A NOTE ON SUBADDITIVITY OF ZERO-UTILITY PREMIUMS

BY

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ABSTRACT

Many papers in the literature have adopted the expected utility paradigm to analyze insurance decisions. Insurance companies manage policies by growing, by adding independent risks. Even if adding risks generally ultimately decreases the probability of insolvency, the impact on the insurer's expected utility is less clear. Indeed, it is not true that the risk aversion toward the additional loss generated by a new policy included in an insurance portfolio always decreases with the number of contracts already underwritten. The present paper derives conditions under which zero-utility premium principles are subadditive for independent risks. It is shown that subadditivity is the exception rather than the rule: the zero-utility premium principle generates a superadditive risk premium for most common utility functions. For instance, all completely monotonic utility functions generate superadditive zero-utility premiums. The main message of the present paper is thus that the zero-utility premium for a marginal policy is generally not sufficient to guarantee the formation of insurance portfolios without additional capital.

KEYWORDS

Expected utility, compensating premium, properness.

1. INTRODUCTION AND MOTIVATION

For decades, actuaries have used indifference arguments, after Bühlmann (1970) introduced the equivalent utility principle in the context of setting insurance premiums. Assume that the insurer acts in order to maximize expected utility and possesses some non-decreasing and concave utility function $u(\cdot)$. Given a random variable Z with distribution function $F_Z(\cdot)$, its expected utility for the insurer is given by

$$\mathbb{U}[Z] = \int_{-\infty}^{+\infty} u(z) \, dF_Z(z) = \mathbb{E}[u(Z)].$$

Now, let L denote the amount paid in execution of some insurance contract. Then, $\mathbb{E}[L]$ is the corresponding pure premium. The insurer is ready to cover $X = \mathbb{E}[L] - L$ for a price at least equal to the solution $\pi[X]$ of the equation

$$\mathbb{U}\big[\kappa + \pi[X] + X\big] = \mathbb{U}[\kappa] = u(\kappa) \tag{1.1}$$

where κ is the amount of capital owned by the insurer. Note that in this paper we consider that the amount of capital κ is held fixed.

The premium charged to the policyholder for covering L is then $\mathbb{E}[L] + \pi[X]$ and is known as the zero-utility premium (the amount of capital in (1.1) is considered as fixed and can without loss of generality be set equal to zero by redefining the utility function). See e.g. Goovaerts et al. (1984) or Kaas et al. (2008). After Pratt (1964), Eeckhoudt & Gollier (2001) called $\pi[X]$ the compensating premium. It is the price to be paid to compensate for the bearing of the zero-mean random variable X, when initially the insurer holds the deterministic capital κ. Condition (1.1) expresses that $\pi[X]$ is fair in terms of utility: the right-hand side of (1.1) represents the utility without X; the left-hand side of (1.1) represents the expected utility of the insurer covering X. Therefore (1.1) means that, provided an amount of $\pi[X]$ is obtained, the expected utility of wealth with X is equal to the utility without X: (1.1) can be interpreted as an equality between the expected utility of the income $\pi[X] + X$ and the utility of not accepting X.

The properties of $\pi[\cdot]$ are inherited from those of the utility function $u(\cdot)$. Assuming that $u(\cdot)$ is non-decreasing and concave then $\pi[\cdot]$ is non-negative. Furthermore, the zero-utility premium principle is known to be additive for linear and exponential utilities and to agree with second-degree stochastic dominance.

In this paper, our aim is to compare $\pi[X] + \pi[Y]$ to $\pi[X + Y]$ when X and Y are independent. With compensation of independent risks in mind, we expect that the inequality $\pi[X] + \pi[Y] \geq \pi[X + Y]$ holds true (at least for identically distributed random variables), a property known as subadditivity. However, we show in this paper that this inequality is generally reversed so that zero-utility premiums generally violate the requirement of subadditivity. In particular, subadditivity often does not hold even for independent and identically distributed centered losses X and Y, the typical situation where aggregating insurance risks is viewed as beneficial because of diversification.

The present paper also raises the following important question: are there economies of scale from diversification in the insurance sector? This question is translated into the following mathematical problem. If an insurance portfolio with a random centered loss X for the insurer is acceptable, is it desirable to accept another insurance portfolio with centered loss Y, where X and Y are independent and identically distributed? Using the expected utility criterion, we show that this is in general false.

The limiting case of additivity for independent risks ($\pi[X] + \pi[Y] = \pi[X + Y]$) justifies premium computation from the top down; see, e.g., Borch (1962) or

Bühlmann (1985). The most general representation of risk measures that are additive for independent random variables is due to Gerber & Goovaerts (1981) and is known as the mixed Esscher principle. Despite the numerous appealing features of the Esscher principle, it does not satisfy the strong monotonicity requirement. Counterexamples have been provided by Van Heerwaarden, Kaas & Goovaerts (1989). This is why Goovaerts, Kaas, Laeven & Tang (2004) provided a new axiomatic characterization of risk measures that are additive for independent random variables, involving an axiom that guarantees monotonicity. The obtained risk measure is a restricted version of the mixed Esscher principle that can be regarded as an ordinary mixture of exponential premiums. In a recent contribution, Goovaerts, Kaas & Laeven (2010) prove that within rank-dependent utility, including expected utility for decision under risk considered here as a special case, the zero-utility premium is additive for independent risks if, and only if, it is an exponential premium.

The paper is organized as follows. In Section 2, we discuss an important characteristic of a utility function, called the acceptance property after Diamond (1984). This concept is closely related to the subadditivity of zero-utility premiums. Then, in Section 3, we show that most utility functions used in the literature do not possess this property. We also stress the importance of the domain of the utility function when acceptance property is considered. Section 4 concludes.

2. Subadditivity of π and acceptance property

2.1. Acceptance property

Many explanations for the creation of insurance portfolios are based on a loose application of the law of large numbers and of the central-limit theorem. Most authors use these fundamental results of probability theory to show that the average loss per policyholder becomes more concentrated around the mean as the size of the portfolio increases. However, the insurer is not so much interested in the average loss per policy, but rather in its total payout. As pointed out by Brockett (1983), large deviations theorems are the appropriate tools to study exceedance probabilities for the insurer's total payout as the portfolio size increases.

Smith & Kane (1994) explained that the insurance is made possible by the inclusion of loadings, i.e. excesses of the premiums paid over the corresponding expected losses. In the case of independent risks, loadings ensure that the insolvency probability becomes negligible as the size of the portfolio is large enough. Contributions in excess of the insured's expected loss create capacity to absorb deviations from the expected outcomes. This explains why insurance is beneficial.

As insurance policies are purchased to protect policyholders against adverse financial contingencies, insolvency risk plays a special role in the insurance industry. Risk capital is held to assure policyholders that claims can be paid even if larger than expected.

Increasing the number n of independent policies is expected to decrease the probability of losing money so that the collective of n policies may be found acceptable. This, however, confuses risk with insolvency. Increasing the size n of the portfolio may increase the risk even though it lessens the probability of insolvency and lowers expected loss. Even if a large gain becomes highly probable, a loss is still possible and its disutility may be considerable. An expected utility maximizer must take the disutility into account. Equating the decreasingness in the insolvency probability to less risk falls into what Samuelson (1963) termed the "fallacy of large numbers". An insurer with $n = 10,000$ policies may be less likely to become insolvent than an insurer with $n = 1,000$ policies but it may also generate a much larger loss. Also, the variance which is a classical measure of risk, grows linearly with the size of the portfolio. It is thus not obvious, as it may first seem to an actuary, that the group of n contracts can be accepted if a single one is rejected. For instance, the probability of insolvency may not decrease fast enough compared to the negative tail of the utility function, for the collective to be accepted.

Samuelson (1963) termed adding risks a "fallacy of large numbers" because it is not true for all risk averse utility functions that the risk aversion toward the nth independent risk is a decreasing function of n. Samuelson (1963) established that if a utility function $u(\cdot)$ rejects a risk X at all wealth levels, then it will also reject any collective $\sum_{i=1}^{n} X_i$ when the X_i's are independent copies of X. This leads to a real world paradox since the probability of insolvency is often found to be decreasing in n. Ross (1999) pointed out that the application of the result of Samuelson (1963) is nevertheless limited since the only utilities rejecting the same risk at all wealth levels are the linear and the exponential utility functions. In the latter case, (1.1) admits the explicit solution

$$\pi[X] = \frac{1}{a} \ln L_X(a) \qquad (2.1)$$

where $a > 0$ reflects absolute risk aversion and $L_X(\cdot)$ is the Laplace transform of X, that is, $L_X(t) = \mathbb{E}[\exp(-tX)]$. We thus see that the compensating premium does not depend on the amount of capital κ owned by the insurance company. In this case, the analysis of Samuelson (1963) applies. Hence, if the premium charged to a policyholder generating a centered loss X is smaller than the compensating premium $\pi[X]$ in (2.1) then no collective $\sum_{i=1}^{n} X_i$ made of independent copies of X will be accepted by an insurer with an exponential utility, no matter the number n of policies. Since (2.1) is additive for independent losses, it is easily seen that such an insurer agrees to increase the size of the portfolio as long as the premium charged to each contract is at least equal to (2.1). In this case, there is no net diversification benefit.

Diamond (1984) was the first to provide conditions under which adding risks is beneficial, that is, the conditions for adding independent risks to reduce insurer's risk aversion, which are the conditions when Samuelson (1963)'s "fallacy of large numbers" is not a fallacy.

Nielsen (1985) found necessary and sufficient conditions for a concave utility function to eventually accept a sequence of bounded independent and identically distributed risks when the insurer charges a loaded premium. Lippman & Mamer (1988) extended this result to the unbounded case. Hellwig (1995) generalized Samuelson's (1963) problem to comparisons of sums of independent and identically distributed bounded risks. Specifically, Hellwig (1995) showed that if a decision-maker's utility function satisfies certain additional conditions at large negative and large positive wealth levels, then the law of large numbers is relevant, and the choice between such sums will be guided by expected values if the numbers of terms in these sums is large enough.

Remark 2.1. (Eventual acceptance) When the acceptance property holds, adding a new contract is beneficial whatever the number of policies. The acceptance property may thus appear quite restrictive, in that the actuary may value diversification only if the size of the portfolio is large enough. A weaker requirement is that this might occur only for sufficiently large portfolios. This leads to the concept of "eventual acceptance", introduced by Ross (1999) and further studied by Hammarlid (2005). The eventual acceptance can be defined as the property for which there exists a finite n (i.e. there exists a sufficiently large portfolio) such that, given the sequence $X_1, X_2, ..., X_n$, the collective $X_1 + ... + X_n$ will eventually be accepted. In relation to Samuelson (1963), Ross (1999) found a class of utility functions, exponentially bounded from below, that eventually accept sequences of good deals. In this work we do not consider this alternative concept and we focus on Diamond's condition, where the requirement is for portfolios of every dimension.

2.2. Link with subadditivity

According to Diamond (1984), adding independent risks provides true diversification if the incremental compensating premium for adding the second risk to the portfolio is lower than for adding the first risk. Consider independent (but not necessarily identically distributed) centered random variables X and Y, and define $\pi[X]$, $\pi[Y]$, and $\pi[X + Y]$ as the solutions of (1.1) for X, Y, and $X + Y$, respectively. Adding risks reduces the incremental compensating premium if

$$\pi[X + Y] < \pi[X] + \pi[Y]. \tag{2.2}$$

This means that the functional $\pi[\cdot]$ is subadditive[1].

   In order to show that this favors adding risks, let us consider independent (but non-necessarily identically distributed) zero-mean random variables X_1, X_2, ... representing (centered) insurance losses. The increase in the compensating

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[1]   As an alternative to the subadditivity property one might study the weaker convexity property. In this paper, we confine our analysis to subadditivity.

premium caused by the addition of a new policy X_{n+1} in a portfolio $\sum_{i=1}^{n} X_i$ of size n is

$$\pi\left[\sum_{i=1}^{n+1} X_i\right] - \pi\left[\sum_{i=1}^{n} X_i\right].$$

If we take $X = \sum_{i=1}^{n} X_i$ and $Y = X_{n+1}$ then (2.2) ensures that

$$\pi\left[\sum_{i=1}^{n+1} X_i\right] - \pi\left[\sum_{i=1}^{n} X_i\right] < \pi[X_{n+1}]. \tag{2.3}$$

Hence, it is less expensive to cover X_{n+1} if the policy is included in an existing portfolio of n independent risks $X_1, X_2, \ldots, X_n$ than to cover X_{n+1} in isolation. This property is called the "acceptance property" of the insurer's utility function.

2.3. Sufficient condition for acceptance

Let us come back to (2.2) and define the indirect utility function $v(\cdot)$ as

$$v(y) = \mathbb{E}\big[u(\kappa + X + y)\big].$$

This is also a utility function and it inherits non-decreasingness and concavity from $u(\cdot)$. The expected utility of the insurer bearing $X + Y$ is $\mathbb{E}[v(Y)]$. Let us denote as $u^{(1)}(\cdot), u^{(2)}(\cdot), \ldots$ the successive derivatives of the utility function $u(\cdot)$ starting with the marginal utility $u^{(1)}(\cdot)$, with similar notations involving $v(\cdot)$. The condition (2.2) holds if $u(\cdot)$ is more risk averse than $v(\cdot)$, that is,

$$-\frac{v^{(2)}(0)}{v^{(1)}(0)} = -\frac{\mathbb{E}\big[u^{(2)}(\kappa + X)\big]}{\mathbb{E}\big[u^{(1)}(\kappa + X)\big]} < -\frac{u^{(2)}(\kappa)}{u^{(1)}(\kappa)}. \tag{2.4}$$

Sufficient conditions for $u(\cdot)$ to be more risk averse than $v(\cdot)$ can be obtained by finding sufficient conditions for (2.4) to hold. As pointed out by Diamond (1984), a set of sufficient conditions obtained by Jensen's inequality is given by $u^{(3)}(x) \geq 0$ and $u^{(4)}(x) \geq 0$ for all x (with at least one strict inequality). In Section 3, we show that some care is needed when imposing these conditions since they restrict the domain of definition of the utility function.

Note that these sufficient conditions are determined only by imposing restrictions on the utility function and do not depend on the distribution of X: if these conditions are satisfied then the acceptance property holds whatever the distribution of X. Of course, we implicitly assume that the distribution of X is such that $\pi[X]$ is well-defined for the utility function $u(\cdot)$.

If $u^{(3)}(x) \leq 0$ and $u^{(4)}(x) \leq 0$ for all x (with at least one strict inequality) then the inequality in (2.4) is reversed and the functional $\pi[\cdot]$ is superadditive.

In this case, defining $\pi[X+Y]$, $\pi[X]$, and $\pi[Y]$ as the solution of the indifference equation (1.1), the superadditivity of $\pi[\cdot]$ means that

$$\pi[X+Y] > \pi[X] + \pi[Y]. \tag{2.5}$$

Thus, utility functions of this type do not positively value the compensation of independent risks. Indeed, inequality (2.3) is reversed in this case. Including a new risk X_{n+1} in a portfolio of size n is, thus, more expensive than covering X_{n+1} in isolation.

3. COMMONLY USED UTILITY FUNCTIONS DO NOT SATISFY THE ACCEPTANCE PROPERTY

In this section, we examine the implications of the acceptance property for some commonly used utility functions. Before doing this, we introduce some economic concepts related to risk aversion which are useful for this goal.

3.1. Decreasing absolute risk aversion

Whereas the sign of successive derivatives indicates the direction of various attitudes towards risks, they are silent about the intensity of such preferences. The coefficient of absolute risk aversion $a_u(\cdot)$ defined as $a_u(x) = -u^{(2)}(x)/u^{(1)}(x)$, measures the strength of risk aversion for utility function $u(\cdot)$.

Recall that the utility function $u(\cdot)$ exhibits decreasing absolute risk aversion (DARA) if the coefficient of absolute risk aversion $a_u(\cdot)$ is decreasing. The decreasingness of risk aversion in wealth is usually considered as a very natural requirement: getting richer may never increase your aversion to risk.

3.2. Decreasing absolute prudence

More recently, several authors used the coefficient of absolute prudence $p_u(\cdot)$ defined as $p_u(x) = -u^{(3)}(x)/u^{(2)}(x)$. Also, $u(\cdot)$ has decreasing absolute prudence (DAP) if the index of absolute prudence $p_u(\cdot)$ is decreasing in x.

DAP often implies DARA. Indeed, both Kimball (1993) and Maggi, Magnani & Menegatti (2006) prove that global DAP implies global DARA. Specifically, under the assumption $u^{(2)}(+\infty) = 0^+$, each local minimum (maximum) of the function $x \mapsto a_u(x)$ is followed (or coincides with) at least one local minimum (maximum) of the function $x \mapsto p_u(x)$. This means that global DAP implies global DARA under this assumption.

3.3. Proper risk aversion

Properness has been defined by Pratt & Zeckhauser (1987) to answer the following question: if an individual considering two independent undesirable

risks is required to take one of them, should he continue to find the other one undesirable? Under proper risk aversion, two independent risks that are separately undesirable are never jointly desirable. Note that this is the opposite view to Ross (1999) who argues that independent risks could be complement: two separately undesirable risks can be jointly desirable. Proper risk aversion is difficult to characterize and it is difficult to determine whether a particular utility function satisfies this condition. A necessary condition for properness is DARA while DARA and DAP together are sufficient for properness. In many cases, DAP alone is sufficient for properness (since DAP often implies DARA, as discussed above). All the commonly used utility functions are proper, including the exponential, the logarithmic, and the power utility functions.

3.4. Violation of the acceptance property

A number of papers have been devoted to the effect of one risk on other independent risks. The literature related to this effect is clearly going against subadditivity. If one is indifferent about risks X and Y in isolation, one should dislike the aggregate risk $X + Y$. To be precise, this is the essence of the notion of properness recalled above, which is satisfied by all standard utility functions (exponential, power, log, quadratic). So, the subadditivity of $\pi[\cdot]$ is an hypothesis which is known to be hopeless from reading the economic literature. In this framework, Eeckhoudt & Gollier (2001) proved that the compensating premium is superadditive in the number of independent and identically distributed risks if the utility function is proper. This result easily extends to independent but non necessarily identically distributed risks.

Under proper risk aversion, defining $\pi[X + Y]$, $\pi[X]$, and $\pi[Y]$ as the solution of the indifference equation (1.1), we thus have

$$\pi[X + Y] \geq \pi[X] + \pi[Y]. \tag{3.1}$$

Such a utility function thus violates the acceptance property. Note that properness is sufficient for (3.1) to hold, but not necessary. Eeckhoudt & Schlesinger (2001) related superadditivity to the class of temperate utility functions (i.e. utility functions $u(\cdot)$ such that $u^{(4)}(x) \leq 0$ for all x).

This leads to the following result, which is a direct consequence of Pratt & Zeckhauser (1987). If the utility function exhibits proper risk aversion then the acceptance property (2.2) is not satisfied. For instance, the acceptance property (2.2) is not satisfied for the following utility functions:

  (i) a power utility function;
 (ii) a logarithimic utility function;
(iii) a HARA utility function of the form $u(x) = a(b + xc^{-1})^{1-c}$ with $c > 0$;
 (iv) an exponential utility function;
  (v) a quadratic utility function.

For the utilities listed above, the zero-utility premium for a marginal policy is, thus, generally not sufficient to guarantee the formation of insurance portfolios at that premuim. However, this does not necessarily preclude the formation of insurance portfolios. After the first policy is issued, more capital may be required in order to issue the second for the same premium. While this is not true for constant absolute risk aversion, it is true for other commonly used utility functions. The main message of this paper is that the zero-utility premium for a marginal policy is generally not sufficient to guarantee the formation of insurance portfolios without additional capital.

Notice that properness does not exclude that a utility function satisfying (2.4) can be obtained for specific distributions of X. Moreover, if the utility function is only defined on $\mathbb{R}^+$ then appropriate feasibility restrictions on L (or on X) are obviously needed.

3.5. Domain of definition of insurers' utility functions

Considering utility functions defined over $\mathbb{R}^+$, Menegatti (2001) established that, once the signs of the first and second derivatives are fixed to be respectively positive and negative, the sign of the fourth derivative cannot be negative for all $x \in \mathbb{R}^+$. This means that decision-makers who are non-satiated and risk averse cannot exhibit $u^{(4)}(x) \geq 0$ for all $x \in \mathbb{R}^+$. This result implies the following one.

Proposition 3.1. *No increasing and concave utility function $u(\cdot)$ defined over $\mathbb{R}^+$ can satisfy Diamond's sufficient conditions for the acceptance property for all $x \in \mathbb{R}^+$.*

Proposition 3.1 implies that the sufficient conditions for the acceptance property (2.2) cannot be satisfied for the whole domain of the utility function if this domain is unbounded.

The result of Proposition 3.1 holds only under the assumption that the utility function u is defined over the domain $[0, +\infty)$. A different conclusion can be obtained if the function is defined over a domain which is bounded above, i.e when $x \in [0, x_1]$. In order to illustrate the case of a bounded domain, we examine the following example.

Example 3.2. Let us consider the following utility function defined on a bounded domain $[0, x_1]$:

$$u(x) = bx - cx^2 + dx^3 + x^a$$

with $x \in [0, x_1]$, $1 < a < 2$ and $b, c, d > 0$. An appropriate choice for contants b, c and d ensures that $u^{(1)}(x) > 0$, $u^{(2)}(x) < 0$ and $u^{(3)}(x) > 0$ while it is easy to see that $u^{(4)}(x) > 0$ for all x. This utility function, thus, satisfies Diamond's sufficient conditions.

With reference to the results in this subsection, it should finally be emphasized that the assumption of bounded domain for the utility function is not very strong for insurance problems. In fact, as insurance companies cover potential losses, the most favorable case is that no claim originates from the portfolio. The terminal wealth of the insurance company then equals the initial capital plus the annual premium income. This amount provides a natural upper bound on the domain of the utility function $u(\cdot)$. This means that our conclusions do not imply a strong constraint for actuarial applications while it gives some important indications on the characterization of the functions to be used in that context.

4. Conclusion

Insurance companies like to add independent (and identically distributed) risks in their portfolio in order to reduce the probability of insolvency. This note discusses the fact that the zero-utility premium principle generates a superadditive risk premium for most common utility functions. While the degree of novelty in this contribution is perhaps limited, it touches upon a key issue in insurance that is not very well understood, and that is presented here in a unifying manner.

There are two types of diversification. In the first case, the risk is subdivided into a number of independent fractions. According to this "risk subdividing" type of diversification, each risk averse economic agent obtains a higher expected utility by investing a fraction $\frac{1}{n}$ of initial wealth in each independent copy $X_1, X_2, \ldots, X_n$ of X than in X itself. This is the situation of a mutual insurer.

The second type of diversification is by adding risks. It is the situation of a private insurer bearing 100% of n independent risks, with diversification occurring as n grows. This is quite different from risk subdivision because the total risk imposed on the insurer rises as n grows, while with subdivision it falls. Samuelson (1963) termed diversification by adding risks a "fallacy of large numbers" because it is not true for all risk averse utility function that the risk aversion toward the nth independent risk is a decreasing function of n. Diamond (1984) was the first to provide conditions under which the second type of diversification is beneficial, that is, the conditions when adding independent risk reduces insurer's risk aversion, which are the conditions when Samuelson (1963)'s "fallacy of large numbers" is not a fallacy. According to Diamond (1984), adding independent risks provides true diversification if it reduces the risk premium, that is, diversification works if the incremental risk premium for adding the second risk to the portfolio is lower than for adding the first risk.

In this paper, we have analyzed in detail the reasons for the opposition between the "acceptance property" and the standard properties of the utility function. It is also shown that the sufficient conditions for adding risks that can be found in the literature need to be refined by restricting the domain of definition of the insurer's utility function. Let us mention that conditions for

the superadditivity of the prices of liabilities under mean-variance hedging were explored in Thomson (2005). See in particular the appendix to that article. Despite some differences inherent to the approaches adopted in the two papers, our findings are consistent with those of Thomson (2005).

In view of the results in the recent contribution by Goovaerts, Kaas & Laeven (2010) mentioned above, one may wonder to what extent the results discussed in this note can be generalized to rank-dependent utility. Together with replacing subadditivity with convexity, these are topics for future research.

Acknowledgements

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