho V^3 SC_{D0} + 2W_2^2k/(\rho VS)}\right)^n$$

$$t_{total} = t_1 + t_2 \qquad \ldots(16)$$

where $W_2 = W(1 - x(f_p)_{battery})$.

Assuming a capacity of 4Ah and a battery weight ratio of 0.3, the endurance when flying with constant velocity and constant lift coefficient are compared in Fig. 7. These results reveal that the disposable UAV can achieve greater endurance under the influence of the Peukert effect in the condition of constant lift coefficient (3.4h), being 10–13% greater than the other situation. When the battery discharge performance is desirable (n is lower), the difference between the two options is tiny. However, under the condition of fixed velocity, the UAV will reach its maximum endurance at lower velocity.

After the battery is dumped, a new balance with gravity can be achieved by changing the velocity or angle-of-attack (lift coefficient). Reducing either the velocity or the angle-of-attack will reduce the drag, as shown in equation (6). However, a reduction in velocity can lead to an extra decrease in the required power and current. The Peukert effect in this situation acts to improve the endurance, as shown in Fig. 5, which means that changing the velocity will lead to an extra improvement. Thus, longer endurance can be obtained by changing the velocity with a constant lift coefficient before or after the battery dumping than by keeping the same velocity. Nevertheless, for an ideal battery with $n = 1.0$, there is no obvious difference between these two strategies, as the Peukert effect has no influence in this situation.

5.0 CONCLUSION

The influence of the Peukert effect on the endurance benefits of battery dumping strategies is studied. The double- and triple-pack battery strategies were studied, with the benefit being found to be significantly influenced by the battery weight ratio, capacity and Peukert constant according to the sensitivity analysis method. The following conclusions can be drawn:

(1) With an appropriate battery weight ratio and Peukert constant, the battery dumping strategy can achieve a desirable endurance extension. The triple-pack battery strategy can achieve a benefit of up to 28% ($n = 1.0$), while the double-back battery strategy can achieve a benefit of up to 20% with $(f_p)_{battery} = 0.4$. However, the current drain severely impacts these benefits and reduces the endurance. The benefits drop to 14% and 18% when $n = 1.1$, and even worse, the two battery splitting strategies show almost the same endurance benefits at higher n, reaching 9% ($n = 1.2$) and 3% ($n = 1.3$).

(2) The benefits in terms of endurance extension are greater for a large battery weight ratio and low Peukert constant because of the low total weight and current drain. Taking the triple-pack battery strategy as an example, the benefit will be 28% if $(f_p)_{battery} = 0.4$ and $n = 1.0$, but only 2% if $(f_p)_{battery} = 0.2$ and $n = 1.1$. The Peukert constant exhibits a critical value above which the battery dumping strategy will lead to a negative effect on the endurance. For both battery splitting strategies, $n_{critical}$ is 1.1 for $(f_p)_{battery} = 0.2$ and 1.2 for $(f_p)_{battery} = 0.3$.

(3) As the current varies with the velocity of the UAV, the Peukert effect can lead to an improvement in the endurance when the battery capacity is larger than the current. For the triple-pack battery strategy ($C = 4$Ah), the improvement (0.1h, 3%) is not so obvious than for the double-pack battery strategy (0.4h, 13%), because the capacity of each battery pack is smaller. However, the endurance at 4Ah remains larger than twice that at 2Ah.

(4) For the double-pack battery strategy, when the battery is a realistic one ($n = 1.2$), longer endurance (10%) can be obtained by changing the velocity with constant lift coefficient before or after the battery dumping due to an extra decline in drag. Meanwhile, for an ideal battery with $n = 1.0$, there is no obvious difference.

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