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BOOK REVIEW

Advanced Techniques in Applied Mathematics. Edited by S. Bullett, T. Fearn and F. Smith. World Scientific, 2016. 196 pages, ISBN 9781786340221.

Advanced Techniques in Applied Mathematics is a compendium of six sets of condensed lecture notes, each of about 35 pages, on topics presented to beginning graduate students at the London Taught Course Centre over the past few years. The stated aim is that each chapter should be an advanced introduction to the topic, with references to further reading. The title of the book is slightly misleading. Although three of the chapters are on techniques, one is on the philosophy of modelling, whilst two are on specific research topics (Resonances in Wave Scattering, and Introduction to Random Matrix Theory (RMT)). However, the book contains lots of interesting material and, with the possible exception of RMT, there are plenty of direct and indirect connections to fluid mechanics.

The logic behind the ordering of the chapters is unclear to me, and I will comment on each in an order that made sense to me.

Modelling – What is it Good For?

This chapter provides an excellent introduction to the philosophy of mathematical modelling. It presents and discusses, with clarity, insight and enthusiasm, several well-known models based on differential equations (traffic flow modelling, the logistic equation, the Lorenz equations). It also includes some models not based on differential equations (the Ising model, self-organized criticality), and provides additional insight into these well-worn examples not usually given in other texts. For example, the Lorenz equations are derived from a simplification of the equations for convective flow between two horizontal plates so that the parameters can be related back to the underlying physics, and the assumptions and approximations that lead to this iconic system of equations examined critically. Overall, I would happily recommend this to students as a good introduction to modelling.

Practical Analytical Methods for Partial Differential Equations

This is a workmanlike introduction to some of the standard ideas for solving partial differential equations, which one would hope the audience for the book would have seen as undergraduates. There is a whistle-stop tour through characteristics for first-order PDEs, classification of second-order PDEs, and separation of variables, followed by a longer section on asymptotic solution techniques. I found some of the explanations and the choice of examples a little idiosyncratic, but as light revision, it works well enough.

Symmetry Methods for Differential Equations

The first 10 pages of this chapter provide an admirably clear introduction to Lie group methods for finding solutions and/or reducing the order of differential equations. This is followed by lots of examples, which seem less interesting, but is probably exactly what the student reader would want to see. There are also signposts to articles on non-classical methods at the end of the chapter, which I think students would find very useful.

Finite Elements

For me, this is a chapter that attempts too much, or at the very least requires too much to be packed into its 32 pages. It is an introduction to the finite element method, which, after a very brief introduction to the ideas, becomes a sequence of theorems and proofs with little explanation. Even the section on implementation is very terse, and presents an algorithm for an adaptive meshing method. For a very well-prepared student, this chapter might provide a useful distillation of key theorems and algorithms, but I would not recommend it as a starting point for a first-year PhD student.

Resonances in Wave Scattering

I'm not sure that this chapter represents a 'technique' as much as an introduction to an area of research, but it is nicely structured, beginning with simple, but illustrative examples of linear vibration and scattering, passing through the general theory of scattering and moving on to quantum scattering. A student with a weak background in complex variables would struggle with some of this material, but that's inevitable given the subject matter, and there are plenty of suggestions for background reading.

Introduction to Random Matrix Theory

I'm ashamed to admit that I knew nothing at all about random matrix theory before reading this chapter, but this may well have made me an ideal test reader. The introductory material is very clear, and gives the reader a good feel for the scope and nature of RMT. As the chapter proceeds to more advanced topics, the material becomes more technical and the explanations more terse. The main challenge seems to lie in approximating complicated multidimensional integrals, but I would need to have delved into the extensive bibliography to have gained more than a superficial knowledge of the subject. Overall, I think that this constitutes a successful introduction to RMT for somebody starting from a position of complete ignorance.

My final impression is that, whilst the tone, level and scope of the writing are rather uneven across the six chapters, the authors are mainly successful in presenting the important ideas of their topics, giving examples to illustrate them and directing the reader to relevant further reading, and therefore that the book is a useful and interesting introductory graduate-level textbook that conveys a good feel for the breadth and depth of applied mathematics.

John Billingham