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A new family of spatial 3-DOF parallel manipulators with two translational and one rotational DOFs

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SUMMARY
This paper proposes a new family of spatial 3-DOF (degree of freedom) parallel manipulators with two translational and one rotational DOFs. The manipulators in this family are the variations of the parallel manipulators, which are capable of very high rotational capability, introduced by X.-J. Liu, J. Wang, and G. Pritschow ("A new family of spatial 3-DoF fully parallel manipulators with high rotational capability," *Mech. Mach. Theory* **40**(4), 475–494, 2005). However, compared with those old manipulators, the new parallel manipulators proposed here have the advantages of simpler kinematics and structure, easier manufacturing, and energy saving.

KEYWORDS: Parallel manipulators; Type synthesis; Degree of freedom; Rotational capability.

1. Introduction

Unlike serial robots, parallel manipulators are usually good for motion/force transmission but not for dexterous manipulation. For such a reason, parallel manipulators are always welcome in the fields where high stiffness, high speed, and large payload capability are needed. Moreover, compared with serial robots, parallel manipulators have the advantages of compact structure, low moving inertia, and low cost. This type of manipulator has been studied intensively for more than 20 years and still attracts much attention from universities and industry.

In the field of parallel manipulators, one of the most important and interesting problems is the structure design, i.e. type synthesis. Up to now, so many parallel manipulators with specified number and type of DOF (degree of freedom) have been proposed (see ref. [1] for most examples). Most recently, several systematic approaches were also proposed for the type synthesis of parallel manipulators, such as methods based on displacement group theory,[2] methods based on screw theory,[3,4] the approach based on units of single-opened chains,[5] and the vector approach.[6] Especially, parallel manipulators with 3 DOFs are becoming more and more popular.[7–11] However, few researchers studied the spatial parallel manipulators with two translational and one rotational DOFs. Such a manipulator is usually capable to have a high rotational capability, from which most spatial mixed-DOF parallel manipulators suffer. The first version of such a manipulator was proposed by Liu *et al.* in 2001.[12] Later, a family of such manipulators was introduced in 2005.[13] In 2006, Kong and Gosselin[14] gave a synthesis method about this type of parallel manipulator based on the screw theory.

The parallel manipulators proposed in this paper are actually the variations of the parallel manipulators presented in ref. [13]. However, compared with the old manipulators, the parallel manipulators introduced in this paper have the obvious advantages in kinematics, architecture, manufacturing, and energy cost.

2. Architecture Description of the New Parallel Manipulator Family

A family of spatial 3-DOF fully parallel manipulators was introduced in ref. [13]. In this family, all manipulators consist of at least one parallelogram in their architectures. The parallelogram will eventually result in the difficulty of manufacturing and assembling. This will affect the accuracy and application of the manipulators. This motivates us to find a solution to overcome it.

2.1. HALF parallel manipulators*

As analyzed in ref. [13], the use of a parallelogram in each manipulator of the family guarantees the unique rotational DOF of the mobile platform. For example, as shown in Fig. 1, the parallelogram in the ***HALF*** manipulator can restrict the rotations about the z- and x-axes.[12] The translation in the O–yz plane of the mobile platform is actually implemented by actuating the sliders of the two legs (denoted as the first and second legs) with identical kinematic chains. The two legs are in a same plane, i.e. the O–yz plane. Therefore, the leg with a parallelogram (referred to as the third leg) acts the role of providing the mobile platform with the active rotation about the axis parallel to the y-axis and the passive translations along the x-, y-, and z-axes. It is noting that the translation along the x-axis is a parasitic motion. Observing the ***HALF*** manipulator, it is not difficult to find out that, at any position (y, z) of the mobile platform, the movement of the mobile platform and the third leg is actually that of a slider-crank mechanism. If the z-coordinate of the mobile platform is specified, the shape change of the parallelogram in the third leg is just conforming to the translation along the y-axis. Then it is feasible for us to replace the third leg of the

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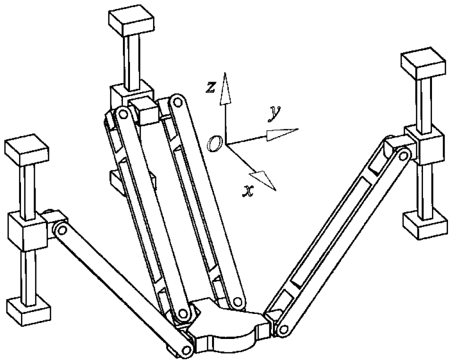


Fig. 1. The ***HALF*** parallel manipulator presented in ref. [12].

Table I. The constraint and DOFs of the ***HALF**** parallel manipulator with prismatic actuators.

	Single leg		Combination of three legs	
No.	Leg type	Constraints	Constraints	Remained DOFs
1	$P_z\ R_z\ U_{xy}$	$\{RO_x, T_x\}$	$\{T_x, RO_x, RO_z\}$	$\{T_y, T_z, RO_y\}$
2	$P_z\ R_x\ U_{xy}$	$\{RO_x, T_x\}$		
3	$P_z\ R_z\ U_{xy}$	$\{RO_x, T_x\}$		

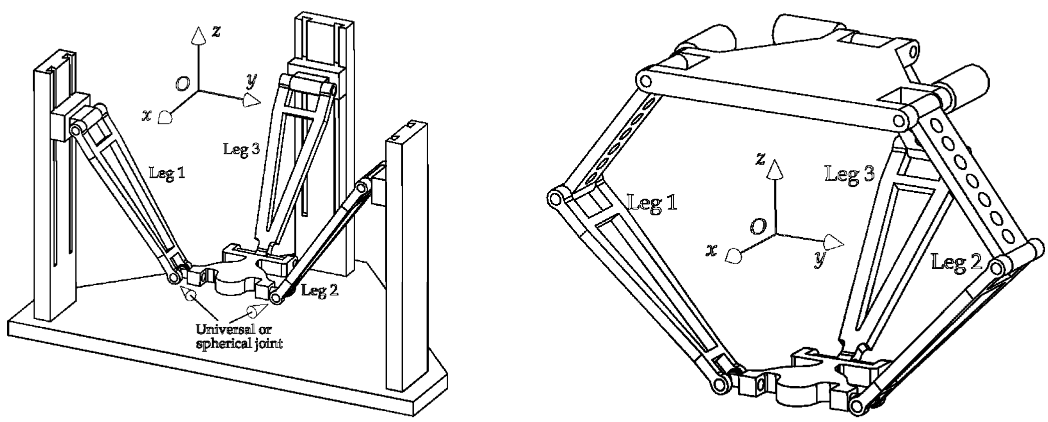
Note: P: prismatic joint; R: revolute joint; C: cylinder joint; T: translation; RO: rotation, in each of which the subscript stands for the DOF.

HALF parallel manipulator with a PRC (P: prismatic joint; R: revolute joint; C: cylinder joint) kinematic chain. The new manipulator, which is referred to as the ***HALF**** parallel manipulator, is shown in Fig. 2(a). In the ***HALF**** parallel manipulator, Legs 1 and 2 have the identical kinematic chains, i.e. PRU (U: universal) chains. Since a U joint is usually composed of two R joints. In the two U joints, the axes of two R joints that are connected to the mobile platform should be collinear. Otherwise, the mobile platform will lose 1 DOF. It is noteworthy that, due to the PRC chain of Leg 3, the two U joints can be replaced by two S (spherical joint) joints. Undoubtedly, the new manipulator also has the same mobility as that of the ***HALF*** manipulator, i.e. the translation in the O–yz plane and the rotation about the axis parallel to the y-axis if the P joints are actuated. To understand the mobility of the new manipulator, Table I shows the description about the constraint and DOFs of the ***HALF**** parallel manipulator.

Although they have the same output, there is a big difference in kinematic motion of the third leg between ***HALF*** and ***HALF**** manipulators. When the z-coordinate of the mobile platform is specified but the platform translates along the y-axis, in order to maintain the orientation of the platform, the slider in the third leg of the ***HALF*** manipulator must be active accordingly, whereas, the slider in the ***HALF**** manipulator must be locked. This indicates that the translation along the y-axis and rotation of the ***HALF**** manipulator are decoupled. This also means that the ***HALF**** manipulator is energy saving. It is well known that there must be input error in the actuator. Therefore, the active input in the ***HALF*** manipulator may lead to rotational error of the mobile platform. Thus, the difference allows the ***HALF**** manipulator having better accuracy than the ***HALF*** manipulator. Additionally, the kinematic problem of the new manipulator will be simpler accordingly (see the example in Section 3).

It is obvious that the ***HALF*** parallel manipulator is more complex than the ***HALF**** manipulator because of the use of a parallelogram. In a planar parallelogram, every two links should be parallel to each other. This needs approving manufacturing accuracy and increases the difficulty of link machining and assembling and the cost. Since there is no parallelogram in the third leg, the architecture of the new manipulator is simpler. The manufacturing will be easier. The cost will be accordingly decreased. As shown in Fig. 2(a), the self-calibration can be implemented by attaching the sensors to the two revolute joints and the cylinder joint that are connected to the mobile platform. Hereby, the accuracy of the ***HALF**** parallel manipulator can be improved. Therefore, compared with the old version, the new manipulator will be more popular in practical applications.

Figure 2(b) shows the ***HALF**** parallel manipulator with revolute actuators, where the R joints fixed to the base platform are active. Notably, the actuating direction of all sliders in the ***HALF**** parallel manipulator with prismatic actuators may be inclined at an α angle with respect to the vertical line as shown in Fig. 3(a). Figure 3(b) illustrates a typical example when the actuating direction is horizontal.



(a) (b)

Fig. 2. ***HALF**** parallel manipulators: (a) with linear actuators and (b) with revolute actuators.

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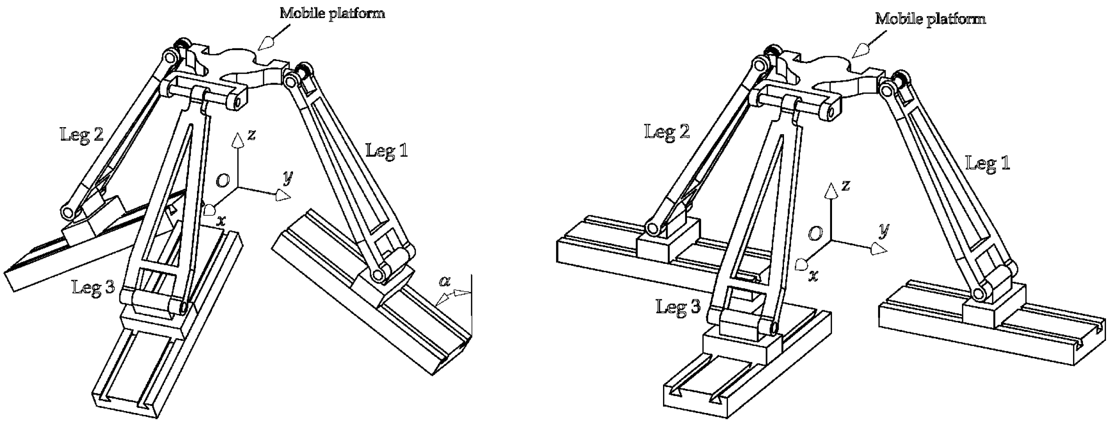


(a) (b)

Fig. 3. ***HALF**** parallel manipulators: (a) with inclined angle α and (b) with horizontal actuators.

For the manipulator shown in Fig. 2(a), as previously mentioned, the collinear axes for the two revolute joints lead to the motivation of redesigning Legs 1 and 2, as shown in Fig. 4. In these two designs, the two revolute joints are combined to one revolute joint. In Fig. 4(a), the first and second legs are connected to the moving platform through one common revolute joint. In Fig. 4(b), the first and second legs have the PRR chains, which are connected to a constant orientation bar that is linked to the mobile platform by a revolute joint. If the kinematics chain for the manipulator shown in Fig. 2(a) is denoted as (2-PRU)-PRC, it will be $(PRR)_2$R-PRC for the two designs shown in Fig. 4. This modification, which has no negative influence on the kinematics and rotational capability of the manipulator, can be also extended to the ***HALF**** parallel manipulator with revolute actuators shown in Fig. 2(b). It is noteworthy that in the ***HALF**** parallel manipulators the universal joints connected to the mobile platform can be replaced by spherical joints.

2.2. *HANA* parallel manipulators*

Figure 5 shows the ***HANA*** parallel manipulator introduced in ref. [15], which is also one member of the family presented in ref. [13]. In the ***HANA*** manipulator, two legs (the first and second legs) consist of parallelogram. The kinematic chain of each of the two legs is the same as that of the third leg in the ***HALF*** parallel manipulator shown in Fig. 1. Therefore, the two legs can be also replaced by the PRC chain. The new version of the ***HANA*** parallel manipulator is illustrated in Fig. 6(a), which is referred to as the ***HANA**** parallel manipulator where the P joints are actuated. The new

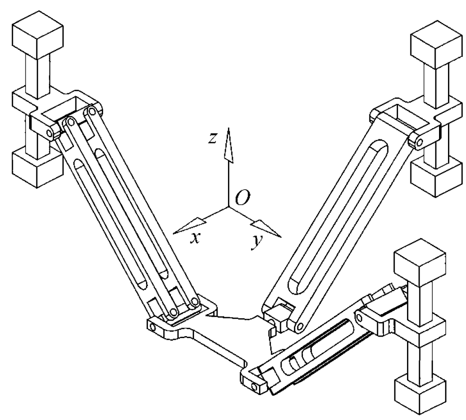
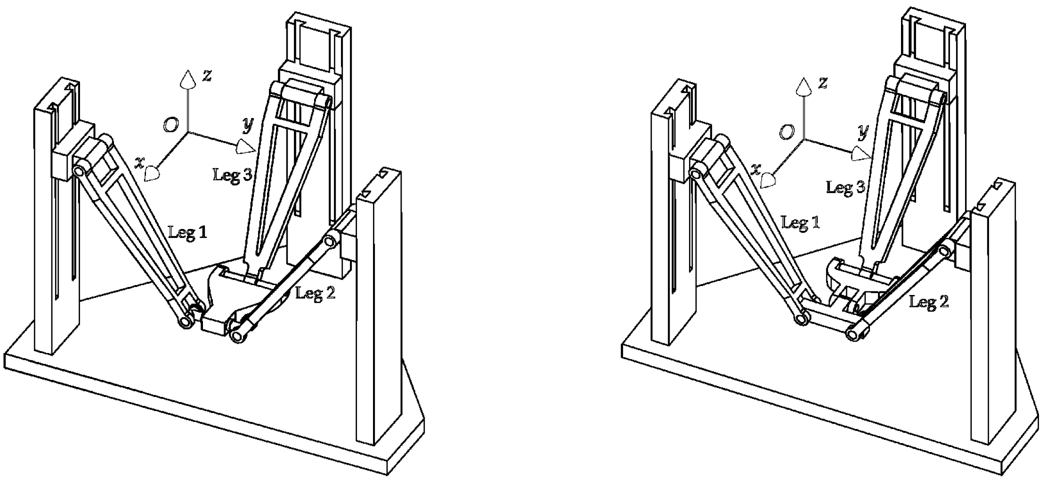


Fig. 5. The ***HANA*** parallel manipulator introduced in ref. [15].



(a) (b)

Fig. 4. Modified versions of the ***HALF**** parallel manipulator with $(PRR)_2$R-PRC chain.

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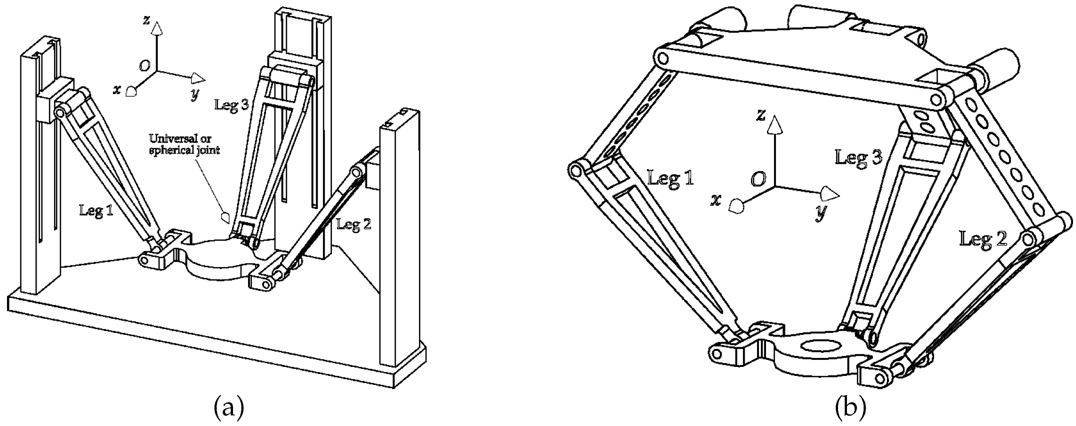

Fig. 6. ***HANA**** parallel manipulators: (a) with linear actuators and (b) with revolute actuators.

manipulator with revolute actuators is shown in Fig. 6(b), where the R joints fixed to the base platform are active.

Then, as shown in Fig. 6(a), the mobile platform of the ***HANA**** manipulator is connected to the base by a PRU or PRS chain (Leg 3) and two PRC chains (Legs 1 and 2). It is noteworthy that the axes of the two C joints in Legs 1 and 2 must be parallel to each other. If Leg 3 uses the PRU chain, the axes must also be parallel to the R joint of the U joint that is connected to the mobile platform. Due to the arrangement of links and joints of the manipulator, the combination of the three legs constrains the rotation of the moving platform with respect to the y- and z-axes and the translation along the y-axis, leaving the manipulator with two translational DOFs in the O–xz plane and one rotational DOF about the axis parallel to the x-axis. Table II shows the description about the mobility of the manipulator. One may see that the ***HALF**** and ***HANA**** have the same mobility. However, there is a remarkable difference between these two manipulators in terms of the rotational DOF. The rotational DOF of ***HANA**** is implemented with the combination of Legs 1 and 2 with the PRC chain. This situation is same to the ***HANA**** with revolute actuators shown in Fig. 6(b). In the ***HALF****, the rotational DOF is reached by actuating only one leg, i.e. Leg 3.

Table II. The constraint and DOFs of the ***HALF**** parallel manipulator with prismatic actuators.

Single leg			Combination of three legs	
No.	Leg type	Constraints	Constraints	Remained DOFs
1	$P_z R_x C_x$	$\{RO_y, RO_z\}$		
2	$P_z R_x R_x$	$\{RO_y, RO_z\}$	$\{T_y, RO_y, RO_z\}$	$\{T_x, T_z, RO_x\}$
3	$P_z R_y U_{xy}$	$\{RO_z, T_y\}$		

Note: P: prismatic joint; R: revolute joint; C: cylinder joint; T: translation; RO: rotation, in each of which the subscript stands for the DOF.

Similarly, the actuating direction of all sliders in the ***HANA**** parallel manipulator with prismatic actuators may be inclined at an α angle with respect to the vertical line as shown in Fig. 7(a). Figure 7(b) illustrates a typical example when the actuating direction is horizontal. They have the same mobility as that of the manipulator shown in Fig. 6.

It is not difficult to find out that, compared with the ***HANA*** manipulators, the ***HANA**** parallel manipulators introduced here also have the advantages in kinematics, architecture, manufacturing, energy cost, accuracy, and assembling for the similar reasons described in Section 2.1.

Since there is no planar parallelogram in each manipulator of the new family, every leg can be designed as a telescopic link. For example, the ***HALF**** and ***HANA**** parallel manipulators with such links are shown in Fig. 8(a) and (b), respectively.

Although, compared with the ***HALF*** and ***HANA*** parallel manipulators, the new manipulators have some advantages, they still have their own disadvantages. For example, the

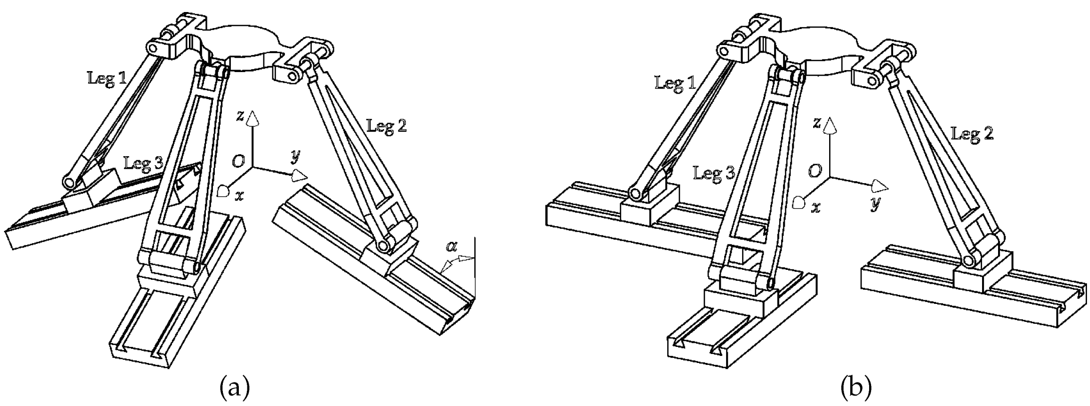

Fig. 7. ***HANA**** parallel manipulators: (a) with inclined angle α and (b) with horizontal actuators.

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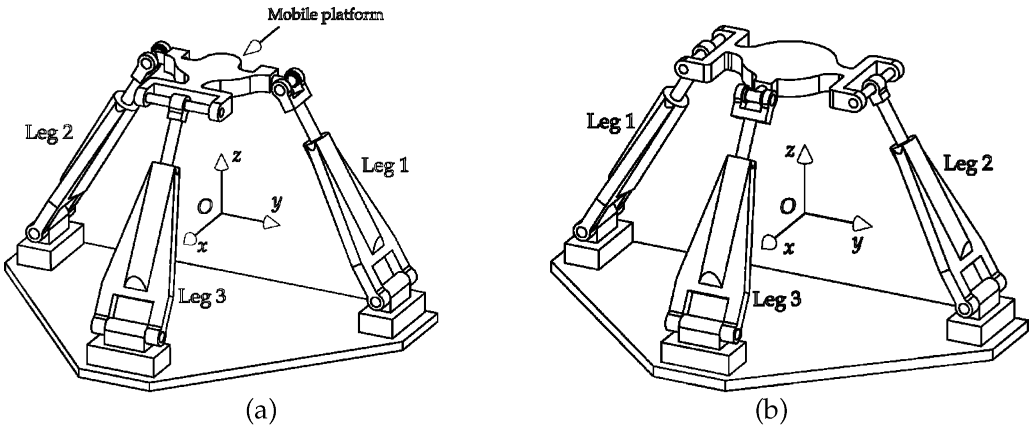

Fig. 8. New parallel manipulators with telescopic legs: (a) ***HALF****; (b) ***HANA****.

adoption of C joint may cause operational failure since it is a passive joint. To avoid this problem, we can apply the passive DOF to the manipulators. For example, in the ***HALF**** manipulators, the joints connected to the moving platform in the first and second legs can be spherical joints, in each of which there is one passive DOF. On the other hand, to decrease the operational failure, the demand on the parallelism of the C axes in the first and second legs should be very high.

3. Kinematic Analysis Example

As mentioned in the last section, the kinematics of the new manipulators proposed in this paper will be simpler than that of the old manipulators introduced in ref. [13]. As an example, we here investigate the inverse kinematic problem of the ***HALF**** parallel manipulator with prismatic actuators shown in Fig. 2(a).

The kinematical scheme of the manipulator is shown in Fig. 9. Vertices of the mobile platform are denoted as platform joints P_i $(i = 1, 2, 3)$; and central points of the three revolute joints attached to the sliders are denoted as B_i $(i = 1, 2, 3)$. A fixed global reference frame $O - xyz$ is located at the center point of the line segment ab with the z-axis normal to the plane abc and the y-axis directed along ab. Another reference

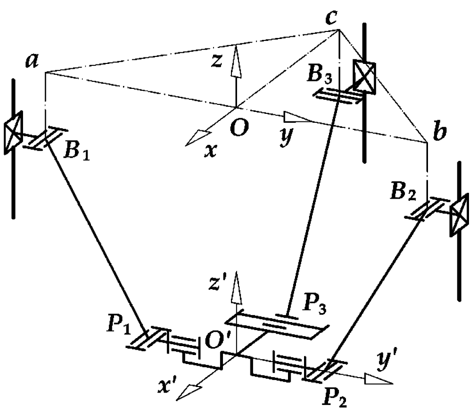

Fig. 9. Kinematic scheme of the ***HALF**** parallel manipulator with prismatic actuators.

frame, called the moving frame $(O' - x'y'z')$, is located at the center of the side P_1P_2. The z'-axis is perpendicular to the moving platform and the y'-axis is directed along P_1P_2. Since the first and second legs are the same in kinematic chains and the third leg is different, the geometric parameters for the first and second legs can be the same, but different from those of the third leg. Then, the geometric parameters will be $O'P_1 = O'P_2 = r$, $B_1P_1 = B_2P_2 = R_2$, $O'P_3 = L_1$, $P_3B_3 = L_2$, the normal distance L_3 from the point O to the straight-line path of the joint point B_3, i.e. $Oc = L_3$, and $Oa = Ob = R$.

The inverse kinematic problem of this manipulator is somewhat similar to that of the ***HALF*** manipulator.[12] Vectors $\boldsymbol{b}_i$ $(i = 1, 2, 3)$ are defined as the position vectors of points B_i in the reference frame O–xyz and can be written as

$$\boldsymbol{b}_1 = (0 \quad -R \quad z_1)^T, \quad \boldsymbol{b}_2 = (0 \quad R \quad z_2)^{\mathrm{T}}, \quad \boldsymbol{b}_3 = (-L_3 \quad 0 \quad z_3)^{\mathrm{T}}. \tag{1}$$

In the reference frame O–xyz, position vectors $\boldsymbol{p}_i$ $(i = 1, 2, 3)$ of points P_i can be written as

$$\boldsymbol{p}_1 = (0 \quad y - r \quad z)^{\mathrm{T}}, \quad \boldsymbol{p}_1 = (0 \quad y + r \quad z)^{\mathrm{T}}, \quad \boldsymbol{p}_3 = (-L_1 \cos\phi \quad 0 \quad z + L_1 \sin\phi)^{\mathrm{T}} \tag{2}$$

where $(y, \ z, \ \phi)$ is the pose of the manipulator and ϕ is the rotating angle of the moving platform about the y'-axis. The kinematic problem of the manipulator can be solved by writing

$$|\boldsymbol{b}_i \boldsymbol{p}_i| = B_i P_i. \tag{3}$$

Then, there are

$$(R - r + y)^2 + (z_1 - z)^2 = R_2^2 \tag{4}$$

$$(R - r - y)^2 + (z_2 - z)^2 = R_2^2 \tag{5}$$

$$(z_3 - z - L_1 \sin\ \phi)^2 + (L_3 - L_1 \cos\ \phi)^2 = L_2^2 \tag{6}$$

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For a given pose $(y, \ z, \ \phi)$, the inputs y_i $(i = 1, 2, 3)$ can be obtained as

$$z_1 = \pm\sqrt{R_2^2 - (R - r + y)^2} + z \tag{7}$$

$$z_2 = \pm\sqrt{R_2^2 - (R - r - y)^2} + z \tag{8}$$

$$z_3 = \pm\sqrt{L_2^2 - (L_3 - L_1 \cos\ \phi)^2} + z + L_1 \sin\ \phi \tag{9}$$

Therefore, there are eight inverse kinematic solutions for the manipulator. The configuration shown in Fig. 9 corresponds to the solution when the "±" signs in Eqs. (7)–(9) are all "+". Observing the kinematic equations of the ***HALF*** and ***HALF**** parallel manipulators, one may see that the kinematics of the ***HALF**** manipulator is relatively simpler.

Equations (4), (5), and (6) can be differentiated with respect to time to obtain the velocity equations, which can be written as

$$\dot{\rho} = \boldsymbol{J}\,\dot{\boldsymbol{p}} \tag{10}$$

where $\dot{\boldsymbol{p}}$ is the vector of output velocities defined as $\dot{\boldsymbol{p}} = (\,\dot{y}\ \dot{z}\ \dot{\phi}\,)^{\mathrm{T}}$, $\dot{\rho}$ is the vector of input velocities defined as $\dot{\rho} = (\dot{z}_1\ \dot{z}_2\ \dot{z}_3)^{\mathrm{T}}$, and $\boldsymbol{J}$ is the Jacobian matrix of the manipulator that can be written as

$$\boldsymbol{J} = \begin{bmatrix} \dfrac{r - R - y}{z_1 - z} & 1 & 0 \\ \dfrac{r - R + y}{z_2 - z} & 1 & 0 \\ 0 & 1 & \dfrac{(z_3 - z)\,L_1 \cos\ \phi - L_1 L_3 \sin\ \phi}{z_3 - z - L_1 \sin\ \phi} \end{bmatrix} \tag{11}$$

For the configuration shown in Fig. 9, letting the "±" signs in Eqs. (7)–(9) be "+", the Jacobian matrix should be

$$\boldsymbol{J} = \begin{bmatrix} \dfrac{r - R - y}{\sqrt{R_2^2 - (R - r + y)^2}} & 1 & 0 \\ \dfrac{r - R + y}{\sqrt{R_2^2 - (R - r - y)^2}} & 1 & 0 \\ 0 & 1 & \dfrac{[\sqrt{L_2^2 - (L_3 - L_1 \cos\ \phi)^2} + L_1 \sin\ \phi]\,L_1 \cos\ \phi - L_1 L_3 \sin\ \phi}{\sqrt{L_2^2 - (L_3 - L_1 \cos\ \phi)^2}} \end{bmatrix} \tag{12}$$

As most kinematic designs of parallel manipulators are based on the workspace and indices defined with respect to the Jacobian matrix, the information in the matrix is very important. From Eq. (12), we may see that there is no z parameter in the matrix, i.e. the Jocabian matrix of the manipulator is independent of the z-coordinate. This means that in the design process we can just consider the performance of the manipulator along the y-axis. This property was actually introduced in ref. [16]. What is more, for elements in the third column of the Jacobian matrix, there is no y parameter involved. This indicates that the performance about the orientational DOF, i.e. ϕ is independent of the positional workspace of the manipulator. Actually, if the z-coordinate is specified, the y parameter is nothing to the orientational DOF of the manipulator. This is obvious by observing Eq. (6).

Letting R - r in Eqs. (7) and (8) be R_1, i.e. $R_1 = R$ - r, there are

$$z_1 = \pm\sqrt{R_2^2 - (R_1 + y)^2} + z \tag{13}$$

$$z_2 = \pm\sqrt{R_2^2 - (R_1 - y)^2} + z \tag{14}$$

from which we may see that the inverse kinematic problem of the first and second legs are actually that of the PRRRP symmetrical parallel manipulator,[17] which is kinematically a planar parallel mechanism. If the position of point O' is specified, the kinematic Eq. (6) is actually that of a slider-crank mechanism.[18] Therefore, in the design process, the manipulator can be considered as the combination of a PRRRP parallel manipulator and a slider-crank mechanism. Observing the ***HANA**** manipulator shown in Fig. 6(a), we will see that, it can be thought of as the combination of a slider-crank mechanism and a PRRP four-bar mechanism. Therefore, compared with those of the manipulators in ref. [13], the kinematics and design of some new manipulators proposed in this paper will become accordingly simpler. Actually, the design of most spatial manipulators in the new family can be divided into two parts, i.e. those of two simple mechanisms.

4. Conclusions

By replacing the kinematic chain of the leg consisting of a parallelogram of the manipulator family introduced in ref. [13] with the PRC chain, a new family of spatial 3-DOF parallel manipulators with two translational and one rotational DOFs is proposed in this paper. It is shown that the architecture, kinematic analysis, and design of the new manipulators are simpler. What is more, compared with the old manipulators, the parallel manipulators introduced in this paper have the obvious advantages in manufacturing and energy cost. Additionally, the translation and rotation of the new manipulators are partly decoupled. For these reasons, the new family will become more popular in industry applications.

Acknowledgements

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