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A revisit to the "Rigorous study of propagation in metallic circular waveguide filled with anisotropic metamaterial"

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Editorial

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Abstract

Sakli et al. previously studied the propagation characteristics of wave modes in a metallic circular waveguide filled with anisotropic metamaterial [*Int. J. Microw. Wirel. Technol.* **9**, 805–813 (2017)]. They derived and analyzed the wave equation and dispersion relations for TE_z and TM_z modes (i.e., TE and TM waves related to the z-axis) within the waveguide. However, they did not verify whether the system actually supports these TE_z and TM_z waves. This work aims to investigate that issue. Our findings indicate that, in general, a metallic circular waveguide filled with anisotropic metamaterial cannot support the propagation of TE_z and TM_z waves. Consequently, the results presented by Sakli et al. are incorrect.

Introduction

Sakli et al. [1] investigated the propagation characteristics of wave modes in a metallic circular waveguide filled with anisotropic metamaterial. In this way, by assuming the existence of TE_z and TM_z waves related to the waveguide axis in the mentioned system, they obtained and analyzed the wave equation and dispersion relations for TE_z and TM_z modes in the waveguide.

Now it is worth asking the question whether circular metal waveguides filled with anisotropic metamaterial can support an electromagnetic wave with TE_z (or TM_z) mode or not. The purpose of this work is to investigate this issue. We should point out that in the general case, hybrid wave propagation should be expected for such systems [2–4], and therefore TE_z and TM_z waves are unable to propagate. This means that in the general case, the results derived by Sakli et al. [1] are incorrect.

The rigorous electromagnetic analysis

Let us consider a z-directional metallic circular waveguide filled with a metamaterial possesses anisotropic material parameters that are described by tensors diagonalized in a cylindrical coordinate system. Taking the coordinate axis to coincide with the waveguide axis, these take the form $\bar{\bar{\varepsilon}} = \bar{\bar{I}}\,(\varepsilon_{rr}, \varepsilon_{r\theta}, \varepsilon_{rz})\,\varepsilon_0$ and $\bar{\bar{\mu}} = \bar{\bar{I}}\,(\mu_{rr}, \mu_{r\theta}, \mu_{rz})\,\mu_0$ in which $\bar{\bar{I}}$ is the identity dyadic [1]. In fact $\bar{\bar{\varepsilon}}$ and $\bar{\bar{\mu}}$ tensors have zero off-diagonal elements (biaxial material). Substituting electric and magnetic fields $\mathbf{E}$ and $\mathbf{H}$ describing a wave traveling in the z-direction

$$\mathbf{E}(r,\theta,z) = \left[\mathbf{e}_r E_r(r,\theta) + \mathbf{e}_\theta E_\theta(r,\theta) + \mathbf{e}_z E_z(r,\theta)\right] e^{-jk_z z}\,, \tag{1}$$

$$\mathbf{H}(r,\theta,z) = \left[\mathbf{e}_r H_r(r,\theta) + \mathbf{e}_\theta H_\theta(r,\theta) + \mathbf{e}_z H_z(r,\theta)\right] e^{-jk_z z}\,, \tag{2}$$

into Maxwell curl equations, i.e., $\nabla \times \mathbf{E} = -j\omega\bar{\bar{\mu}} \cdot \mathbf{H}$ and $\nabla \times \mathbf{H} = j\omega\bar{\bar{\varepsilon}} \cdot \mathbf{E}$, we can determine E_r, E_θ, H_r, and H_θ with respect to partial derivatives of components E_z and H_z, as shown by Eqs. (5)–(8) in [1]. By substituting E_r, E_θ, H_r, and H_θ into z-components of Eqs. (1) and (2) in [1], one may obtain the following coupled equations for components E_z and H_z as

$$\frac{\partial^2 H_z}{\partial r^2} + \frac{1}{r}\frac{\partial H_z}{\partial r} + \left(\frac{K_{c.\theta}\sqrt{\mu_{r\theta}}}{K_{c.r}\sqrt{\mu_{rr}}}\right)^2 \frac{1}{r^2}\frac{\partial^2 H_z}{\partial \theta^2} + \left(\frac{\sqrt{\mu_{rz}}}{\sqrt{\mu_{rr}}}K_{c.\theta}\right)^2 H_z = \frac{k_z k_0^2}{r\omega\mu_0\mu_{rr}}\,\frac{\mu_{r\theta}\varepsilon_{rr} - \mu_{rr}\varepsilon_{r\theta}}{K_{c.r}^2}\,\frac{\partial^2 E_z}{\partial r\partial\theta}\,, \tag{3}$$

$$\frac{\partial^2 E_z}{\partial r^2} + \frac{1}{r}\frac{\partial E_z}{\partial r} + \left(\frac{K_{c.r}\sqrt{\varepsilon_{r\theta}}}{K_{c.\theta}\sqrt{\varepsilon_{rr}}}\right)^2 \frac{1}{r^2}\frac{\partial^2 E_z}{\partial \theta^2} + \left(\frac{\sqrt{\varepsilon_{rz}}}{\sqrt{\varepsilon_{rr}}}K_{c.r}\right)^2 E_z = \frac{k_z k_0^2}{r\omega\varepsilon_0\varepsilon_{rr}}\frac{\mu_{r\theta}\varepsilon_{rr}-\mu_{rr}\varepsilon_{r\theta}}{K_{c.\theta}^2}\frac{\partial^2 H_z}{\partial r\partial\theta}\ , \qquad (4)$$

where $K_{c.r}^2 = k_0^2\varepsilon_{rr}\mu_{r\theta} - k_z^2$, $K_{c.\theta}^2 = k_0^2\varepsilon_{r\theta}\mu_{rr} - k_z^2$, and $k_0^2 = \omega^2\varepsilon_0\mu_0$, and all other parameters in the equations of the present work were defined in [1].

The TE_z and TM_z modes are defined by assuming $E_z = 0$ and $H_z = 0$, respectively. Now, let us explore the possibility of TE_z and TM_z wave propagation in the system. If we assume $E_z = 0$, then Eqs. (3) and (4) reduce to

$$\frac{\partial^2 H_z}{\partial r^2} + \frac{1}{r}\frac{\partial H_z}{\partial r} + \left(\frac{K_{c.\theta}\sqrt{\mu_{r\theta}}}{K_{c.r}\sqrt{\mu_{rr}}}\right)^2 \frac{1}{r^2}\frac{\partial^2 H_z}{\partial \theta^2} + \left(\frac{\sqrt{\mu_{rz}}}{\sqrt{\mu_{rr}}}K_{c.\theta}\right)^2 H_z = 0\ , \qquad (5)$$

$$\frac{k_z k_0^2}{r\omega\varepsilon_0\varepsilon_{rr}}\frac{\mu_{r\theta}\varepsilon_{rr}-\mu_{rr}\varepsilon_{r\theta}}{K_{c.\theta}^2}\frac{\partial^2 H_z}{\partial r\partial\theta} = 0\ . \qquad (6)$$

The TE_z waves must satisfy Eqs. (5) and (6). In Sakli et al. work [1], the authors obtained Eq. (5) for TE_z waves (see Eq. (12) in [1]) but Eq. (6) was neglected. This means that the calculations for TE_z modes in Sakli et al. paper [1] do not satisfy Eq. (6), and therefore there is an obvious and serious error in [1]. In physics, the presence of such a significant error in the investigation of a problem is unacceptable and inevitably leads to incorrect results [5].

Note that Eqs. (5) and (6) cannot be simultaneously satisfied by the same value of k_z unless in two particular cases. In the first case, material parameters must satisfy the condition

$$\mu_{r\theta}\varepsilon_{rr} = \mu_{rr}\varepsilon_{r\theta}\ . \qquad (7)$$

In the second case, decoupling occurs if

$$\frac{\partial H_z}{\partial r} = 0, \quad \text{or} \quad \frac{\partial H_z}{\partial \theta} = 0\ . \qquad (8)$$

These conditions reduce Eq. (6) to zero. Hence, in general TE_z modes cannot propagate in a metallic circular waveguide filled with anisotropic metamaterial. However, from Eq. (7) it is easy to conclude that TE_z and TM_z modes can propagate separately in a metallic circular waveguide when it is filled with uniaxial media, i.e.,

$$\varepsilon_{rr} = \varepsilon_{r\theta}, \quad \text{and} \quad \mu_{r\theta} = \mu_{rr}\ . \qquad (9)$$

In other words, the analysis by Sakli et al. [1] is valid for uniaxial cases. Similarly, if we assume that $H_z = 0$ (TM_z modes), then Eqs. (3) and (4) reduce to

$$\frac{\partial^2 E_z}{\partial r^2} + \frac{1}{r}\frac{\partial E_z}{\partial r} + \left(\frac{K_{c.r}\sqrt{\varepsilon_{r\theta}}}{K_{c.\theta}\sqrt{\varepsilon_{rr}}}\right)^2 \frac{1}{r^2}\frac{\partial^2 E_z}{\partial \theta^2} + \left(\frac{\sqrt{\varepsilon_{rz}}}{\sqrt{\varepsilon_{rr}}}K_{c.r}\right)^2 E_z = 0\ , \qquad (10)$$

$$\frac{k_z k_0^2}{r\omega\mu_0\mu_{rr}}\frac{\mu_{r\theta}\varepsilon_{rr}-\mu_{rr}\varepsilon_{r\theta}}{K_{c.r}^2}\frac{\partial^2 E_z}{\partial r\partial\theta} = 0\ , \qquad (11)$$

Again, Eqs. (10) and (11) cannot be simultaneously satisfied unless material parameters satisfy the condition $\mu_{r\theta}\varepsilon_{rr} = \mu_{rr}\varepsilon_{r\theta}$, or when we have either $\partial E_z/\partial r = 0$, or $\partial E_z/\partial\theta = 0$. Thus, one may conclude that in the general case, a metallic circular waveguide filled with anisotropic metamaterials cannot support the propagation of TM_z modes. This means that the results for TM_z modes, derived by Sakli et al. (see Eq. (18) in [1]) are also incorrect.

Conclusion

In summary, we have demonstrated that, in general, a metallic circular waveguide filled with anisotropic metamaterial cannot support the propagation of TE_z and TM_z waves. Consequently, the results derived by Sakli et al. are incorrect. However, their calculations are valid for uniaxial cases (where $\varepsilon_{rr} = \varepsilon_{r\theta}$ and $\mu_{r\theta} = \mu_{rr}$). In fact, uniaxial media represent a specific case of the condition shown in Eq. (7).

Competing interests. The authors declare no conflicts of interest/competing interests.

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