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COMPLIANCE AND COMMAND II, IMPERATIVES AND DEONTICS

KIT FINE
Philosophy Department, New York University

Abstract. I extend the previously given truth-maker semantics and logic for imperatives to deontic statements.

In this part of the article, I am interested in providing a semantics and logic for deontic sentences and working out their connection with the previous semantics and logic for imperatives.

The standard approach to deontic logic is in terms of possible worlds. It is supposed that with each world is associated a set of ideal worlds. A statement O(A) to the effect that A is obligatory is then taken to be true if A is true in all ideal worlds and a statement P(A) to the effect that A is permissible is taken to be true if A is true in some ideal world.

I believe the possible worlds approach to be fundamentally misguided. The main problem is that obligation and permission relate most directly to action. At the end of the day, what we want to know is what it is obligatory or permissible *to do*. But the intensional treatment of the embedded clause A prevents the deontic statements from providing, in this way, a guide to action. For A merely represents the possible outcomes of some action or actions. But as I have argued in Fine (2014), given a set of outcomes, there is, in general, no satisfactory way to determine the actions from which they arose.

The present approach, by contrast, follows the lead of those such as Segerberg (1990) and Barker (2010) in being action- rather than outcome-oriented. The deontic operators are taken to apply directly to expressions that indicate a range of possible actions rather than a range of possible outcomes or worlds. Indeed, under the semantics for imperatives in part I, an imperative will indicate a range of actions, those in compliance with the imperative; and so we may take the deontic operators to have direct application to imperatives. To say when a deontic sentence is true we must therefore distinguish, not a preferred set of worlds but a preferred set of actions, something I call a 'code of conduct'; and the conditions under which a deontic sentence is true relative to a code of conduct will then be somewhat different from the conditions under which such a sentence is true relative to a sphere of worlds.

The plan of the article is as follows. I begin by making some distinctions and stipulations which will be useful in the rest of the article (§1); I introduce and explain the key notion of a code of conduct, relative to which deontic formulas are to be interpreted (§2); I give the clauses for when a deontic formula is true or false relative to a code of conduct (§3) and spell out some of the consequences of these clauses, especially in regard to the contrast with the standard possible worlds semantics for deontic logic (§4); I consider various ways of reformulating the criterion of validity for deontic formulas and point, in particular, to a very close connection between this criterion and the criterion of validity for imperative inference proposed in part I (§5); I consider some of the characteristic inferences that are or fail to

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be valid (§6) and outline a system of deontic logic within the truthmaker approach (§7); I show how one might deal with the problem of deontic updating within the truthmaker framework (§8); and I conclude with a brief formal appendix. Of perhaps special note is the account of free choice permission and obligation (§6), the treatment of moral dilemmas (also in §6), the semantically marked distinction between implicit and explicit permission (§4), the intimate connection forged between deontic logic and the logic of imperatives (§5), and the development of a logic for explicit and free choice permission (§7).

I assume the reader is familiar with the basic material from part I, including the truthmaker semantics for imperatives and the definition of validity for imperative inference; and it would also be helpful to have some knowledge of the standard possible worlds semantics for deontic logic.

§1. Preliminaries. I begin with some remarks on syntax. The reader will recall from part I that imperative formulas are constructed from imperative atoms $\alpha_1, \alpha_2, \ldots$ using the usual array of connectives—$\vee$, $\wedge$ and $\neg$—and also the verum constant $\top$. We will now take the deontic operators O and P, for obligation and permission, to have application to imperatives. Thus when X is an imperative, O(X) and P(X) will be deontic sentences. Thus if X is the imperative ‘stop’, then O(X) might be taken to be the sentence ‘It is obligatory to stop’ and P(X) to be the sentence ‘it is permitted to stop’.

This account of the logical grammar of the deontic formulas O(X) and P(X) is not meant to have any implications for the correct grammatical analysis of the corresponding sentences of natural language. One might, of course, see the sentences ‘it is obligatory to go’ (or ‘you ought to go’) and ‘it is permitted to go’ (or ‘you may go’) as containing the imperative ‘go’. But there is no need to regard them in this way and many other grammatical analyses of these sentences are possible. The present account of the logical grammar merely allows us to relate imperative to deontic sentences in an especially convenient way; and it would be straightforward to modify the account to accommodate other views on how they are related.

However, one aspect of our account is more significant. For the deontic operators cannot sensibly be taken to apply to arbitrary indicative sentences, as in standard deontic logic, or even to clauses that are not indicative of actions. Only actions, on the present view, can properly be said to obligatory or permitted; and if one wants to apply the deontic operators to clauses that are not directly indicative of actions, then it must be by way of some transformation of those clauses into ones that are. Thus to make sense of such sentences as ‘You may own a dog’ or ‘You should know who packed your luggage’ (which relate to states rather than to actions), we should take them to mean something like ‘You may get a dog if you do not already own one’ or ‘You should find out who packed your luggage if you do not already know’ (or, more generally, you may (should) put oneself in the state if you are not already in that state).[1]

This has the consequence that, in contrast to standard systems of deontic logic, iterative statements of obligation and permission, such as O(O(X)) or P(O(X)) or P(P(X)), will not be well-formed, since the operators O and P can only properly be taken to have application to imperative sentences. However, we may still form truth-function compounds of deontic sentences, as in $O(X) \vee O(Y)$ or $O(X) \supset P(X)$, just as we may form truth-functional compounds of imperative sentences.

[1] Thanks to one of the referees for raising this issue.

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There are a number of (more or less familiar) ways to interpret deontic sentences, the distinction between which will be important in what follows.

There is, in the first place, the distinction between the performative and descriptive interpretation of these locutions. Under the performative interpretation, 'you ought to go' is used to *place* one under an obligation and 'you may go' is used to *grant* a permission whereas, under the descriptive interpretation, 'you ought to go' is merely used to state that one is under an obligation and 'you may go' is used merely to state that one is so permitted (the descriptive use goes more naturally with the past tense, as in 'you were permitted (obliged) to go'). For the most part, I shall be concerned with the descriptive use of these sentences but, in the section on updates, I shall also be interested in their performative use, in which they are more akin to imperatives. I shall also assume, pace the emotivists and some expressivists, that there is a straightforward sense in which the deontic sentences, under their descriptive use, are capable of being true or false.

There is, in the second place, a familiar distinction between a strict and weak sense of permission (as in von Wright (1963), p. 90). Roughly speaking, an action is permitted in the weak sense if it is not forbidden, whereas something more is required for an action to be permitted in the strict sense. It must have been *singled out* as being permitted—by being expressly permitted or perhaps in some other, less direct, way. The semantics I provide is most naturally taken to be for the strict sense of permission, although it can also be applied or extended to the weak sense of permission.

I shall assume that if a compound action, such as turning on the gas and lighting the stove, is permitted then so are the component actions, of turning on the gas and of lighting the stove. There is perhaps a sense in which one is not permitted to turn on the gas, since one is not permitted to turn on the gas without doing anything else, which is to say that one is obliged to do something else, viz. light the stove, if one turns on the gas. But we shall find it convenient to use permission in the less restrictive sense, so that an action can be permitted even though its performance requires one to do something else.

Finally, there is also a distinction, in a somewhat different sense, between a strict and weak sense of obligation. Suppose one is obliged to shut the door. Then does it follow that one is obliged to shut the door or burn the house down? There is a sense of 'obliged' in which it does follow. This is a sense in which it is consistent to say that one is obliged to shut the door or burn the house down but not permitted to burn the house down. However, there would also appear to be a sense in which it does not follow. This is a sense in which one's being obliged to shut the door or burn the house down entails that one is permitted to burn the house down. I call the first sense of obligation *limited-choice* and the second *free-choice*. In what follows I shall be interested in giving a semantics for both sense of obligation.

§2. Codes of conduct. We shall take a deontic statement to be true or false relative to a code of conduct. For our purposes, we may take a code of conduct to be a prescription, i.e., a set of actions. But which actions?

There are two relevant conditions. The first is that each action should be *permissible* according to the code (in either the weak or in the strict sense, depending upon which notion of permission is in question). The second is that each action should be *adequate*, i.e., the performance of the action should be sufficient to discharge all of one's obligations in regard to the code. The first condition (Permissibility) means that the action should not contain too much, it should not encroach on what is impermissible, and the second (Adequacy) means that the action should not contain too little, it should not fall short of

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what is obligatory. Thus an action meeting both conditions will conform to what one might call the 'Goldilocks' principle', it will strike a middle course between containing enough and not containing too much (although perhaps Aristotle, not Goldilocks, should be given credit for the principle).

An action meeting both conditions will be said to be *ideal*. An ideal action is the analogue of a deontic alternative in the possible worlds semantics for deontic logic; for a deontic alternative is an ideal world, one in which all obligations are discharged. However, for us, there is no requirement that an ideal action should be a complete action, let along a complete state of the world, and there is not even any requirement that it should be consistent. Moreover, for us, the two conditions (of Permissibility and Adequacy) are independent whereas, in the standard modal setting, a world will be permissible just in case it is a world in which all one's obligations are discharged.

We might say that a code *sanctions* those actions which are ideal with respect to the code; and a code of conduct may be identified with the set of the actions that it sanctions. The actions sanctioned by a code will often be an action-stream, composed of many individual actions, and, for this reason, we might talk instead of a 'course of action'. However, the space of actions is closed under fusion; and so, strictly speaking, a course of action is just another action.

A code of conduct is the analogue of the 'sphere' of deontic alternatives within the standard possible worlds semantics for deontic logic. Indeed, with each code of conduct may be associated a deontic sphere, consisting of all those worlds which are compatible with some course of action sanctioned by the code. If we think of each course of action sanctioned by the code in terms of its intensional content, i.e., as the set of worlds containing the action, then the associated deontic sphere will be the union or 'disjunction' of all these contents.

However, the correspondence is far from being one-one. Thus one code might consist of the action of my eating an apple and another code of the action of my eating an apple and the compound action of my eating an apple and a pear. These are distinct codes, since the second sanctions the compound action of my eating an apple and a pear while the first does not, but the corresponding deontic spheres are the same—all of the worlds in which I eat an apple are worlds in which I eat or an apple or eat an apple and a pear. Thus codes of conduct are much more fine-grained than the corresponding deontic spheres and can be expected—at least, in principle—to deliver different results about what is obligatory or permissible.

An interesting question is whether we intuitively wish to distinguish between distinct codes of conduct. Consider the codes $\{b, r\}$and $\{b, r, c\}$, where b is picking a black card, r picking a red card, and c picking a card.[2] Do we wish to distinguish them? My inclination is to say that, in this case, b, r, and c cannot all properly belong to the same space of actions, since doing c is simply a matter of doing b and r and so is not an appropriately determinate action. But consider now the case in which one code is $\{\square\}$and the other is $\{r, \overline{r}\}$, for $\overline{r}$ the action of not picking a red card. As before, $\{\square\}$and $\{r, \overline{r}\}$correspond to the same set of possible worlds. However, in this case we may want to distinguish between permitting the null action and (explicitly) permitting someone to either take or not take the red card. Of course, someone who did not wish to make these distinctions could insist that the codes should be suitably enlarged. Thus a code containing b, r would also have to contain c since c is necessarily equivalent, as it were, to the disjunction of b and r.

2 I owe this question and this example to one of the referees.

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There are a number of different ‘consistency’ conditions one might wish to impose on codes of conduct:

> Nonemptiness. Each code should sanction at least one action.

If this were not so then there would be no action (even impossible or necessary) sanctioned by the code. This condition does not mean that the code must have any real substance since it is met by the ‘minimal’ code $\{\Box\}$, under which it is guaranteed that one will do what is permissible (via the performance of the null action $\Box$) and thereby discharge all of one’s obligations.

Given that a code of conduct is nonempty, we might also want to restrict its content:

> Nonanarchy. The full action is not sanctioned by any code of conduct.

If this condition were violated it would then mean that everything whatever (even the impossible) was permitted.

Two other consistency conditions, along with some of the other conditions listed below, require that we distinguish between possible and impossible actions (and hence are working within a ‘modalized’ action space):

> Consistency. Each code should sanction at least one possible course of action.

If this condition were violated then the code of conduct would either be empty or consist entirely of impossible courses of action. A stronger condition still (in the presence of Nonemptiness) is

> Complete Consistency. Each action sanctioned by a code is possible.

Barcan-Marcus (1980) is well-known for endorsing the first of these consistency constraints, though not the second; a moral code should be obeyable in some possible world though not necessarily in any possible world.

It has sometimes been thought that consistency constraints of this sort are normative rather than logical in character. This may be so. But their failure will restrict the capability of a code to serve as a guide to conduct. If a code sanctions no action, then it can provide no guidance at all; if a code sanctions only impossible actions, then it can only serve as a guide in so far as one is capable of performing some consistent part of a action that it sanctions; and if a code sanctions some impossible action, then one is no longer capable of performing all of the action that it sanctions. It is only when the conditions of Nonemptiness and Complete Consistency are both satisfied, that the code can straightforwardly serve as a guide to conduct.

Say that an action b *lies between* two others, a and c, if a is a part of b and b a part of c. Then the fact that codes of conduct conform to the Goldilocks’ principle means that

> Convexity. Any action that lies between two courses of action sanctioned by a code is also sanctioned by the code.

For suppose that both a and c are ideal, i.e., permissible and adequate. Then b is permissible since it is a part of a permissible action c; and b is adequate since it contains an adequate action a.

It might be thought to be implausible that all codes of conduct should conform to Convexity. Suppose, for example, that the code sanctions pressing button A and also pressing

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buttons A, B, and C. Then why should it also sanction pressing button A and B? But we may appeal here to the considerations from §5 of Part I. If pressing buttons A and B is not ideal, then that can only be because it is not pressing button A that is ideal, but pressing A without B, in which case the convexity condition will have no application.

Say that a (course of) action is *complete* if it is possible and every action is either incompatible with it or part of it. Thus a complete course of action is the action-theoretic counterpart of a possible world. A further closure condition one might then wish to impose is

> Completeness. Every possible action sanctioned by a code is included in complete course of action sanctioned by the code.

This condition is plausible when weak permission is in question; if the action *c* is implicitly permitted then some completion of the action must also be implicitly permitted. However, it has no plausibility for strict (explicit) permission. Suppose that the null action □ is sanctioned. Then this tells us nothing about whether shutting the door or leaving it open is strictly permitted.

Let us note, finally, that we might define a natural relation of part-whole among codes. For we may suppose that one code is a part of another if it is analytically entailed by the other, i.e., if every action sanctioned by the first is part of an action sanctioned by the second and every action sanctioned by the second contains an action sanctioned by the first. It is readily shown, given that codes are convex sets, that the relation of part-whole is a partial order and that the least upper bound, or fusion, of any codes is also a code. Thus the space of codes will form a state space and, as we shall see, the codes may themselves be regarded as truthmakers or falsemakers for deontic statements.

§3. Truthmaker semantics for deontic logic. We provide a truthmaker semantics for deontic statements. The atomic deontic statements will be of the form O(X) or P(X), for X an imperative. Recall from part I that the prescriptive content *X* of X is the set of actions in compliance with X. For the purpose of providing a semantics, we suppose given a code of conduct *C* and then go on to specify when O(X) or P(X) is true in terms of an appropriate relation between the content *X* and the code *C*. Recall the notions of subsumption and subservience from part I. Given prescriptive contents *X* and *Y*, *X* will subsume *Y* if every action in compliance with *X* contains an action in compliance with *Y* and *Y* will subserve *X* if every action in compliance with *Y* is contained in an action in compliance with *X*. I would now like to suggest the following account for when statements of obligation or permission are true:

> (i) O(X) is true iff *C* subsumes *X*;
> (ii) P(X) is true iff *X* subserves *C*.

Or to state the clauses explicitly:

> (i)′ O(X) is true iff every ideal course of action contains an action in compliance with X;
> (ii)′ P(X) is true if every course of action in compliance with X is contained in an ideal action.

These clauses may be extended to truth-functional compounds of atomic deontic statements in the standard (classical) way; and we might then say that the deontic statements $S_1, S_2, \ldots$ (*classically*) *entail* the deontic statement T if T is true whenever (i.e., in any model in which) $S_1, S_2, \ldots$ are true.

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We may informally justify each of these clauses as follows:

First, in regard to the right-to-left direction of (i), suppose that C subsumes X, i.e., that each ideal action $c_1, c_2, \ldots$ contains an action $a_1, a_2, \ldots$ in compliance with X. Now surely one is obliged to discharge one's obligations in a permissible manner, i.e., one is obliged to perform some one of $c_1, c_2, \ldots$. But then surely one is obliged to perform some one of their parts $a_1, a_2, \ldots$.

Second, for the other direction, suppose that X is obligatory. Consider any ideal action c. Thus one may discharge one's obligations by performing c. Suppose now that no action in compliance with X is a part of c. How then can X be obligatory, given that the performance of an action in compliance with X will go no way towards discharging this particular way of discharging one's obligations?

Third, in regard to the right-to-left direction of (ii), suppose X subserves C, i.e., that every action a in compliance with X is part of an ideal action c. Then since c is permissible, so is a (under the loose understanding of permission which we have adopted).

Finally, for the other direction, suppose X is permissible. Then it is plausible (at least for those of us who have Ross-type intuitions!) that every action in compliance with X is permissible. Assuming:

(*) Every permissible action is part of an ideal action

we can infer that every action in compliance with X is part of an ideal action.

The above justifications are not entirely unproblematic. But there is one major line of questioning which I believe may be resisted. For one might question assumption (*) on the grounds that, when the obligations within a code of conduct conflict, there will be no ideal action and so, a fortiori, an action (such as the null action) may be permissible without being contained in an ideal action. However, the only reason to deny that there is any ideal action in this case is that one accepts the Complete Consistency condition above, that any action sanctioned by the code should be consistent. But it seems to me that in the case of conflicting obligations, we might simply allow a code of conduct to contain inconsistent courses of action. It will not then follow, within our framework as it does within the standard possible worlds framework, that everything whatever is obligatory, since an inconsistent action will not, in general, contain every other action.[3]

Another advantage of the present approach is that it can provide us with a unified account of what is obligatory and what is permitted by reference to a single code of conduct. However, once we give up assumptions, such as (*) above, it may be necessary to 'fracture' a code of conduct into two components which separately specify what is relevant to the permissibility or obligatoriness of a given content (this is the approach of Anglberger, Korbmacher & Faroldi (2016)). This provides a more general approach; and one is then able to deal with certain anomalous cases, though at an expense in elegance and simplicity.

§4. Some remarks on the semantics. We make some remarks on the form of the clauses, how they might be extended to other deontic operators, and how they compare with the standard possible worlds clauses.

4.1. The quantificational form of the clauses. From a formal point of view, the clauses for the operators O and for P are both $\forall\exists$ (for-all/for-some) in form.[4] This is in marked

[3] The logical issues raised by the existence of moral dilemmas are further discussed in 6, under §4 of 'O-Inference'.

[4] Aloni (2007, 76) provides a similar (albeit modal form) of the $\forall\exists$ clause for permission but adopts the quantificational dual $\exists\forall$-form for obligation.

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contrast to the clauses for the operators in the standard possible worlds semantics, according to which:

> O(S) is true iff S true in all deontic alternative worlds
> P(S) is true iff S is true in some deontic alternative world,

which are, respectively, universal and existential in form. Indeed, the quantifier relevant to the embedded clause X in (i)′& (ii)′ is existential in the case of obligation and universal in the case of permission; and so one might claim, with some justice, that the standard treatment gets the correspondence with the quantifiers completely backwards!

We also lose the duality in the clauses (i)′& (ii)′ that parallels the duality in the two quantifiers. But oddly enough, the clauses are dual in another respect, since there is a reversal of mereological role and order in going from the one clause to the other.

4.2. Free-choice obligation. Clauses (i) and (ii) above are appropriate for *limited-choice* obligation and for *strict* permission. The clause for free-choice obligation may be obtained by combining the two clauses:

> (iii)$^+$ O^P(X) is true if C subsumes X and if X subserves C;

or, more explicitly:

> (iii)$^+$ O^P(X) is true if every ideal course of action contains an action in compliance with X and every action in compliance with X is contained in an ideal course of action.

Thus O^P(X) will have the same truth-conditions as the conjunction O(X) ∧ P(X) and a free-choice statement of obligation will serve a dual purpose, both stating what one is obliged to do and also specifying the permissible actions by which the obligation might be discharged.[5] There is an interesting question as to whether ordinary language ‘ought’ and its cognates are to be understood in a limited or free-choice sense. Certainly, ‘you should post the letter or destroy it’ in some sense implies ‘it is permissible to destroy it’. But I am inclined to think of this as some kind of pragmatic implication. There seems to be no contradiction involved in saying ‘you should post the letter or destroy it and, since you should not destroy it, you should post it.’ If this is right, then O^P captures a pragmatically strengthened meaning of ought-statements rather than their strict literal meaning. However, in contrast to a number of linguists (e.g., Kratzer & Shimoyama (2002), Alonso-Ovalle (2005), and Fox (2007)), I am do not hold a corresponding view of performative statements of permission. For me, the inference from ‘you may have an apple or an orange’ to ‘you may have an apple’ is semantic.

We should also note that the two conditions, that C subsumes X and that X subserves C, are the two conditions required for the content X to be part of the content C. Thus, under the proposed semantics, a free-choice statement of obligation will play the role of specifying part of the content of the code of conduct. Given our previous semantics for imperatives, we may also see the truth-conditions for free-choice obligation as relating directly to imperative inference since, for an obligation statement O^P(X) to be true is for the embedded imperative X to follow from the implicit code of conduct.

5 Aloni (2007, 86) introduces an imperative analogue of O^P, for which she adopts a somewhat similar clause.

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4.3. Weak permission. Recall from Part I that two actions are compatible (within a modalized action space) if their fusion is a possible action. The clause for weak permission (for which we use the symbol P^+) may then be obtained by replacing talk of part-whole with talk of compatibility:

> (ii)″ $P^+(X)$ is true if every action in compliance with X is compatible with a course of action sanctioned by the code.

Thus strict permission requires that the condoned actions should be ruled *in* by the code of conduct whereas weak permission only requires that they not be ruled *out*.

Clause (ii)″ would appear to be a variant rather than a special case of clause (ii). But, as I have already suggested, there is a way of seeing it as a special case. For suppose we replace a code of conduct C by the set C^* of complete courses of action compatible with some member of C.[6] Then for an action to be compatible with a conduct sanctioned by C is simply for it to be a part of a course of action sanctioned by C^*. Thus we can also see the semantics for weak permission as arising from a conception of codes in which only complete courses of action are sanctioned.

Under the present approach, strict permission would appear to be the more straightforward notion; weak permission must be obtained by replacing mereological with modal relationships or by imposing special conditions on codes of conduct. This is in contrast to the standard possible worlds approach, in which permission can only be understood as weak permission and under which it is difficult even to see how strict permission might be defined.

4.4. Weak obligation. Just as we might give a modal clause for weak permission, so we might also for 'weak' obligation:

> (i)″ $O^+(X)$ is true if C necessitates X, i.e., if it is impossible for a course of action in C to be performed without an action in X being performed.

Thus under the weak modal criterion, if one is obliged to square the circle then one is obliged to do anything whatever, whereas this will not follow under the strong mereological criterion.

As before, we may see the modal criterion as a special case of the mereological criterion, obtained by replacing the code of conduct C with the corresponding set C^* of completions. For C will necessitate X just in case every member of C^* contains a member of X.

Clause (i)″ corresponds, of course, to the standard possible worlds clause, with C^* the deontic sphere of alternatives. Thus the operator O^+, so understood, will be intensional; $O^+(X)$ and $O^+(X')$ will have the same truth value whenever X and X′ are necessarily co-enacted.

The same is not true of clause (ii)′ for weak permission, since it embodies a free-choice effect; the permissibility of X requires, not simply that X be compatible with C, but that every member of X be compatible with C. We could get a counterpart to the standard clause by replacing (ii)″ with:

> (ii)‴ $P^+(X)$ is true if *some* action in compliance with X is compatible with a course of action sanctioned by the code

and the resulting notion of permission would then also be intensional.

6 This presupposes that we are working within a W-space, as defined in the formal appendix.

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In either case, we can state the clauses for permission and obligation by reference simply to the *permissible* courses of action since, as already noted, any complete permissible action will automatically be ideal.[7]

4.5. The reduction to alethic logic. There is a familiar reduction of deontic to alethic modal logic, deriving from Anderson (1966) and Kanger (1957). Although they have standard versions of modal logic in mind, their general method of reduction may be applied, perhaps somewhat surprisingly, to our own semantical approach.

Let us begin with permission. Each code of conduct C may be closed under part to give what one might call the *code of permission* $C\downarrow$. Thus an action is sanctioned by $C\downarrow$ just in case it is part of a course of action sanctioned by C. Intuitively, $C\downarrow$ consists of the actions permitted by the code; and the only aspect of C required to state the clause for permission is given by $C\downarrow$.

Let us now introduce a constant 'OK' for the imperative 'do something alright (permitted)'. The actions in compliance with OK will be the members of $C\downarrow$. Let us also introduce an implicational connective $\Rightarrow$ for exact entailment between imperatives; X $\Rightarrow$ Y is to be true if every action in compliance with X is in compliance with Y. It is then evident that P(X) will be true just in case X $\Rightarrow$ OK is true. To say that X is permitted is to say that each action in compliance with X is permitted, which is to say that X exactly entails the imperative 'do something permitted'.

Alternatively, we might introduce a constant 'A-OK' for the imperative 'do something completely alright (ideal)'. The actions in compliance with A-OK will then be the members of C. Say that the action a is in *sub-compliance with* the imperative X if it is part of an action in compliance with X and define an analogue of exact entailment by saying that X *sub-entails* the imperative Y if any action in sub-compliance with X is in sub-compliance with Y. The statement P(X) of permission will then be true just in case X sub-entails A-OK or, using $\Rightarrow_*$ for sub-entailment, P(X) will be true just in case X $\Rightarrow_*$ A-OK is true.

It should be noted that both $\Rightarrow_*$ and $\Rightarrow$ have the property that $X \vee Y \Rightarrow_{(*)} Z$ implies $X \Rightarrow_{(*)} Y$ and $Y \Rightarrow_{(*)} Z$ even though neither has the property that $X \Rightarrow_{(*)} Z$ implies $X \wedge Y \Rightarrow_{(*)} Z$. Thus the reduction will give us the desirable result that $P(X \vee Y)$ implies P(X) and P(Y) without giving us the undesirable result that P(X) implies $P(X \wedge Y)$.

Let us turn to obligation. In the case of free-choice obligation, $O^P(X)$ will, of course be true just in case A-OK *analytically* entails X. For the case of limited-choice obligation, we might say that the action a is in *super-compliance with* the imperative X if it contains an action in compliance with X and that X *super-entails* the imperative Y if any action in super-compliance with X is in super-compliance with Y, which is to say that any action in compliance with X is in super-compliance with Y (this is what in other works I have called *inexact* entailment). The statement O(X) will then be true just in case A-OK super-entails X.

We see that by suitably interpreting the 'sanction' constant and the conditional we may obtain Anderson/Kanger type reductions for both obligation and permission, although the interpretation of either the constant or of the conditional must be different in each case.[8]

[7] It is perhaps worth noting that although Aloni and Ciardelli (2013) have option sets, which correspond to our actions and are identified with sets of worlds, their counterpart to our code of conduct is a set of worlds and so they do not avail themselves of the more general notion of a code of conduct.

[8] Some rather different attempts at reduction using a nonstandard conditional are to be found in Asher & Bonevac (2005) and Barker (2010).

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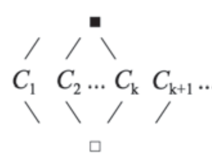
We can, of course, provide a reduction for obligation under the standard possible worlds account by interpreting O(X) as C → X, where C stands for the set of deontic alternatives and → for strict implication. But it is hard to see how a corresponding reduction for statements of permission might go. Hence Nute's claim, "We cannot reduce any interesting notion of permission to conditionals no matter how we interpret conditionals" (Nute, 1985, 179). However, this claim is somewhat of an over-statement in the present context.

4.6. Truthmakers for deontic statements. We have so far given conditions for when a deontic statement of the form O(X) or P(X) is true but we have not specified when a state verifies or falsifies such a statement. This would be required if we wished, for example, to say when one deontic statement analytically entailed another.

There are a number of different ways in which this might be done. The most straightforward is as follows. Each code of conduct C is understood to be the state that consists in its members $c_1, c_2, \ldots$ being all *and only* the ideal courses of action. We might say, in this case, that the code C *prevails*; and so the code is, in effect, being identified with the state that it prevails.

No code, as so conceived, will be a part of any other code, and so the space of codes will have the following 'flat' structure:



with the full state on top, the null state at the bottom, and all of the codes of conduct in between. Given that the codes $C_1, C_2, \ldots$ are all possible, the only impossible state will be the full state and any two codes will be incompatible. In effect, each code is a mini-world, completely settling which deontic statements do and do not hold.

We may then adopt the following clauses for when an atomic deontic statement is verified or falsified by a code C:

(i)$^+$ C verifies O(X) iff C subsumes X;
(i)$^-$ C falsifies O(X) iff C does not subsume X;
(ii)$^+$ C verifies P(X) iff X subserves C;
(ii)$^-$ C falsifies P(X) iff X does not subserve C.

These clauses may then be extended to truthfunctional compounds of atomic deontic statements in the usual way.

However, we might wish to provide a less demanding account of what verifies or falsifies a deontic statement, making clearer what it is about the code that is responsible for the statement's being true or false. To this end, we might associate two states with each code C: the state C^* of C *upwardly prevailing* in the sense that the prevailing code contains C as a part (the prevailing code is *at least* C); and the state C_* of C's *downwardly prevailing* in the sense that the prevailing code is a part of C (the prevailing code is *at most* C). The state C^* will be part of the state D^* just in case C is a part of D; the state C_* will be a part of D_* just in case D is a part of C; and if we identify a code C with the state of its prevailing, then $C = C^* \sqcup C_*$.

We might now adopt the following clauses for the atomic deontic statements:

(i)$^+$ s verifies O(X) if s is of the form C^* and C subsumes X;
(i)$^-$ s falsifies O(X) if s is of the form C_* and C does not subsume X;

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(ii)$^+$ s verifies P(X) if s is of the form C^* and X subserves C;
(ii)$^-$ s falsifies P(X) if s is of the form C_* and X does not subserve C.

Thus it is an upwardly prevailing state (the code is at least this) that will verify a statement of permission or obligation and a downwardly prevailing state (the code is at most this) that will falsify a statement of permission or obligation.

Once we combine obligation and permission statements, we will need to combine states of the form C^* and C_*. We may do this by taking each state to be of the form (C^*, D_*), where what was previously C^* is now $(C^*, \square)$ and what was previously C_* is now $(\square, C_*)$. Part-whole is defined "point-wise". Thus (C^*, D_*) is a part of (E^*, F_*) if C^* is a part of E^* and D_* is a part of F_*. And the state (C^*, D_*) will be possible if some code E lies between C and D. Thus when previously we took C to verify a deontic statement, we now take this to be the special, and highly demanding, case in which (C^*, C_*) verifies the statement.

§5. Imperatival validity revisited. I would like to consider various different ways of formulating the criterion of validity for imperative inference in the light of the previous semantics for deontic logic. The reformulations are technically trivial and more of philosophical than technical interest.

Recall that the imperative inference X/Y was taken to be valid if two conditions were met: X subsumes Y; and Y subserves X. This definition does not take the usual form, in which the validity of an inference is a matter of some value or values being preserved in the transition from premiss to conclusion; and it is natural to wonder whether a definition of this sort can be given.

There is a way in which the answer to this question is trivial. For let us suppose that entailment (the relation that holds between X and Y when the inference X/Y is valid) is both reflexive and transitive. Then X will entail Y just in case for every Z, Z entails Y whenever Z entails X. For the left-to-right direction will follow from transitivity; and the right-to-left direction will follow from reflexivity upon setting Z = X. Taking the values in question to be properties of the form 'entailed by Z' will then give us an account of entailment in terms of the preservation of these values.

Let us now apply this 'cheap trick' to the validity of imperative inference. Say that the imperative X *accords with* the code of conduct C if X subserves and is subsumed by C. Then the cheap trick will give us that X entails Y just in case Y accords with any code of conduct with which X accords.[9]

But this gives us a new way to understand the force of imperative inference. For we may suppose that prior to the stipulation of the premisses, there is a prevailing code of conduct (as given by the moral code, a body of law, some previous imperatives, or the like). We can then understand the imperative inference as telling us that the conclusion will accord with the prevailing code as long as the premisses do.

This is in line with thinking of there being an underlying warrant for the imperative in terms of a reason or norm and of the validity of an imperative inference as then consisting in the preservation of such a warrant (as in Vranas (2011), for example). For we may take the

[9] Strictly speaking, a code of conduct is a convex set. But it is readily shown that this is a harmless addition, since X will subserve (or be subsumed by) a set of actions C if and only if it is subserves (or is subsumed by) the convex closure of C. For the purposes of the application, I have also switched from imperatives to their contents.

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reason or norm to be a code of conduct and we can then take the warrant of the imperative to consist in its according with the reason or norm. But of course, from our own point of view, this way of thinking does not give us an actual criterion of validity since the relevant notion of validity is already presupposed in what it is to have a warrant.

The present criterion of imperative validity also makes evident the connection between imperative and deontic reasoning:

> The Imperative-Deontic Link. The imperative inference X/Y is analytically valid iff the deontic inference $O^P(X)/O^P(Y)$ is classically valid.

For the classical validity of the deontic inference simply amounts to Y according with any code of conduct with which X accords. In effect, one reasons, in imperative inference, from the obligation implicit in the premisses to the obligation implicit in the conclusion. Thus there is a sense in which imperative inference is classical after all but under a suitable modal understanding of the premisses and conclusion; and if someone (such as Kaufman (2012)) felt tempted to construe imperatives as themselves statement of obligation then imperative inference would be straightforwardly classical although not, of course, under the usual possible worlds account of obligation!

The criterion also provides us with yet another understanding of imperative validity in terms of 'updating'. For we might think of an imperative, not as something that accords with a prevailing code of conduct, but as a means of updating the code. Thus if the prevailing code is C and the imperative premiss is X, then the result $C[\mathrm{X}]$ of updating C with X will be $C \wedge X$. We simply combine the actions sanctioned by C with the actions in compliance with X. The inference X/Y will then be valid if and only if Y accords with the updated code $C[\mathrm{X}]$. This is something like the definition of validity in the update semantics of Veltman (1996) though, as previously noted (in §5 of Part I), we cannot assume, in the usual way, that Y's according with $C[\mathrm{X}]$ is a matter of $C[\mathrm{X}]$ being identical with $C[\mathrm{X}][\mathrm{Y}]$.

This particular connection is relevant, I believe, to the inferential interface between deontic and imperative statements. For it is natural to suppose that from an imperative X (such as 'Shut the door'), we can infer the corresponding free-choice statement of obligation $O^P(X)$ ('so you ought to shut the door and you may shut the door'). We do not yet have a notion of validity for which this is so since the inference involves a mix of imperative and indicative statements. Such mixed inferences will be discussed at greater length in Part III, but let us note that the present conception of updating provides us with a handle for dealing with this particular case. For we may take the inference from an imperative X to a deontic statement S to be valid if, for any code of conduct C, S is true in $C[X]$; and the inference from X to $O^P(X)$ will then be valid. Thus each imperative has the effect of updating the code of conduct and, since one's obligations are understood by reference to the code of conduct, the inference from X to $O^P(X)$ will automatically be secured.[10]

§6. Some special inferences. I should like to discuss some cases of valid or invalid deontic inference of special interest, dealing first with inferences involving obligation and permission separately and then with inferences involving both together. In each case, I have paid particular attention to the comparison with standard deontic logic and have given somewhat informal proofs of the various claims of validity and invalidity.

[10] Cf. Portner (2007), which attempts to justify such inferential connections on the basis of general pragmatic principles.

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6.1. P-inference. 1. We should note right away that the rule of Simplification will hold, i.e., P(X ∨ Y) will entail P(X) and P(Y), in contrast to the standard semantics for deontic logic. For if every action in compliance with X ∨ Y is contained in an ideal action (one sanctioned by the code of conduct) then a fortiori every action in compliance with X or in compliance with Y will be contained in an ideal action.

However, P(α) will not entail P($\alpha \wedge \beta$), for even if every action in compliance with α is contained in an ideal action that is no reason to assume that any action (let alone every action) in compliance with $\alpha \wedge \beta$ is contained in an ideal action. Thus the most immediate logical problem posed by the existence of free choice permission is solved.

This solution is in line with the solution provided by others (such as Aloni & Ciardelli (2013)) within the 'alternatives' tradition, even though the underlying framework is somewhat different.

2. P(X ∧ Y) will entail P(X), in conformity with the standard semantics. For suppose that every action in compliance with X ∧ Y is contained in an ideal action. Take now any action a in compliance with X. Under the semantics, there will be an action in compliance with any imperative and hence an action b in compliance with Y. But then $a \sqcup b$, and hence a, will be contained in an ideal action.

However, P(X) will not entail P(X ∧ X) in marked contrast to the standard semantics. This is because actions a and b in compliance with X may be part of an ideal action even though the action $a \sqcup b$ in compliance with X ∧ X is not part of an ideal action.

Let us say that the imperative X is *definite* (with respect to a model) if there is exactly one action in compliance with X and is otherwise *indefinite*. Then for definite X, P(X) will entail P(X ∧ X). This is but one of many cases in which a principle not valid for all imperatives is valid for definite imperatives.

3. The deontic formula P(⊤), according to which the null action is permissible, will be valid if we assume Nonemptiness, i.e., that the code of conduct contains at least one action, since the single action, □, in compliance with ⊤ will be contained in that action. But dropping that assumption, i.e., allowing for the empty code of conduct, will render P(⊤) invalid, since then no action sanctioned by the code will contain □. Thus P(⊤) tells us, in effect, that some action is sanctioned by the code of conduct.

However, the deontic formula P(X) is not valid for an *arbitrary* tautology X, in contrast to standard deontic logic. For example, P($\alpha \vee \neg\alpha$) is not valid since otherwise, by Simplification, both P(α) and P($\neg\alpha$) would be valid. Indeed, it can be shown that P(X) will not be valid for any formula X that contains an imperative atom (and even under a restricted class of codes of conduct, as long as at least one of them does not contain ■).

The deontic formula ¬P(⊥) is also not valid, since it will be false when the sole action sanctioned by the code of conduct is ■. However, assuming Nonemptiness and Complete Consistency (that every action sanctioned by the code of conduct is consistent), then ¬P(X) will be valid for any classically inconsistent formula X, since only inconsistent actions will then be in compliance with X.

If ¬P(X) is taken to be valid for a classically inconsistent formula X, then, given Simplification, it will render ¬P(X) valid for many formulas X which are *not* classically inconsistent, in marked contrast to the standard case. For example, ¬P(¬X ∧ (X ∨ Y)) will be valid, since P(¬X ∧ (X ∨ Y)) will entail P((¬X ∧ X) ∨ (¬X ∧ Y)), which, by Simplification, will entail P(¬X ∧ X).

Even if ¬P(X) is *not* taken to be valid for classically inconsistent X, we should note that P(⊥) will still entail P(Y)—if the completely impossible is permitted then everything whatever is permitted. For if P(⊥) is true then ■ will be an ideal action, and P(X) will then

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be true since every action is contained in ■. However, we will not in general have that P(X) entails P(Y) for X a classical contradiction, since a code of conduct may sanction an action in compliance with X (even though it is intuitively inconsistent) without sanctioning ■ and so, in particular, P(X) will not in general entail P(⊥).

6.2. O-inference. 1. O(X) will entail O(X ∨ Y), in line with standard deontic logic. For if every ideal action contains an action in compliance with X then, a fortiori, it will contain an action in compliance with X ∨ Y. Similarly, O(X ∧ Y) will entail O(X). For if every ideal action contains an action in compliance with X ∧ Y (of the form $a \sqcup b$ for a an action in compliance with X and b an action in compliance with Y) then it will thereby contain an action (viz. a) in compliance with X.

On the other hand, O^P(X) will *not* entail O^P(X ∨ Y). For suppose a is the sole action in compliance with X and b the sole action in compliance with Y; and suppose a is the sole action sanctioned by the code of conduct. Then O^P(X) will be true. But there is no reason why O^P(X ∨ Y) should be true. For its truth would require the truth of P(X ∨ Y), which, in its turn, would require the truth of P(Y). It is no doubt the free-choice interpretation of obligation that stands in the way of accepting the inference from 'you ought to post the letter' to 'you ought to post the letter or burn the house down', since the latter seems to grant permission to burn the house down.

2. We have the standard principles that O(X) ∧ O(Y) entails O(X ∧ Y) and that O(X ∧ Y) entails O(X) ∧ O(Y). For if every ideal action contains an action a in compliance with X and an action b in compliance with Y then it will contain an action $a \sqcup b$ in compliance with X ∧ Y, and conversely.

We also have the principle that O^P(X ∧ Y) entails O^P(X) ∧ O^P(Y) for free-choice obligation, since O(X ∧ Y) entails O(X) ∧ O(Y) and P(X ∧ Y) entails P(X) ∧ P(Y). However, we do not have the principle that O^P(X) ∧ O^P(Y) entails O^P(X ∧ Y) for free-choice obligation. Indeed, we do not even have that O^P(X) entails O^P(X ∧ X) for, when X is an indefinite imperative, O^P(X) may be true while P(X ∧ X) is false.

3. The deontic formula O(⊤) is valid since any ideal action will contain the single action □ in compliance with ⊤; and the formula P(⊤), and hence the formula O^P(⊤), will be valid if we assume Nonemptiness, since □ will be a part of some ideal action. But dropping the assumption will render P(⊤), and hence O^P(⊤), invalid, since then no ideal action will contain □.

The deontic formula O(X) is not valid for an *arbitrary* tautology X, in contrast to standard deontic logic. For example, O($\alpha \vee \neg\alpha$) is not valid, since there is no reason, in general, to think that a code of conduct will contain an action in compliance with or in contravention to α. Indeed, it can be shown that O(X) will not be valid for any formula X that contains an imperative atom. Of course, matters will be different if we suppose that the courses of action sanctioned by the code of conduct are all complete.

The formula ¬O(⊥) is also not valid, since it will be false when the sole action sanctioned by the code of conduct is ■. However, assuming Consistency, that some ideal action is consistent, ¬O(X) will be valid for any classically inconsistent formula X, since only inconsistent actions will then be in compliance with X.

O(⊥) entails O(Y)—if the completely impossible is obligatory then everything whatever is obligatory. For if O(⊥) is true then every ideal action will contain ■ and hence be identical to ■, in which case every ideal action will contain an action in compliance with Y.

However, as before, we will not in general have that O(X) entails O(Y) for X a classical contradiction, since a code of conduct may sanction a contradiction without sanctioning ■.

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4. In the case of a moral—or, more generally, a normative—dilemma, we will have both O(X) and O(¬X) true or both O(X) and O(Y) true, with X incompatible with Y. This possibility is, of course, of great relevance to accommodating the existence of moral dilemmas within the framework of deontic logic.

The usual attitude to cases of this sort (granted that they arise) is to question the inference from O(X) and O(¬X) to O(X ∧ ¬X) or, more generally, the inference from O(X) and O(Y) to O(X ∧ Y) when X is incompatible with Y. Our attitude, on the other hand, is to allow the inference but to deny that 'explosion' thereby results, with everything whatever being obligatory, as would be the case in more standard versions of deontic logic. Suppose, for example, that the code of conduct consists of a single action which is the fusion of *Leave* (to fight the Resistance) and *Stay* (to look after one's ailing mother). Then it will be obligatory to leave (O(Leave)), obligatory to stay (O(Stay)) and even obligatory to leave and to stay (O(Leave ∧ Stay)); and each of these actions will also be permissible. However, it is worth noting that we could also follow the standard route by allowing there to be several codes of conduct, perhaps not all compatible with one another, and then taking O(X) to be true under this class of codes if it true under one of the codes (much as in van Fraassen (1973)).

6.3. O/P inference.

1. Under the present semantics, $O(\alpha)$ does not entail $P(\alpha)$. For suppose a and b are in compliance with α but that a is the only action sanctioned by the code of conduct. $O(\alpha)$ will then be true but there is no reason, in general, why b should be part of a. To take an ordinary example, it may be obligatory to post the letter and hence obligatory to post the letter or burn the house down and yet not permissible to post the letter or burn the house down.

We do, of course, have the principle that O^P(X) entails P(X) for free-choice obligation. And we also have it for limited-choice obligation and definite imperatives X, given Nonemptiness. For if O(X) is true then every ideal action contains the single action a_0 in compliance with X and so, given that some action is ideal, every action in compliance with X, viz. a_0, will be contained in an ideal action. So this is another case in which a generally accepted principle holds for definite but not indefinite imperatives. There is no doubt that any obligatory action is permitted; and within the usual possible worlds approach to deontic logic, no sensible distinction can be drawn between this truism for actions and the corresponding logical principle O(A) ⊃ P(A) for propositions. However, within an approach like our own, in which P but not O receives a free-choice interpretation, the inference to the corresponding logical principle O(X) ⊃ P(X) will no longer be warranted.

2. We also do not have the standard principle that O(X) and P(Y) entails P(X ∧ Y). Indeed, if we were to set Y = ⊤, then this principle would yield, as a special case, that O(X) and P(⊤) entails P(X ∧ ⊤), which, in its turn, given Nonemptiness, would yield O(X) entails P(X).

However, as it to be expected, the principle that O(X) and P(Y) entails P(X ∧ Y) will hold when X and Y are definite or, indeed, when X alone is definite. For suppose that O(X) is true, so that every ideal action contains the single action a_0 in compliance with X, and that P(Y) is true, so that every action b in compliance with Y is contained in an ideal action. Take now any action, of the form $a_0 \sqcup b$, in compliance with X ∧ Y. Then b is contained in an ideal action c, which must contain a_0; and so c must contain $a_0 \sqcup b$.

3. In standard deontic logic, the deontic operators are dual in the sense that O(X) is equivalent to ¬P(¬X) and P(X) to ¬O(¬X). These duality principles fail in the present context because of two separate reasons—one arising from the strict sense of permission and the other from indefiniteness in the imperative X.

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O(X) will entail ¬P(¬X) as long as the code of conduct is completely consistent. For suppose, for reductio, that O(X) and P(¬X) are both true. Then some action c sanctioned by the code of conduct will contain an action a in compliance with ¬X and c in its turn will contain an action in compliance with X; and so c will be inconsistent.[11]

However, $\neg P(\alpha)$ will not entail $O(\neg\alpha)$. For suppose that the sole action sanctioned by the code of conduct is the null action $\square$, that a is a non-null action in compliance with α, and that the null action is in contravention to α. Then $\neg P(\alpha)$ is true since no ideal action contains a and $O(\neg\alpha)$ is false since the ideal action $\square$ contains no action in compliance with $\neg\alpha$. As is clear, the entailment may fail to go through even when there is a single action in compliance with and a single action in contravention to α. The reason intuitively speaking, is that P expresses strict permission. Thus under the null code of conduct, no action (beyond $\square$) will be permitted and no X which does not allow null compliance will be obligatory.

Suppose, however, that we take the courses of action sanctioned by the code of conduct to be complete (as under a modal interpretation of the deontic operators). Then the entailment may still fail to go through since what is not permitted may be indefinite. Thus consider the entailment from $\neg P(\alpha \vee \beta)$ to $O\neg(\alpha \vee \beta)$, equivalent to the entailment from $\neg P(\alpha) \vee \neg P(\beta)$ to $O(\neg\alpha \wedge \neg\beta)$. If this latter entailment were to hold then it would require that $\neg P(\alpha)$ entail $O(\neg\alpha \wedge \neg\beta)$, which clearly is not so.

However, the entailment *will* go through if we insist both that the courses of action sanctioned by the code of conduct be complete and that the imperative α be definite. For suppose $P(\alpha)$ is false. Then the sole action a_0 in compliance with α will not be contained in an ideal (and complete) action sanctioned by the code. But this then means that every action sanctioned by the code must contain an action in contravention to α; and so $O(\neg\alpha)$ will be true.[12] Indeed, in this case, there is, in effect, no quantification over the actions in compliance with α and so our clauses will reduce to the familiar clauses from the possible worlds semantics.

4. Although we cannot, in general, define P(X) as ¬O(¬X), there is a definition of P in terms of $O^P(X)$, though not, I suspect, of O^P in terms of P. For in line with the permissive sense of the imperative, we may define P(X) as $O^P(X \vee \top)$, as long as the code of conduct is nonempty. For then every ideal action will contain an action in compliance with $X \vee \top$, viz. $\square$, while $P(X \vee \top)$ will be equivalent to $P(X) \wedge P(\top)$, which will be equivalent to P(X) given Nonemptiness.

It is not altogether clear to me that we have much use for O as opposed to O^P. Certainly, the most informative and useful way to state an obligation is by indicating the ways of realizing the obligation that are permissible. And if this is so, then we may formulate the language of deontic logic in the present setting by using a single deontic operator, just as in the standard formulations, but with the difference that the operator must be the operator for obligation rather than for permission.

§7. An axiom system. I briefly outline a system of deontic logic appropriate for the deontic formulas which are valid within the truthmaker approach.

We take a *deontic formula* to be a truth-functional compound of formulas of the form O(X) and P(X) for X an imperative formula. Such a formula is then taken to be *valid* if it

11 We here presuppose that the models are classical in the sense of the appendix to Part I.

12 We again presuppose that the models are classical.

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is true in all models whose code of conduct is nonempty. We have found it convenient, in formulating a natural system of axioms, to suppose that the interpretation of the imperative atoms should be doubly definite—for each imperative atom α, there should be a single action in compliance with α and a single action in contravention to α.

Now take the logic IL(>) of §6 of part and add the axiom $\lambda > \lambda\lambda$ for any literal (i.e., atom or its negation). Call the resulting system $IL^+(>)$. We say two imperatives X and Y are *analytically equivalent* (*in* $IL^+(>)$) if both X >Y and Y>X are provable in $IL^+(>)$. We shall use the system $IL^+(>)$ as a basis for formulating our deontic logic.[13]

An imperative formula X is said to be (*syntactically*) *definite* if it is a conjunction of literals. Clearly, each syntactically definite formula X will be semantically definite, there will be a single action in compliance with X (though not necessarily in contravention to X).

We lay down the following axioms:

Implication.
$O(X) \supset O(X')$
$P(X) \supset P(X')$

whenever X is analytically equivalent to X′.

Distribution.
$O(X \wedge Y) \equiv O(X) \wedge O(Y)$
$P(X \vee Y) \equiv P(X) \wedge P(Y)$

Weakening.
$O(X) \supset O(X \vee Y)$
$P(X \wedge Y) \supset P(X)$

Triviality.
$O(\top)$, $\neg O(\bot)$
$P(\top)$, $\neg P(\bot)$

Mixture.
$O(X \vee Y) \wedge \neg P(X) \supset O(Y)$, for definite X
$O(X_1 \vee X_2 \vee \ldots \vee X_n) \wedge P(Y) \supset P(X_1 \wedge Y) \vee P(X_2 \wedge Y) \vee \ldots \vee P(X_n \wedge Y)$,
for definite $X_1, X_2, \ldots, X_n$ and Y.

Let us call the resulting system DL (for 'deontic logic'). Its theorems are all the truth-functional consequences of these various axioms. We may also define two subsystems DL(P) and DL(O). In DL(P) the formulas are restricted to the P-formulas and the only axioms are Equivalence, Distribution, Weakening and Triviality for P. Similarly, in DL(O) the formulas are restricted to the O-formulas and the only axioms are Equivalence, Distribution, Weakening and Triviality for O. Part of the interest of the full system is that it provides for a characteristic interaction between the obligation and permission operators even though neither is definable in terms of the other.

13 This logic is close to, but not the same as, the logic of analytic equivalence in Fine (2016). From a semantical point of view, we *remove* closure under fusion and *add* the requirement that the atoms should be doubly definite. Thus we will have the equivalence of $X \wedge X$ to X for literals, though not for arbitrary formulas, and we will not have the equivalence of $X \vee Y$ to $X \vee Y \vee (X \wedge Y)$.

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Proofs of soundness and completeness for these various systems are sketched in the formal appendix. The proofs of completeness use normal forms and show, in the case of the full system DL, how one might provide a complete description of a code of conduct using P- and O-formulas. Each such description essentially consists of a single P-formula P(X), a single O-formula O(Y), and a bunch of other formulas to the effect that P(X) is a maximal description of what is permitted and O(Y) is a maximal description of what is obligatory.

The resulting logic are not, of course, closed under substitution. Thus even though $P(\alpha) \supset P(\alpha \wedge \alpha)$ is a theorem, $P(\alpha \vee \beta) \supset P((\alpha \vee \beta) \wedge (\alpha \vee \beta))$ is not. The restrictions on Mixture point to the utility of distinguishing between definite and indefinite statements in reasoning that combines consideration of what is obligatory and what is permissible. However, it would be of interest to determine which system we would obtain when the interpretation of the atoms was unrestricted or when we admitted two kinds of atoms, either with or without a restricted interpretation. One might also consider the various extensions of these systems which result from imposing further conditions on the codes of conduct. One could, for example, get something closer to standard deontic logic if one insisted upon Complete Consistency. The standard principles, $\neg O(X \wedge \neg X)$ and $O(X) \wedge O(\neg X \vee Y) \supset O(Y)$ would then both be valid.

§8. Updating. I should like to make a few remarks concerning the problem of deontic updating. The topic calls for much more extensive discussion, especially in regard to its connection with belief revision and reasoning from inconsistent premisses. But, at the very least, the present discussion will indicate how very different the problem looks from the present perspective as opposed to the usual possible worlds perspective and how my approach differs from other approaches that trade in a "sphere of permissibility" for a "to-do list".

In a given context, we may suppose, certain deontic statements directed to a given agent are true and the others false. Suppose now someone in authority, perhaps myself, tells the agent what he ought to do or what he may do, where this is not something that was previously true. The problem of deontic updating is then the problem of explaining which deontic statements will then be true.

Of course, if I tell the person that he ought to do something (or may do something), it will then be true in the new context that he ought to do it (or may do it), even though this was not true before. We may also assume that the change in context is *determinate*—which is to say that it must still hold, in the new context, that each deontic statement is either true or false. But this means, on pain of inconsistency, that some other deontic statements must change from true to false or from false to true; and so the problem is to say which they are (and why).

If we approach the problem from a possible worlds perspective,[14] then the set of true and false deontic statements will, in effect, be given by a 'sphere' of ideal worlds. On being told that A is obligatory or permissible, the agent must change the deontic sphere so that A is obligatory or permissible. This means, in the case of obligation, that every world in the new deontic sphere should be an A-world and, in the case of permission, that some A-world must belong to the new deontic sphere (even though neither was so before). We presumably want the change to be minimal—we want in some sense that is

[14] As is done in the article of Lewis (1979), which introduces the problem.

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not altogether clear to minimize the change in the truth-value of the deontic statements; and this presumably translates into the change in the deontic sphere being minimal—we want, again in a sense that is not altogether clear, to minimize the change in which worlds belong to the deontic sphere.

It might be thought that the solution to this problem is straightforward in the case of obligation. For the new sphere can simply be the intersection of the old sphere with the set of A-worlds. In other words, we may restrict the former ideal worlds to the A-worlds. However, there is one special case in which this strategy does not work. For the intersection may be empty, there may be no A-worlds that were previously ideal (even though there are A-worlds); and in this case—if we insist that the deontic sphere be nonempty, i.e., that something be permissible—then it is not at all clear what the change in the deontic sphere should be.

In the case of permission, the problem seems generally hopeless unless one brings some further information to bear upon what the change should be. For let us suppose that A was not originally permissible, i.e., that none of the A-worlds belong to the original deontic sphere. Then, in the absence of any further information, there is no reason to prefer the addition of one of the A-worlds to the sphere as opposed to any other (and at least one must be added if A is to become permissible). One could, of course, add all the A-worlds to the sphere, since this does not discriminate between them, but this would then result, as a rule, in all sorts of monstrous worlds being rendered permissible. The solution to the problem, in the case of permission, therefore requires some further basis for distinguishing between the worlds that might be added to the original sphere.

Let us now consider the same problem from the truthmaker perspective.[15] In this case, the deontic base will not be a deontic sphere, the set of ideal worlds, but a code of conduct, the set of ideal actions. The problem then takes the form of how to modify the code of conduct so as to make the given prescription X obligatory or permitted. This means, in the case of obligation, that every action sanctioned by the new code of conduct should contain an X-action and, in the case of permission, that every X-action should be contained in an action sanctioned by the new code of conduct and, in the case of free-choice obligation, that both requirements should be met.

It is perhaps rather odd to update with a limited-choice statement of obligation. If I tell the agent that he is obliged to wear a jacket or a tie, then it would normally be supposed that he was being permitted to do either. So let us first consider the case of free-choice obligation and only then turn to the cases of limited-choice obligation and of permission.

There is a way in which the update problem in this case may be trivial. For it may not be my intention to override the original code of conduct. Suppose the original code of conduct contains the action a of fighting in the Resistance and suppose that the agent is told 'you ought to stay at home and look after your mother', where this is a matter of performing an action b incompatible with a. Then the result of the update may simply be a code of conduct consisting of that action $a \sqcup b$ of fighting in the Resistance and staying at home. On this way of looking at the matter, the result $C{:}\mathrm{O}^P(\mathrm{X})$ of updating a code of conduct C with the prescription $\mathrm{O}^P(\mathrm{X})$ will simply be the conjunction $C \wedge X$ of C and X, i.e.,

15 Yablo (2011) also proposes a solution to the problem from the truthmaker perspective, but he still conceives of the problem in terms of a change to the sphere of permissible worlds and, given this and other differences of framework, it is not altogether clear how his solution relates to mine.

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the code of conduct that consists of all the actions of the form $c \sqcup a$ with c in C and a in X.[16]

In this case, of course, some of the actions in the new code of conduct may be inconsistent. So let us suppose, as is plausible, that the original code of conduct is completely consistent and that the intent behind the updating is that the new code of conduct should also be completely consistent. How then should the code of conduct be updated? An obvious solution is to apply a consistency filter. The new code of conduct should consist of all those actions in $C \wedge X$ which are consistent.

But if a given action a in compliance with X is not compatible with any action sanctioned by the original code of conduct, the resulting code of conduct will sanction no action containing a and so X will not be permitted. So what should we do when no action sanctioned by the original code of conduct is compatible with a? When presented with a similar problem, under the possible worlds approach, of what to do when the new deontic sphere was empty (no ideal A-worlds), there seemed nothing sensible to say—the update simply failed. But in the present case, we have further resources with which to deal with the problem.

If a is already inconsistent, the requirement of complete consistency cannot be preserved and so the update fails, as before. But suppose a *is* consistent. In such a case, what we would like to do is to keep a in its entirety and as much of c as is compatible with a.

To see how this might work, let us suppose that there exists a greatest part c' of c compatible with a, i.e., c' is compatible with a and contains any part of c compatible with a. In this case, we may let $c{:}a$, the result of updating the action c with a, be $a \sqcup c'$ and then add $c{:}a$ rather than $a \sqcup c$ to C. But even when the greatest part c' does not exist, there may still exist maximal parts c' of c compatible with a, i.e., parts of c compatible with a which are not proper parts of any other part of c compatible with a; and in this case, we can add $a \sqcup c'$ for each of the maximal parts c' to C. It is not altogether clear to me how far in this direction we may proceed. But it seems clear that there may be cases in which $a \sqcup c$ is inconsistent and yet there is no reason, on the basis of the given information, for preferring one consistent fusion $a \sqcup c'$, for c' a non-null part of c, over any other. In such a case, we should add $a \sqcup c'$ to C for *each* non-null part c' of c compatible with a (or, if there is no such c', we should simply add a).

In any case, let us suppose, if only for the sake of simplicity, that an update of the form $c{:}a$ always exists. Instead of taking the update $C{:}\mathrm{O}^P(\mathrm{X})$ to be $C \wedge X = \{c \sqcup a \colon c \in C$ and $a \in X\}$, we can take it to be $\{c{:}a \colon c \in C$ and $a \in X\}$.

There is also something to be said for taking this to be the general definition of the update and not just for the case in which no member of X is compatible with each member of C. It makes for uniformity in the definition (which is of help in establishing general principles). And it is not unintuitive. For suppose the code of conduct is given by the fact that you should either have eggs and bacon or porridge for breakfast ($C = \{e \sqcup b, p\}$). If you are then told not to have bacon (X = ¬b), you might think that would still leave one with the option of having eggs without bacon ($C{:}\mathrm{O}^P(\mathrm{X}) = \{e \sqcup \not{b}, p \sqcup \not{b}\}$) rather than being obliged to have porridge ($C{:}\mathrm{O}^P(\mathrm{X}) = \{p \sqcup \not{b}\}$).

[16] If codes of conduct are subject to certain closure conditions, such as convexity, it will then also be necessary to subject the resulting code $C{:}\mathrm{O}^P(\mathrm{X})$ to these conditions.
The present operation C: S of updating is somewhat different from the previous operation C[X] from Section 5, since it is not meant to provide a semantic explanation of S. The content or semantic value of S is already presupposed.

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We may apply a similar strategy to defining updates with limited-choice statements of obligation or with permission statements. For a limited-choice statement of obligation, it is not necessary that each action in compliance with X be permissible and so we might simply take C:O(X) to be the set $\{c \sqcup a: c \in C, a \in X$ and $c \sqcup a$ is consistent$\}$. However, the resulting code may be empty; and so if we wish to insist that the code be nonempty, we should take C:O(X) to be $\{c \sqcup a: c \in C, a \in X$ and $c \sqcup a$ is consistent$\}$ when this set is nonempty, as before, and otherwise take it to be $\{c{:}a: c \in C$ and $a \in X\}$, since in this case there is no reason to prefer any particular $c \in C$ to any other.

Updating with a statement of permission can be regarded as a special case of updating with a free-choice statement of obligation, given the equivalence of P(X) to $\mathrm{O}^P(\top \vee \mathrm{X})$. Thus we will have $C{:}\mathrm{P(X)} = C{:}\mathrm{O}^P(\top \vee \mathrm{X}) = C \cup C{:}\mathrm{O}^P(\mathrm{X})$. In this case, we will wish to retain each member of the original code of conduct C, since we are not *required* by P(X) to perform any of the actions in compliance with X. But we will also wish each action a in compliance with X to be contained in an action sanctioned by the new code of conduct and this will then be guaranteed by the presence of $C{:}\mathrm{O}^P(\mathrm{X})$.

We have considered a number of different options for defining the update. But perhaps the simplest and most natural is one in which we suppose that, for any actions c and a from the action space, there is a largest action contained in a compatible with c. Taking this action to be $c{:}a$, we may then define $C{:}\mathrm{O}^P(\mathrm{X})$ to be $\{c{:}a: c \in C$ and $a \in X\}$and define $C{:}P(\mathrm{X})$ to be $C \cup \{c{:}a: c \in C$ and $a \in X\}$. This gives rise to a nice set of principles (somewhat analogous to the standard principles governing belief revision) and seems to be very much in conformity with our intuitive judgements.

The present account of updating has two significant advantages over the possible worlds approach. One is that the updating is ultimately done at the level of the actions themselves, which is then projected upwards to the level of propositions; and it is much easier, given the mereological structure of actions, to see how an action should be modified rather than a set of worlds. Another, less obvious, advantage, arises from the fact that we are working with a strict notion of permission. When something is not strictly permitted it is relatively easy to see how, through updating, it might become strictly permitted. Under the possible worlds approach, on the other hand, we work with a weak notion of permission; and it is much harder to see how what is not weakly permitted might, through updating, become weakly permitted. From this point of view, the possible worlds approach fails to take full advantage of the performative character of permission statements. For in permitting something it thereby becomes strictly permitted; and by only taking account of the weak content of the performative utterance, the possible world approach can take no advantage of this relatively straightforward form of updating. Our own approach, by contrast, updates on what is strictly permitted. It can still achieve a change in what is weakly permitted, but only indirectly, as a consequence of a change in the code of conduct.

It might also be helpful to compare my approach to some other approaches in the literature that work off a ‘to-do’ list. My own conception of a to-do list is somewhat different from the usual one: in place of a set of actions I have a single action, which is their fusion. My general reasons for preferring this conception have already been given in §8 of part I; and the mereological structure of the fusion, which might not be fully apparent from a list, also plays an important role in defining the update.

A number of authors (such as van Rooij (2000), Portner (2012), Torre & Tan (1998)) use something like a to-do list as a basis for defining a preference relation and then appeal to the preference relation in explaining how one should update. Our approach is very different. We do not appeal, either implicitly or explicitly, to a preference relation and simply define

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the update on the basis of the mereological structure of the actions on which the update is being performed. My own view is that appeal to a preference relation is either idle or philosophically suspect. It is idle, if preferring doing a to not doing a is simply a way to represent that a is to be done (whether as the result of an command or the statement of an obligation); and it is philosophically suspect if what is to be done is to be *understood* in terms of what is best, since there are any number of respectable philosophical positions which reject this point of view.

My approach is therefore more in the tradition of those who, like Kamp (1973), Kratzer (1977), Mastop (2005) and Yablo (2011), think of an update as operating directly on the to-do list without any detour through a preference relation. What my approach brings to the table, as opposed to some of these alternative accounts, are two things. First, a sensitivity to hyperintensional considerations. Thus updating with X is in general different from updating with X ∨ (X ∧ Y), even though they are truth-functionally equivalent, since the former only permits the actions in compliance with X while the latter also permits the actions in compliance with X ∧ Y. Second, the mereological framework provided by an action space. This enables us to conceive the problem of updating in its full generality without tying the solution down to any particular conception or representation of what the actions on the to-do list might be and it provides the means by which a natural definition of updating in mereological terms might be given.

§9. Formal appendix.

9.1. Syntax and semantics. Recall from part I that imperative formulas (or what we might also call *action* or A-*formulas*) are constructed from imperative atoms $\alpha_1, \alpha_2, \ldots$ by means of negation (¬), conjunction (∧), disjunction (∨) and the verum constant ⊤. We now take O(X) and P(X) to be the *deontic atoms*, for X any A-formula; *deontic formulas* are constructed in the usual way from the deontic atoms by means of the usual truth-functional connectives, O-*formulas* from the O-atoms O(X), and P-*formulas* from the P-atoms. We use α, β, γ and the like for arbitrary imperative atoms, X, Y, Z and the like for arbitrary imperative formulas, and S, T, U and the like for arbitrary deontic formulas.

Recall the definitions of a state space and of a modalized state space from Part I. A state s of a modalized space $\boldsymbol{M} = (S, S^{\diamond}, \sqsubseteq)$ is said to be a *world-state* if it is consistent and if any consistent state is either a part of s or incompatible with s; and the space $\boldsymbol{M}$ itself is said to be a *W-space* if every consistent state of $\boldsymbol{M}$ is part of a world-state. An action space is just a state space under another name.

A *normative action space* **A** is a structure of the form $(A, C, \sqsubseteq)$, where $(A, \sqsubseteq)$ is an action space and C (code of conduct) is a nonempty convex subset of A which does not contain ■. A *normative action model* $\boldsymbol{M}$ is a structure $(A, C, \sqsubseteq, |\bullet|)$, where $(A, C, \sqsubseteq)$ is a normative action space and $|\bullet|$ is a bilateral valuation of the usual sort (and similarly when modalized models are in play). We assume that $|\alpha|^+$ and $|\alpha|^-$ are both nonempty for any atom α but place no other restrictions on models. From this it follows that $|X|^+$ and $|X|^-$ are both nonempty for any imperative formula X.

Relative to a normative action model $(A, C, \sqsubseteq, |\bullet|)$, we stipulate the following truth-theoretic clauses for the various deontic operators:

(i) $\models$O(X) if C subsumes X;
(ii) $\models$P(X) if X subserves C;
(iii) $\models$O^P(X) if C subsumes X and X subserves C.

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For the modal construal of these operators, we must suppose we are working within a modalized space $(A, C, \sqsubseteq, A^{\Diamond})$. Relative to such a space, we say that every subset Y of A *modally subserves* the subset X if every $b \in Y$ is compatible with an $a \in X$ and that X *modally subsumes* (*necessitates*) Y if any consistent extension $a^{+} \sqsupseteq a$ of an action $a \in X$ is compatible with an action $b \in Y$. The corresponding clauses, relative to a modalized model $(A, C, \sqsubseteq, A^{\Diamond}, |\bullet|)$ are then:

(i) $\models$O(X) if C modally subsumes X (the standard definition)
(ii) $\models$P(X) if X modally subserves C
(iii) $\models$O^{P}(X) if C modally subsumes X and X modally subserves C.

Soundness and completeness. We shall find it helpful to work up to the completeness proof for the full system DL by considering the subsystems DL(P) and DL(O).

A-Normal Forms. In the definitions below, we shall suppose that the formulas under consideration occur in a fixed order, that a conjunction $F_1 \wedge F_2 \wedge \cdots \wedge F_n$ of formulas, for $n \geq 0$, is $\top$ when $n = 0$, is F_1 when $n = 1$, and is $((\ldots ((F_1 \wedge F_2) \wedge F_3) \wedge \cdots \wedge F_{n-1}) \wedge F_n)$, with association from left to right, when $n > 0$, and that, likewise, a disjunction $F_1 \vee F_2 \vee \cdots \vee F_n$ of formulas, for $n \geq 0$, is $\bot$ when $n = 0$, is F_1 when $n = 1$, and is $((\ldots ((F_1 \vee F_2) \vee F_3) \vee \cdots \vee F_{n-1}) \wedge F_n)$, with association from left-to-right, when $n > 0$. We shall also suppose that the conjuncts or disjuncts of such a conjunction or disjunction occur in the fixed order, with F_1 preceding F_2, F_2 preceding F_3, and so on. In this way the conjunction or disjunction of a finite set of formulas is always unique.

An *action* or A-*literal from* the imperative atom α is α or its negation; and an A-*literal* is a literal from some atom. An A-*description in* the distinct atoms $\alpha_1, \alpha_2, \ldots, \alpha_n$, for $n \geq 0$, is a conjunction $\lambda_1 \wedge \lambda_2 \wedge \cdots \wedge \lambda_m$ of literals from $\alpha_1, \alpha_2, \ldots, \alpha_n$, with $0 \leq m \leq n$.

Suppose that $\lambda_1 \wedge \lambda_2 \wedge \cdots \wedge \lambda_m$ is an A-description and take a selection $\mu_1, \mu_2, \ldots, \mu_k$ of the literals $\lambda_1, \lambda_2, \ldots, \lambda_n$. Then $\mu_1 \wedge \mu_2 \wedge \cdots \wedge \mu_k$ (suitably ordered) is also an A-description and an A-description obtained in this way is said to be a *sub-description of* the original A-description $\lambda_1 \wedge \lambda_2 \wedge \cdots \wedge \lambda_m$.

A (*disjunctive*) *normal* A-*form in* the atoms $\alpha_1, \alpha_2, \ldots, \alpha_n$ is a disjunction $X_1 \vee X_2 \vee \cdots \vee X_m$ of A-descriptions $X_1, X_2, \ldots, X_m$ in $\alpha_1, \alpha_2, \ldots, \alpha_n$, with $m \geq 0$. A normal A-form $X_1 \vee X_2 \vee \cdots \vee X_m$, is said to be a *standard* A-*form* if it contains the A-description Y whenever X_i is a sub-description of Y and Y a sub-description of X_j for some disjuncts X_i and X_j of $X_1 \vee X_2 \vee \cdots \vee X_m$ (this condition is the syntactic analogue of convexity). Thus $\alpha_1 \vee (\alpha_1 \wedge \alpha_2 \wedge \alpha_3)$ is not standard while $\alpha_1 \vee (\alpha_2 \wedge \alpha_2) \vee (\alpha_1 \wedge \alpha_3) \vee (\alpha_2 \wedge \alpha_2 \wedge \alpha_3)$ is. The normal A-form $Y_1 \vee Y_2 \vee \cdots \vee Y_k$ is said to be the *full expansion of* the normal A-form $X_1 \vee X_2 \vee \cdots \vee X_m$ in the atoms $\alpha_1, \alpha_2, \ldots, \alpha_n$ if the disjuncts of $Y_1 \vee Y_2 \vee \cdots \vee Y_m$ are all and only those A-descriptions in $\alpha_1, \alpha_2, \ldots, \alpha_n$ of which some disjunct of $X_1 \vee X_2 \vee \cdots \vee X_m$ is a sub-description. Thus $\alpha_1 \vee (\alpha_2 \wedge \alpha_3) \vee (\alpha_1 \wedge \alpha_2) \vee (\alpha_1 \wedge \alpha_3) \vee (\alpha_1 \wedge \alpha_2 \wedge \alpha_3)$ is the full expansion of $\alpha_1 \vee (\alpha_2 \wedge \alpha_3)$ (relative to the atoms $\alpha_1, \alpha_2, \alpha_3$).

A *conjunctive normal* A-*form* is a conjunction of disjunctions of A-literals. As before, we allow null conjunctions and disjunctions and insist that the conjuncts and disjuncts conform to the fixed order.

Lemma 1. Any A-formula X in the atoms $\alpha_1, \alpha_2, \ldots, \alpha_n$ is analytically equivalent in $IL^{+}(>)$ to a standard A-form in the atoms $\alpha_1, \alpha_2, \ldots, \alpha_n$.

Proof. A straightforward modification of the proof of Lemma 17 of Fine (2016).

P-Normal Forms. A P-*atom in* the atoms $\alpha_1, \alpha_2, \ldots, \alpha_n$, $n \geq 0$, is a formula of the form P(X), where X is an A-description in $\alpha_1, \alpha_2, \ldots, \alpha_n$; and a P-*literal* in $\alpha_1, \alpha_2, \ldots, \alpha_n$ is

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either a P-atom in $\alpha_1, \alpha_2, \ldots, \alpha_n$ or its negation. A P-*description in* $\alpha_1, \alpha_2, \ldots, \alpha_n$ is then a conjunction $S_1 \wedge S_2 \wedge \cdots \wedge S_m$, $m \geq 0$, of P-literals in $\alpha_1, \alpha_2, \ldots, \alpha_n$.

The P-description $S_1 \wedge S_2 \wedge \cdots \wedge S_m$ in $\alpha_1, \alpha_2, \ldots, \alpha_n$ is said to be *classical* if, for each A-description X in $\alpha_1, \alpha_2, \ldots, \alpha_n$, either P(X) or else ¬P(X) is a conjunct of $S_1 \wedge S_2 \wedge \cdots \wedge S_m$; and the P-description $S_1 \wedge S_2 \wedge \cdots \wedge S_m$ is said to be *standard* if (i) it is classical, (ii) contains at least one conjunct P(X), and (iii) contains the conjunct P(Y) whenever it contains the conjunct P(X) and Y is a sub-description of X. Thus a classical P-description is consistent and complete with respect to the formulas P(X) for X an A-description in $\alpha_1, \alpha_2, \ldots, \alpha_n$. Note that any standard P-description will contain P(⊤), since ⊤ is a sub-description of any A-description X. Finally, a *normal* P-*form in* $\alpha_1, \alpha_2, \ldots, \alpha_n$ is a disjunction $S_1 \vee S_2 \vee \cdots \vee S_m$ of P-descriptions in $\alpha_1, \alpha_2, \ldots, \alpha_n$ and a *standard* P-*form* is a normal P-form all of whose disjuncts are standard.

Lemma 2. Each P-formula S in some atoms is provably equivalent in DL(P) to a standard P-form in those atoms.

Proof. By Lemma 1, each A-formula X in $\alpha_1, \alpha_2, \ldots, \alpha_n$ is analytically equivalent to a standard A-form $X_1 \vee X_2 \vee \cdots \vee X_m$ in $\alpha_1, \alpha_2, \ldots, \alpha_n$. In case $m = 0$, P(X) is provably equivalent in DL(P) to P(⊥), which, by P-Triviality, is provably equivalent to the standard P-form ⊥. In case $m > 0$, it follows by P-distribution that P(X) is equivalent to $P(X_1) \wedge P(X_2) \wedge \cdots \wedge P(X_m)$. Hence, by classical logic, S is equivalent to some normal P-form $X_1 \vee X_2 \vee \cdots \vee X_m$. Each disjunct X_i will contain P(⊤) as a conjunct, since otherwise it will contain ¬P(⊤) as a conjunct, contrary to P-Triviality. Also, if a disjunct contains P(X) without containing P(Y) for Y a sub-description of X, then it will contain ¬P(Y) and hence, by P-Weakening, the whole disjunct will be contradictory. Applying these simplifications, we may convert $X_1 \vee X_2 \vee \cdots \vee X_m$ into a standard P-form.

There is a way of providing a more compact formulation of a standard P-description $S_1 \wedge S_2 \wedge \cdots \wedge S_m$. For let $P(X_1), P(X_2), \ldots, P(X_k)$ be all of the P-atoms P(X) which occur as a conjunct in $S_1 \wedge S_2 \wedge \cdots \wedge S_m$. Then, given P-Distribution, the conjuncts $P(X_1), P(X_2), \ldots, P(X_k)$ may be replaced by the single conjunct $P(X_1 \vee X_2 \vee \cdots \vee X_k)$. Moreover, by P-Weakening, ¬P(X) may be dropped as a conjunct from $S_1 \wedge S_2 \wedge \cdots \wedge S_m$ if ¬P(X′) is a conjunct for some sub-description X′ of X. Accordingly, given A-descriptions $X_1, X_2, \ldots, X_k$ in $\alpha_1, \alpha_2, \ldots, \alpha_n$, let us use $P^M(X_1 \vee X_2 \vee \cdots \vee X_k)$ ('M' for maximal) as an abbreviation for the conjunction of $P(X_1 \vee X_2 \vee \cdots \vee X_k)$ and all of the P-literals ¬P(Y), where Y is the conjunction of a set of P-literals $\{\lambda_1, \lambda_2, \ldots, \lambda_k\}$ in $\alpha_1, \alpha_2, \ldots, \alpha_n$ which are such that, for each $i = 1, 2, \ldots, k$, λ_i is not a conjunct of X_i (the $\lambda_1, \lambda_2, \ldots, \lambda_k$ need not be distinct). Say that S is a *simplified* P-*form* in $\alpha_1, \alpha_2, \ldots, \alpha_n$ if it is a disjunction of maximal formulas $P^M(X_1 \vee X_2 \vee \cdots \vee X_k)$ in $\alpha_1, \alpha_2, \ldots, \alpha_n$. Thus what we have shown is

Lemma 3. Each P-formula S in the atoms $\alpha_1, \alpha_2, \ldots, \alpha_n$ is provably equivalent in DL(P) to a simplified P-form in $\alpha_1, \alpha_2, \ldots, \alpha_n$.

The System DL(P).

Theorem 4. The system DL(P) is sound and complete.

Proof. The proof of soundness is straightforward, though it is perhaps worth commenting on P-Equivalence and P-Weakening. We wish to show $P(X) \equiv P(X')$ is valid for X analytically equivalent to X′. Suppose P(X) is true in a model. Then X subserves C. But given the equivalence of X and X′, it follows that X' subserves X. Hence X' subserves C and P(X′) is true in the model. We also wish to show that $P(X \wedge Y) \supset P(X)$ is valid. Suppose $P(X \wedge Y)$ is true in a model and take an action $a \in X$. Y is nonempty (by the

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corresponding restriction on models) and hence contains an action b. So $a \sqcup b \in X \wedge Y$ is a part of an action in C and hence a is also a part of an action in C.

For completeness, we use normal forms. Suppose the P-formula S is consistent. By Lemma 2, it is provably equivalent to a simplified P-form $S_1 \vee S_2 \vee \cdots \vee S_m$, for $m > 0$, in the atoms $\alpha_1, \alpha_2, \ldots, \alpha_n$ of S (we might also have used standard P-forms). We can now read a normative action model $\boldsymbol{M} = (A, C, \sqsubseteq, |\bullet|)$ for S off from S_1, which is of the form $P^M(X_1 \vee X_2 \vee \cdots \vee X_k)$, $k > 0$:

$A = \{L$: L is a set of literals from $\alpha_1, \alpha_2, \ldots, \alpha_n\} \cup \{\blacksquare\}$, for some new element $\blacksquare$;
$C = \{L \in A$: L is the set of literals in some conjunction $X_i\}$;
$\sqsubseteq = \{(L, K) \in A^2: L \subseteq K\} \cup \{(L, \blacksquare): L \in A\}$;
$|\alpha| = (\{\alpha\}, \{\neg\alpha\})$.

It is readily established that $\boldsymbol{M}$ is indeed a normative action model; and it can be shown that S_1 and hence S is true in $\boldsymbol{M}$. The critical point is that if $\neg P(Y)$ is a conjunct of S_1 then, for each i …, k, Y will contain a literal as a conjunct which is not a conjunct of X_i and hence $\neg P(Y)$ will be true in $\boldsymbol{M}$.

O-Normal Forms. In the logic $IL^+(>)$: $X \wedge X$ is not in general equivalent to X; nor is $X \vee (Y \wedge Z)$ in general equivalent to $(X \vee Y) \wedge (X \vee Z)$, with the consequence that A-formulas cannot always be put into conjunctive normal form. However, results of this sort do hold *within* the context of the O-operator:

Lemma 5. (i) $O(X \wedge X)$ is provably equivalent in DL(O) to O(X);
(ii) O(X) is provably equivalent to $O(X')$ for some full expansion X' of X;
(iii) O(X) is provably equivalent to $O(X')$ for some conjunctive normal form X';
(iv) O(X) provably implies $O(X')$ in DL(O) whenever X analytically implies X' in the logic $IL^+(>)$.

Proof. (i) $O(X \wedge X)$ is equivalent to $O(X) \wedge O(X)$, by Distribution, which is equivalent to O(X).

(ii) By Lemma 2, we may suppose X is in (disjunctive) normal A-form $X_1 \vee X_2 \vee \cdots \vee X_m$. O(X) is provably equivalent to $O(X \wedge X \wedge \cdots \wedge X)$ (for at least m occurrences of X). Applying Distribution, $X \wedge X \wedge \cdots \wedge X$ is analytically equivalent to $X_1 \vee X_2 \vee \cdots \vee X_m \vee (X_1 \wedge X_2 \wedge \cdots \wedge X_m) \vee Y_1 \vee Y_2 \vee \cdots \vee Y_k$ (where the Y_j are conjunctions of some of $X_1, X_2, \ldots, X_m$), which is analytically equivalent to the full expansion Z of $X_1 \vee X_2 \vee \cdots \vee X_m$. By part (i), O(X) is equivalent in DL(O) to $O(X \wedge X \wedge \cdots \wedge X)$, which, by Equivalence, is equivalent to O(Z).

(iii) We may suppose X is in (disjunctive) normal A-form. Form the conjunctive normal A-form X' from X in the usual way and then the disjunctive normal A-form X'' of X'. It can be shown that the full expansions of X and X'' are the same and so, by part (ii), O(X) is provably equivalent to $O(X')$.

(iv) Suppose X analytically implies X'. Then $X \wedge X$ is analytically equivalent to $X \wedge X'$. So $O(X \wedge X)$ is provably equivalent in DL(O) to $O(X \wedge X')$. But O(X) provably implies $O(X \wedge X)$ and $O(X \wedge X')$ provably implies $O(X')$.

We turn to the normal forms in O. An O-*atom in* the atoms $\alpha_1, \alpha_2, \ldots, \alpha_n$, $n \geq 0$, is a formula of the form O(X), where X is a standard A-form in $\alpha_1, \alpha_2, \ldots, \alpha_n$ distinct from $\bot$; and an O-*literal* in $\alpha_1, \alpha_2, \ldots, \alpha_n$ is either an O-atom in $\alpha_1, \alpha_2, \ldots, \alpha_n$ or its negation. Note that X here is not an A-description but a normal A-form, i.e., a disjunction of A-descriptions; and note also that we do not allow $O(\bot)$.

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An O-*description in* $\alpha_1, \alpha_2, \ldots, \alpha_n$ is now a conjunction $S_1 \wedge S_2 \wedge \cdots \wedge S_m$, $m \geq 0$, of O-literals in $\alpha_1, \alpha_2, \ldots, \alpha_n$. The O-description $S_1 \wedge S_2 \wedge \cdots \wedge S_m$ is said to be *classical* if, for each O-description X in $\alpha_1, \alpha_2, \ldots, \alpha_n$, either O(X) or else ¬O(X) is a conjunct of $S_1 \wedge S_2 \wedge \cdots \wedge S_m$. Say that the normal A-form $Y_1 \vee Y_2 \vee \cdots \vee Y_k$ *falls under* the normal A-form $X_1 \vee X_2 \vee \cdots \vee X_m$ if each disjunct X_i of $X_1 \vee X_2 \vee \cdots \vee X_m$ has some disjunct Y_j of $Y_1 \vee Y_2 \vee \cdots \vee Y_k$ as a sub-description (some disjuncts Y_j may not be sub-descriptions of any X_i). We then say that an O-description $S_1 \wedge S_2 \wedge \cdots \wedge S_m$ is *standard* if it (i) is classical and (ii) contains a "maximal" conjunct O(X), which is such that, for any standard A-form Y in $\alpha_1, \alpha_2, \ldots, \alpha_n$, O(Y) is a conjunct of $S_1 \wedge S_2 \wedge \cdots \wedge S_m$ if and only if Y falls under X. Note that any standard O-description will contain O(⊤), since ⊤ is a sub-description of any A-description X. Also note that the maximal conjunct O(X) need not be unique. Thus if O(X ∨ Y) is maximal then so is O(X ∨ Y ∨ (X ∧ Y)) (allowing for reorder). Finally, a *normal* O-*form in* $\alpha_1, \alpha_2, \ldots, \alpha_n$ is a disjunction $S_1 \vee S_2 \vee \cdots \vee S_m$ of O-descriptions and a *standard* O-*form* is a normal P-form all of whose disjuncts are standard.

Lemma 6. Each O-formula S in some atoms is provably equivalent in DL(O) to a standard O-form in those atoms.

Proof. By Lemma 1, each A-formula X is analytically equivalent to a standard A-form $X_1 \vee X_2 \vee \cdots \vee X_m$ (in the given atoms $\alpha_1, \alpha_2, \ldots, \alpha_n$). In case $m = 0$, O(X) is provably equivalent in DL(O) to O(⊥), which, by O-Triviality, is provably equivalent to the standard O-form ⊥. So we may suppose $m > 0$. By classical logic, S is equivalent to a normal O-form $S_1 \vee S_2 \vee \cdots \vee S_m$. Each disjunct S_i will contain O(⊤) as a conjunct, since otherwise it will contain ¬O(⊤) as a conjunct, contrary to O-Triviality. Let $Y_1, Y_2, \ldots, Y_k$ be all the A-descriptions Y for which O(Y) is a conjunct of S_i. By O-Distribution, $O(Y_1) \wedge O(Y_2) \wedge \cdots \wedge O(Y_k)$ implies $O(Y_1 \wedge Y_2 \wedge \cdots \wedge Y_k)$ in DL(O). It follows that O(Z) must be a conjunct of S_i for some standard A-form Z analytically equivalent to $Y_1 \wedge Y_2 \wedge \cdots \wedge Y_k$. Z is obtained by Distribution (of ∧ over ∨) from $Y_1 \wedge Y_2 \wedge \cdots \wedge Y_k$ and it is readily shown that each Y_k falls under Z. Moreover, it follows from O-Weakening and Lemma 5(iv) that, whenever a standard A-form Y in $\alpha_1, \alpha_2, \ldots, \alpha_n$ falls under Z, O(Z) will provably imply O(Y) and hence O(Y) will be a conjunct of X_i.

There is a way of providing a more compact formulation of a standard O-description $S_1 \wedge S_2 \wedge \cdots \wedge S_m$. For pick O(Z) as in the proof of Lemma 6, $Z = Z_1 \vee Z_2 \vee \cdots \vee Z_k$. Then each O(Y) that occurs as a conjunct in $S_1 \wedge S_2 \wedge \cdots \wedge S_m$ is provably implied by O(Z) and may therefore be dropped. Suppose ¬O(Y) is a conjunct of $S_1 \wedge S_2 \wedge \cdots \wedge S_m$, for $Y = Y_1 \vee Y_2 \vee \cdots \vee Y_l$. Then Y does not fall under Z and so, for some Z_j, no Y_i is a sub-description of Z_j. For each Y_i, pick a literal conjunct λ_i of Y_i that is not a conjunct of Z_j. Then $\lambda_1 \vee \lambda_2 \vee \cdots \vee \lambda_l$ does not fall under Z and so $\neg O(\lambda_1 \vee \lambda_2 \vee \cdots \vee \lambda_l)$ is a conjunct of $S_1 \wedge S_2 \wedge \cdots \wedge S_m$. Moreover, $\neg O(\lambda_1 \vee \lambda_2 \vee \cdots \vee \lambda_l)$ provably implies $\neg O(Y_1 \vee Y_2 \vee \cdots \vee Y_l)$ and so $\neg O(Y_1 \vee Y_2 \vee \cdots \vee Y_l)$ may be dropped from $S_1 \wedge S_2 \wedge \cdots \wedge S_m$. Accordingly, given a standard A-descriptions $X_1, X_2, \ldots, X_k$ in $\alpha_1, \alpha_2, \ldots, \alpha_n$, let us use $O^M(X_1 \vee X_2 \vee \cdots \vee X_k)$ as an abbreviation for the conjunction of $O(X_1 \vee X_2 \vee \cdots \vee X_k)$ and all the O-literals of the form ¬O(Y), where Y is the disjunction of a set of literals $\{\lambda_1, \lambda_2, \ldots, \lambda_l\}$ in $\alpha_1, \alpha_2, \ldots, \alpha_n$ which are such that, for some X_j, $\lambda_1, \lambda_2, \ldots, \lambda_l$ are all the literals from $\alpha_1, \alpha_2, \ldots, \alpha_n$ which do not occur as a conjunct in X_j. Say that S is a *simplified* O-*form* in $\alpha_1, \alpha_2, \ldots, \alpha_n$ if it is a disjunction of maximal O-formulas $O^M(X_1 \vee X_2 \vee \cdots \vee X_k)$ in $\alpha_1, \alpha_2, \ldots, \alpha_n$. Thus what we have shown is:

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Lemma 7. Each O-formula S in some atoms is provably equivalent in DL(O) to a simplified O-form in those atoms.

The System DL(O).

Theorem 8. The system DL(O) is sound and complete.

Proof. The proof of soundness is straightforward, though it is perhaps worth commenting on O-Equivalence. We wish to show $O(X) \equiv O(X')$ is valid for X analytically equivalent to X′. Suppose O(X) is true in a model then C subsumes X. But given the equivalence of X and X′, it follows that X subsumes X'. Hence C subsumes X' and O(X′) is true in the model.

We turn to completeness. Suppose the O-formula S is consistent. By Lemma 7, it is provably equivalent to a simplified O-form $S_1 \vee S_2 \vee \cdots \vee S_m$, for $m > 0$, in the atoms α_1, $\alpha_2, \ldots, \alpha_n$ of S. We can now read a normative action model $\boldsymbol{M} = (A, C, \sqsubseteq, |\bullet|)$ for S off from S_1, which is of the form $O^M(X_1 \vee X_2 \vee \cdots \vee X_k)$, $k > 0$. We set

$A = \{L$: L is a set of literals from $\alpha_1, \alpha_2, \ldots, \alpha_n\} \cup \{\blacksquare\}$, for some new element $\blacksquare$;
$C = \{L \in A$: L is the set of literals in some conjunction $X_i\}$;
$\sqsubseteq = \{(L, K) \in A^2: L \subseteq K\} \cup \{(L, \blacksquare): L \in A\}$;
$|\alpha| = (\{\alpha\}, \{\neg\alpha\})$.

It is readily established that $\boldsymbol{M}$ is indeed a normative action model; and it can be shown that S_1 and hence S is true in $\boldsymbol{M}$. The critical point in the proof is that if ¬O(Y) is a conjunct of S_1 for $Y = \lambda_1 \vee \lambda_2 \vee \cdots \vee \lambda_l$ then, for some X_i, no λ_j will be a conjunct of X_i and hence ¬O(Y) will be true in $\boldsymbol{M}$.

D-Normal Forms. We take a D-*description in* the atoms $\alpha_1, \alpha_2, \ldots, \alpha_n$, $n \geq 0$, to be a conjunction of the form $P^M(X_1 \vee X_2 \vee \cdots \vee X_m) \wedge O^M(Y_1 \vee Y_2 \vee \cdots \vee Y_l))$ and we take a D-*normal* form to be a disjunction of D-descriptions. By combining the proofs of Lemmas 3 and 7, we obtain

Lemma 9. Each D-formula is provably equivalent in DL to a normal D-form.

The P- and O-formulas in a D-description will interact. Suppose we are given a D-description of the form $P^M(X_1 \vee X_2 \vee \cdots \vee X_k) \wedge O^M(Y_1 \vee Y_2 \vee \cdots \vee Y_l)$. By the second Mixture Axiom, $P(X_i) \wedge O(Y_1 \vee Y_2 \vee \cdots \vee Y_l)$ will imply $P(X_i \wedge Y_1) \vee P(X_i \wedge Y_2) \vee \cdots \vee P(X_i \wedge Y_l)$. But each $P(X_i \wedge Y_j)$ implies $P(X_i)$; so $\bigwedge_i [P(X_i \wedge Y_1) \vee P(X_i \wedge Y_2) \vee \cdots \vee P(X_i \wedge Y_l)]$ implies $P^M(X_1 \vee X_2 \vee \cdots \vee X_k)$; and so $P^M(X_1 \vee X_2 \vee \cdots \vee X_k)$ may be dropped in favor of $\bigwedge_i [P(X_i \wedge Y_1) \vee P(X_i \wedge Y_2) \vee \cdots \vee P(X_i \wedge Y_l)]$. Distributing through, we see that we can require that, in the original D-description $P^M(X_1 \vee X_2 \vee \cdots \vee X_k) \wedge O^M(Y_1 \vee Y_2 \vee \cdots \vee Y_l)$, each X_i has some Y_j as a sub-description. Suppose now that some Y_j is not a sub-description of any X_i. Then $\neg P(Y_j)$ is a conjunct of $P^M(X_1 \vee X_2 \vee \cdots \vee X_k)$. By the first Mixture Axiom, $O(Y_1 \vee Y_2 \vee \cdots \vee Y_l) \wedge \neg P(Y_j)$ implies $O(Y_1 \vee Y_2 \vee \cdots \vee Y_{j-1} \vee Y_{j+1} \vee \cdots \vee Y_l)$ and, by Weakening, $O(Y_1 \vee Y_2 \vee \cdots \vee Y_{j-1} \vee Y_{j+1} \vee \cdots \vee Y_l)$ implies $O(Y_1 \vee Y_2 \vee \cdots \vee Y_l)$. Thus in this case, $O(Y_1 \vee Y_2 \vee \cdots \vee Y_l)$ may be replaced by $O(Y_1 \vee Y_2 \vee \cdots \vee Y_{j-1} \vee Y_{j+1} \vee \cdots \vee Y_l)$. We may therefore assume that, in the original D-description $P^M(X_1 \vee X_2 \vee \cdots \vee X_k) \wedge O^M(Y_1 \vee Y_2 \vee \cdots \vee Y_l)$, each Y_j is a sub-description of some X_i. Accordingly, let us say that a D-description $P^M(X_1 \vee X_2 \vee \cdots \vee X_k) \wedge O^M(Y_1 \vee Y_2 \vee \cdots \vee Y_l)$ is a *simplified* D-description if (i) each X_i has some Y_j as a sub-description and (ii) each Y_j is a sub-description of some X_i (this is the syntactic analogue of the requirement that the content of the obligation should be part of the content of the permission).

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We have therefore proved:

Lemma 10. Each D-formula in some atoms is provably equivalent in DL to a simplified D-form in those atoms.

Theorem 11. The system DL is sound and complete.

Proof. We comment on the validity of the Mixture Axioms. First consider $O(X \vee Y) \wedge \neg P(X) \supset O(Y)$ for definite X, and suppose $O(X \vee Y)$ and $\neg P(X)$ are true in a given model. Take any action $c \in C$. Then $c \sqsupseteq a$ for some $a \in X \cup Y$. But $a \notin Y$ since, given that X is definite, a would otherwise be the sole member of X and so P(X) would then be true. So $a \in X$ and consequently O(X) is true. Now consider $O(X_1 \vee X_2 \vee \cdots \vee X_n) \wedge P(Y) \supset P(X_1 \wedge Y) \vee P(X_2 \wedge Y) \vee \cdots \vee P(X_n \wedge Y)$, for definite $X_1, X_2, \ldots, X_n$ and Y, and suppose $O(X_1 \vee X_2 \vee \cdots \vee X_n)$ and P(Y) are both true. Then where b is the sole member of Y, $b \sqsubseteq c$ for some $c \in C$. But $c \in a$ for some $a \in X_1 \cup X_2 \cup \cdots \cup X_n$. Given that $X_1, X_2, \ldots, X_n$ are definite, a is the sole member of some X_i and $a \sqcup b$ is the sole member of $X_i \wedge Y$; and so $P(X_i \wedge Y)$ is true.

We turn to completeness. Suppose the D-formula S is consistent. By Lemma 2, it is provably equivalent to a simplified D-form $S_1 \vee S_2 \vee \cdots \vee S_m$, for $m > 0$, in the atoms $\alpha_1, \alpha_2, \ldots, \alpha_n$ of S. We can now read a normative action model $\boldsymbol{M} = (A, C, \sqsubseteq, |\bullet|)$ for S off from S_1, which is of the form $P^M(X_1 \vee X_2 \vee \cdots \vee X_k) \wedge O^M(Y_1 \vee Y_2 \vee \cdots \vee Y_l)$, $k, l > 0$. We set:

> $A = \{L$: L is a set of literals from $\alpha_1, \alpha_2, \ldots, \alpha_n\} \cup \{\blacksquare\}$, for some new element $\blacksquare$;
> $C = \{L \in A$: L is the set of literals in some conjunction X_i or in some conjunction $Y_j\}$;
> $\sqsubseteq = \{(L, K) \in A^2: L \subseteq K\} \cup \{(L, \blacksquare): L \in A\}$;
> $|\alpha| = (\{\alpha\}, \{\neg\alpha\})$.

It is readily established that $\boldsymbol{M}$ is indeed a normative action model; and it can be shown that S_1 and hence S is true in $\boldsymbol{M}$. In establishing the truth of $O(Y_1 \vee Y_2 \vee \cdots \vee Y_l)$, it is critical that each X_i has some Y_j as a sub-description; and in establishing the truth of a conjunct $\neg P(Z)$, it is critical that each Y_j be a sub-description of some X_i.

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PHILOSOPHY DEPARTMENT
NEW YORK UNIVERSITY
NEW YORK, NY 10003, USA
E-mail: kf14@nyu.edu

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