

# Peculiarities of surface plasmons in quantum plasmas

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(Received 13 December 2012; accepted 3 January 2013; first published online 22 February 2013)

**Abstract.** Surface plasmons (SP) in a semi-bounded quantum plasma with degenerate electrons (e.g. a metal) are considered, and some interesting consequences of electron Pauli blocking for the SP dispersion and temporal attenuation are discussed. In particular, it is demonstrated that a semi-bounded degenerate plasma with a sharp boundary supports *two* types of SP with distinct frequencies and qualitatively different temporal attenuation, in contrast to a non-degenerate hot plasma that only supports one type of SP.

## 1. Introduction

Surface plasmons (SP) are a type of collective oscillations that are supported by bounded media, and propagate along an interface of two media with different signs of the real part of dielectric response function. Their qualitative difference from the volume plasmons (which can propagate in both unbounded and bounded media) is that their field is localized near the interface along which they propagate. They also have spectral and attenuation properties different from volume plasmons (Alexandrov et al. 1984).

Surface plasmons have been studied extensively since their theoretical prediction (Ritchie 1957) and experimental detection (Powell and Swan 1959a,b; Kretschmann and Raether 1968; Otto 1968) in the 1950s and 1960s. There has been a significant advance in theoretical and experimental investigations of surface plasmons and their applications in various bounded plasma structures, both in the field of plasma science (see Vladimirov et al. 1994 and references therein) and in the fields of condensed matter and surface science (see e.g. a review by Pitarke et al. 2007). Currently, there is a renewed interest in surface plasmons due to their ability to concentrate light in sub-wavelength structures, enabling the creation of surface plasmon-based circuits that can couple photonics and electronics at nanoscale. This offers a route to faster and smaller devices, and opens up possibilities to new technologies employing surface plasmons (Brongersma and Shalaev 2010). For example, one of the recent interesting advances in the new area of quantum nanoplasmonics is the development of the concept of spaser (a surface plasmon 'laser'; Bergman and Stockman 2003), followed by its further development into a lasing spaser (Zheludev et al. 2008), and by an experimental demonstration of a

spaser-based nanolaser (Garcia-Vidal and Moreno 2009; Noginov et al. 2009).

These developments require a solid understanding of SP properties in various bounded metallic and semi-conductor structures. The properties of surface plasmons in bounded structures are defined, among other things, by the dielectric properties of the medium that sustains them. The latter are often (e.g. in metals, for which the electrons are strongly degenerate) significantly affected by the quantum nature of the charge carriers and their interaction in the medium. This can affect the properties of SP in a non-trivial way, via modification of analytic properties of the medium response. In particular, quantum effects (due to Pauli blocking and overlapping wave functions of free charge carriers in the medium; Shukla and Eliasson 2011), when significant, can modify the dispersion, damping (Tyshetskiy et al. 2012b) and spatial attenuation of SP (Vladimirov 1994) supported by a bounded medium.

Recently, the properties of surface plasmons in a semi-bounded degenerate plasma have been analyzed using quantum hydrodynamical approach (Lazar et al. 2007) and a more rigorous kinetic approach (Tyshetskiy et al. 2012b). In particular, the effects of quantum recoil and quantum degeneracy of plasma electrons on SP properties have been analyzed. In this paper, we show another consequence of quantum degeneracy of electrons on SP properties, exemplified by a simple case of SP in a semi-bounded collisionless plasma with degenerate electrons. In particular, we show that such a system supports *two* types of SP, with different frequencies and qualitatively different temporal attenuation, in contrast to the case of a non-degenerate semi-bounded plasma that only supports one type of SP (Guernsey 1969).

## 2. Method

### 2.1. Model and assumptions

We consider a semi-bounded, non-relativistic collisionless plasma with degenerate mobile electrons ( $T_e \ll \epsilon_F$ , where  $T_e$  is the electron temperature in energy units,  $\epsilon_F = \hbar^2(3\pi^2 n_e)^{2/3}/2m_e$  is the electron Fermi energy), and immobile ions; the equilibrium number densities of electrons and ions are equal,  $n_{0e} = n_{0i} = n_0$  (quasi-neutrality). The plasma is assumed to be confined to a region  $x < 0$ , with mirror reflection of plasma particles at the boundary  $x = 0$  separating the plasma from a vacuum at  $x > 0$ .

We look at SPs in the non-retarded limit, when their phase velocity is small compared with the speed of light. In this limit, the SP field is purely electrostatic; hence, we can restrain ourselves to considering only electrostatic oscillations in the considered system. Following the discussion of Tyshetskiy et al. (2012b), we adopt here the quasi-classical kinetic description of plasma electrons in terms of the 1-particle distribution function  $f(\mathbf{r}, \mathbf{v}, t) = f(x, \mathbf{r}_\parallel, v_x, \mathbf{v}_\parallel, t)$  (Vladimirov and Tyshetskiy 2011) (where  $\mathbf{r}_\parallel$  and  $\mathbf{v}_\parallel$  are, respectively, the components of  $\mathbf{r}$  and  $\mathbf{v}$  parallel to the boundary, and  $x$  and  $v_x$  are the components of  $\mathbf{r}$  and  $\mathbf{v}$  perpendicular to the boundary), whose evolution is described by the Vlasov equation coupled with the Poisson's equation for the electrostatic potential.

### 2.2. Initial-value problem

We introduce a small initial perturbation  $f_p(x, \mathbf{r}_\parallel, v_x, \mathbf{v}_\parallel, t = 0)$  to the equilibrium electron distribution function  $f_0(v)$ ,  $|f_p(x, \mathbf{r}_\parallel, v_x, \mathbf{v}_\parallel, t = 0)| \ll f_0(v)$ , and use the kinetic equation to study the resulting evolution of the system's charge density  $\rho(x, \mathbf{r}_\parallel, t) = e[\int f(x, \mathbf{r}_\parallel, \mathbf{v}, t) d^3\mathbf{v} - n_0]$ , and hence of the electrostatic potential  $\phi(x, \mathbf{r}_\parallel, t)$  defined by the Poisson's equation. Introducing the dimensionless variables  $\Omega = \omega/\omega_p$ ,  $\mathbf{K} = \mathbf{k}\lambda_F$ ,  $\mathbf{V} = \mathbf{v}/v_F$ ,  $X = x/\lambda_F$ ,  $\mathbf{R}_\parallel = \mathbf{r}_\parallel/\lambda_F$ ,  $\lambda_F = v_F/\sqrt{3}\omega_p$ ,  $v_F = (2\epsilon_F/m_e)^{1/2}$ ,  $\omega_p = (4\pi e^2 n_0/m_e)^{1/2}$ , and following Guernsey (1969), the solution of the formulated initial-value problem for  $\rho(X, \mathbf{R}_\parallel, T)$  with the specified boundary condition is

$$\rho(X, \mathbf{R}_\parallel, T) = en_0 \tilde{\rho}(X, \mathbf{R}_\parallel, T), \tag{2.1}$$

where

$$\tilde{\rho}(X, \mathbf{R}_\parallel, T) = \frac{1}{(2\pi)^3} \int_{-\infty}^{+\infty} dK_x e^{iK_x X} \int d^2\mathbf{K}_\parallel e^{i\mathbf{K}_\parallel \cdot \mathbf{R}_\parallel} \tilde{\rho}_\mathbf{k}(T), \tag{2.2}$$

$$\tilde{\rho}_\mathbf{k}(T) = \frac{1}{2\pi} \int_{i\sigma-\infty}^{i\sigma+\infty} \tilde{\rho}(\Omega, \mathbf{K}) e^{-i\Omega T} d\Omega, \text{ with } \sigma > 0. \tag{2.3}$$

The integration in (2.3) is performed in the complex  $\Omega$  plane along the horizontal contour that lies in the upper half-plane  $\text{Im}(\Omega) = \sigma > 0$  above all singularities of the function  $\tilde{\rho}(\Omega, \mathbf{K})$ . The function  $\tilde{\rho}(\Omega, \mathbf{K})$ , defined as the

Laplace transform of  $\tilde{\rho}_\mathbf{k}(T)$ ,

$$\tilde{\rho}(\Omega, \mathbf{K}) = \int_0^\infty \tilde{\rho}_\mathbf{k}(T) e^{i\Omega T} dT, \tag{2.4}$$

is found to be

$$\begin{aligned} \tilde{\rho}(\Omega, \mathbf{K}) = & i \frac{I(\Omega, \mathbf{K})}{\epsilon(\Omega, K)} + \frac{iK_\parallel}{2\pi\zeta(\Omega, K_\parallel)} \left[ 1 - \frac{1}{\epsilon(\Omega, K)} \right] \\ & \times \int_{-\infty}^{+\infty} \frac{I(\Omega, \mathbf{K}')}{K'^2 \epsilon(\Omega, K')} dK'_x, \end{aligned} \tag{2.5}$$

where the functions  $I(\Omega, \mathbf{K})$  and  $I(\Omega, \mathbf{K}')$  are fully defined by the initial perturbation (Guernsey 1969),  $K_\parallel = |\mathbf{K}_\parallel|$ ,  $K = |\mathbf{K}|$ ,  $\mathbf{K} = (K_x, \mathbf{K}_\parallel)$ ,  $K' = |\mathbf{K}'|$ , and  $\mathbf{K}' = (K'_x, \mathbf{K}_\parallel)$ . The functions  $\epsilon(\Omega, K)$  and  $\zeta(\Omega, K_\parallel)$  in (2.5) are defined (for  $\text{Im}(\Omega) > 0$ ) as follows:

$$\epsilon(\Omega, K) = 1 - \frac{1}{\sqrt{3}K^2} \int \frac{\mathbf{K} \cdot \partial \tilde{f}_0(\mathbf{V})/\partial \mathbf{V}}{\Omega - \sqrt{3}\mathbf{K} \cdot \mathbf{V}} d^3\mathbf{V}, \tag{2.6}$$

$$\zeta(\Omega, K_\parallel) = \frac{1}{2} + \frac{K_\parallel}{2\pi} \int_{-\infty}^{+\infty} \frac{dK_x}{K^2 \epsilon(\Omega, K)}, \tag{2.7}$$

with

$$\tilde{f}_0(\mathbf{V}) = \frac{v_F^3}{n_0} f_0(\mathbf{V}) = \frac{v_F^3}{n_0} f_0(v)|_{\mathbf{v}=v_F\mathbf{V}}.$$

For fully degenerate electrons, the function  $\epsilon(\Omega, K)$  becomes (for  $\text{Im}(\Omega) > 0$ ) (Gol'dman 1947; Alexandrov et al. 1984)

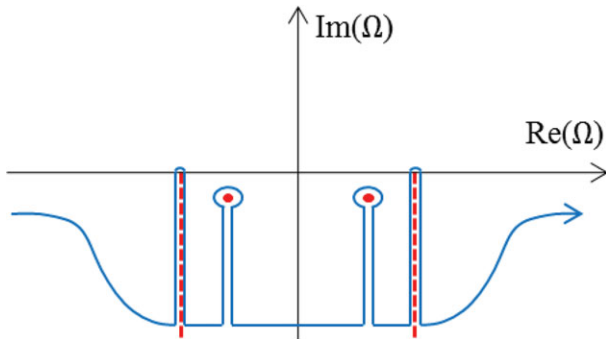
$$\begin{aligned} \epsilon(\Omega, K) = & 1 + \frac{1}{K^2} \left[ 1 - \frac{\Omega}{2\sqrt{3}K} \ln \left( \frac{\Omega + \sqrt{3}K}{\Omega - \sqrt{3}K} \right) \right], \\ & \text{Im}(\Omega) > 0, \end{aligned} \tag{2.8}$$

where  $\ln(z)$  is the principal branch of the complex natural logarithm function.

Note that the solution (2.5) differs from the corresponding solution of the transformed Vlasov–Poisson system for infinite (unbounded) uniform plasma only in the second term involving  $\zeta(\Omega, K_\parallel)$ ; indeed, this term appears due to the boundary at  $x = 0$ .

The definition (2.4) of the function  $\tilde{\rho}(\Omega, \mathbf{K})$  of complex  $\Omega$  has a sense (i.e. the integral in (2.4) converges) only for  $\text{Im}(\Omega) > 0$ . Yet the long-time evolution of  $\tilde{\rho}_\mathbf{k}(T)$  is obtained from (2.3) by displacing the contour of integration in the complex  $\Omega$  plane from the upper half-plane  $\text{Im}(\Omega) > 0$  into the lower half-plane  $\text{Im}(\Omega) \leq 0$  (Landau 1946). This requires the definition of  $\tilde{\rho}(\Omega, \mathbf{K})$  to be extended to the lower half-plane,  $\text{Im}(\Omega) \leq 0$ , by analytic continuation of (2.5) from  $\text{Im}(\Omega) > 0$  to  $\text{Im}(\Omega) \leq 0$ . Hence, the functions  $I(\Omega, \mathbf{K})$ ,  $\epsilon(\Omega, K)$  and  $\zeta(\Omega, K_\parallel)$  that make up the function  $\tilde{\rho}(\Omega, \mathbf{K})$  must also be analytically continued into the lower half-plane of complex  $\Omega$ , thus extending their definition to the whole complex  $\Omega$  plane. With thus continued functions, the contributions to the inverse Laplace transform (2.3) are of three sources (Guernsey 1969):

- (a) Contributions from the singularities of  $I(\Omega, \mathbf{K})$  in the lower half of the complex  $\Omega$  plane; with some



**Figure 1.** (Colour online) A sketch of the deformed integration path in (2.3) in the complex  $\Omega$  plane, with contributions of poles (solid circles) and branch cuts (dashed lines) of  $1/\zeta(\Omega, K_{\parallel})$ . The singularities due to  $1/\varepsilon(\Omega, K)$  are not shown, but they also contribute to (2.3), yielding volume plasmons.

simplifying assumptions about the initial perturbation (Guernsey 1969), these contributions are damped in a few plasma periods and can be ignored.

- (b) Contribution of singularities of  $1/\varepsilon(\Omega, K)$  in the lower half of the complex  $\Omega$  plane, of two types: (i) residues at the poles of  $1/\varepsilon(\Omega, K)$ , which give the volume plasma oscillations (Guernsey 1969); and (ii) integrals along branch cuts (if any) of  $1/\varepsilon(\Omega, K)$  in the lower half-plane of complex  $\Omega$ , which can lead to non-exponential attenuation of the volume plasma oscillations (Hudson 1962; Krivitskii and Vladimirov 1991).
- (c) Contribution into (2.3) of singularities of  $1/\zeta(\Omega, K_{\parallel})$  in the lower half of the complex  $\Omega$  plane, of two types: (i) residues at the poles of  $1/\zeta(\Omega, K_{\parallel})$ , corresponding to the surface wave solutions of the initial-value problem in the considered system (Guernsey 1969; Tyshetskiy et al. 2012b); and (ii) integrals along branch cuts (if any) of  $1/\zeta(\Omega, K_{\parallel})$  in the lower half-plane of complex  $\Omega$ .

Below, we consider the latter contributions from poles and branch cuts of  $1/\zeta(\Omega, K_{\parallel})$  in the lower half-plane of complex  $\Omega$ , as illustrated in Fig. 1, and show that they yield two types of electrostatic surface oscillations with different frequencies and qualitatively different temporal attenuation.

### 3. Two types of surface oscillations

#### 3.1. Contribution of poles of $1/\zeta(\Omega, K_{\parallel})$

The contribution of poles of  $1/\zeta(\Omega, K_{\parallel})$  into (2.3) leads to exponentially damped surface oscillations (Tyshetskiy et al. 2012b)

$$\tilde{\rho}_{\mathbf{K}}^{(\text{poles})}(T) \propto e^{-|\Gamma_s|T} \cos(\Omega_s T), \quad (3.1)$$

with frequency  $\Omega_s = \omega_s/\omega_p$  and damping rate  $\Gamma_s = \gamma_s/\omega_p$  obtained from the dispersion equation  $\zeta(\Omega, K_{\parallel}) = 0$ . The frequency asymptotes are

$$\Omega_s \approx \frac{1}{\sqrt{2}}(1 + 0.95K_{\parallel}), \quad \text{for } K_{\parallel} \ll 1, \quad (3.2)$$

$$\Omega_s \approx \sqrt{3}K_{\parallel}(1 + 2 \exp[-2 - 4K_{\parallel}^2]),$$

$$\text{for } K_{\parallel} \gg 1 \text{ (zero sound)}. \quad (3.3)$$

We see that at large  $K_{\parallel}$ , the frequency of these surface oscillations asymptotically approaches the frequency of zero sound – an intrinsically quantum mode in a Fermi gas (Lifshitz and Pitaevskii 1981).

The absolute value of the damping rate is a non-monotonic function of  $K_{\parallel}$ . At small  $K_{\parallel}$ , it increases linearly with  $K_{\parallel}$ ,

$$|\Gamma_s(K_{\parallel})| \approx 2.1\sqrt{3} \times 10^{-2}K_{\parallel}, \quad (3.4)$$

reaches maximum  $|\Gamma_s| \approx 6.2 \times 10^{-3}$  at  $K_{\parallel} \approx 0.4$ , and then quickly decreases at  $K_{\parallel} > 0.4$ . Since the maximum growth rate is small, the surface oscillations due to the poles of  $1/\zeta(\Omega, K_{\parallel})$  are weakly damped at all wavelengths (Tyshetskiy et al. 2012b).

#### 3.2. Contribution of branch cuts of $1/\zeta(\Omega, K_{\parallel})$

For degenerate plasma, the analytically continued function  $\zeta(\Omega, K_{\parallel})$  has two branching points on the real axis of the complex  $\Omega$  plane at  $\Omega = \pm\Omega_v(K_{\parallel})$ , where  $\Omega_v(K_{\parallel}) \in \mathbb{R}$  is the solution of equation

$$\varepsilon(\Omega, K_{\parallel}) = \varepsilon(\Omega, K)|_{K_x=0} = 0, \quad (3.5)$$

with the corresponding branch cuts going down into the  $\text{Im}(\Omega) < 0$  part of the complex  $\Omega$  plane, as schematically shown in Fig. 1. Let us consider the contribution of the integration along these branch cuts into the inverse Laplace transform (2.3). The branching points lie above the poles  $\Omega_s - i|\Gamma_s|$  of  $1/\zeta$  (since the latter lie below the real axis of the  $\Omega$  plane); therefore, we can expect the contribution of the integration along the branch cuts into (2.3) to be at least as important as the contribution of the poles, if not to exceed it.

At large times  $T \gg 1$ , the main contribution into the integrals along the branch cuts comes from the small vicinity of the branching points, so it suffices to approximate the second term of (2.5) near the branching points in the lower semi-plane of complex  $\Omega$ . This can be done in two steps:

- (a) approximate  $\tilde{\rho}(\Omega, \mathbf{K})$  defined by (2.5) in the upper vicinities of the branching points, in terms of elementary functions; the approximate function should have the same branching points as the original one;
- (b) analytically continue these approximations into the lower vicinities of the branching points, choosing the branch cuts to go down.

Then, we can perform the integration of thus obtained approximations along the branch cuts in the vicinity of the branching points. This procedure of integration along the branch cuts (the details of which will be published elsewhere) yields an intrinsically quantum type of surface oscillations, with

$$\tilde{\rho}_{\mathbf{K}}^{(\text{cuts})}(T) \propto \frac{\cos(\Omega_v T)}{T^{3/2}} + O(T^{-5/2}), \quad \text{for } K_x \neq 0, \quad (3.6)$$

$$\tilde{\rho}_{\mathbf{K}}^{(\text{cuts})}(T) \propto \frac{\cos(\Omega_v T)}{T^{1/2}} + O(T^{-3/2}), \quad \text{for } K_x \rightarrow 0, \quad (3.7)$$

$$\tilde{\rho}_{\mathbf{K}}^{(\text{cuts})}(T) \propto \frac{\cos(\Omega_v T)}{T} + O(T^{-3/2}),$$

$$\text{for } K_x \rightarrow \sqrt{\Omega_v^2/3 - K_{\parallel}^2}. \quad (3.8)$$

The frequency of these oscillations is equal to the frequency of volume plasma waves with the same wavelength,  $K = K_{\parallel}$ , and thus exceeds the frequency  $\Omega_s$  of the surface oscillations (3.1) due to the poles of  $1/\zeta$ . The temporal attenuation of  $\tilde{\rho}_{\mathbf{K}}^{(\text{cuts})}(T)$  (power law) is qualitatively different from that of  $\tilde{\rho}_{\mathbf{K}}^{(\text{poles})}(T)$  (exponential).

#### 4. Discussion

We thus see that our system supports two distinct types of surface oscillations, with different frequencies and temporal attenuation:

- (a) exponentially damped surface oscillations (3.1) with frequency  $\Omega_s(K_{\parallel})$ , due to the poles of  $\tilde{\rho}(\Omega, \mathbf{K})$  (Tyshetskiy et al. 2012b);
- (b) power-law attenuated surface oscillations (3.6)–(3.8) with frequency  $\Omega_v(K_{\parallel}) > \Omega_s(K_{\parallel})$ , due to the branch cuts of  $\tilde{\rho}(\Omega, \mathbf{K})$ . Since the power-law attenuation is slower than the exponential attenuation, these oscillations should become dominant at large times, and should become experimentally observable.

It is interesting to note that, as seen from (3.6)–(3.8), different  $K_x$  components in the wave packet making up the surface oscillation of this type are attenuated at a different rate. Since  $T^{-1/2}$  decays slower than  $T^{-1}$  or  $T^{-3/2}$ , the small- $K_x$  part of the wave packet becomes dominant over the large- $K_x$  part at large times, which corresponds to the penetration of the perturbation away from the surface and deeper into plasma.

The present analysis relies on several assumptions discussed in detail by Tyshetskiy et al. (2012b), of which perhaps the most critical ones are the assumptions of collisionless plasma and of the sharp perfectly reflecting boundary confining the plasma. While relaxing the former assumption does not change the results qualitatively (Tyshetskiy et al. 2012a), relaxing

the latter assumption may somewhat change the results. First, the smooth boundary leads to a new resonant damping of surface oscillations, significantly increasing the exponential damping rate  $|\Gamma_s|$  in (3.1) (Marklund et al. 2008). Second, allowing for boundary smoothness (with a simultaneous account for the quantum tunneling, as they both have the same spatial scales) should change the analytic properties of  $\tilde{\rho}(\Omega, \mathbf{K})$  in the lower semi-plane  $\text{Im}(\Omega) < 0$  of the complex  $\Omega$  plane, and thus may change its branch cuts and their contribution to (2.3).

#### References

- Alexandrov, A. F., Bogdankevich, L. S. and Rukhadze, A. A. 1984 *Principles of Plasma Electrodynamics*. Berlin: Springer.
- Bergman, D. J. and Stockman, M. I. 2003 *Phys. Rev. Lett.* **90**, 027402.
- Brongersma, M. L. and Shalaev, V. M. 2010 *Science* **328**, 440.
- Garcia-Vidal, F. J. and Moreno, E. 2009 *Nature* **461**, 604.
- Goldman, I. I. 1947 *Zh. Eksp. Teor. Fiz.* **17**, 681.
- Guernsey, R. L. 1969 *Phys. Fluids* **12**, 1852.
- Hudson, J. F. P. 1962 *Math. Proc. Camb. Philos. Soc.* **58**, 119.
- Kretschmann, E. and Raether, H. 1968 *Z. Naturf. A* **23**, 2135.
- Krivitskii, V. S. and Vladimirov, S. V. 1991 *Zh. Eksp. Teor. Fiz.* **100**, 1483.
- Landau, L. D. 1946 *J. Phys. (USSR)* **10**, 25.
- Lazar, M., Shukla, P. K. and Smolyakov, A. 2007 *Phys. Plasmas* **14**, 124501.
- Lifshitz, E. M. and Pitaevskii, L. P. 1981 *Physical Kinetics*. Oxford: Pergamon Press.
- Marklund, M., Brodin, G., Stenflo, L. and Liu, C. S. 2008 *Europhys. Lett.* **84**, 17006.
- Noginov, M. A. et al. 2009 *Nature* **460**, 1110.
- Otto, A. 1968 *Z. Phys.* **216**, 398.
- Pitarke, J. M., Silkin, V. M., Chulkov, E. V. and Echenique, P. M. 2007 *Rep. Prog. Phys.* **70**, 1.
- Powell, C. J. and Swan, J. B. 1959a *Phys. Rev.* **115**, 869.
- Powell, C. J. and Swan, J. B. 1959b *Phys. Rev.* **116**, 81.
- Ritchie, R. H. 1957 *Phys. Rev.* **106**, 874.
- Shukla, P. K. and Eliasson, B. 2011 *Rev. Mod. Phys.* **83**, 885.
- Tyshetskiy, Y., Vladimirov, S. V. and Kompaneets, R. 2012a *Phys. Plasmas* **19**, 112107.
- Tyshetskiy, Y., Williamson, D. J., Kompaneets, R. and Vladimirov, S. V. 2012b *Phys. Plasmas* **19**, 032102.
- Vladimirov, S. V. 1994 *Phys. Scr.* **49**, 625.
- Vladimirov, S. V. and Tyshetskiy, Yu. O. 2011 *Phys. Usp.* **54**, 1313.
- Vladimirov, S. V., Yu, M. Y. and Tsytovich, V. N. 1994 *Phys. Rep.* **241**, 1.
- Zheludev, N. I., Prosvirnin, S. L., Parasimakis, N. and Fedotov, V. A. 2008 *Nature Photon.* **2**, 351.