International Journal of Microwave and Wireless Technologies

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An acceleration method employing sparse sensing matrix for fast analysis of the wide-angle electromagnetic problems based on compressive sensing

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Research Paper

Cite this article: Qi Q, Cao X, Liu Y, Kong M, Kuang X, Chen M (2024) An acceleration method employing sparse sensing matrix for fast analysis of the wide-angle electromagnetic problems based on compressive sensing. *International Journal of Microwave and Wireless Technologies* **16**(4), 645–650. https://doi.org/10.1017/S1759078724000291

Received: 18 May 2023
Revised: 4 February 2024
Accepted: 17 February 2024

Keywords:
compressive sensing; measurement matrix; method of moments; sparse sensing matrix

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Abstract

The electromagnetic scattering problem over a wide incident angle can be rapidly solved by introducing the compressive sensing theory into the method of moments, whose main computational complexity is comprised of two parts: a few calculations of matrix equations and the recovery of original induced currents. To further improve the method, a novel construction scheme of measurement matrix is proposed in this paper. With the help of the measurement matrix, one can obtain a sparse sensing matrix, and consequently the computational cost for recovery can be reduced by at least half. The scheme is described in detail, and the analysis of computational complexity and numerical experiments are provided to demonstrate the effectiveness.

Introduction

Method of moments (MoM) possesses the advantage of high accuracy in solving the electromagnetic (EM) scattering problems [1]. However, it will cost a huge computational amount when the incident wave is from a wide-angle range, since the procedure needs to be repeatedly implemented at every angle increment. To accelerate the solution process, many methods have been proposed, such as asymptotic waveform evaluation [2], model-based parameter estimation [3], etc., but these methods show some shortcomings [4]. Recently, a fast method based on MoM conjunction with compressive sensing (CS) theory has been put forward [5]. In this method, a kind of new excitation sources containing abundant information of different incident angles is built firstly. Then, the induced currents over a wide-angle can be solved by means of the sparse transform and the recovery algorithm from the measurement results, which are obtained by a few calculations of traditional MoM with the new sources. The computational complexity of the fast method mainly consists of two parts: one is the measurement, i.e., the calculations of MoM; the other is to acquire the projections of induced currents in sparse domain.

In order to further improve the fast method, much effort has been devoted to the research on these two parts, and many effective schemes, such as Bayesian CS method [4], efficient basis function [6], and two-dimensional CS method [7], have been devised. In this paper, a novel scheme for designing the measurement matrix is raised, by which the sensing matrix shows remarkable sparsity when the orthogonal basis is taken as the sparse transform. Accordingly, the computational complexity for acquire the projections by using recovery algorithm can be sharply decreased. The principle and the complexity analysis are presented, and the effectiveness is validated by the numerical experiments, in which several typical orthogonal bases, such as the fast Fourier transform (FFT) basis, are taken as the sparse transforms, respectively.

Theory

Fast method based on CS

The wide-angle EM scattering problem solving by the traditional MoM can be described as a matrix equation with multiple right-hand sides

$$\mathbf{Z}\left[\mathbf{I}_1\mathbf{I}_2\cdots\mathbf{I}_n\right] = \left[\mathbf{V}_1\mathbf{V}_2\cdots\mathbf{V}_n\right], \tag{1}$$

in which, $\mathbf{Z}$ is the impedance matrix, $\mathbf{V}_1$ to $\mathbf{V}_n$ are the excitation vectors at n different incident angles, and $\mathbf{I}_1$ to $\mathbf{I}_n$ represent the n corresponding induced current vectors.

In the fast method, M new excitation vectors based on CS theory are constructed as

$$\mathbf{V}_i^{\mathrm{CS}} = c_{i1}\mathbf{V}_1 + c_{i2}\mathbf{V}_2 + \cdots + c_{in}\mathbf{V}_n \quad (i = 1, 2, \cdots, M), \tag{2}$$

where c_{ij} is the element in the measurement matrix $\mathbf{\Phi}$. In CS theory, a measurement matrix must satisfy the restricted isometry property (RIP) [8], which ensures the accurate reconstruction of original signal. In general, Gaussian random matrix is often used as $\mathbf{\Phi}$.

Substituting equation (2) to equation (1), one can obtain M current vectors under the new excitations by

$$\mathbf{Z}\left[\mathbf{I}_1^{\mathrm{CS}}\mathbf{I}_2^{\mathrm{CS}}\cdots\mathbf{I}_M^{\mathrm{CS}}\right] = \left[\mathbf{V}_1^{\mathrm{CS}}\mathbf{V}_2^{\mathrm{CS}}\cdots\mathbf{V}_M^{\mathrm{CS}}\right]. \tag{3}$$

Due to the linearity of the problem, $\mathbf{I}_1^{\mathrm{CS}}$ to $\mathbf{I}_M^{\mathrm{CS}}$ can be written as

$$\mathbf{I}_i^{\mathrm{CS}} = c_{i1}\mathbf{I}_1 + c_{i2}\mathbf{I}_2 + \cdots + c_{in}\mathbf{I}_n \quad (i = 1, 2, \cdots, M). \tag{4}$$

The M current vectors can be regarded as the results of M measurements of the original induced current vectors $\mathbf{I}_1$ to $\mathbf{I}_n$. If the original induced current vectors have a sparse representation, equation (4) can be described as

$$\mathbf{\Phi}\left[\mathbf{I_1 I_2}\cdots\mathbf{I_n}\right]^{\mathrm{T}} = \mathbf{\Phi\Psi}\left[\boldsymbol{\alpha}_1\boldsymbol{\alpha}_2\cdots\boldsymbol{\alpha}_N\right] = \left[\boldsymbol{I}_1^{CS}\boldsymbol{I}_2^{CS}\cdots\boldsymbol{I}_M^{CS}\right]^{T}, \tag{5}$$

in which $\mathbf{\Psi}$ is the sparse transform, N is the number of the basis functions, and $\boldsymbol{\alpha}_1$ to $\boldsymbol{\alpha}_N$ are the projections of each column of $[\mathbf{I}_1\ \mathbf{I}_2 \ldots \mathbf{I}_n]^{\mathrm{T}}$ in sparse domain. In the wide-angle EM scattering problems, FFT basis, Hermite basis, discrete wavelet transform (DWT) [9] basis, or other orthogonal bases are often selected as $\mathbf{\Psi}$ [7, 10].

By utilizing the recovery algorithm (e.g., orthogonal matching pursuit (OMP) [11]), the projections can be approximated by

$$\begin{aligned}\left[\hat{\alpha}_1\hat{\alpha}_2\cdots\hat{\alpha}_N\right] &= \operatorname{argmin}\left\|\left[\hat{\alpha}_1\hat{\alpha}_2\cdots\hat{\alpha}_N\right]\right\|_L \ \ s.t.\ \ \Theta\left[\alpha_1\alpha_2\cdots\alpha_N\right] \\ &= \left[\mathbf{I}_1^{\mathrm{CS}}\mathbf{I}_2^{\mathrm{CS}}\cdots\mathbf{I}_M^{\mathrm{CS}}\right]^T,\end{aligned} \tag{6}$$

where $\mathbf{\Theta}$ is the sensing matrix and $\mathbf{\Theta}=\mathbf{\Phi\Psi}$.

Then, the original induced current vectors are reconstructed by

$$\left[\hat{\mathbf{I}}_1\hat{\mathbf{I}}_2\cdots\hat{\mathbf{I}}_n\right]^T = \Psi\left[\hat{\boldsymbol{\alpha}}_1\hat{\boldsymbol{\alpha}}_2\cdots\hat{\boldsymbol{\alpha}}_N\right]. \tag{7}$$

The computational complexity for solving equation (6) by OMP is $O(nKMN)$, where K is the sparsity of original induced current vectors in sparse domain, and the inner products of the columns in sensing matrix and the measurement results are the dominant computational cost.

Construction scheme of measurement matrix

To reduce the complexity of recovery, one can make the sensing matrix sparse to decrease the cost of inner products. For the purpose, a novel construction scheme of measurement matrix is proposed as follows:

First, by randomly extracting P columns from the sparse transform $\mathbf{\Psi}$, one can obtain ψ_1 to ψ_P, where ψ represents the column of $\mathbf{\Psi}$.

Afterward, to better satisfy RIP, a linear superposition of these P vectors is implemented as

$$\varphi_1 = d_{11}\Psi_1 + d_{12}\Psi_2 + \cdots + d_{1P}\Psi_P. \tag{8}$$

Finally, repeating the above two steps M times and a new measurement matrix $[\varphi_1\ \varphi_2 \ldots \varphi_M]^{\mathrm{T}}$ is established, in which

$$\varphi_i = d_{i1}\Psi_1 + d_{i2}\Psi_2 + \cdots + d_{iP}\Psi_P \quad (i = 1, 2, \cdots, M). \tag{9}$$

Obviously, when an orthogonal basis is taken as $\mathbf{\Psi}$, the inner products of φ_i and the $(n - P)$ columns in $\mathbf{\Psi}$ that are not extracted in the first step are zero, respectively. In other words, there are only P non-zero elements in the ith row of $\mathbf{\Theta}$. A sparse $\mathbf{\Theta}$ with MP

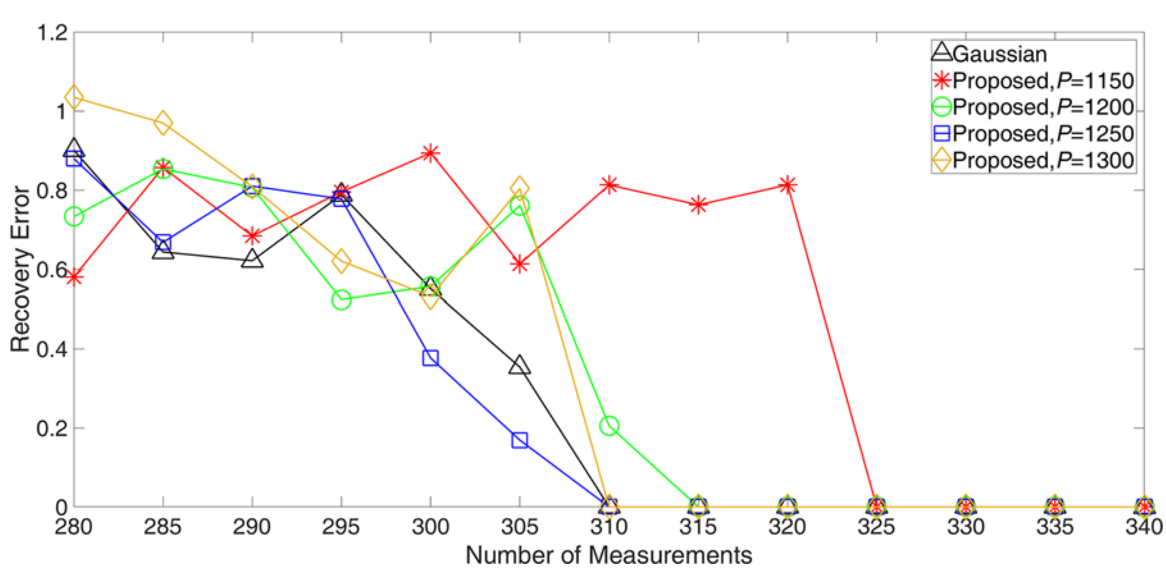

Figure 1. Relationship between the recovery error and the number of measurements in the case of FFT basis.

https://doi.org/10.1017/S1759078724000291 Published online by Cambridge University Press

Table 1. Computing time of recovery for sphere (unit: s)

Measurement matrix	FFT ($M = 310$)	Hermite ($M = 245$)	DWT ($M = 275$)
Gaussian	8273.0	6513.7	6987.1
Proposed	3085.1 ($P = 1250$)	2974.2 ($P = 1550$)	2990.2 ($P = 1400$)

non-zero elements is obtained by using the proposed measurement matrix, and the operations of inner products in recovery algorithm are significantly accelerated. Accordingly, the computational complexity of solution to equation (6) is decreased to $O(\eta nKMN)$, where η is the proportion of non-zero elements in $\boldsymbol{\Theta}$ and $\eta = P/n$. Generally, P is much less than n.

Using the proposed measurement matrix and an orthogonal basis as $\boldsymbol{\Phi}$ and $\boldsymbol{\Psi}$ respectively, equation (5) can be transformed as

$$[\varphi_1\varphi_2\cdots\varphi_M]^T\Psi\,[\alpha_1\alpha_2\cdots\alpha_N] = \mathbf{S}\Psi^T\Psi\,[\alpha_1\alpha_2\cdots\alpha_N] = \mathbf{S}\,[\alpha_1\alpha_2\cdots\alpha_N] = \left[\mathbf{I}_1^{\mathrm{CS}}\mathbf{I}_2^{\mathrm{CS}}\cdots\mathbf{I}_M^{\mathrm{CS}}\right]^T, \tag{10}$$

in which, the proposed measurement matrix is expressed as the multiplication of a sparse random matrix $\mathbf{S}$ and the transposed sparse transform $\boldsymbol{\Psi}$. The ith row of $\mathbf{S}$ has P random coefficients d_{i1} to d_{iP} in the columns corresponding to the randomly extracted columns from $\boldsymbol{\Psi}$ in the ith measurement, and other entries in $\mathbf{S}$ are all zero.

So, equation (10) can be considered as measuring the sparse signals $\boldsymbol{\alpha}_1$ to $\boldsymbol{\alpha}_N$ directly with the sparse random matrix $\mathbf{S}$, which has been proved to satisfy a different form of RIP, so-called RIP(p) for p equal (or very close) to 1 [12, 13]. Therefore, equation (6) can produce an accurate solution with high probability by using the proposed measurement matrix. It is interesting to note that, if the random coefficients are all set to 1 in (10), the sparse random matrix is simplified to a sparse binary matrix (SBM) [14], which consists of only 0 and 1. SBM is often applied as the measurement matrix in wireless sensor networks, since it is easy to be implemented on hardware and has low complexity [15].

Numerical results

Two numerical experiments with perfect electrical conductor objects of different shapes are presented in this section to validate the proposed scheme, in which the electric field integral equation is established to solve the problems, and OMP is taken as the recovery algorithm. For the convenience of comparison, we define the recovery error as

$$\Delta = \frac{\left\|\left[\hat{\mathbf{I}}_1\hat{\mathbf{I}}_2\cdots\hat{\mathbf{I}}_n\right] - [\mathbf{I}_1\mathbf{I}_2\cdots\mathbf{I}_n]\right\|_2}{\|[\mathbf{I}_1\mathbf{I}_2\cdots\mathbf{I}_n]\|_2}. \tag{11}$$

Sphere

A sphere with the radius of 5 m illuminated by the plane waves of 300 MHz is considered, who contains 12,620 Rao-Wilton-Glisson (RWG) basis functions. The waves are set in the *xoy* plane, and the incident angle is divided into 0.1°, 0.2°, ..., 360°.

FFT basis is used as the sparse transform. As is shown in Fig. 1, the similar precision can be achieved at the same number (310) of measurements by applying the Gaussian random matrix and the proposed measurement matrix respectively, while the number of extracting columns P is larger than 1250. It means that one can get a sensing matrix with non-zero elements accounting for 1250/3600 (η) in the best case. Thus, the computational cost for acquiring the projections of original induced currents in sparse domain is cut by about two-thirds ($1 - \eta$) by using the proposed measurement matrix rather than Gaussian random matrix; meanwhile, the computational complexity of measurement for both is the same. The comparison of the computing time for acquiring the projections is presented in Table 1, which further proves the high efficiency.

To show the universality of the proposed technique for orthogonal bases, FFT basis is, respectively, replaced by Hermite basis and DWT basis. The corresponding results are shown in Figs. 2 and 3. It is evident that, for both Hermite basis and DWT basis, much less computational complexity for acquiring the projections is available with the proposed measurement matrix than with Gaussian matrix under the same condition of measurement. The comparisons of

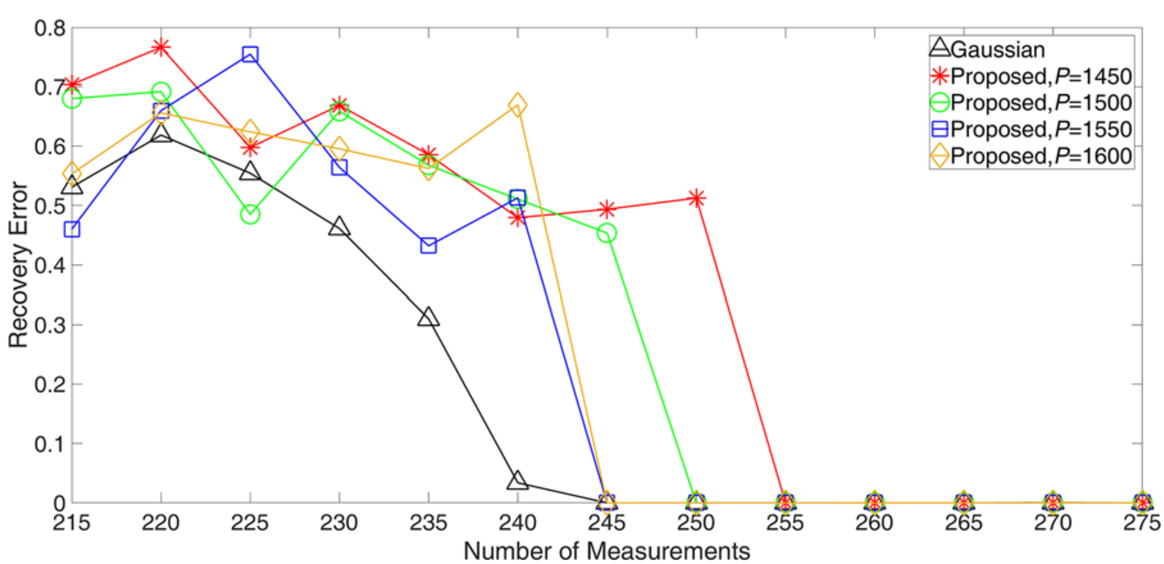

Figure 2. Relationship between the recovery error and the number of measurements in the case of Hermite basis.

https://doi.org/10.1017/S1759078724000291 Published online by Cambridge University Press

computing time in the case of two bases are also provided in Table 1.

Missile model

Then, consider a missile model who contains 1963 RWG basis functions, and the angle increment is set as 1°. The second kind of Chebyshev basis and Legendre basis is chosen as the sparse transforms, respectively. Other experimental parameters are the same with the previous one. Both Figs. 4 and 5 indicate that the proposed scheme is also validity for the complex-shaped objects.

By using the second kind of Chebyshev basis and the proposed measurement matrix ($M = 90$, $P = 100$), and setting the elements less than 10^{-14} in the sensing matrix to zero, there are only 9000 non-zero elements in the sensing matrix, which is consistent with the expected number MP. Hence, the solution to equation (6) with the help of OMP is accelerated, which is demonstrated in Table 2. The bistatic radar cross section (RCS) of the missile model illuminated at a random incident angle (take 77° as an example) is also provided in Fig. 6, which agrees well with the result solved by the traditional MoM. From Table 2 and Fig. 6, we can see clearly that the computing time can be significantly reduced while the high accuracy is kept by using the proposed measurement matrix.

Conclusion

A novel scheme for constructing the measurement matrix has been developed. One can get a sparse sensing matrix by adopting the proposed measurement matrix in the solution to wide-angle EM scattering problem based on MoM conjunction with CS. In addition, the number of measurements required for both Gaussian random matrix and the proposed one is the same. Consequently, the computational complexity for acquiring the projections by

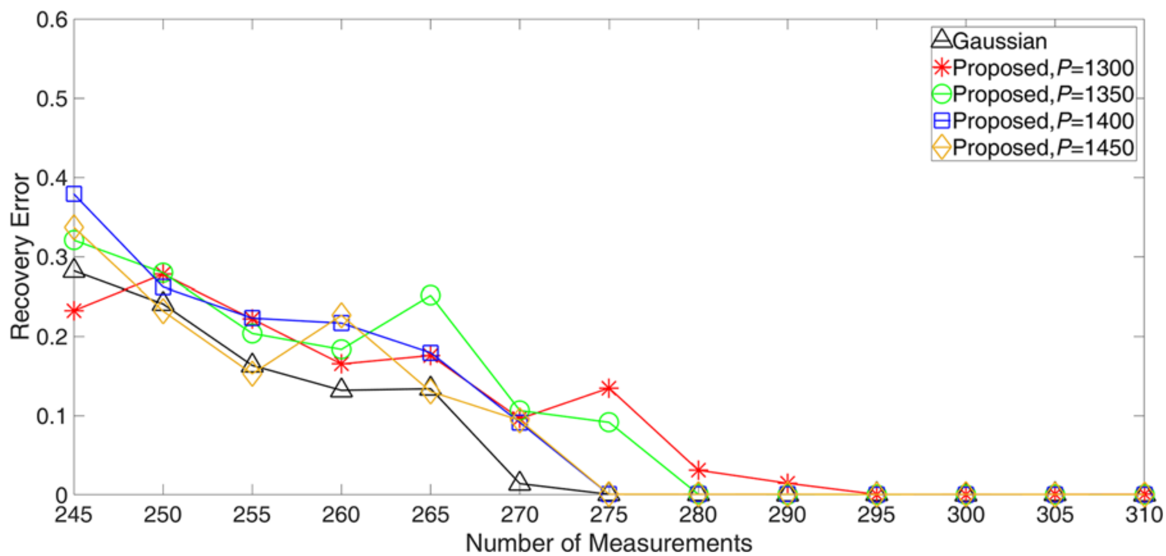

Figure 3. Relationship between the recovery error and the number of measurements in the case of DWT basis.

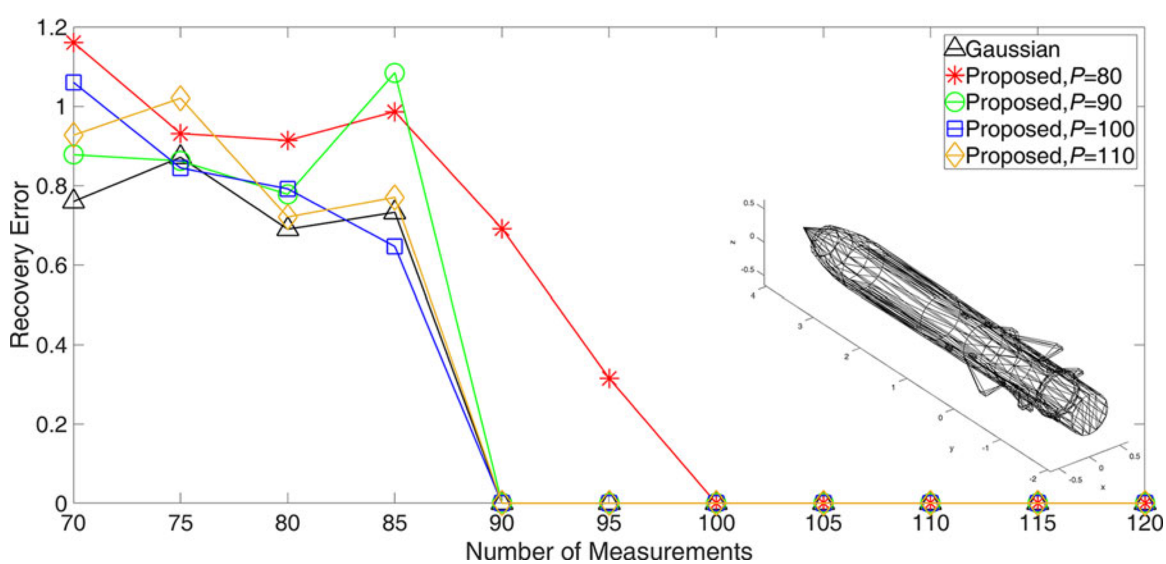

Figure 4. Relationship between the recovery error and the number of measurements in the case of the second kind of Chebyshev basis.

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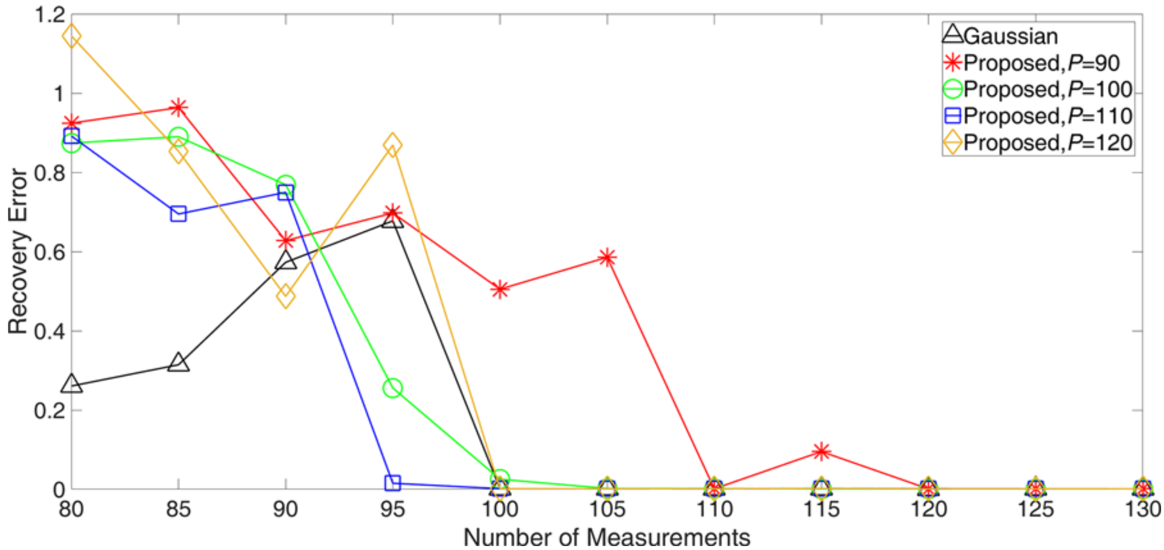

Figure 5. Relationship between the recovery error and the number of measurements in the case of Legendre basis.

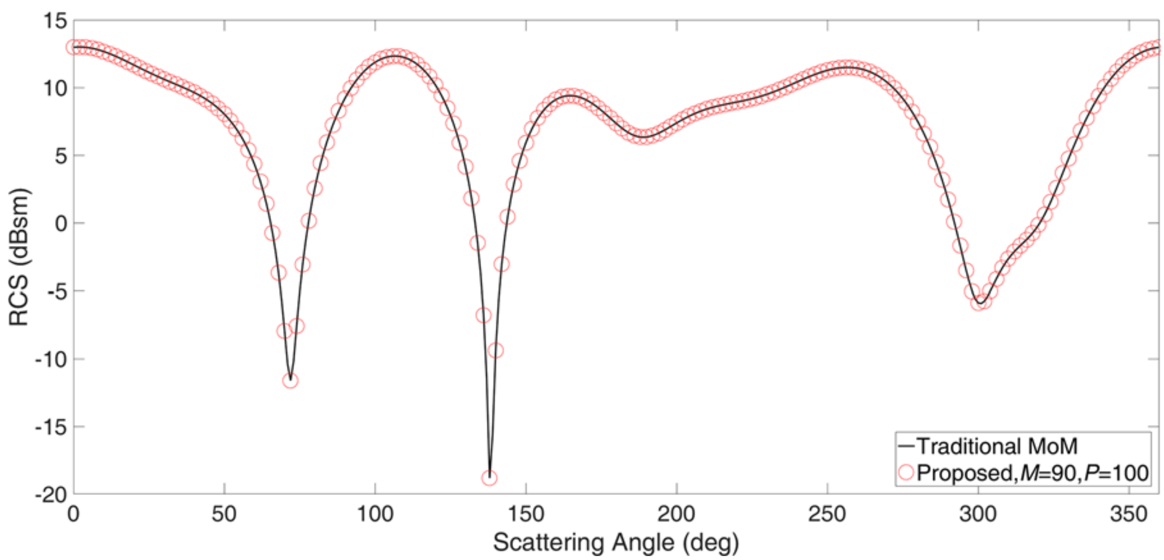

Figure 6. Comparison of RCS for the missile model illuminated at a random incident angle.

Table 2. Computing time of recovery for missile model (unit: s)

Measurement matrix	2nd Chebyshev ($M = 90$)	Legendre ($M = 100$)
Gaussian	120.8	181.2
Proposed	38.7 ($P = 90$)	59.9 ($P = 110$)

using recovery algorithm can be significantly reduced under the condition that the computational cost for measurement remains unchanged.

Acknowledgements. This work was supported by the Anhui Provincial Natural Science Foundation of China under Grant 2208085QF183, the National Natural Science Foundation of China under Grant 61701163, the Universities Natural Science Foundation of Anhui Province of China under Grant KJ2021A0905 and Grant KJ2020A0102, the Anhui Science and Technology Project under Grant 202104a05020004, the University Synergy Innovation Program of Anhui Province of China under Grant GXXT-2021-090.

Competing interests. The authors report no conflict of interest.

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