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HOW DO EXCAVATED MANUSCRIPTS AND TRANSMITTED CANONS AND COMMENTARIES SHED LIGHT ON EACH OTHER? AN OUTLOOK FROM MATHEMATICS

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Abstract

Before the mathematical manuscript titled *Writings on Mathematical Procedures* (*Suanshu shu* 筭數書) was found at Zhangjiashan, historians of mathematics could trace mathematics in early imperial China only on the basis of the received canonical literature, notably *The Nine Chapters on Mathematical Procedures* (*Jiuzhang suanshu* 九章算術). After the Zhangjiashan and other mathematical manuscripts were found, they were mainly compared with *The Nine Chapters*, in the belief that these were all early imperial mathematical works and therefore adequate objects of comparison. As such, *The Nine Chapters* was transmitted with layers of commentaries and subcommentaries. This article argues that *Writings on Mathematical Procedures* presents important parallels with the commentarial literature on *The Nine Chapters*. This sheds light on how such exegeses were composed. The article further demonstrates that examination of these commentaries and subcommentaries allows us to perceive parallels between *Writings on Mathematical Procedures* and *The Nine Chapters* that to date have not been considered.

Introduction

In 656, Li Chunfeng 李淳風 (602?–670) submitted to the throne an annotated edition of *The Ten Canonical Texts of Mathematics* (*Suanjing shishu* 算經十書) that, upon imperial order, he had prepared with a team of

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scholars.[1] In line with the editorial project carried out under the responsibility of Kong Yingda 孔穎達 (574–648), which led in 653 to the publication of *The Proper Meaning of the Five Canonical Texts* (*Wujing zhengyi* 五經正義), Li Chunfeng and his colleagues selected earlier commentaries on some of the canonical texts for mathematics, and sometimes, depending on the individual case, they added their own commentaries and subcommentaries on the source material.[2] The oldest work devoted solely to mathematics in the collection—the Han composition *The Nine Chapters on Mathematical Procedures* (*Jiu zhang suanshu* 九章算術, hereafter *The Nine Chapters*)—attests to this editorial procedure.[3] For this work, which was mainly composed of problems and of procedures solving these problems, Li Chunfeng and his team selected Liu Hui's 劉徽 commentary (*zhu* 注), completed during the Wei dynasty, in 263, to which they added a "commentary and explanation" (*zhu shi* 注釋).[4] Subsequently, judging from the extant editions, *The Nine Chapters* was transmitted with these two layers of exegesis. And before the Zhangjiashan discovery, in

1. All dates are C.E. unless otherwise noted. Recent critical editions of this anthology include *Suanjing shi shu* 算經十書, ed. Qian Baocong 錢寶琮, 2 vols. (Beijing: Zhonghua, 1963); *Suanjing shi shu* 算經十書, ed. Guo Shuchun 郭書春 and Liu Dun 劉鈍, 2 vols. (Shenyang: Liaoning jiaoyu, 1998).

2. For an outline of the context and the project, see David McMullen, *State and Scholars in T'ang China* (Cambridge: Cambridge University Press, 1988), 67–112. A precise description of the canonical texts in mathematics, the selected commentaries, and the state of the surviving editions is given in Karine Chemla and Zhu Yiwen 朱一文, "Contrasting Commentaries and Contrasting Subcommentaries on Mathematical and Confucian Canons: Intentions and Mathematical Practices," in *Mathematical Commentaries in the Ancient World: A Global Perspective*, ed. Karine Chemla and Glenn W. Most (Cambridge: Cambridge University Press, 2022).

3. To make this assertion, I set aside *The Gnomon of the Zhou* (*Zhoubi* 周髀), which was probably completed in the first century, before *The Nine Chapters*, as this work provides the mathematical knowledge needed for cosmography and the calendar. Since it does not present any significant parallel with excavated texts, it does not allow me to consider the issue addressed in this article. For *The Gnomon of the Zhou*'s date of completion, and an English translation, see Christopher Cullen, *Astronomy and Mathematics in Ancient China: The Zhou Bi Suan Jing* (Cambridge: Cambridge University Press, 1996). The date of completion is still debated, however, as summarized in Feng Ligui 馮禮貴, "*Zhoubi suanjing* chengshu niandai kao" 《周髀算經》成書年代考, *Guji zhengli yanjiu xuekan* 古籍整理研究學刊 1986, 37–41.

4. About this translation, see Chemla and Zhu, "Contrasting Commentaries." A critical edition and a French translation of *The Nine Chapters*, Liu Hui's commentary, and the exegetical material prepared under Li Chunfeng's editorial supervision, appear in Karine Chemla and Guo Shuchun 郭書春, *Les Neuf Chapitres: Le Classique Mathématique de la Chine Ancienne et ses Commentaires* (Paris: Dunod, 2004). In what follows, unless otherwise specified, I rely on the critical edition published in this book. I have argued that *The Nine Chapters* was completed in the first century (Chemla, "Présentation du chapitre 6," in Chemla and Guo, *Neuf Chapitres*, 475–78).

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the early 1980s, our knowledge of Han mathematics was entirely dependent upon Tang editorial work.

During the winter of 1983–1984, the first excavated example of a Han mathematical manuscript—*Writings on Mathematical Procedures* (*Suanshu shu* 筭數書, hereafter *Writings*)—was found in a tomb sealed in c. 186 B.C.E.[5] Like *The Nine Chapters* and most of the other canonical texts in mathematics, *Writings* contains mainly problems and procedures, but also a significant number of tables.[6] Given the strong "family resemblance" between the contents of the Zhangjiashan manuscript, as seen in Peng Hao's critical edition of 2001, and *The Nine Chapters*,[7] historians of mathematics unsurprisingly first used the excavated document to shed light on *The Nine Chapters* and more broadly on *The Ten Canonical Texts of Mathematics*.

The main questions that dominated the early discussions were these: How could the manuscript help us date the pieces of mathematical

5. This interpretation of the title derives from the analysis of the document given in Daniel Morgan and Karine Chemla, "Writing in Turns: An Analysis of Scribal Hands in the Bamboo Manuscript *Suan shu shu* 筭數書 (Writings on Mathematical Procedures) from Zhangjiashan Tomb No. 247," *Silk and Bamboo* 1.1 (2018), 152–90. The first annotated critical edition of this manuscript was given in Peng Hao 彭浩, *Zhangjiashan Han jian "Suanshu shu" zhushi* 張家山漢簡《算數書》注釋 (Beijing: Kexue, 2001). Unless otherwise noted, I rely on this edition. The work of critical edition continues, and, where necessary, I will mention important contributions. See Guo Shuchun 郭書春, "*Suanshu shu* jiaokan" 筭數書校勘, *Zhongguo keji shiliao* 中國科技史料22.3 (2001), 202–19; Guo Shirong 郭世榮, "*Suanshu shu* kanwu" 算術書勘誤, *Neimenggu shida xuebao ziran kexue (Han wen) ban* 內蒙古師大學報–自然科學 (漢文)版30.3 (2001), 276–85. A new critical edition, with translations into modern Chinese and Japanese, appears in Chōka zan Kankan *Sansū sho* kenkyūkai, 張家山漢簡『算數書』研究会, *Kankan* Sansû *sho* 漢簡算數書 (Kyoto: Hōyū, 2006). The same year, an annotated translation into modern Chinese, with another critical edition, was published: Horng Wann-sheng 洪萬生, Lin Cangyi 林倉億, Su Huiyu 蘇惠玉, and Su Junhong 蘇俊鴻, *Shu zhi qiyuan* 數之起源 (Taipei: Taiwan shangwu, 2006). An English translation, based on Peng Hao's edition (modified), appeared in Christopher Cullen, *The Suan Shu Shu* 筭數書 *'Writings on Reckoning': A Translation of a Chinese Mathematical Collection of the Second Century BC, with Explanatory Commentary* (Cambridge: Needham Research Institute, 2004). Another English translation is Joseph Dauben, "算數書 *Suan Shu Shu* (*A Book on Numbers and Computations*). English Translation with Commentary," *Archive for History of Exact Sciences* 62 (2008), 91–178. More recently, Rémi Anicotte, *Le Livre sur les Calculs Effectués avec des Bâtonnets. Un Manuscrit du —IIe Siècle Excavé à Zhangjiashan* (Paris: Presses de l'Inalco, 2019) has provided a French translation.

6. Karine Chemla, "Numerical Tables in Chinese Writings Devoted to Mathematics: From Early Imperial Manuscripts to Printed Song-Yuan Books," *EASTM (East Asian Science, Technology and Medicine)* 44 (2016), 69–123, esp. 83–89.

7. The expression is used in Christopher Cullen, "The *Suan Shu Shu* 筭數書 'Writings on Reckoning': Rewriting the History of Early Chinese Mathematics in the Light of an Excavated Manuscript," *Historia Mathematica* 34.1 (2007), 10–44, 28.

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knowledge contained in the anthology of canonical works, and especially in *The Nine Chapters*?[8] How could *Writings* help date the emergence of the standard format with which problems and procedures were presented in canonical texts?[9] How did *Writings* compare with canonical texts of mathematics,[10] and more specifically, with *The Nine Chapters*?[11] A tacit assumption underlies all early publications on *Writings*: the manuscript records a "book,"[12] and this assumption informs several other issues that were addressed, such as, Was *Writings* a prototype from which *The Nine Chapters* derived, or, put differently, one of the books on which the authors of *The Nine Chapters* relied to compose the canonical text?[13] Which kind of book was *Writings* in comparison with *The Nine Chapters*?[14] How could *Writings* shed light on the process

8. The prominence of this question is acknowledged in Cullen, "The *Suan Shu Shu*: Rewriting," 40.

9. Guo Shuchun 郭書春, "Shilun *Suanshu shu* de shuxue biaoda fangshi" 試論《筭數書》的數學表達方式, *Zhongguo lishi wenwu* 中國歷史文物 2003.3, 28–38.

10. Guo Shuchun 郭書春, "*Suanshu shu* yu *Suanjing shishu* bijiao yanjiu" 《算數書》與《算經十書》比較研究, *Ziran kexueshi yanjiu* 自然科學史研究 23.2 (2004), 106–20.

11. Comparative remarks are pervasive in the footnotes and annotations given in the critical editions and translations. Sections are devoted to them in, e.g., Horng et al., *Shu zhi qiyuan*, 104–26, as well as in Dauben, "*Suan Shu Shu*," 94–100.

12. Material analyses of the document led Morgan and me to suggest rather that *Writings* assembled notes produced by several hands, in a context of mathematical education. Importantly, we discerned two separate hands (Hand B and Hand A), with the former guiding the learning process of the latter. For part of the argument, see Mo Zihan 墨子涵 (Daniel Morgan), and Lin Lina 林力娜 (Karine Chemla), "Ye you lunzhe xie de: Zhangjiashan Han jian *Suanshu shu* xieshou yu pianxu chutan" 也有輪著寫的：張家山漢簡《筭數書》寫手與篇序初探, *Jianbo* 簡帛 12 (2016), 235–52. See also Morgan and Chemla, "Writing in Turns," 181–82.

13. Anicotte, *Le Livre sur les Calculs*, 18, mentions this issue. Peng, *Zhangjiashan Han jian*, 25–32, actually describes how *The Nine Chapters* was produced using *Writings*. In Guo Shuchun 郭書春, "Shilun *Suanshu shu* de lilun gongxian yu bianzuan" 試論算數書的理論貢獻與編纂, *Faguo hanxue* 法國漢學 (French Sinology) 6 (2002), 505–37, Guo Shuchun opposes views of this kind. For him, *The Nine Chapters* and *Writings* are intimately connected but in no way is the latter a basis on which the former was composed (see n. 14). For Zou, the converse holds true: *The Nine Chapters* existed as a pre-Qin canon, and *Writings* derived from it; Zou Dahai 鄒大海, "Cong *Suanshu shu* yu *Jiuzhang suanshu* de guanxi kan suanfashi shuxue wenxian zai shanggu shidai de liuchuan" 從《算數書》與《九章算術》的關係看算法式數學文獻在上古時代的流傳, *Gannan shifan xueyuan xuebao* 贛南師範學院學報 2004.6, 6–10.

14. Peng, *Zhangjiashan Han jian*, 26, and Dauben, "*Suan Shu Shu*," 94–95, both see in *Writings* a reference work for administrators. I return to this point below. Guo considers that, with the diversity of its procedures and the luxuriance of its terminology, *Writings* derives from mathematical sources that reflect the disunity of pre-Qin mathematics, whereas *The Nine Chapters* is a product of early imperial China order and of the standardization that was applied to every subject (Guo Shuchun, "Shilun *Suanshu shu* de lilun gongxian yu bianzuan," 530). For Zou, *Writings* belonged to a set of works that

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of composition of works such as *The Nine Chapters*?[15] For some scholars, the key point was to determine how these documents might enable historians to discuss pre-Qin mathematics. In particular, on the basis of the history of *The Nine Chapters* as recounted in Liu Hui's preface, some historians relied on the newly found manuscript to argue that *The Nine Chapters* could be deemed a pre-Qin work. In brief, as Joseph Dauben put it, historians focused on "the ways in which mathematics developed and changed from pre-Qin to Eastern Han times."[16]

Such issues and a similar focus have persisted as other new documents have come to light since the publication of *Writings*, including the Shuihudi 睡虎地 (Hubei) manuscript entitled *Mathematical Procedures* (*Suanshu* 算術), excavated from tomb M77, which was sealed before 157 B.C.E.,[17] and the manuscript purchased on the antiquities market, entitled simply *Mathematics* (*Shu* 數), which, to date, is the only other published mathematical manuscript.[18]

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were not canonical (he sees two types of books of this type: some specialized for professionals, and others practical), whereas, in pre-Qin times, *The Nine Chapters* belonged to the set of canonical works, and it represents pre-Qin mathematics more faithfully than *Writings*; see Zou Dahai 鄒大海, "Cong *Suanshu shu* yu *Jiuzhang suanshu* de guanxi" and "Zai lun *Suanshu shu* yu *Jiuzhang suanshu* de guanxi" 再論《算數書》與《九章算術》的關係. *Xin fajia* 新法家, January 26, 2007, www.xinfajia.net/2830.html, accessed on September 3, 2021.

15. This is a major point addressed in Cullen, "The *Suan Shu Shu*: Rewriting," 27, 35–38. Cullen's thesis is that *The Nine Chapters* was put together during Wang Mang's reign, on the basis of "textlets" such as those forming the sections of *Writings*, to fit the *jiu shu* 九數 (Nine Fundamental Procedures) in the *Zhou li* 周禮 (*Zhou Rites*). On other views, see n. 14. Karine Chemla, "Documenting a Process of Abstraction in the Mathematics of Ancient China," in *Studies in Chinese Language and Culture—Festschrift in Honor of Christoph Harbsmeier on the Occasion of His 60th Birthday*, ed. C. Anderl and H. Eifring (Oslo: Hermes Academic Publishing and Bookshop A/S, 2006), 169–94, addresses the question of the composition of *The Nine Chapters* from a different angle.

16. See Dauben's "The Evolution of Mathematics in Ancient China: From the Newly Discovered *Shu* and *Suan Shu Shu* Bamboo Texts to the *Nine Chapters on the Art of Mathematics*," *Notices of the ICCM (International Congress of Chinese Mathematicians)* 2.2 (2014), 29. This is also the focus in Cullen, "The *Suan Shu Shu*: Rewriting."

17. Hubei sheng wenwu kaogu yanjiusuo 湖北省文物考古研究所 and Yunmeng xian bowuguan 雲夢縣博物館, "Hubei Yunmeng Shuihudi M77 fajue jianbao" 湖北雲夢睡虎地M77發掘簡報, *Jiang Han kaogu* 江漢考古 109 (2008), 31–37 and Plates 11–16. NB: photos of only ten slips have been published to date. For an edition and an annotated translation of these slips, see Karine Chemla and Ma Biao 馬彪, "Interpreting a Newly Discovered Mathematical Document Written at the Beginning of Han Dynasty in China (before 157 B.C.E.) and Excavated from Tomb M77 at Shuihudi 睡虎地," *Sciamvs* 12 (2011), 159–91.

18. This document, acquired in December 2007 on the Hong Kong antiquities market, is now housed in the Yuelu Academy 嶽麓書院 (Hunan University). In its editors' view, *Mathematics* was probably composed no later than 212 B.C.E. On the date,

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The previous overview shows that the focus of discussions about the manuscripts—and particularly about *Writings*—has been on their relationship to the canonical works—particularly *The Nine Chapters*. The ancient commentaries with which *The Nine Chapters* was transmitted has seldom received extended attention, except in relation to some alternative procedures added to the canon and already included in *Writings*.[19]

In this article, I would like to address precisely the issue of the light shed by *Writings* and by the earlier commentaries and subcommentaries for *The Nine Chapters* on each other. Some might consider this an odd question as, at first sight, everything sets *Writings* and the exegetical material on *The Nine Chapters* in opposition to one another, and this might explain why this topic has not yet been discussed. As noted, many historians view *Writings* as a practical reference work useful for government officials for three main reasons.[20] First, the Zhangjiashan tomb occupant is believed to have been a low-level official, active at a county level.[21] Second, *Writings* contains many data and problems that an official of this kind might have needed in his official capacity. (The manuscript quotes a Qin regulation about grain, for example.)[22] Third, *Writings* reflects Qin and Han administrative statutes.[23] Not unimportantly, the Zhangjiashan manuscript also shares this feature with *The Nine Chapters*, and indeed, discussions about the official management of

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see Zhu Hanmin 朱漢民 and Chen Songchang 陳松長 eds., *Yuelu shuyuan cang Qin jian (er)* 嶽麓書院藏秦簡（貳） (Shanghai: Shanghai cishu, 2011), 3, and Xiao Can 蕭燦, "Yuelu shuyuan cang Qin jian *Shu* yanjiu" 嶽麓書院藏秦簡《數》研究 (PhD diss., Hunan University, 2010), 16. They all argue in favor of this *terminus ante quem*. The title is on the verso of slip 0956 (editors' number 1 verso), and its photograph is reproduced in *Yuelu shuyuan cang Qin jian*, 3.

19. See *Zhangjiashan Han jian*, 24, 29, 47, for only a few examples.

20. "The *Suan shu shu* was almost certainly designed as a work of ready reference for government bureaucrats of the Qin and early Han dynasties"; Dauben, "*Suan Shu Shu*," 92. See also Peng, *Zhangjiashan Han jian*, 26.

21. Peng, *Zhangjiashan Han jian*, 6–12. On this level of the administration, see chapter 3, "Provincial and Local Government," in Michael Loewe, *The Government of the Qin and Han Empires* (Indianapolis: Hackett Publishing Company, 2006), 37–55. About this kind of official, and their need of mathematical knowledge, see chapter 6, "The Officials," in ibid., 71–85.

22. This was first noticed by Peng Hao, *Zhangjiashan Han jian*, 80–81 (slips 88–90). See the parallel regulation on the slips 41–43 of *Shuihudi Qin mu zhujian* 睡虎地秦墓竹簡, ed. Shuihudi Qin mu zhujian zhengli xiaozu 睡虎地秦墓竹簡整理小組 (Beijing: Wenwu, 1990), 29–30. Translation and analysis are given in A. Hulsewé, *Remnants of Ch'in Law* (Leiden: Brill, 1985), 42.

23. Peng Hao, "Official Salaries and State Taxes as Seen in Qin-Han Manuscripts, with a Focus on Mathematical Texts," in *Mathematics, Administrative and Economic Activities in Ancient Worlds*, ed. Cécile Michel and Karine Chemla (Cham: Springer Nature, 2020), 125–55. See also Horng et al., *Shu zhi qiyuan*, 141–53.

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grain in early China have regularly relied on *The Nine Chapters*, precisely because the canon is presumed to reflect administrative realities.[24]

The commentaries and subcommentaries discuss almost nothing about administrative procedures, except for clarifying measurement units and computations of the values of π involved in the manufacturing of such measurement standards as Wang Mang's bronze vessel. Generally speaking, the commentaries and subcommentaries, unlike *The Nine Chapters* and *Writings*, dwell on the proofs for the correctness of the procedures specified in the canon; explain the meaning of certain technical terms; and present philosophical discussions relating to the mathematical topics under discussion (e.g., they discuss the power attached to transformations in mathematics against the backdrop of transformations at play in the world more broadly). Moreover, in contrast with the text they comment upon, exegetes occasionally make explicit some of the epistemological values they prize.

Seen from the perspective of their style and content—and as far as we can judge from the remaining documents—there seems to be a clear gap between the Han texts and the later commentaries, not withstanding the fact that sometimes the later commentaries offer alternative procedures, and some of these procedures have parallels in *Writings*. Yet, even if we rely only on the extant evidence, I argue that this interpretation requires rethinking. To begin with, not everybody agrees that this newly discovered Han mathematical manuscript and *The Nine Chapters* are merely practice-oriented. As a rule, there is no reason why a text with practical utility should be devoid of theoretical dimensions or rhetorical sophistication (see Luke Habberstad's article in this journal issue). The validity of this opposition between theory and practice, which has often been projected onto ancient documents, is questionable.

Christopher Cullen adopts a different approach when, in relation to *Writings*, he asks: "Is this material simply a practical handbook of reckoning, or is it written in part by and for people whose interest in mathematics goes beyond its uses for administrative purposes?"[25] In answering this question, Cullen emphasizes that in some sections of *Writings*, "the main interest is in problem-solving structures rather than any conceivable administrative reality."[26] For him, these sections suggest that there were "Western Han scholars" who "were at least part of the time interested in displaying their ability to create (or at least to pass on) problems and methods of solution whose interest lay in their

24. Michael Loewe, "The Measurement of Grain During the Han Period," *T'oung Pao*, 2nd ser. 49.1–2 (1961/62), 64–95.

25. Cullen, "The *Suan Shu Shu*: Rewriting," 38.

26. Cullen, "The *Suan Shu Shu*: Rewriting," 38.

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structure and ingenuity rather than in their practical value."[27] In Cullen's view, this further means that there were people at the time who "were to some extent interested in clever methods of calculation for reasons other than their straightforward usefulness in administration."[28] Likewise, for some historians, some of the topics discussed in *Writings* attest to a development of mathematics for its own sake. Guo Shuchun goes further, when he rejects the claim that *Writings* is a manual or a reference text for administrators and puts forward the thesis that *Writings* displays theoretical dimensions and practices of abstraction.[29]

In earlier work on the practices of abstraction to which *Writings* attests, proceeding with a notion of abstraction derived from an observer's viewpoint, I have argued for continuities between the testimony found in the *Writings* manuscript and in *The Nine Chapters*.[30] In the present article, I intend to move closer to actors' categories and practices (as they can be approached through the available documents) in considering how, for questions of this kind, commentaries and subcommentaries, on the one hand, and *Writings*, on the other, illuminate each other. Interestingly, such a method will enable us to highlight forms of continuity between the two Han documents that, to my knowledge, have not yet been discussed. After all, the exegeses attached to the names of Liu Hui and Li Chunfeng both explicitly refer to earlier documents that they used for their commentarial work. Thus considering the relationships between the exegetical material and the Zhangjiashan manuscript may also shed light on the kinds of Han documents a commentator like Liu Hui or exegetes like those working under Li Chunfeng's supervision perused when writing their annotations.

Comparing *Writings* and *The Nine Chapters* in the Light of the Commentaries

As we have seen, *The Nine Chapters*, like *Writings*, mainly contains problems and procedures. What were the stylistic and technical constraints ruling the way these textual units were written up, and how were they read and interpreted early on? To date, the earliest readers whose approach to problems and procedures we might observe are precisely the two commentators on *The Nine Chapters*. Admittedly, they lived centuries after the composition of the canonical text. However,

27. Cullen, "The *Suan Shu Shu*: Rewriting," 38.
28. Cullen, "The *Suan Shu Shu*: Rewriting," 38.
29. Guo Shuchun, "Shilun *Suanshu shu* de lilun gongxian yu bianzuan," 530, 510–12, respectively.
30. See Chemla, "Documenting a Process of Abstraction."

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they represent the best guide we have at our disposal to observe early reading practices. If we manage to establish connections between their reading and key features of the Han texts, we may consider that their testimony captures something relevant about Han theories and practices. Let us first establish a connection between exegetes' reading of mathematical problems and the type of problems contained in *Writings*. To do so, I will begin by summarizing conclusions I derived earlier.[31]

Practices with Problems in *The Nine Chapters*

The commentary ascribed to Liu Hui contains a critical piece of evidence about actors' reading of and practice with mathematical problems. It occurs after problem 18 of chapter 6— I refer to this problem as 6.18—, which reads as follows:

> Suppose that five persons share five cash in such a way that what the two superior persons obtain is equal to what the three inferior persons obtain. One asks how much each obtains.
>
> 今有五人分五錢，令上二人所得與下三人等，問各得幾何。[32]

This passage is representative of most mathematical problems in *The Nine Chapters* and *Writings*. The statement uses a specific situation and gives explicit numerical values for the quantities that are taken as data. It concludes with a question, which, here, asks for the different amounts to be received by five persons respectively. The related procedure prescribes the operations to be carried out to solve this problem. What makes this passage unusual, indeed unique, is that the procedure that follows the problem solves it correctly, but is not general. This tells us something about *The Nine Chapters*.

In his commentary to this problem, Liu Hui first accounts for why the procedure answers the question correctly. Meanwhile, he highlights the fact that the procedure makes use of two specific features of the situation: first, it puts into play the fact that the set of inferior persons has only one more person than the set of superior persons; second, it relies on the fact that the sum of the coefficients attached to the superiors (Liu Hui makes all these coefficients explicit, since the passage does not give them) is greater than that for the inferiors, whereupon the procedure subtracts the latter from the former to arrive at the right solution. By doing this, the

31. Karine Chemla, "Generality above Abstraction: The General Expressed in Terms of the Paradigmatic in Mathematics in Ancient China," *Science in Context* 16.3 (2003), 413–58.

32. *Neuf Chapitres*, 526–27. I use small caps to distinguish the text of the canon from that of the commentaries.

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commentator's argument establishing the correctness of the procedure indicates that the procedure is not general, since it extends only to problems presenting the similar specifics. Immediately after, he formulates another problem that satisfies none of these two features. The commentator's problem and his analysis of the situation read as follows:

> Suppose seven persons share seven cash and they want to do this in such a way that [what] the two superior persons [obtain] is equal to [what] the five inferior persons [obtain].
>
> Then, the groups of superiors and inferiors present a difference of three persons.[33]
>
> Summing the [coefficients corresponding to] the group of superiors makes thirteen. [Summing the coefficients corresponding to] the group of inferiors makes fifteen. The inferiors are more numerous than the superiors, and there is not enough for the [sum of coefficients corresponding to the] inferiors to be subtracted from the [sum of coefficients corresponding to the] superiors.
>
> 假令七人分七錢，欲令上二人與下五人等，則上、下部差三人。并上部爲十三，下部爲十五。下多上少，下不足減上.[34]

The commentator thus emphasizes that the problem he introduces, despite its similarity with the one found in *The Nine Chapters*, fails to satisfy the two specific features on which, as he has shown, the procedure of the canon relies. Accordingly, the procedure of *The Nine Chapters* cannot be used for this new problem. The commentator then goes on to explain how this procedure can be changed to be valid for all the problems similar to the one in *The Nine Chapters*.[35]

Plainly, this piece of evidence shows that Liu Hui does not expect that a procedure placed after a problem solves *only* that single problem. Indeed, the procedure of *The Nine Chapters* does just this, and he as commentator finds this unsatisfactory. In his view, the procedure should in fact solve all the problems similar to the problem contained in *The Nine Chapters*. In other words, although this problem is formulated with a specific situation and specific numerical values, it should stand in this

33. The group of superiors has two persons and that of inferiors, five: the "difference" 差 is now of three persons (five minus two), and no longer "one," as above.

34. *Neuf Chapitres*, 528–29. I have inserted paragraphs in the translation to make the argument clearer.

35. On the commentators' understanding of the similarity between problems, see Chemla, "Qu'est-ce qu'un problème dans la tradition mathématique de la Chine ancienne?" *Extrême-Orient, Extrême-Occident* 19 (1997), 91–126.

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way for a broad class of similar problems. The commentator thus articulates an expectation with respect to generality. He reads the problem as a paradigm, just as, when we learn French, we know that the conjugation of the verb *chanter* is not a simple example, but the statement of the conjugation of all verbs falling into the same group. The fact that all the problems and procedures of *The Nine Chapters*, except this very one, obey this rule seems to indicate that this was the practice with problems that was embraced by the authors of the canon. Moreover—and this is a crucial point—the key to exploring the generality of the statement lay within the procedure itself: we have seen the commentator determining the extension of its validity while establishing its correctness. Presumably, this annotation glossing the canonical text gives clues as to how readers went about exploring the class of problems for which a given problem and its related procedure stood.

Commentators' Practices with Problems and Visual Tools

Notably, the commentators attest a similar practice with problems. Indeed, it regularly happens that a commentator introduces the outline of a problem for the sake of his exegesis,[36] as is illustrated by the commentary on the general procedure that enables someone to carry out a sharing into unequal parts; this operation in *The Nine Chapters* has the name "Parts weighted in function of the degree" (*cui fen* 衰分). In contrast with the procedure quoted above, this procedure is given at the start of the chapter, so that it does not follow any problem. To highlight the meaning of the operations composing this procedure, the commentary ascribed to Liu Hui introduces a problem and uses it to account for the correctness of the procedure, as follows:

> Suppose that a first family has three persons, a second family, two persons, the third family, one person—altogether six persons—and that they share twelve together. That makes that each person obtains two. If one wants to make it again according to families, then one must put in a row the quantities of persons [in each family] and multiply them by what a single person obtains. Now, in this procedure [i.e., the procedure commented upon], one multiplies first and then divides [in contrast with the newly introduced procedure in which one divides first, and then multiplies].
>
> 假令甲家三人，乙家二人，丙家一人，并六人，共分十二，爲人得二也。欲復作逐家者，則當列置人數，以一人所得乘之。今此術先乘而後除也。[37]

36. Chemla, "Qu'est-ce qu'un problème."
37. *Neuf Chapitres*, 282–83, and related endnotes.

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In fact, Liu Hui first puts forward a situation, which—just as his problem above—he introduces using the term *jialing* 假令, in contrast with the usual formulation of a supposition at the beginning of each problem in *The Nine Chapters* (using the phrase "suppose we have" [*jin you* 今有], as in problem 6.18 quoted above).[38] Liu Hui then considers two questions: "How much does each person get?"— made manifest by the use of the term *de* 得, when he formulates the answer—and "How much does each family get?"—to which the concluding operation corresponds. The first question is answered by a succession of an addition—to get the number of people—and a division, and thereby highlights the meaning of the succession of an addition and a division in the general procedure. The second question, which leads from an equal sharing to an unequal sharing, requires that the above division be followed by a multiplication, which likewise clarifies the meaning of the multiplication within the general procedure. Liu Hui can then indicate that the general procedure is obtained from the procedure whose meaning has been highlighted by simply swapping multiplication and division. This is one way in which he establishes the correctness of the general procedure. The principal point is that, to do so, Liu as commentator uses a situation in a problem (three families and 1, 2, 3, as respectively the number of their members) that is quite simple, so that the meaning of the operations becomes crystal clear. However, this way of conducting the discussion in no way detracts from the generality of what is presented in this fashion. Liu Hui uses the situation articulated in the problem in the same way as he uses that of problem 6.18 to discuss the related procedure.

The foregoing analysis demonstrates two points. First, the commentator uses the situation in the problem to formulate the "meaning" of the operations of the procedure; the exegetes use the term *yi* 意 to designate this "meaning." Second, the commentator relies on a situation that involves the simplest possible numerical values, without ever taking advantage of the singularity of the values to simplify the procedure or the discussion of why it is correct. His treatment preserves the general validity of the operations under consideration. Exactly the same two features characterize the commentators' use of visual tools: they use "diagrams" (*tu* 圖) for situations in the plane and "blocks" (*qi* 棊) for those in space. In this respect, a piece of evidence strikingly confirms our conclusions.

This document occurs in the commentary on the pyramid of the *Yang ma* 陽馬 type, which is the pyramid treated in *The Nine Chapters*.[39] Without going into mathematical details here, and limiting ourselves to the structure of the commentary ascribed to Liu Hui, we see that the commentator

38. On these terms, see Karine Chemla, "Glossaire des expressions techniques," in Chemla and Guo, *Neuf Chapitres*, 936–37, 940.

39. *Neuf Chapitres*, 428–33, plus the endnotes.

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first considers the case where the three dimensions of the pyramid (height, length and width) are equal. Liu Hui asserts that combining three identical pyramids of this kind makes a cube. It is thus clear that in this case, the procedure is correct: multiplying the length and width and height by one another makes the volume of the cube, and dividing by three yields that of the pyramid in question. The procedure stated in *The Nine Chapters* is thus proven to be correct. However, the commentator continues, this argument does not extend to cases where the dimensions of this pyramid differ from each other. In the latter cases, combining three identical pyramids will not make a simple shape. One must, Liu Hui asserts, develop another argument for these other situations, and this is what he does in the rest of his commentary. Clearly, the commentator is not content with an argument that exploits the specific features of a singular case—this is exactly what he rejected in the procedure given for problem 6.18. Rather, he aims at an argument that holds in every case. Significantly, Liu Hui develops this argument in the case of a pyramid whose three sides have equal length (2 *chi* 尺), using standard blocks that also have equal dimensions (1 *chi* 尺). Liu Hui makes use of four types of standard blocks to compose the pyramid and then transform it into other solids that will enable him in the end to reach the conclusion with full generality. Liu Hui perfectly knows that for a pyramid of this kind there exists a direct and quicker reasoning, since he has just presented it only to discard it. However, he nevertheless relies on this singular pyramid to offer a general argument, valid for any pyramid of the *Yangma* kind. In other words, here again, the generality is in the operations and the reasoning, and not in the situation *stricto sensu*.

Was this a practice with visual tools specific to the post-Han commentators? Or did this practice exist in Han times? It is impossible to tell on the basis of *The Nine Chapters*, which itself contains no reference to any visual tool. However, the opening section of *The Gnomon of the Zhou* contains the statement of a procedure and the description of a graphical process to argue for its correctness. Both are formulated with respect to the simplest paradigm possible.[40]

With these remarks, we have reached a point where the practices just described echo what the problem and corresponding procedure contained in the section titled "Chu" 除 of *Writings* reveal about the practice of problems related to the manuscript. The section under consideration deals with a solid that is also treated in *The Nine Chapters* under the name *yanchu* 羨除.[41] This identification is confirmed not only by the

40. Karine Chemla, "Geometrical Figures and Generality in Ancient China and Beyond: Liu Hui and Zhao Shuang, Plato and Thabit Ibn Qurra," *Science in Context* 18.1 (2005), 123–66. The article contains a full bibliography on the topic.

41. See problem 5.17, *Neuf Chapitres*, 432–37.

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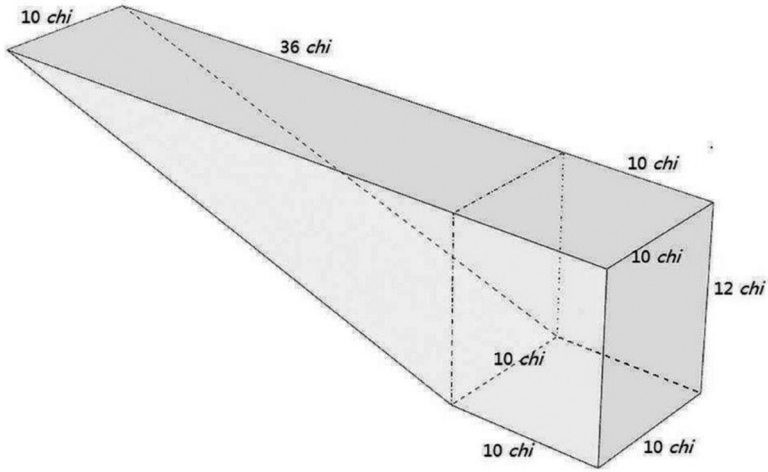

Figure 1. The figure of the solid dealt with. Reproduced from Anicotte, *Le Livre sur les Calculs*, 252, with the kind permission of Rémi Anicotte, whom I thank.

terms used—to which we will return—but also by the fact that the procedure begins with summing the three widths, a step characteristic of the procedure for the solid *yanchu*. The text of the procedure in *Writings* is, however, damaged in ways that require some care. What matters at this juncture is that that passage contains an important clue about the practice of problems to which *Writings* attests and that this clue is independent of the editorial treatment chosen. We will first discuss the editorial problems, before revisiting the clue itself.

Discussing an Editorial Problem in *Writings*

In my view, there are two admissible ways to restore the text of the correct procedure, even though the first one is supported by a larger body of evidence. According to this first way of restoring the text, the corresponding slips (141–142) read as follows (see also Figure 1):[42]

> Pathway (*chu*): A pathway to a tomb (*yanchu*), having a frontal (cavity) (*ding*) of square (section) of side one *zhang* and height one *zhang* two *chi*, and having a path (*chu*) of width one *zhang*, length three *zhang* six *chi*,

42. Two pieces of information are needed to read. First, as length measurement units, 1 *zhang* 丈 = 10 *chi* 尺. Second, the unit *chi* is also used to state volumes, with the following convention: A volume of two *chi* is the volume of a right cuboid (or rectangular parallelepiped) with two *chi* height and a square face of one *chi* side. The square base does not change, while the height will express the designated amount of volume.

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and no height at the other end, has a volume of three thousand three hundred sixty *chi*. Procedure: The widths [of the *chu*] being accumulated, thirty *chi*, [one multiplies this [the result] by the height and the length and [the result] is divided by six, hence the [volume of the]] path (*chu*). The height, one multiplies this by the corresponding width and length [of the *ding*], hence the [volume of the] frontal [cavity] (*ding*).

除[43] 美[羨][44] 除，其定(頂)[45] 方丈，高丈二尺，其除廣丈、袤三丈六[46]尺，其一旁毋高，積三千三百六十尺。[47]朮（術）曰：廣積卅（三

43. *Zhangjiashan Han jian*, 101–3. For other editorial remarks and suggestions of restoring of the text, see Guo Shirong, "*Suanshu shu* kan wu," 283; Guo Shuchun, "*Suanshu shu* jiaokan," 214; Cullen, *The Suan Shu Shu: A Translation*, 138; *Kankan Sansû sho*, 34–35 as well as Plate 6, editors' number 14; Horng et al., *Shu zhi qiyuan*, 70–72, 257. Note, however, that the research group Chōkazan Kankan *Sansū sho* Kenkyūkai worked on photos of the manuscript taken after Peng Hao had completed his transcription and that for them, after the first two characters, the greatest part of the procedure at the bottom of slip 141 was illegible (see Plate 6).

44. *Zhangjiashan Han jian*, 101n1, considers that 美 is a copy mistake for 羨. *Kankan Sansū sho*, 34, note 1, suggests it is simply an abridged form. Peng (*Zhangjiashan Han jian*, 101) discusses the evidence enabling us to interpret the restored expression as a path towards a tomb.

45. *Zhangjiashan Han jian*, 101n2, suggests reading 定 as 頂. Cullen, *The Suan Shu Shu: A Translation*, 138n83, follows, without, however, translating in agreement with this interpretation (p. 89). The term has been interpreted in two ways: either as designating the most advanced face of the solid, which contains its deepest part (Peng, *Zhangjiashan Han jian*, 101, for example, interprets in this way), or as referring to a solid that is contrasted with the *chu*, the two making the *yanchu*. To my knowledge, this suggestion was first put forward by Su Yiwen 蘇意雯, Su Junhong 蘇俊鴻, Su Huiyu 蘇惠玉, Chen Fengzhu 陳鳳珠, Lin Cangyi 林倉億, Huang Qingyang 黃清陽, and Ye Jihai 葉吉海, "*Suanshu shu* jiaokan" 《算數書》校勘, *HPM Tongxun* 通訊 3.11 (Nov. 2000), 1–20 (16n152). In my view, it better fits the way the text describes the solid. The authors suggest the solid described was a combination of a half-parallelepiped (the *chu*, in this case) and a parallelepiped (*ding*). However, their publication depended on Peng's incorrect reading of a numerical value in *Zhangjiashan Han jian*, 101 (see below), and the authors thought their interpretation did not correspond to the value of the volume given in *Writings*. *Kankan Sansū sho*, 34n3, gives the correct reading of the value, thereby highlighting that the hypothesis put forward by Su et al. in fact fits with the volume as stated in the manuscript. *Kankan Sansū sho*, 34, adopts the latter interpretation of the solid. Anicotte, *Le Livre sur les Calculs*, 250, also follows this interpretation and translates *ding* "fond," signaling the meaning of "extrémité."

46. Here, *Zhangjiashan Han jian*, 101, reads "九." *Kankan Sansū sho*, 34n3, corrects the reading to "六."

47. For those who followed the first interpretation of *ding* (see n. 45), the solid described was only the left part of what Figure 1 shows. They thus considered that either the volume or one of the data given was wrong. *Zhangjiashan Han jian*, 101n4, suggests correcting the result of the volume. Dauben, "*Suan Shu Shu*," 153, also adopts this solution. However, in note d (p. 154), he points out the alternative interpretations. Cullen, *The Suan Shu Shu: A Translation*, 138n84, suggests correcting the value of the

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十）尺。[高、袤乘之，六而一，即][48]除。高以其/141/廣、袤乘之，即定（頂）[49]。/142/.

The evidence supporting the emendation of the text that I adopt here is this: The solid described by the outline of the problem clearly has two parts—a *ding* (the right part in Figure 1) and a *chu* (the left part)— that together constitute the shape dealt with. The statement of the dimensions of the *ding* and of the *chu* indicate without ambiguity that the *ding* is a right cuboid, whereas the *chu* corresponds to the same shape as the *yanchu* in *The Nine Chapters* (see below).[50] The beginning of the procedure corresponds to the beginning of the procedure for the latter shape in *The Nine Chapters* and is unique to the *chu*. The volume computed by the procedure restored in this way, which consists of two parts—the one related to the *chu*, and then the one dealing with the *ding*—corresponds to the volume actually stated (3360 *chi*).

However, I would also have readers consider a second way of restoring the text, although in my opinion, the second possibility is less likely to have been the case. This second hypothesis rests on the assumption that the text of the procedure given in *Writings* dealt only with the *chu* part of the solid, as announced in the title (i.e., the part that corresponds to the similar body in *The Nine Chapters*). If one adopts this assumption, an alternative emendation of the procedure would result, as follows:[51]

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length as "五丈六尺." Guo Shirong, "*Suanshu shu* kan wu," 283, points out that one might indifferently correct one or the other. No correction is needed with an interpretation of the solid as shown on Figure 1.

48. That a scribe dropped a portion of the procedure, which I restore here, has been suggested in Horng et al., *Shu zhi qiyuan*, 70–72, 257; they presumed that the passage "以高及袤乘之，六而一, 即" was omitted. I suggest a slightly different emendation, whose rationale can be summarized as a *saut du même au même*. See, below, in the paragraph after the translation, for further evidence supporting my emendation.

49. *Zhangjiashan Han jian*, 101n5, considers that, in contrast with the previous one, this 定 *ding* should not be understood as referring to 頂 *ding*. As a result, the two occurrences of the same character on the same slip would have two different meanings. This is probably incorrect. Anicotte, *Le Livre sur les Calculs*, 252, suggests that the two occurrences should be interpreted alike. I adopt this interpretation. However, I do not adopt the way he understands the whole sentence. Clearly, from the emendation of the text in Horng et al., *Shu zhi qiyuan*, 70–72, 257, Horng et al., consider that the two *ding* have the same meaning. In-text slip numbers are enclosed in slashes.

50. On the fact that the terms designating the fundamental dimensions of a solid state its shape, see Chemla, Chapter D, in Chemla and Guo, *Neuf Chapitres*, 101–4. This is yet another practice that *Writings* shares with *The Nine Chapters*.

51. The alternative emendation suggested here relies on a proposition put forward by Guo Shirong, "*Suanshu shu* kan wu," 283, who corrects the text of the procedure as follows (numbers 1–3 in brackets indicate numbered comments below): "朮（術）曰：廣積卅（三十）尺，(1) 以其/141 [高] (2)、袤乘之，[六而一] (3)，即定。/142/." (1) Here, Guo Shirong considers 除高 as an interpolation, whereas I suggest alternatively

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> Procedure: The widths (of the *chu*) being accumulated, thirty *chi*, one multiplies this (the result) by the height and the length of the *chu* [and (the result) is divided by six], hence the result is determined.
>
> 术（術）曰：廣積卅（三十）尺。除，以其/141/高、袤乘之，[六而一]，即定。52/142/

The drawback of this way of restoring the text is that the volume computed—which is only that of the *chu*—does not correspond to the global volume stated.

Practices with Problems in *Writings*

Whichever alternative we adopt, the procedure of *Writings* begins with an addition of the three widths—this is a crucial clue—and thus it was dealing with a *chu*. This conclusion is confirmed by the description of the body, which makes use of dimensions that identify the solid. On one face, that contiguous with the *ding*, the *chu* has an upper width and a lower width, as well as a height. At the other extreme, it has a third width (let us call it the "end-width"), which corresponds to the upper face whose length is also given in the outline of the problem. At this other extreme, however, it has no height. These are precisely the dimensions characteristic of the *yanchu* in *The Nine Chapters*, seen in Figure 2.[53] This explains why in *The Nine Chapters*, as well as in the procedure of *Writings*, the first step is to add the three widths, before multiplying the result by the depth and the length, and dividing by 6. Let me insist on the fact that only this body has three widths of this kind in Han mathematical texts.

Interestingly, when the commentary ascribed to Liu Hui discusses the correctness of the procedure given for the *yanchu*, it considers all kinds

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that only 高 was interpolated. (2) Here, Guo Shirong considers that 廣 was erroneously written instead of 高. (3) Finally, Guo Shirong suggests that the division by 6 (六而一) was omitted. Guo Shuchun, "*Suanshu shu* jiaokan," 214, considers the mistake occurred following another scenario. Here is the text as he corrects it, into which again, I have inserted note references to describe the scenario: "术（術）曰：廣積卅（四十）尺，以除高 (1) /141/袤乘之，[六而一] (2)，即定。/142/." (1) Here, Guo Shuchun suggests that the three characters 以其/141/廣 were interpolated. (2) Guo also considers that the division by 6 was omitted. Dauben, "*Suan Shu Shu*," 153, translates a text modified along these lines, opting for a formulation closer to that of the procedure in *The Nine Chapters*. However, a typo led him to repeat the multiplication by the width erroneously.

52. Note that in the context of this alternative suggestion, 定 is not read as 頂 here. However, there is no other such occurrence of the character 定 with the intended meaning in Han mathematical texts. This remark undermines the likelihood of this emendation.

53. *Neuf Chapitres*, 432–33. In modern terms, this solid is a pentahedron with a trapezoidal base and a face perpendicular to the base. Note that in *Writings*, the solid has a "height" and not a "depth," as in *The Nine Chapters*. Why exactly this is the case needs to be clarified.

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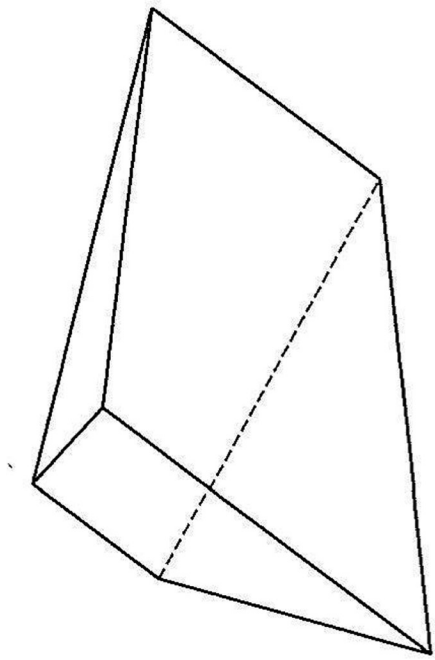

Figure 2. The *yanchu* dealt with in *The Nine Chapters*. Illustration reproduced from *Les Neuf Chapitres*, 825, Figure 5.8.

of possibilities for the shape of the solid in question, depending on how the three widths relate to each other, whether the lower width is smaller than the upper or not, whether the lower width is smaller than the end-width or not, and so on. The resulting shapes vary (see Figure 3), but the fundamental dimensions and the procedure remain the same, since the various shapes are characterized by their three widths and the vanishing of their height at one end, and the procedure is characterized by the initial sum of the three widths. These are the distinctive features that enable us to compare the *chu* of *Writings* and the shapes dealt with in *The Nine Chapters* and its commentaries.

With these pieces of information in mind, we can return to the practice of problems attested in *Writings*. Indeed, the *chu* solid described in slips 141–142 is notably specific: its three widths are equal, and so it is simply half a right cuboid. If the procedure aimed at solving only this problem, and not the class of the problems it stands for, it could simply multiply the three dimensions—to compute the volume of the right cuboid—and divide by 2. However, *Writings* instead uses the general—and thus more cumbersome—procedure, and as a result, the procedure is valid for all solids with the same kinds of dimensions. Again, in this case, instead of

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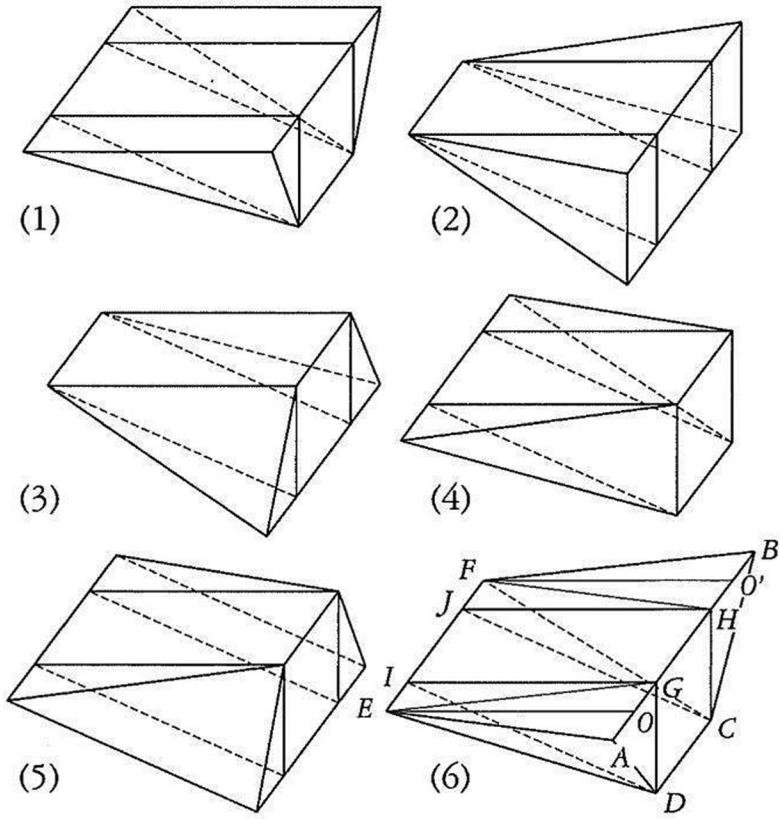

Figure 3. The various cases of *yanchu* dealt with in the commentary on *The Nine Chapters* ascribed to Liu Hui. Illustration reproduced from *Les Neuf Chapitres*, 824, Figure 5.28.

solving only the problem itself, and relying on its singularities to do so in the simplest possible way, the procedure given is apparently meant to work for a broader class of solids, which the solid of the specific problem represents; consequently, the procedure extends to the general *yanchu*. Evidently, problem and procedure form a paradigm, in the same way that Liu Hui expected for problem 6.18 in *The Nine Chapters*: the procedure determines the generality of the class of situations for which the problem stands. Moreover, the general procedure is presented in relation not to a "generic" case, but to a "degenerate" one (that of half a right cuboid), much like Liu Hui's account of the *Yangma* pyramid. Thus, we find in *Writings* the same practice that the commentator expects from *The Nine Chapters* and whose aberration Liu denounces in the only case where *The Nine Chapters* diverges from the practice. This is also

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the practice followed by the commentators. The evidence provided by slips 141–142 in *Writings* suggests that the practice of problems taught to those who studied mathematics in this context was consistent with the practice to which *The Nine Chapters* and its commentaries attest.

Note, however, that the formulation of the problem in the section *chu* of *Writings* that we have just considered differs from that of problem 6.18 in *The Nine Chapters* (see above). All problems in the canon share a standard form: after the outline of a situation and numerical values for some of its magnitudes are given in the form of a supposition, a question is put forward (*wen . . . jihe* 問 . . . 幾何). The answer(s), introduced by the expression *da yue* 答曰, is (are) given explicitly, and followed by the statement of the procedure yielding this or these answer(s). In contrast with the *Nine Chapters* standard form, problems in *Writings* do not all follow the same pattern. Here, the problem and answer are formulated as the description of a situation, with some numerical data given, and the statement of a value, which corresponds to the result of the procedure placed immediately after and which we can thus consider as "the answer." We will encounter below problems of *Writings* formulated in the *Nine Chapters* way, and problems formulated differently.[54]

Note, meanwhile, that we have relied on the commentators' reading of *The Nine Chapters* to perceive this continuity in the practice of problems. This is a first instance of what we can gain from considering the Han *Writings* in relation to the Wei and Tang *Nine Chapters* commentaries.

Interestingly, the *chu* solid is also dealt with in a procedure contained in the other recently found manuscript from Qin times, titled *Mathematics*.[55] In this context, the *chu* solid also seems to be half a right cuboid. (This is what I deduce from the procedure, which apparently is given outside the context of any problem.) In that case, however, the procedure employs the more straightforward way and thus the less general one. One might thus suggest that in contrast with *Writings* and *The Nine Chapters,* which share a similar practice with mathematical problems, *Mathematics* might reflect a different practice. The practice common to *Writings* and *The Nine Chapters* suggests that for the actors who produced these documents, generality was a cardinal value and that it was understood in a specific way.[56] Texts of procedures contained in these two writings support this hypothesis. To establish this point, we will

54. Guo Shuchun, "Shilun *Suanshu shu* de lilun gongxian yu bianzuan," is devoted to describing the lack of "standardization" of *Writings* and to interpreting this feature from the standpoint of the history of mathematics in early imperial China (see n. 14).

55. Slip 0977, editors' number 193, Xiao Can, "Yuelu shuyuan cang Qin jian," 96; *Yuelu shuyuan cang Qin jian*, 27. Also see Anicotte, *Le Livre sur les Calculs*, 253.

56. On this specificity, see Karine Chemla, *The Practices of Generality in Various Epistemological Cultures* (Uppsala: Salvia Småskrifter, 2019), www.idehist.uu.se/

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concentrate on two similar texts of procedures that we find in our documents. At first sight—we return to this point below—they both aim to enable readers to multiply quantities of the type of integers to which fractions are added (like $a + b/c$).

Comparing Han Practices of Procedures

Let us first read the text of the procedure in *Writings*.[57] It is placed after the statement of a problem that, despite the damage of the slip, appears to be formulated exactly as was the problem of the section *chu* discussed above. The problem gives the numerical values of the length (*zong* 縱) and width (*guang* 廣) of a rectangle[58] and states the value of the corresponding area. The two numerical values being given with the measurement unit *bu* 步, one might assume that the statement refers to a field (*tian* 田), as in virtually all area problems in all known Han mathematical documents. However, the text does not specify this point.[59] The numerical values of length and width are given under the form of an integer and a fraction (we will note them, respectively, $a + b/c$ and $a' + b'/c'$), and the procedure thus deals with the multiplication of such quantities with each other. It reads as follows:

> Procedure: one places the length and width [on the calculating surface]. For each, with the corresponding denominator of the parts one multiplies the corresponding whole [number of] *bu* above and makes the numerator of the parts join this [i.e., the results of the previous operation]. One makes [them, i.e., the result of the previous operations] multiply by each other, which makes the dividend. ㄴ Furthermore, one respectively (?) makes the denominators of the parts multiply each another, which makes the divisor. ㄴ [Each time one takes out of the dividend a number] like the divisor, this yields one *bu* [for the quotient, that is, dividing by the divisor yields the quotient in *bu*]. As for the *bu*s that do not fill up [the divisor, that is, the remaining *bu* in the dividend], one names them with the divisor.
>
> 術曰：直(置)廣從(縱)而各以其分母/183/乘其上全步，令分子從之，令相乘也為實ㄴ，有（又）各令分母相乘為法。如法得一步ㄴ。不盈步以法命之。/184/

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57. Slips 183–84, *Zhangjiashan Han jian*, 123–24. *Kankan Sansū sho*, 9–10 and pl. 3.

58. As above (see n. 50), the shape of the rectangle is indicated by the names of the fundamental dimensions of the figure.

59. *Kankan Sansū sho*, 9, considers that one of the damaged characters in the statement of the problem might be *tian* 田.

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Table 1. The successive stages of the multiplication of $a+b/c$ and $a'+b'/c'$, on the calculating surface (using modern symbols instead of actual numerical values)

a	a'		ca	$c'a'$		$ca+b$	$c'a'+b'$		$(ca+b).(c'a'+b')$
b	b'	—>	b	b'	—>			—>	
c	c'		c	c'		c	c—>		$c.c'$

Before analyzing the formulation of the procedure, let us illustrate the successive computations prescribed by the text, as they occurred on the calculating surface on which numbers were represented using calculating rods (see Table 1). The two values to be multiplied ($a+b/c$ and $a'+b'/c'$) were placed on the surface, as indicated to the left. Since the text refers to the integral parts of the numerical values (a and a') as placed "above," we assume that such was the way in which the two values were placed. Each of the denominators (c and c') then multiplied the integer corresponding to it, yielding in turn ac and $a'c'$, to which the numerators b and b' were, respectively added. The term used for the addition ("join") implies that the numerators disappear from the surface, once they have "joined" the numbers to which they are added. Multiplying the top positions and the lower positions with each other, respectively, gave the values of the dividend and the divisor whose division yielded the result.

In modern terms, the computation can be represented as follows:

$$\left(a+\frac{b}{c}\right).\left(a'+\frac{b'}{c'}\right)=\frac{ac+b}{c}.\frac{a'c'+b'}{c'}=\frac{\left(ac+b\right).\left(a'c'+b'\right)}{cc'}$$

The formulation of the procedure in *Writings* explicitly refers to two distinct quantities, a length and a width. However, once they are placed on the calculating surface, the text prescribes the sequence of operations to be applied to both, using one and the same list of clauses. Each clause (e.g., "with the corresponding denominator of the parts one multiplies the corresponding whole [number] of *bu* above") thus refers to two executions of the same operation, one relating to the length, the other to the width. The text of the procedure thereby at the same time highlights and states the identity of, or the symmetry between, the processes of transformation undergone by each of the values multiplied. Also note that the description of the procedure relies on the assumption that, if the terms of an operation are not made explicit, the operation prescribed is applied to the result(s) of

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the operation placed immediately before in the text (even when the latter operation corresponds to two actual executions, each in one of the parallel lines of the computation).

The Nine Chapters contains a text of procedure that carries out exactly the same operation and is placed after a sequence of problems dealing the area of rectangular fields, whose length and width are likewise of the form $a+b/c$ and $a'+b'/c'$. Moreover, the formulation of the procedure shares exactly the same features as those underlined above, which points to a continuity between the two mathematical documents under consideration in the practice of writing up procedures. Indeed, the procedure of the canon reads as follows (without translating the pieces of commentary inserted between its sentences):

> Procedure: The denominators of the parts respectively multiply the integer to which they correspond; the numerators of the parts join these (i.e., the results of the previous operations); multiplying (them, i.e., the results of the previous operations) by each another makes the dividend. The denominators of the parts being multiplied by each other make the divisor. Dividing the dividend by the divisor yields the result.
>
> 術曰：分母各乘其全，分子從之，相乘爲實。分母相乘爲法。實如法而一。[60]

By comparison, we see that the text no longer makes an explicit reference to the calculating surface and to specific positions, even though, to be sure, computations were executed on it. Nor does the text mention explicitly that the terms of the operation carried out are a length and a width, thereby detaching the formulation of the procedure from a specific topic. From our observers' perspective, it seems to gain in abstraction. On the other hand, the text of the procedure makes use of the same technical terms as *Writings* and the computations are exactly the same. Moreover, the passage is characterized by the same intention to integrate the lists of operations to be applied to each term of the multiplication, even though the integration is carried out to an even higher degree in *The Nine Chapters*. From an observers' viewpoint, this is one type of generality that the text of the procedure appears to put into play. However, my point here is that in the subcommentary composed under Li Chunfeng's supervision, another form of generality is highlighted.

60. *Neuf Chapitres*, 170–73.

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Writings and *The Nine Chapters*' Commentaries: Mutual Elucidation

The Practice of Generality

The kind of generality that the Tang exegetes read into this procedure is made clear in the annotation that the subcommentators attach to the name of the procedure, which I have not yet mentioned. They write:

> Your servant, [Li] Chunfeng et al., comment respectfully: As for the field with the greatest generality: in the procedure at the beginning [of the chapter], there were only integral [numbers of, literally: whole] *bu* and there were no remaining parts [i.e., fractions]. In a following procedure, there were only remaining parts and there was no integral [number of] *bu*. In this procedure, there appear first integral [number of] *bu*, and subsequently there are remaining parts. It enables *uniting more generally the three procedures*, and this is why one says: "the *greatest generality*."
>
> 臣淳風等謹按：大廣田知，初術直有全步而無餘分，次術空有餘分而無全步，此術先見全步，復有餘分，可以廣兼三術，故曰大廣.[61]

What Li Chunfeng and his team emphasize here is this: if the positions where one puts the numerators (*b* and *b'*) are left empty—this means one multiplies integers with one another—the procedure under consideration becomes simply and precisely a procedure to multiply integers by one another. Similarly, if the top positions in which one places the integers (*a* and *a'*) are left empty, the terms become pure fractions, and the very same procedure, applied to the three-line configuration, becomes that for multiplying pure fractions.[62] This shows that, for the exegetes, the procedure was devised to be general and uniformly cover all possible cases.[63] Several important conclusions derive from this piece of evidence.

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61. *Neuf Chapitres*, 172–73, emphasis added. The similarity between the two procedures was noted notably by *Zhangjiashan Han jian*, 124, Cullen, *The Suan Shu Shu: A Translation*, 109–10, Dauben, "*Suan Shu Shu*," 95.

62. This property of procedures is analyzed in greater detail in Karine Chemla, "Describing Texts for Algorithms: How They Prescribe Operations and Integrate Cases. Reflections Based on Ancient Chinese Mathematical Sources," in *Texts, Textual Acts and the History of Science*, ed. Karine Chemla and Jacques Virbel (Dordrecht: Springer, 2015), 317–84, esp. 341–43, 359–60.

63. For them, this was precisely what its name expresses, and so I follow their interpretation and translate *da guang* 大廣 as "the greatest generality." I thus interpret here *guang* 廣 as "general," and not as "width," which in my view does not make sense here. In particular, *da guang* has been considered as opposite to *shao guang* 少廣. However, this does not hold either—*da* 大 is opposed to *xiao* 小, and *shao* 少 is opposed to *duo* 多—and hence the names of these two operations need not be translated as a pair. I therefore take *da* as enhancing the generality to which *guang* 廣 refers. Both

footnote continued on next page

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First, in the subcommentators' opinion, generality was an epistemological value that was meaningful for the Han authors, and this, in a quite specific way (see n. 56). Liu Hui's commentary on the procedure for the operation titled "the greatest generality" also insists on this feature. We had seen above one facet of the valuing of generality that the exegetes lent to Han practitioners of mathematics with the practice of problems. We now see another, with that of texts for procedures. Why, if not to comply with the value of generality, would actors need to solve three different problems with a single, uniform procedure, to the point that the solution of two of them would become more cumbersome than necessary? Seen in this light, the constraint bearing on procedures resembles what we have seen above about problems and general procedures. However, in this case, with the title given to the operation, the Han canon gives us a crucial clue that this may have been the way Han actors thought about this issue (and not just later commentators).

Second, in both cases, *Writings* shares the same practice as *The Nine Chapters*. Indeed, we have seen that the procedure to execute multiplication presented the same features in both documents. Additionally, the section of *Writings* in which the procedure is found and the operation in the manuscript are both entitled "the greatest generality." Judging from the evidence, the mode of reading displayed by the exegetes on the Han mathematical canon underlines a theoretical facet in the practice with procedures that is shared by *Writings* and *The Nine Chapters*. So while both documents might have been connected with practical activities, this does not seem to have prevented people from developing theoretical interests. A view of any of these Han texts as purely practical would fail to capture these dimensions, which we can approach through the commentaries and subcommentaries on *The Nine Chapters*.

Finally, it is worth noting this: the name of the operation "the greatest generality" that Li Chunfeng et al. comment upon is precisely the name as it is found in *Writings*, not the one contained in the received text of *The Nine Chapters*, where its full name reads: "Field with the greatest generality" (*da guang tian* 大廣田). The exegetes hence might

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options lead me to translate differently from Cullen, *The Suan Shu Shu: A Translation*, 109–10 ("The greater breadth") and from Dauben, "*Suan Shu Shu*," 165–66 ("General widths"). One can find a compilation of all the translations of this expression that have been offered in Guo Shuchun 郭書春, Joseph W. Dauben 道本周, and Xu Yibao 徐義保, *Nine Chapters on the Art of Mathematics. With the Annotations by Liu Hui [State of Wei] and Notes and Annotations of Li Chunfeng and Associates [Tang Dynasty]* 九章算術，魏劉徽注，唐李淳風注釋. *A Critical Edition and English Translation Based upon a New Collation of the Ancient Text and Modern Chinese Translation by Guo Shuchun. English Critical Edition and Translation, with Notes by J. Dauben and Xu Yibao* 郭書春校勘并譯注。道本周，徐義保英譯并注, 3 vols. (Shenyang: Liaoning jiaoyu, 2013), vol. 1, 79.

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have had a version of the title in line with the title found in the Han manuscript, and not in the received version of the canon.[64] Interestingly, the text commented upon by the exegetes might thus be closer to the Han manuscript than the text of *The Nine Chapters* to which all earlier editions testify. Apparently, the alleged opposition between the Han documents (supposedly practical) and the commentarial literature (supposed theoretical) seems in need of revision. If this opposition does not hold, what can we say about the various types of theoretical continuities between *Writings* and *The Nine Chapters*, and also between *Writings* and the commentaries on the canon? We will examine these two points in turn.

Theoretical Continuities and Discontinuities between *Writings* and *The Nine Chapters*

Let me emphasize the fact that by "continuity," I do not mean here identity. Beyond continuities, importance differences can be noted between the Han manuscript under consideration and the canon. A clear-cut example is this: both documents explicitly refer to the use of rods placed on a calculating surface. Indeed, the texts of procedures they contain both use the prescription "one places" (*zhi* 置). The term *suan* 算 designating the representation of numbers with calculating rods and computations with them features in both writings. Finally, they both sometimes refer to precise positions on the calculating surface, as with the aforementioned position "above." However, I claim that the ways in which calculating rods and positions were used in the contexts in which *Writings* and *The Nine Chapters* were written differ.[65] I argue *The Nine Chapters* attests to the fact that the use of positions on the calculating

64. Exactly the same situation recurs with the name of the operation *jing fen* 徑分 (directly sharing). This is the title in *Writings* (*Zhangjiashan Han jian*, 48) and the one glossed by Li Chunfeng et al.; see *Neuf Chapitres*, 166–67. However, in all the ancient editions of *The Nine Chapters*, the name of the operation is recorded as *jing fen* 經分. Rarely have authors addressed how *Writings* could help us assess the reliability of the information about *The Nine Chapters* given in the editions of the work that have come down to us. The comparison between the manuscript and the commentaries might help us restore states of parts of the canon closer to what the ancient commentators saw.

65. In this, I differ from the assumption tacitly adopted by most historians (e.g., Cullen, *The Suan Shu Shu: A Translation*, 24–25; Anicotte, *Le Livre sur les Calculs*, 69–74). My argumentation is sketched in Karine Chemla, "Observing Mathematical Practices as a Key to Mining Our Sources and Conducting Conceptual History. Division in Ancient China as a Case Study," in *Science after the Practice Turn in Philosophy, History, and the Social Studies of Science*, ed. Léna Soler et al. (New York: Routledge, 2014), 238–68, esp. 242–48, 257–62.

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surface had undergone a major shift, the positions being at the time a crucial tool of theoretical exploration. In my view, the use of a decimal place-value system—that is, a numeration system using positions with a mathematical meaning, as is the case when in the inscription "123," where 1 means a hundred, and 2, twenty, in relation to their positions in the sequence—adheres to this context. In contrast, there is no such use of position in *Writings* and notably no place-value numeration system. This thesis receives support from the fact that all manuscripts contain tables to multiply between powers of ten that have disappeared in *The Nine Chapters* and the other canons.[66]

Commentaries and subcommentaries allow us to perceive a second admixture of continuity and discontinuity in another theoretical practice with procedures that *Writings* and *The Nine Chapters* partly share. We will discuss this point on the basis of the different procedures to carry out unequal sharing found in the two Han texts. It is precisely in relation to a problem and procedure falling under this category that Cullen asserts: "It is striking how often one can find quite close parallels between problems of the *Suàn shù shū* and in the Nine Chapters."[67]

For the sake of comparison, let us quote a procedure of this kind from *Writings*, with the problem that it solves:[68]

> 共買材　三人共[69]材，以賈(價)一人出五錢，一人出三[70]∟，一人出二錢。●今有贏（盈）四錢，欲以錢数衰分之。出五者得二錢，出三者 /32/
>
> 得一錢五分錢一，出二者得五分錢四。术（術）曰：并三人出錢数以為法，即以四錢各乘所出錢数，如法得一錢。/33/
>
> Buying timber together: Three persons [buying] timber together, according to the price, one of the persons pays five cash, the other pays three ∟, and the third pays two cash●. Suppose there is an excess of four cash: one wants to share this, weighing in function of the degree (*cuifen zhi*) according to the amounts of cash [each person paid].[71] The [person] who paid five gets two cash, the one who paid three gets one cash and one fifth of a cash, and the one who paid two gets four fifths

66. See Chemla and Ma, "Interpreting a Newly Discovered Mathematical Document," and Chemla, "Numerical Tables."

67. Cullen, "The *Suan Shu Shu*: Rewriting," 28.

68. *Zhangjiashan Han jian*, 50–52. *Kankan Sansū sho*, 147–48 and plate 22.

69. *Zhangjiashan Han jian*, 51n2, considers that after 共， 買 was omitted.

70. *Zhangjiashan Han jian*, 51n3, suggests that after 三， 錢 was omitted.

71. That is: giving back the money in proportion of what each paid. On the name of this operation, see below.

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> of a cash. Procedure: one sums the quantities of cash paid by the three persons, which is taken as divisor. Hence, multiplying, respectively, by four cash, the quantities of cash paid, and dividing [the results] by the divisor yields the results in cash.

The mathematical meaning of procedures such as these was discussed above. Here, I focus on its formulation. Clearly, its text makes use of ordinary arithmetical terminology ("sum," "multiply," etc.), and not of theoretical terms like those specifically introduced in *The Nine Chapters* for these problems—for example, *lie cui* "array of coefficients for weighting in function of the degree."[72] Moreover, this procedure is expressed making reference to the terms of the problem. In *The Nine Chapters*, similar problems are followed by procedures formulated more or less in the same way. However, the main point is that before all these problems, the canon inserts another procedure, whose text is of an entirely different nature. It reads as follows:

> Parts weighted in function of the degree.
>
> Procedure: one places respectively [on the calculating surface] the array of coefficients for weighting in function of the degree (*cui*) and, in an auxiliary [position], sums them to make the divisor. One multiplies by that which is shared, the [weights in function of the degree that one had] before they were summed, which respectively makes the dividends. One divides the dividends by the divisor.
>
> 衰分
>
> 術曰：各置列衰；副并為法。以所分乘未并者各自為實。實如法而一。

As underlined above, both texts of procedures refer to parallel computations using a single sentence. Clearly, compared to a text of procedure such as the one quoted above, this second formulation is abstract,[73] in that it indicates exactly the same operations as the more specific procedures prescribed, but refers to them using theoretical terms. This correlates with the fact that the text is placed outside the context of any problem, and, more importantly, the text introduces

72. Cullen, "The *Suan Shu Shu*: Rewriting," 22, notes this fact.

73. I argue the text embodies an understanding of abstraction shared by Han actors in Karine Chemla, "Writing Abstractly in Mathematical Texts from Early Imperial China," in *Technical Arts in the Han Histories: Tables and Treatises in the "Shiji" and "Hanshu,"* ed. Mark Csikszentmihalyi and Michael Nylan (Albany: State University of New York Press, 2021), 307–38.

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terms that commentators and subcommentators use to prove the correctness of the related specific procedures. Crucially, in *Writings* we find procedures with the same property as this abstract one. However, the ways these abstract procedures are formulated in the manuscript and in *The Nine Chapters* differ. We thus have here a theoretical practice shared by the manuscript and the canon. However, the canon seems to reflect a higher kind of abstraction. This is not always the case.

Theoretical Continuities and Discontinuities between *Writings* and the Exegeses

We have actually already encountered a case in which *Writings* attests to a theoretical practice also found in the exegeses, but not in *The Nine Chapters*. The text of the procedure of *The Nine Chapters* just quoted begins with the name of the operation it allows practitioners to execute: "parts weighted in function of the degree." In fact, Liu Hui's and Li Chunfeng's annotations regularly use such names of operations verbally, followed by *zhi* 之, exactly as we have encountered above: "one multiplies this" (*cheng zhi* 乘之). For instance, exegetes use expressions such as *jin you zhi* 今有之, which I interpret as "applying [the operation] 'suppose' to this."[74] A practice of this kind attests the actors' interest in fundamental operations beyond the arithmetical operations. It is thus striking that the problem from *Writings* just quoted asks to carry out the operation *cuifen* using exactly the same kind of syntax "share this, weighing in function of the degree" (*cuifen zhi*), whereas these expressions never occur in *The Nine Chapters*.

This gives a first example of a theoretical parallel between *Writings* and the exegetical layers on *The Nine Chapters* that is absent in the canon, but importantly, this parallel is by no means the only one. The procedures given in the canon and the manuscript to add fractions attests to a similar phenomenon. *The Nine Chapters* text reads as follows:

> Gathering parts.
>
> Procedure: The denominators multiply the numerators that do not correspond to them; one takes the sum as the dividend. The denominators being multiplied by one another make the divisor. One divides the dividend by the divisor. That which does not fill the divisor one names with the divisor. If the corresponding

74. The operation "suppose" is the form of "rule of three" contained in *The Nine Chapters* (*Neuf Chapitres*, 222–23). The expression occurs in both Liu Hui's commentary and Li Chunfeng's subcommentary, after problem 6.24 (*Neuf Chapitres*, 538–39).

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DENOMINATORS ARE EQUAL, ONE ONLY MAKES THEM [THE NUMERATORS] JOIN EACH OTHER。

合分

術曰：母互乘子，并以為實。母相乘為法。實如法而一。不滿法者，以法命之。其母同者，直相從之。[75]

By contrast, one of the procedures given in *Writings* for the same operation reads as follows:[76]

> Procedure: If the denominators are *of the same category* as one another, the numerators join one another; if the denominators are *not of the same category* one as the other [. . . first operations . . .]. If the [resulting] denominators are *of the same category* as one another, the numerators join one another. If the [resulting] denominators are *not of the same category* as one another, the denominators being multiplied by one another make the divisor. The denominators multiply the numerators that do not correspond to them; one sums and takes this as the dividend. The denominators being multiplied by one another make the divisor.

術曰：母相類，子相從；母不相類，[. . . first operations. . .] 母相類，/21/

者，子相從。其不相類者，母相乘為法，子互乘母并以為實 … 。/22/

The procedure above gives a central role to the concept of "category" (*lei* 類) of a fraction. Two fractions belong to or become of the same category, if their denominators are or become the same. This concept, which does not feature in *The Nine Chapters*, nonetheless plays a theoretical part in Liu Hui's commentary on the addition of fractions as dealt with in the canon. I say this, because, to begin with, the operation "gathering parts" is placed in the first chapter of the canon, titled "Rectangular Field" ("Fangtian" 方田). In line with the positioning of the operation in this chapter, after establishing the correctness of the procedure, Liu Hui's commentary on "gathering parts" goes on to quote the *Xici zhuan* 繫辭傳 attached to the *Changes* classic, which reads:

> *Rectangles* are assembled according to their *category*; existing things are divided in groups.

方以類聚，物以群分。.[77]

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75. *Neuf Chapitres*, 158–59.
76. Emphasis added. *Zhangjiashan Han jian*, 45–47.
77. My emphasis. Here and below, I rely on *Neuf Chapitres*, 158–59, references being indicated in the related footnotes.

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The quotation also gives the concept of category a central role. The context invites the understanding that for the commentator, fractions are associated with rectangles (their "body," *ti* 體), and their transformations correspond to a reshaping of the related rectangle that changes its form (*xing* 形) while retaining the original area. Accordingly, rectangles can be transformed in ways that allow the practitioner to join the resulting rectangles together: the corresponding fractions, being of the same category, can thus be added. Here is Liu Hui's related theoretical development, in which the concept of category is central:

> When quantities are *of the same category*, they are not far from each other; when they are *of different categories*, they are not close to each other. If they are far from each other but one makes their body communicate, even if they are at different places, they *join* each other. If they are close to each other but differ in form, even if they are in a same place, they repel each other.
>
> 數同類者無遠；數異類者無近。遠而通體知，雖異位而相從也；近而殊形知，雖同列而相違也。

Thus, Liu Hui as commentator uses the same concept and terms that feature in the procedure of *Writings*, notably the concept of category, with exactly the same meaning,[78] giving these ideas much greater extension in his commentary. In this respect, his commentary displays strong continuity with abstract terms occurring in *Writings*, but not in *The Nine Chapters*.

This is not an isolated example, as is plain when we examine the concept of "restoring" *fu* 復. In the commentaries and subcommentaries, *fu* refers to the property of fractions and irrational numbers such as "square root of 2" to allow practitioners to "restore" the number one started from, using the operation opposed to the one that produced said fraction or irrational number. This concern plays a central role when Liu Hui accounts for the introduction of irrational numbers as results of square root extractions in the canon;[79] it is crucial to a type of proof conducted in the subcommentaries.[80] However, the concept of *fu* does not feature in *The Nine Chapters*, but occurs several times in *Writings*.[81]

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78. On this, see Chemla, "Glossaire des expressions techniques," 948–49. As a rule, the commentators use the concept of "category" in two main ways: they either discuss how mathematical objects sharing the same category can interact with each other or how they can be all known in the same respect in virtue of sharing the same category. The uses of the concept in *Writings* and in the commentary on "Gathering parts" both fall under the former type of use.

79. *Neuf Chapitres*, 364–66.

80. *Neuf Chapitres*, 456–57, as well as 36–39.

81. Chemla, "Glossaire des expressions techniques," 924–25.

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Conclusion

As stated in the introduction, knowledge of *The Nine Chapters* once depended entirely on the editorial work carried out in the first decades of the Tang dynasty under Li Chunfeng's supervision. From this perspective, understanding better how the commentators worked and which type of documents they relied upon thus appears as a central issue. From our overall argument in this article, several conclusions follow.

First, given the extant evidence, if we only had the manuscripts and *The Nine Chapters* without the commentaries, we would not be able to understand what lies behind the use of concepts like "category" or "restoring" in *Writings*. Here, as in the reading of problems and procedures, indirectly exegeses make us aware that something deeper is at stake than we might otherwise suppose, if we remained at the surface of the extant text.

Second, in some respects theoretical facets that seemed specific to commentaries and subcommentaries (e.g., those attached to concepts such as "category" or "restoring") derive from Han mathematical work (or work carried out even earlier). For their annotations, exegetes hence most probably relied on documents that had family resemblance with *Writings*. This conclusion is supported by two types of evidence. As noted previously, exegetes of *The Nine Chapters* mention using earlier procedures and explanations, and indeed the exegeses and *Writings* share procedures that are not contained in *The Nine Chapters*. What my article shows is that this observation extends to theoretical dimensions of commentaries and subcommentaries. These remarks have a bearing on our understanding of the exegetical activity. But they also have implications for our knowledge of the canon.

Third, the previous comparisons show why treating the Han manuscript as purely practical is controversial. Is it enough to emphasize, as Cullen does, that *Writings* reflects an early Western Han interest "in clever methods of calculation?" In my view, reading the manuscript with the interpretative techniques displayed in the exegeses on *The Nine Chapters* highlights other facets. With respect to the epistemological value of generality, for instance, the Wei and Tang exegetes helped us identify theoretical dimensions in the canon that were also present in *Writings*. This conclusion recasts how we understand mathematical activity in Han times.

Given the highly fragmentary evidence we have for Han mathematics, much is thus at stake in considering newly found manuscripts in dialogue not merely with the canons, but also with their commentaries and subcommentaries. Notably, however, while the Han Zhangjiashan

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manuscript and Wei-Tang exegeses sometimes echo each other, the Yuelu Academy manuscript devoted to mathematics diverges from the canonical literature in this respect. This remark invites us to see not only continuities where we were tempted to think in terms of opposition, but also differences where we might be tempted to assume continuities, as among Han mathematical manuscripts that at first sight might have looked similar.

從數學的角度試論出土文獻和經典與其一起流傳下來的注和注釋如何相互闡明

林力娜

提要

在張家山漢簡《筭數書》被發掘出來之前, 數學史家只能在傳世文獻, 尤其是《九章算術》的基礎上研究秦、漢數學。《筭數書》這部漢簡出土後, 主要用來與《九章算術》進行比較。作為一部經典,《九章算術》是與劉徽注和唐朝李淳風等注釋一起流傳下來的。筆者認為,《筭數書》與《九章算術》的注和注釋之間有重要的相似之處, 這為注和注釋如何形成提供了一些有趣的線索。筆者還認為, 注和注釋使我們能夠察覺到《筭數書》與《九章算術》之間某些至今尚未為人考慮的相似之處。

Keywords: *Writings on Mathematical Procedures* (*Suanshu shu* 筭數書), *The Nine Chapters on Mathematical Procedures* (*Jiuzhang suanshu* 九章算術), commentary, subcommentary, generality, practice of mathematics, mathematical problem, mathematical procedure
《筭數書》，《九章算術》，注, 注釋，一般性，數學實作，數學問題，術

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