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A NEW STOPPING PROBLEM AND THE CRITICAL EXERCISE PRICE FOR AMERICAN FRACTIONAL LOOKBACK OPTION IN A SPECIAL MIXED JUMP-DIFFUSION MODEL

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A new stopping problem and the critical exercise price of American fractional lookback option are developed in the case where the stock price follows a special mixed jump diffusion fractional Brownian motion. By using Itô formula and Wick-Itô-Skorohod integral a new market pricing model is built, and the fundamental solutions of stochastic parabolic partial differential equations are deduced under the condition of Merton assumptions. With an optimal stopping problem and the exercise boundary, the explicit integral representation of early exercise premium and the critical exercise price are also derived. Numerical simulation illustrates the asymptotic behavior of this critical boundary.

Keywords: asymptotic behavior, fundamental solutions, mixed jump-diffusion fractional brownian motion, optimal stopping problem, wick-itô-skorohod integral

1. INTRODUCTION

As is known that lookback options are path-dependent options whose payoff depends on the maximum or the minimum of the underlying asset price attained over a lookback period. An American lookback call (put) option allows it to be exercised at any time prior to expiry and gives the holder the right to buy (sell) at the historical minimum (maximum) of the underlying asset price on exercising the option. A more general and less expensive variant called fractional or partial lookback option, where the strike is fixed at some fraction over (for a call) or below (for a put) the extreme value. Specifically, the payoffs for European lookback call and put with fractional floating strikes and maturity date T are given, respectively, by $(S_T - \mathbb{a}\mathbb{m}_T)^+$ and $(\mathbb{b}\mathbb{M}_T - S_T)^+$, where $\mathbb{a}$ and $\mathbb{b}$ are positive constants, allowing flexible adjustment of option premiums. In this paper, we consider a new stopping problem and the critical exercise price of American fractional lookback option under the environment of mixed jump-diffusion fractional Brownian motion(MJD-fBm).

The mixed fractional Brownian motion(mfBm) is a family of Gaussian processes, which is a linear combination of Brownian motion and fractional Brownian motion(fBm). As the

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fact that Black–Scholes model [22] is inadequate to describe the asset returns and the behavior of the option markets. [6] proposed a jump-diffusion process with Poisson jump to match the abnormal fluctuation of stock price. Several authors [1,2,4,5,19,26,34] also considered the problem of pricing options under a jump-diffusion environment in a larger setting. Actually, various empirical studies on the statistical properties of log-returns show that the log-returns are not necessarily independent and also not Gaussian [7,8,13]. One way to a more realistic modeling is to change the geometric Brownian motion to a geometric fBm: the dependence of the log-return increments can now be modeled with the Hurst parameter of the fBm. It can be said that the properties of financial return series are non-normal, non-independent and nonlinear, self-similar, with heavy-tails, in both autocorrelations and cross-correlations, and volatility clustering [7,10,11,17,24,35]. Since fBm has two substantial features such as self-similarity and long-range dependence, thus using it is more applicable to capture behavior from a financial asset [35]. Some scholars proposed a mfBm version of a option pricing Merton model [23,29]. [29] assume that the value of the firm obeys to a geometric mfBm, the result shows that the mixed-fractional model to simulate credit risk pricing is a reasonable one. [7,8] derived a European call pricing option on an asset driven by a linear combination of a Brownian motion and an independent fBm. Cheridito had proved that, for $H \in (3/4, 1)$, the mfBm was equivalent to Brownian motion and hence it was arbitrage-free. The case even for the fBm with arbitrary Hurst parameter and Wick products of the fractional Black–Scholes model have been proposed as an improvement of the classical Black-Scholes model [3,17,32]. In virtue of the fBm is neither a Markov process nor a semi-martingale, we cannot apply the common stochastic calculus to analyze it. Fortunately, Hu *et al.* [17] employed the Wick product rather than the pathwise product to define a fractional stochastic integral whose mean is indeed zero. This property was very convenient for both theoretical developments and practical applications. Further, in [32], it was stated that if one uses the Wick-Itô-Skorohod integral, then can obtain an arbitrage-free model, while Wick integration leads to no-arbitrage, the definition of the corresponding self-financing trading strategies is quite restrictive. Therefore, the fractional market based on Wick integrals is considered which is a beautiful mathematical construction but with restricted applicability in finance. In recent years, many researchers investigate the mfBm (see [16,28,35]) derived explicit pricing formulas for European currency options when the valuation models were governed by mfBm. However, continuous assumptions on the dynamics of assets ignore sudden shocks to asset returns due to the arrival of important information, since the financial crisis and significant business always result in sudden changes in firm values, which cannot be captured by continuous sample paths. To get around this problem and to take into account the long memory property, it is reasonable to use the mfBm with jumps model to capture fluctuations of the financial asset (see [27,30,31,33,36]).

Further, to capture jumps or discontinuous, fluctuations problem or take into account of long memory property, we present here a new stopping problem and the exercise boundary to solve the American fractional lookback option pricing problem in a MJD-fBm environment. It is different from the Shokrollahi's and Rao's model, we establish MJD-fBm model which is a linear combination of Brownian motion, fBm, and Poisson process. By using Itô formula and fractional Wick-Itô-Skorohod integral a new market pricing model is built, and the fundamental solutions of stochastic parabolic partial differential equations are deduced under the condition of Merton assumptions. With an optimal stopping problem and the exercise boundary, the explicit integral representation of early exercise premium and the critical exercise price is also derived. Based on some researches of lookback options pricing and early exercise premium in the literature [12,14,15,18,20,21,25], to achieve quick and accurate pricing for practical purposes, this paper adopts the critical exercise price to valuing American fractional lookback options, and numerical simulation illustrate some notable features of American fractional lookback options.

The remainder of this paper is structured as follows. In Section 2, we present some basic lemmas and preliminary results of our special mixed jump-diffusion pricing model and Wick-Itô-Skorohod integral which will be used throughout this paper. In Section 3, we turn to the exercise boundary formulation of special optimal stopping problem described, and the critical exercise price is given by a Volterra integral equation. In Section 4, the asymptotic behaviors of the critical exercise prices are presented, some simulation results and notable features are also provided. The paper is ended with conclusive remarks in the last section.

2. A SPECIAL MIXED JUMP-DIFFUSION MODEL

In this section, we construct mixed Poisson jump-diffusion processes as a suitable alternative to fBm.

DEFINITION 2.1 [7,28,35]: *A mfBm of parameters α, β, and H is a linear combination of Brownian motion and fBm, defined on the probability space $(\Omega, \mathfrak{F}, \mathbb{P})$ for any $t \in \mathbb{R}^+$ by:*

$$M_t^H = \alpha B_t + \beta B_t^H, \tag{1}$$

where B_t is a Brownian motion, B_t^H is an independent fBm with $H \in (0, 1)$, α, β are two real constants such that $(\alpha, \beta) \neq (0, 0)$.

Now we list some properties in [23] by the following proposition.

PROPOSITION 2.1: *The mfBm M_t^H satisfies the following properties:*

(i) *M_t^H is a centered Gaussian process and not a Markovian one for all $H \in (0, 1)\backslash 1/2$;*

(ii) *$M_0^H = 0$ $\mathbb{P}-$almost surely;*

(iii) *the covariation function of $M_t^H(\alpha, \beta)$ and $M_s^H(a, b)$ for any $t, s \in \mathbb{R}^+$ is given by*

$$Cov(M_t^H, M_s^H) = \alpha^2(t \wedge s) + \frac{\beta^2}{2}(t^{2H} + s^{2H} - |t-s|^{2H}),$$

where $\wedge$ denotes the minimum of two numbers;

(iv) *the increments of $M_t^H(\alpha, \beta)$ are stationary and mixed-self-similar for any $h > 0$*

$$M_{ht}^H(\alpha, \beta) \triangleq M_t^H(\alpha h^{1/2}, \beta h^H),$$

where $\triangleq$ means "to have the same law";

(v) *the increments of M_t^H are positively correlated if $1/2 < H < 1$, uncorrelated if $H = 1/2$ and negatively correlated if $0 < H < 1/2$;*

(vi) *the increments of M_t^H are long-range dependent if, and only if $H > 1/2$;*

(vii) *for all $t \in \mathbb{R}^+$, we have*

$$E[(M_t^H(\alpha, \beta))^n] = \begin{cases} 0, & n = 2l+1 \\ \dfrac{(2l)!}{2^l l!}(\alpha^2 t + \beta^2 t^{2H})^l, & n = 2\,l. \end{cases}$$

PROOF: These properties are easily obtained based on [7,23]. ∎

Now, let $(\Omega, \mathfrak{F}, \mathbb{P})$ be a probability space such that B_t is a Brownian motion with respect to $\mathbb{P}$ and B_t^H is an independent fBm with respect to $\mathbb{P}$. Some results presented that is needed for the following Lemma (see [7,28,35]). We assume hereafter that the index $H > 3/4$ which ensures that the probability measure generated by the process M_t^H is equivalent to the Wiener measure.

LEMMA 2.1: *For every* $0 < t < T$ *and* $\sigma \in \mathbb{C}$ *we have*

$$\widetilde{E}_t[e^{\sigma(B_T+B_T^H)}] = e^{\sigma(B_T+B_T^H)+1/2(T-t)\sigma^2+1/2\sigma^2(T^{2H}-t^{2H})},$$

where $\widetilde{E}_t$ *denotes the quasi-conditional expectation with respect to the risk-neutral measure.*

LEMMA 2.2: *Let* f *be a measurable function such that* $\widetilde{E}_t[f(B_T, B_T^H)] < \infty$. *Then for every* $0 < t \leq T$ *and* $\sigma \in \mathbb{C}$,

$$\widetilde{E}_t\left[f\left(\sigma B_T + \sigma B_T^H\right)\right] = \int_{\mathbb{R}} \frac{1}{\sqrt{2\pi[\sigma^2(T-t+T^{2H}-t^{2H})]}} \times \exp\left[-\frac{(x-\sigma B_t-\sigma B_t^H)^2}{2\sigma^2(T-t+T^{2H}-t^{2H})}\right]\cdot f(x)\mathrm{d}x.$$

LEMMA 2.3: *Let* $A \in \mathfrak{B}(\mathbb{R})$, $f(x) = \mathbf{1}_A(x)$. *Then*

$$\widetilde{E}_t[\mathbf{1}_A(\sigma B_T + \sigma B_T^H)] = \int_{\mathbb{R}} \frac{1}{\sqrt{2\pi[\sigma^2(T-t+T^{2H}-t^{2H})]}} \times \exp\left[-\frac{(x-\sigma B_t-\sigma B_t^H)^2}{2\sigma^2(T-t+T^{2H}-t^{2H})}\right]\mathbf{1}_A(x)\mathrm{d}x.$$

Let $\sigma_1, \sigma_2 \in \mathbb{R}$. *Consider the process*

$$\mathbb{Z}_t^* = \sigma_1(B_t^H)^* + \sigma_2 B_t^* = \sigma_1 B_t^H + \sigma_1^2 B_t^{2H} + \sigma_2 B_t + \sigma_2^2 t, \quad 0 \leq t \leq T;$$

where σ_1 *and* σ_2 *are two real constants such that* $(\sigma_1, \sigma_2) \neq (0, 0)$. *From fractional Girsanov's theorem* ([24], *Lemma* 1.7), *there exists a measure* $\mathbb{P}^*$, *such that* $\mathbb{Z}_t^*$ *is a new mfBm. We denote* $E_t^*[\cdot]$ *the quasi-conditional expectation with respect to* $\mathbb{P}^*$ *as following*

$$E_t^*[e^{\sigma_1(B_T^H)^*+\sigma_2 B_T^*}] = e^{\sigma_2(B_T+B_T^H)+1/2\sigma_2^2(T-t)+1/2\sigma_1^2(T^{2H}-t^{2H})}. \tag{2}$$

LEMMA 2.4: *Let* f *be a measurable function such that* $\widetilde{E}_t[f(\sigma_1 B_T^H + \sigma_2 B_T)] < \infty$. *Let* $X_t = \exp(-\sigma_1 B_t^H - \sigma_1^2/2t^{2H} - \sigma_2 B_t - \sigma_2^2/2t)$. *Then for every* $t \leq T$, *we have*

$$\widetilde{E}_t^*[f(\sigma_1 B_T^H + \sigma_2 B_T)] = \frac{1}{X_t}\widetilde{E}_t[f(\sigma_1 B_T^H + \sigma_2 B_T)X_T]. \tag{3}$$

LEMMA 2.5: *The price at every* $t \in [0, T]$ *of a bounded* F_T^H-*measurable claim* $F \in L^2$ *is given by*

$$F_t = e^{-r(T-t)}\widetilde{E}_t[F],$$

where r *represents the constant riskless interest rate.*

The proof of Lemma 2.1–2.5 can be seen [7,28,35].

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2.1. MJD-fBm Pricing Model

Consider a continuous-time financial market in $[0, T]$. It can be described by a filtered complete probability space $\{\Omega, \mathfrak{F}, \mathfrak{F}_t, \mathbb{P}\}$. $\{\mathfrak{F}_t\}_{0\le t\le T} \equiv \mathbb{F}$ is a natural $\sigma-$filtration generated by a standard Brownian motion B_t, fBm B_t^H, and a Poisson process P_t. Here P_t is an $(\mathfrak{F}_t, \mathbb{P})$-Poisson jump process with intensity λ, independent of B_t and B_t^H. j_t is jump percent at time t and i.i.d.; j_t satisfy Merton assumptions $\ln(1+j_t) \sim \mathbb{N}[\ln(1+\mu_j) - 1/2\sigma_j^2, \sigma_j^2]$, where μ_j is the unconditional expectation of j_t, and σ_j^2 is the variance of $\ln(1+j_t)$. Notice that the unconditional expectation μ_j is deterministic and can be calculated, j_t is a bounded functions of t, so we assume that $\sigma_3 = \ln(1+\mu_j)$ is a constant. Then we can denote the process

$$\mathscr{Z}_t^* = \sigma_1(B_t^H)^* + \sigma_2 B_t^* + \sigma_3 P_t^* = \sigma_1 B_t^H + \sigma_1^2 B_t^{2H} + \sigma_2 B_t + \sigma_2^2 t + \sigma_3 P_t + \sigma_3^2 t, 0 \le t \le T;$$

where $(\sigma_1, \sigma_2, \sigma_3) \neq (0, 0, 0)$. By fractional Girsanov's theorem ([24], Lemma 1.7), there exists a measure $\mathscr{P}^*$, such that $\mathscr{Z}_t^*$ is a new mfBm. And the quasi-conditional expectation with respect to $\mathscr{P}^*$ can be denoted as following

$$E_t^*[\mathrm{e}^{\sigma_1(B_T^H)^* + \sigma_2 B_T^* + \sigma_3 P_T^*}] = \mathrm{e}^{\sigma_2(P_T + B_T + B_T^H) + 1/2\sigma_3^2(T-t) + 1/2\sigma_2^2(T-t) + 1/2\sigma_1^2(T^{2H} - t^{2H})}.$$

Let f be a measurable function and satisfying $\widetilde{E}_t[f(\sigma_1 B_T^H + \sigma_2 B_T + \sigma_3 P_T)] < \infty$, let $\mathbb{X}_t = \exp(-\sigma_1 B_t^H - \sigma_1^2/2t^{2H} - \sigma_2 B_t - \frac{\sigma_2^2}{2}t - \sigma_3 P_t - \sigma_3^2/2t)$, for every $t \le T$, by Lemma 2.4, we have

$$\widetilde{E}_t^*[f(\sigma_1 B_T^H + \sigma_2 B_T + \sigma_3 P_T)] = \frac{1}{\mathbb{X}_t}\widetilde{E}_t[f(\sigma_1 B_T^H + \sigma_2 B_T + \sigma_3 P_T)\mathbb{X}_T].$$

LEMMA 2.6: *Suppose $V_t = V(S_t, t)$ is a binary differential function if stochastic process S_t suitable for the following equation*

$$\mathrm{d}S_t = \mu S_t \mathrm{d}t + \sigma_1 S_t \mathrm{d}B_t^H + \sigma_2 S_t \mathrm{d}B_t + \sigma_3 S_t \mathrm{d}P_t, \tag{4}$$

then

$$\mathrm{d}V_t = \left[\frac{\partial V}{\partial t} + \mu S_t \frac{\partial V}{\partial S} + \left(H\sigma_1^2 S_t^2 t^{2H-1} + \frac{1}{2}\sigma_2^2 S_t^2 + \frac{1}{2}\lambda\sigma_3^2 S_t^2\right)\frac{\partial^2 V}{\partial S^2}\right]\mathrm{d}t$$
$$+ \sigma_1 S_t \frac{\partial V}{\partial S}\mathrm{d}B_t^H + \sigma_2 S_t \frac{\partial V}{\partial S}\mathrm{d}B_t + \sigma_3 S_t \frac{\partial V}{\partial S}\mathrm{d}P_t + \lambda\mathbb{E}[V(S(1+j_t), t) - V(S, t)]\mathrm{d}t,$$

where $\sigma_3 S_t \partial V/\partial S \mathrm{d}P_t$ is the change volume of Poisson-jump process within dt *for $\partial V/\partial S$, $\lambda\mathbb{E}[V(S(1+j_t), t) - V(S, t)]\mathrm{d}t$ is the change volume of Poisson-jump process within* dt, *and $\mathbb{E}$ is the expectation operator of V.*

PROOF: See the Appendix A. ■

2.2. Wick-Itô-Skorohod Integral and Mixed Jump-Diffusion Fractional Stochastic Differential Equation

DEFINITION 2.2.1 ([3]): *For $Y\colon \mathbb{R} \to (\delta)^*$ is a given function such that $Y_t \diamond B_t^H$ is* dt-*integrable in $(\delta)^*$. Then the Wick-Itô-Skorohod integral of Y_t with respect to B_t^H by*

$$\int_{\mathbb{R}} Y_t \mathrm{d}B_t^H := \int_{\mathbb{R}} Y_t \diamond W_t^H \mathrm{d}t,$$

where $\diamond$ is the Wick product and W_t^H is the fractional Gaussian noise.

LEMMA 2.2.1 (Fractional Girsanov formula I)[3]: *Let* $\psi \in L^p(\mathbb{P}^H)$*, for some* $p > 1$ *and let* $\gamma \in L^2_\phi(\mathbb{R}) \cap C(\mathbb{R}) \subset \delta'(\mathbb{R})$*. Let* $\widetilde{\gamma}$ *be defined by* $\widetilde{\gamma} = \int_{\mathbb{R}} \phi(t, s)\gamma(s)\mathrm{d}s$*. Then the map* $\omega \to \psi(\omega + \widetilde{\gamma})$ *belongs to* $L^\rho(\mathbb{P}^H)$*, for all* $\rho < p$ *and*

$$\int_{\delta'(\mathbb{R})} \psi(\omega + \widetilde{\gamma})\mathrm{d}\mathbb{P}^H(\omega) = \int_{\delta'(\mathbb{R})} \psi(\omega) \cdot \exp^{\diamond}(\langle \omega, \gamma \rangle)\mathrm{d}\mathbb{P}^H(\omega).$$

where equip $\delta(\mathbb{R})$ *with the inner product*

$$\langle \omega, \gamma \rangle_H := \int_{\mathbb{R}} \int_{\mathbb{R}} \omega(s)\gamma(t)\phi(s, t)\mathrm{d}s\mathrm{d}t, \quad \omega, \gamma \in \delta(\mathbb{R}),$$

and

$$\phi(s, t) = \phi_H(s, t) = H(2H - 1)|s - t|^{2H-2}, \quad s, t \in \mathbb{R}.$$

LEMMA 2.2.1* (Fractional Girsanov formula II)[3]: *Let* $T > 0$*, and let* γ *be a continuous function with supp* $\gamma \subset [0, T]$*. Let* K *be a function with supp* $K \subset [0, T]$ *and such that*

$$\langle K, f \rangle_H = \langle \gamma, f \rangle_{L^2(\mathbb{R})}, \quad \textit{for all} \quad f \in \delta(\mathbb{R}), \quad \textit{supp} \quad f \subset [0, T],$$

that is,

$$\int_{\mathbb{R}} K(s)\phi(s, t)\mathrm{d}s = \gamma(t), \quad 0 \le t \le T.$$

On the $\sigma-$*algebra* $\mathfrak{F}^H_T$ *generated by* $\{B^H_s : 0 \le t \le T\}$*, define a probability measure* $\mathbb{P}^{H,\gamma}$ *by*

$$\frac{\mathrm{d}\mathbb{P}^{H,\gamma}}{\mathrm{d}\mathbb{P}^H} = \exp^{\diamond}(-\langle \omega, K \rangle),$$

then $\widehat{B}^H(t) = B^H_t + \int_0^t \gamma_s \mathrm{d}s$*,* $0 \le t \le T$ *is a fBm under* $\mathbb{P}^{H,\gamma}$*.*

LEMMA 2.2.2 (Wick products on different white noise spaces) [3]: *Let* $P = \mathbb{P}^H$*,* $Q = \mathbb{P}^{H,\gamma}$*, and* $\widehat{B}^H(t) = B^H_t + \int_0^t \gamma_s \mathrm{d}s$*. Let the Wick products corresponding to* P *and* Q *be denoted by* $\diamond_P$ *and* $\diamond_Q$*, respectively. Then*

$$F \diamond_P G = F \diamond_Q G,$$

for all $F, G \in (\delta)^*_H$*.*

The proof of Lemma 2.2.1–2.2.2 can be seen [3].

Suppose the price of the risky asset S_t and interest rate r_t satisfy the following equation

$$\begin{cases} \mathrm{d}S_t = r_t S_t \mathrm{d}t, \\ S_0 = 1. \end{cases}$$

By Definition 2.2.1–Lemma 2.2.2, we can consider the mixed jump-diffusion fractional stochastic differential equation

$$\begin{cases} \mathrm{d}S_t = S_t \diamond [(r - q - \lambda\sigma_3)\mathrm{d}t + \sigma_1 \mathrm{d}B^H_t + \sigma_2 \mathrm{d}B_t + \sigma_3 \mathrm{d}P_t], \\ S_t = S. \end{cases}$$

where r is the instantaneous expected return, $q \ge 0$ is the continuous dividend rate, σ_1 and σ_2 are the same as Lemma 2.3, σ_3 is the unconditional expectation of j_t. Assume that B^H_t, B_t and P_t are independent.

Suppose J is the path-dependent variable, for lookback put option on the maturity T, the stock price S_t satisfies $S_t \leq J_t = \max_{0\leq t\leq T} S_t$; for lookback call option on the maturity T, the stock price S_t satisfies $S_t \geq J_t = \min_{0\leq t\leq T} S_t$. Thus, the lookback put option value $V(S_t,\, J_t,\, t)$ is the function of S, J, and t. Then we construct a riskless portfolio $\Pi = V - \nabla \diamond S$, choose the appropriate variable ∇ makes the investment portfolio Π in the interval $(t,\, t+\mathrm{d}t)$ on risk-free. Then $\mathrm{d}\Pi = r(V - \nabla \diamond S)\mathrm{d}t$, according to the Itô formula we have

$$
\begin{aligned}
\mathrm{d}\Pi &= \mathrm{d}V - \nabla \diamond \mathrm{d}S - \nabla q \diamond S\mathrm{d}t \\
&= \frac{\partial V}{\partial t}\mathrm{d}t + \frac{\partial V}{\partial J}\mathrm{d}J + \frac{1}{2}\lambda\sigma_3^2 S^2\frac{\partial^2 V}{\partial S^2}\mathrm{d}t + \frac{\partial V}{\partial S}\mathrm{d}S + H\sigma_1^2 S^2 t^{2H-1}\frac{\partial^2 V}{\partial S^2}\mathrm{d}t \\
&\quad + \frac{1}{2}\sigma_2^2 S^2\frac{\partial^2 V}{\partial S^2}\mathrm{d}t + \sigma_3\frac{\partial V}{\partial S}\mathrm{d}P_t - \nabla \diamond \mathrm{d}S - \nabla q \diamond S\mathrm{d}t \\
&= \left[\frac{\partial V}{\partial t} + \left(H\sigma_1^2 t^{2H-1} + \frac{1}{2}\sigma_2^2 + \frac{1}{2}\lambda\sigma_3^2\right) S^2\frac{\partial^2 V}{\partial S^2} - \nabla q \diamond S\right]\mathrm{d}t + \frac{\partial V}{\partial J}\mathrm{d}J + \sigma_3\frac{\partial V}{\partial S}\mathrm{d}P_t \\
&\quad + \left(\frac{\partial V}{\partial S} - \nabla\right)\diamond \mathrm{d}S.
\end{aligned}
$$

Note that J_t is nondifferentiable about t, by $J_n(t) = [\frac{1}{t}\int_0^t (S_\tau)^n \mathrm{d}\tau]^{1/n}$, such that approximate amount $J_n(t)$ is differentiable about t and satisfy

$$
nJ_n^{n-1}(t)\frac{\mathrm{d}J_n}{\mathrm{d}t} = \frac{S_t^n - J_n^n(t)}{t}.
$$

Hence S_t is continuous function with t, we have $\lim_{n\to\infty} J_n(t) = \max_{0\leq t\leq T} S_t = J_t$, and

$$
\begin{aligned}
\mathrm{d}\Pi &= \left[\frac{\partial V}{\partial t} + \left(H\sigma_1^2 t^{2H-1} + \frac{1}{2}\sigma_2^2 + \frac{1}{2}\lambda\sigma_3^2\right) S^2\frac{\partial^2 V}{\partial S^2} - \nabla q \diamond S\right]\mathrm{d}t + \frac{\partial V}{\partial J}\mathrm{d}J \\
&\quad + \sigma_3\frac{\partial V}{\partial S}\mathrm{d}P_t + \left(\frac{\partial V}{\partial S} - \nabla\right)\diamond \mathrm{d}S \\
&= \left[\frac{\partial V}{\partial t} + \left(H\sigma_1^2 t^{2H-1} + \frac{1}{2}\sigma_2^2 + \frac{1}{2}\lambda\sigma_3^2\right) S^2\frac{\partial^2 V}{\partial S^2} + \frac{\partial V}{\partial J_n}\frac{dJ_n}{dt} - \nabla q \diamond S\right]\mathrm{d}t \\
&\quad + \sigma_3\frac{\partial V}{\partial S}\mathrm{d}P_t + \left(\frac{\partial V}{\partial S} - \nabla\right)\diamond \mathrm{d}S.
\end{aligned}
$$

By $\nabla = \frac{\partial V}{\partial S}$, then

$$
\begin{aligned}
&\frac{\partial V}{\partial t} + \left(H\sigma_1^2 t^{2H-1} + \frac{1}{2}\sigma_2^2 + \frac{1}{2}\lambda\sigma_3^2\right) S^2\frac{\partial^2 V}{\partial S^2} + \frac{(S/J_n)^{n-1}S - J_n}{nt}\frac{\partial V}{\partial J_n} \\
&\quad + (r - q - \lambda\sigma_3)S\frac{\partial V}{\partial S} + \lambda\mathbb{E}[V(S(1+j_t),t) - V(S,t)] - (r+\lambda)V = 0,
\end{aligned}
$$

where $S \leq J_n$ and $\mathbb{E}$ is the expectation operator of V.

For fixed (J, t), when $n \to \infty$, then $(S/J_n)^{n-1}S - J_n/nt \to 0$, since we obtain the general equations of lookback option under the environment of MJD-fBm as follows:

$$\frac{\partial V}{\partial t} + \left(H\sigma_1^2 t^{2H-1} + \frac{1}{2}\sigma_2^2 + \frac{1}{2}\lambda\sigma_3^2\right) S^2 \frac{\partial^2 V}{\partial S^2} + (r - q - \lambda\sigma_3) S \frac{\partial V}{\partial S} + \lambda \mathbb{E}[V(S(1+j_t), t) - V(S,t)] - (r+\lambda)V = 0, \tag{5}$$

$$V(S, J, t) = J - S, \tag{6}$$

where $0 \le S \le J < \infty$, $0 \le t \le T$, and the terminal condition is given by

$$\left.\frac{\partial V}{\partial J}\right|_{S=J} = 0. \tag{7}$$

3. THE OPTIMAL STOPPING PROBLEM

In this section, we turn to the exercise boundary formulation of special optimal stopping problem described, and the critical exercise price is given by a Volterra integral equation.

3.1. Fundamental Solution Derivation of the General Equations

Define $\mathfrak{G}(S,\, t; \xi,\, T)$ is the fundamental solution of the following solution problem:

$$\begin{cases} \dfrac{\partial V}{\partial t} + \left(H\sigma_1^2 t^{2H-1} + \dfrac{1}{2}\sigma_2^2 + \dfrac{1}{2}\lambda\sigma_3^2\right) S^2 \dfrac{\partial^2 V}{\partial S^2} + (r - q - \lambda\sigma_3) S \dfrac{\partial V}{\partial S} + \lambda \mathbb{E}[V(S(1+j_t), t) \\ \qquad - V(S,t)] - (r+\lambda)V = 0, & (8) \\ V(S, T) = \delta(S - \xi), & (9) \end{cases}$$

where $0 < S < \infty$, $0 < \xi < \infty$, $0 < t < T$, $\delta(x)$ is Dirac function.

Lemma 3.1.1 (see [24],Theorem 4.3): *The price of a derivative on the stock price with a bounded payoff $f(S_t)$ is given by $D(t,\, S_t)$, where $D(t,\, S)$ is the solution of the PDE:*

$$\begin{cases} \dfrac{\partial D}{\partial t} + H\sigma^2 S^2 t^{2H-1} \dfrac{\partial^2 D}{\partial S^2} + rS\dfrac{\partial D}{\partial S} - rD = 0, \\ D(S, T) = f(S). \end{cases}$$

Lemma 3.1.2: *The solution of equations (8) and (9) is*

$$\begin{aligned} \mathfrak{G}(S, t; \xi, T) = \sum_{n=0}^{\infty} \left\{ \frac{\lambda^n (T-t)^n \exp[-\lambda(T-t)]}{n!} \mathcal{E}_n \right\} \\ \cdot \frac{\exp[-r(T-t)]}{\xi\sqrt{2\pi[2\sigma_1^2(T^{2H} - t^{2H}) + \sigma_2^2(T-t) + \lambda\sigma_3^2(T-t)]}} \\ \cdot \exp\left\{ - \frac{1}{2\sigma_1^2(T^{2H} - t^{2H}) + \sigma_2^2(T-t) + \lambda\sigma_3^2(T-t)} \right. \\ \cdot \left[\ln \frac{S\prod_{i=1}^{n}(1+j_{t_i})}{\xi} + (r - q - \lambda\sigma_3)(T-t) - \sigma_1^2(T^{2H} - t^{2H}) \right. \\ \left.\left. - \frac{1}{2}(\sigma_2^2 + \lambda\sigma_3^2)(T-t) \right]^2 \right\}, \end{aligned}$$

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where $\mathcal{E}_n$ *denotes the expectation operator over the distribution of* $\prod_{i=1}^{n}(1+j_{t_i})$.

Proof: See the Appendix B. ■

Theorem 3.1.3: *The fundamental solution* $\mathfrak{G}(S, t; \xi, T)$ *consider to be the function of* ξ *and* η, *and* $\mathfrak{G}(S, t; \xi, T)$ *satisfies the adjoint equation of* (8) *and* (9). *If we note*

$$\mathfrak{G}(S, t; \xi, \eta) = v(\xi, \eta),$$

then $v(\xi, \eta)$ *satisfy*

$$\begin{cases} -\dfrac{\partial v}{\partial t} - \left(H\sigma_1^2 t^{2H-1} + \dfrac{1}{2}\sigma_2^2 + \dfrac{1}{2}\lambda\sigma_3^2\right) S^2 \dfrac{\partial^2 v}{\partial S^2} - (r - q - \lambda\sigma_3) S \dfrac{\partial v}{\partial S} \\ \quad - \lambda\mathbb{E}[v(S(1+j_t), t) - v(S, t)] - (r+\lambda)v = 0, & \textbf{(10)} \\ v(\xi, \eta) = \delta(\xi - S), & \textbf{(11)} \end{cases}$$

where $0 < S < \infty$, $0 < \xi < \infty$, $t < \eta$.

Proof: See the Appendix C. ■

Corollary 3.1.4: *By Lemma* 3.1.1, 3.1.2 *and* 3.1.3, *if we note* $\mathfrak{G}^*(\xi, \eta; S, t)$ *is the fundamental solution of* (10) *and* (11), *then*

$$\mathfrak{G}(S, t; \xi, \eta) = \mathfrak{G}^*(\xi, \eta; S, t).$$

3.2. Optimal Stopping Problem of American Lookback Option

Conze A and Viswanathan [9] introduced a fractional or partial lookback option, where the strike is fixed at some fraction over (for a call) or below (for a put) the extreme value. Specifically, the payoffs for European lookback call and put with fractional floating strikes and maturity date T are given, respectively, by $(S_T - \mathbb{a}\mathbb{m}_T)^+$ and $(\mathbb{b}\mathbb{M}_T - S_T)^+$, where $\mathbb{a}$ and $\mathbb{b}$ are positive constants, allowing flexible adjustment of option premiums. To reduce option premiums, we assume that $\mathbb{a} \geq 1$ and $0 < \mathbb{b} \leq 1$. Given a finite time horizon $T > 0$, let $\mathcal{C} = \mathcal{C}(t, S, \mathbb{m})$ be the value of the American fractional lookback call option at time $t \in [0, T]$. Note that the values of American and European call options are equal if the underlying asset pays no dividends. In the absence of arbitrage opportunities, the value $\mathcal{C}(t, S, \mathbb{m})$ is a solution of an optimal stopping problem

$$\mathcal{C}(t, S, \mathbb{m}) = \sup_{T_t \in [t, T]} \mathbb{E}[\exp\{-r(T_t - t)\}(S_{T_t} - \mathbb{a}\mathbb{m}_{T_t})^+ | S_t = S, \mathbb{m}_t = \mathbb{m}], \qquad \textbf{(12)}$$

where T_t is a stopping time of the filtration $\mathfrak{F}$ and the conditional expectation is calculated under the risk-neutral probability measure $\mathbb{P}$. The random variable $T_t^* \in [t, T]$ is called an optimal stopping time if it gives the supremum value of the right-hand side of (12). It is clear from (12) that $\mathcal{C}$ is nondecreasing in S and nonincreasing in t, $\mathbb{m}$, and $\mathbb{a}$. Solving the optimal stopping problem (12) is equivalent to finding the points $(t, S_t, \mathbb{m}_t)$ for which early exercise before maturity is optimal. Let

$$\mathfrak{D} = \{(t, S, \mathbb{m}) \in [0, T] \times [\mathbb{m}, \infty) \times \mathbb{R}_+\}$$

be the whole domain, and let $\mathfrak{E}$ and $\mathfrak{C}$ denote the exercise region and continuation region, respectively. In terms of the value function $\mathcal{C}(t,\ S,\ \mathbb{m})$, the exercise region $\mathfrak{E}$ is defined by

$$\mathfrak{E} = \{(t, S, \mathbb{m}) | \mathcal{C}(t, S, \mathbb{m}) = (S - a\mathbb{m})^+\},$$

for which the optimal stopping time T_t^* satisfies

$$T_t^* = \inf\{\eta \in [t, T] | (\eta, S_\eta, \mathbb{m}_\eta) \in \mathfrak{E}\}.$$

The continuation region $\mathfrak{C}$ is the complement of $\mathcal{S}$ in $\mathfrak{D}$, such as

$$\mathfrak{C} = \{(t, S, \mathbb{m}) | \mathcal{C}(t, S, \mathbb{m}) > (S - a\mathbb{m})^+\}.$$

The boundary that separates $\mathcal{S}$ from $\mathfrak{C}$ is referred to as the early exercise boundary, which is defined by

$$\bar{S}(t, \mathbb{m}) = \sup\{S \geq \mathbb{m} | (t, S, \mathbb{m}) \in \mathfrak{C}\}, \quad t \in [0, T].$$

At the early exercise boundary $[\bar{S}(t,\ \mathbb{m})]_{t\in[0,T]}$, the American fractional lookback call option would be optimally exercised. In terms of $S(t,\ \mathbb{m})$, the continuation region $\mathfrak{C}$ can be represented as

$$\mathfrak{C} = \{(t, S, \mathbb{m}); \mathbb{m} \leq S < \bar{S}(t, \mathbb{m})\}.$$

Let $V(S,\ J,\ t)$ be the lookback option price at time t with stock price S and path-dependent variable J. Using argument similar to Section 2.2, it can be shown that the American fractional lookback option price solves the following stochastic partial differential equations:

$$0 = \frac{\partial V}{\partial t} + \left(H\sigma_1^2 t^{2H-1} + \frac{1}{2}\sigma_2^2 + \frac{1}{2}\lambda\sigma_3^2\right) S^2 \frac{\partial^2 V}{\partial S^2} + (r - q - \lambda\sigma_3) S \frac{\partial V}{\partial S} - (r + \lambda) V$$
$$+ \begin{cases} \lambda\mathbb{E}[V(S(1 + j_t), \min\{J, S(1 + j_t)\}, t)], & \text{for lookback call option,} \\ \lambda\mathbb{E}[V(S(1 + j_t), \max\{J, S(1 + j_t)\}, t)], & \text{for lookback put option,} \end{cases} \quad \mathbf{(13)}$$

$$V(S, J, t) = J - S, \quad \mathbf{(14)}$$

where $0 \leq t \leq T$, and satisfy an order continuous differentiable on domain

$$\Sigma = \begin{cases} \{(S, J) : 0 < J \leq S < \infty\}, & \text{for lookback call option,} \\ \{(S, J) : 0 < S \leq J < \infty\}, & \text{for lookback put option,} \end{cases} \quad \mathbf{(15)}$$

the terminal condition is given by

$$\left.\frac{\partial V}{\partial J}\right|_{S=J} = 0. \quad \mathbf{(16)}$$

We define the differential operator $\mathcal{L}_{t,S}$ by

$$\mathcal{L}_{t,S} = \frac{\partial}{\partial t} + \left(H\sigma_1^2 t^{2H-1} + \frac{1}{2}\sigma_2^2 + \frac{1}{2}\lambda\sigma_3^2\right) S^2 \frac{\partial^2}{\partial S^2} + (r - q - \lambda\sigma_3) S \frac{\partial}{\partial S} - (r + \lambda).$$

Then the free boundary problem can be written in a linear complementary form as

$$\begin{cases} [\mathcal{L}_{t,S}\mathcal{C}(t,S,\mathbb{m})]\cdot[\mathcal{C}(t,S,\mathbb{m})-(S-\mathbb{a}\mathbb{m})^+]=0, \\ \mathcal{L}_{t,S}\mathcal{C}(t,S,\mathbb{m})\le 0, \\ \mathcal{C}(t,S,\mathbb{m})-(S-\mathbb{a}\mathbb{m})^+\ge 0 \end{cases}$$

together with auxiliary conditions

$$\mathcal{C}(T,S,\mathbb{m})-(S-\mathbb{a}\mathbb{m})^+=0 \quad \text{and} \quad \lim_{S\downarrow\mathbb{m}}\frac{\partial\mathcal{C}}{\partial\mathbb{m}}=0.$$

For the free boundary $[\bar{S}(t,\,\mathbb{m})]_{t\in[0,T]}$, this problem is equivalent to solving the Black–Scholes–Merton partial differential equations

$$\mathcal{L}_{t,S}\mathcal{C}(t,S,\mathbb{m})=0, \quad \mathbb{m}\le S<\bar{S}(t,\mathbb{m}), \tag{17}$$

together with the boundary conditions

$$\begin{cases} \lim_{S\uparrow\bar{S}}\mathcal{C}(t,S,\mathbb{m})=\bar{S}(t,\mathbb{m})-\mathbb{a}\mathbb{m}, \\ \lim_{S\uparrow\bar{S}}\frac{\partial\mathcal{C}}{\partial S}=1, \quad \text{and} \quad \lim_{S\downarrow\mathbb{m}}\frac{\partial\mathcal{C}}{\partial\mathbb{m}}=0. \end{cases} \tag{18}$$

and the terminal condition

$$\mathcal{C}(T,S,\mathbb{m})=(S-\mathbb{a}\mathbb{m})^+. \tag{19}$$

In the same way as in the call case, by (12)–(19), we can formulate the put case: Let $\mathcal{P}=\mathcal{P}(t,\,S,\,\mathbb{m})$ be the value of the American fractional lookback call option at time $t\in[0,\,T]$. The value $\mathcal{P}(t,\,S,\,\mathbb{M})$ is a solution of an optimal stopping problem

$$\mathcal{P}(t,S,\mathbb{M})=\sup_{T_t\in[t,T]}\mathbb{E}[\exp\{-r(T_t-t)\}(\mathbb{b}\mathbb{M}_{T_t}-S_{T_t})^+|S_t=S,\mathbb{M}_t=\mathbb{M}], \tag{20}$$

And $\mathcal{P}(t,\,S,\,\mathbb{M})$ satisfies the same PDE as (17), then

$$\mathcal{L}_{t,S}\mathcal{P}(t,S,\mathbb{M})=0, \quad \underline{S}(t,\mathbb{M})<S\le\mathbb{M}, \tag{21}$$

where $[\underline{S}(t,\,\mathbb{M})]_{t\in[0,T]}$ is the early exercise boundary for put. The boundary conditions for put are

$$\begin{cases} \lim_{S\downarrow\underline{S}}\mathcal{P}(t,S,\mathbb{M})=\mathbb{b}\mathbb{M}-\underline{S}(t,\mathbb{M}), \\ \lim_{S\downarrow\underline{S}}\frac{\partial\mathcal{P}}{\partial S}=-1, \quad \text{and} \quad \lim_{S\uparrow\mathbb{M}}\frac{\partial\mathcal{P}}{\partial\mathbb{M}}=0. \end{cases} \tag{22}$$

and the terminal condition is given by

$$\mathcal{P}(T,S,\mathbb{M})=(\mathbb{b}\mathbb{M}-S)^+. \tag{23}$$

3.3. The Critical Exercise Price

It is well known that the value of an American fractional option can be represented as the sum of the value of the corresponding European option and the early exercise premium. For American fractional lookback options, Lai and Lim [20] proved that the value has such a decomposition and that the premium has an integral representation. Applying the same

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solution method as in Theorem 3.1.3 to the PDE (23) for $\mathcal{P}(t, S, \mathbb{M})$, we can obtain the critical exercise price under our MJD-fBm environment, which is shown in the following theorem.

THEOREM 3.2.1: *Let $V(t, S, \mathbb{M})$ is American fractional lookback put option, then*

$$V(t, S, \mathbb{M}) = V_E(t, S, \mathbb{M}) + e(t, S, \mathbb{M}), \tag{24}$$

where $V_E(t, S, \mathbb{M})$ is the price of the hedging portfolio of the equivalent European option, and

$$\begin{aligned} V_E(t, S, \mathbb{M}) = \sum_{n=0}^{\infty} \left\{ \frac{\lambda^n (T-t)^n \exp[-\lambda(T-t)]}{n!} \mathcal{E}_n \right\} \\ \cdot \left[\mathbb{b}\mathbb{M} \exp\{-r(T-t)\} \mathbb{N}(-\hat{d}_2) - S \prod_{i=1}^{n} (1 + j_{t_i}) \right. \\ \left. \exp\{-(q + \lambda\sigma_3)(T-t)\} \mathbb{N}(-\hat{d}_1) \right], \end{aligned} \tag{25}$$

where $\mathcal{E}_n$ denotes the expectation operator over the distribution of $\prod_{i=1}^{n}(1 + j_{t_i})$, $\mathbb{M}$ is contractual strike price, $\mathbb{b}$ is positive constants, $\mathbb{N}(\cdot)$ is the cumulative normal distribution function and

$$\begin{aligned} \hat{d}_1 = {} & \left[\ln \frac{S \prod_{i=1}^{n}(1 + j_{t_i})}{\xi} + (r - q - \lambda\sigma_3)(T-t) + \sigma_1^2 (T^{2H} - t^{2H}) + \frac{1}{2}(\sigma_2^2 + \lambda\sigma_3^2) \cdot (T-t) \right] \\ & \cdot \left(\sqrt{2\sigma_1^2 (T^{2H} - t^{2H}) + \sigma_2^2 (T-t) + \lambda\sigma_3^2 (T-t)} \right)^{-1}, \end{aligned} \tag{26}$$

$$\hat{d}_2 = \hat{d}_1 - \sqrt{2\sigma_1^2 (T^{2H} - t^{2H}) + \sigma_2^2 (T-t) + \lambda\sigma_3^2 (T-t)}, \tag{27}$$

$e(t, S, \mathbb{M})$ is early exercise premium, and its explicit integral can be represented by

$$e(t, S, \mathbb{M}) = \int_t^T \mathrm{d}\eta \int_0^{S_\eta} [\mathbb{b}\mathbb{M} r - (q + \lambda\sigma_3)\xi] \cdot \mathfrak{G}(S, t; \xi, \eta) \mathrm{d}\xi. \tag{28}$$

PROOF: See the Appendix D. ■

Remark 1: If the critical exercise price is given by $S = S_t$, then pricing of American fractional lookback Options can be represented as (24)–(28).

Remark 2: This result shows that American fractional lookback options is equal to the option of a hedging portfolio: a risk premium associated with the European values plus the early-exercise premium. Using the similar techniques, the critical exercise price can be given by the following Corollary.

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COROLLARY 3.2.2: *The critical exercise price of American fractional lookback put options is defined by $S = S_t$, $0 \le t < T$, then $S = S_t$ satisfy the following Volterra integral equation*

$$
\begin{aligned}
S_t = {} & \mathbb{bM} + S\exp\{-(q+\lambda\sigma_3)(T-t)\} \\
& \cdot \mathbb{N}\Bigg\{-\Bigg[-\ln\frac{S\prod_{i=1}^{n}(1+j_{t_i})}{\mathbb{bM}} + (r-q-\lambda\sigma_3)(T-t) + \sigma_1^2(T^{2H}-t^{2H}) \\
& + \frac{1}{2}(\sigma_2^2+\lambda\sigma_3^2)(T-t)\Bigg]\cdot\Bigg[\sqrt{2\sigma_1^2(T^{2H}-t^{2H})+\sigma_2^2(T-t)+\lambda\sigma_3^2(T-t)}\Bigg]^{-1}\Bigg\} \\
& - \mathbb{bM}\exp\{-r(T-t)\}\sum_{n=0}^{\infty}\left\{\frac{\lambda^n(T-t)^n\exp[-\lambda(T-t)]}{n!}\mathcal{E}_n\right\} \\
& \cdot \mathbb{N}\Bigg\{-\Bigg[-\ln\frac{S\prod_{i=1}^{n}(1+j_{t_i})}{\mathbb{bM}} + (r-q-\lambda\sigma_3)(T-t) - \sigma_1^2(T^{2H}-t^{2H}) \\
& - \frac{1}{2}(\sigma_2^2+\lambda\sigma_3^2)(T-t)\Bigg]\cdot\Bigg[\sqrt{2\sigma_1^2(T^{2H}-t^{2H})+\sigma_2^2(T-t)+\lambda\sigma_3^2(T-t)}\Bigg]^{-1}\Bigg\} \\
& - \mathbb{bM}r\int_t^T \exp\{-r(\eta-t)\}\sum_{n=0}^{\infty}\left\{\frac{\lambda^n(\eta-t)^n\exp[-\lambda(\eta-t)]}{n!}\mathcal{E}_n\right\} \\
& \cdot\Bigg\{1-\mathbb{N}\Bigg[\Bigg(\ln\frac{S\prod_{i=1}^{n}(1+j_{t_i})}{S_\eta} + (r-q-\lambda\sigma_3)(\eta-t) - \sigma_1^2(\eta^{2H}-t^{2H}) \\
& - \frac{1}{2}(\sigma_2^2+\lambda\sigma_3^2)(\eta-t)\Bigg)\cdot(\sqrt{2\sigma_1^2(\eta^{2H}-t^{2H})+\sigma_2^2(\eta-t)+\lambda\sigma_3^2(\eta-t)})^{-1}\Bigg]\Bigg\}\mathrm{d}\eta \\
& + (q+\lambda\sigma_3)S\int_t^T \exp\{-(q+\lambda\sigma_3)(\eta-t)\}\sum_{n=0}^{\infty}\left\{\frac{\lambda^n(\eta-t)^n\exp[-\lambda(-t)]}{n!}\mathcal{E}_n\right\} \\
& \cdot\Bigg\{1-\mathbb{N}\Bigg[\Bigg(\ln\frac{S\prod_{i=1}^{n}(1+j_{t_i})}{S_\eta} + (r-q-\lambda\sigma_3)(\eta-t) + \sigma_1^2(\eta^{2H}-t^{2H}) \\
& + \frac{1}{2}(\sigma_2^2+\lambda\sigma_3^2)(\eta-t)\Bigg)\cdot\Bigg(\sqrt{2\sigma_1^2(\eta^{2H}-t^{2H})+\sigma_2^2(\eta-t)+\lambda\sigma_3^2(\eta-t)}\Bigg)^{-1}\Bigg]\Bigg\}\mathrm{d}\eta,
\end{aligned}
\tag{29}
$$

where $\mathbb{N}(\cdot)$ *is the cumulative normal distribution function and* $S_\eta \ge \mathbb{M} > 0$, $0 < \eta \le T$.

PROOF: See the Appendix E. ■

In practice, it is very difficult to calculate nonlinear integral in (29), we only show that this assumption holds in Volterra integral equation for the critical exercise price, by the relation (29) and Theorem 3.2.1, asymptotic expression of the critical exercise price $S = S_t$ is obtained near the maturity $t = T$. Comparing with Theorem 3.2.1 and Corollary 3.2.2, the critical exercise price and Volterra integral equation of American fractional lookback call option are obvious as follows.

Corollary 3.2.3: *Let $V^*(t, S, \mathbb{m})$ is American fractional lookback call option, then*

$$V^*(t,S,\mathbb{m}) = V_E^*(t,S,\mathbb{m}) + e^*(t,S,\mathbb{m}),$$

where $V_E^*(t, S, \mathbb{m})$ is price of the hedging portfolio of the equivalent European option, and

$$V_E^*(t,S,\mathbb{m}) = \sum_{n=0}^{\infty}\left\{\frac{\lambda^n(T-t)^n\exp[-\lambda(T-t)]}{n!}\mathcal{E}_n\right\} \cdot \left[S\prod_{i=1}^{n}(1+j_{t_i})\exp\{-(q+\lambda\sigma_3)(T-t)\}\mathbb{N}(-\hat{d}_1) - \mathrm{a}\mathbb{m} \times \exp\{-r(T-t)\}\mathbb{N}(-\hat{d}_2)\right],$$

where $\mathcal{E}_n$ denotes the expectation operator over the distribution of $\prod_{i=1}^{n}(1+j_{t_i})$, $\mathrm{a}\mathbb{m}$ is contractual strike price, $\mathbb{N}(\cdot)$ is the cumulative normal distribution function and

$$\hat{d}_1 = \left[\ln\frac{S\prod_{i=1}^{n}(1+j_{t_i})}{\xi} + (r-q-\lambda\sigma_3)(T-t) + \sigma_1^2(T^{2H}-t^{2H}) + \frac{1}{2}(\sigma_2^2+\lambda\sigma_3^2)\cdot(T-t)\right] \times\left(\sqrt{2\sigma_1^2(T^{2H}-t^{2H})+\sigma_2^2(T-t)+\lambda\sigma_3^2(T-t)}\right)^{-1},$$

$$\hat{d}_2 = \hat{d}_1 - \sqrt{2\sigma_1^2(T^{2H}-t^{2H})+\sigma_2^2(T-t)+\lambda\sigma_3^2(T-t)},$$

$e^*(t, S, \mathbb{m})$ is early exercise premium, and its explicit integral can be represented by

$$e^*(t,S,\mathbb{m}) = \int_t^T \mathrm{d}\eta\int_0^{S_\eta}[(q+\lambda\sigma_3)\xi - \mathrm{a}\mathbb{m}r]\cdot\mathfrak{G}(S,t;\xi,\eta)\mathrm{d}\xi.$$

4. ASYMPTOTIC RESULTS

In the whole domain $\mathfrak{D} = \{(t, S, \mathbb{M}) \in [0, T] \times [\mathbb{M}, \infty) \times \mathbb{R}_+\}$ of the American fractional floating strike lookback put option model, the American put is alive when $S < \mathbb{M}$ and becomes dead when $S > \mathbb{M}$. The boundary which divides the continuation region (option remains alive) and the stopping region (option becomes dead) is time-dependent, that is, S is a function of t. Similar to the usual argument for American floating strike lookback put options that the critical exercise price should be denoted as

$$\underline{S}(t,\mathbb{M}) = \frac{S_t(t,S,\mathbb{M})}{\mathbb{M}}.$$

As deduced from (29), at a given time t, the optimal exercise boundary $S_t(t, S, \mathbb{M})$ increases linearly with $\mathbb{M}$. Since both $\underline{S}(t, \mathbb{M})$ and $\mathbb{M}$ are increasing function of time t, and so $S_t(t, S, \mathbb{M})$ increases as time is approaching expiration.

Similarly, the critical exercise price for the American fractional floating strike lookback call option should be denoted as

$$\overline{S}(t,\mathbb{m}) = \frac{S_t(t,S,\mathbb{m})}{\mathbb{m}},$$

where $\overline{S}(t, \mathbb{m})$ is a monotonically decreasing function of time t. The plots of $\underline{S}(t, \mathbb{M})$ and $\overline{S}(t, \mathbb{m})$ against time t with varying interest rate are shown in Figure 1, where

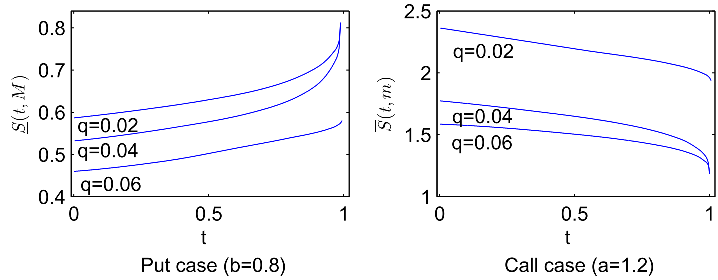

FIGURE 1. Plot of $\underline{S}$ and $\overline{S}$ against time t for American fractional lookback options with q.

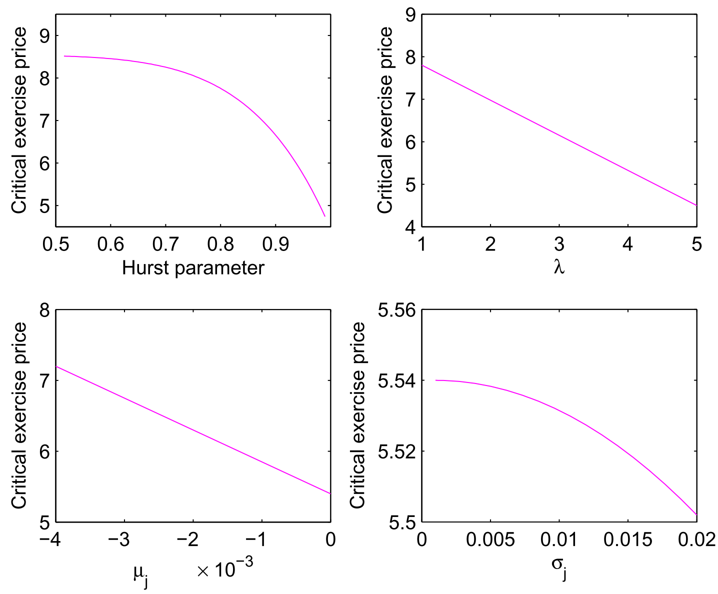

FIGURE 2. The critical exercise prices of the floating strike lookback put option.

$\mathbb{m} = \mathbb{M} = 100$, $T = 1$, $r = 0.05$, $q = 0.02, 0.04, 0.06$, $\sigma_1 = 0.2$, $\sigma_2 = 0.3$, $\sigma_3 = 0.071$, $\lambda = 2.68$, $\mathbb{a} = 1.2$ and $\mathbb{b} = 0.8$. Since the lower value of interest rate leads to the loss of the time value of the floating strike price to be smaller when the American fractional floating strike lookback call option is exercised prematurely, then the critical exercise price decreases when the interest rate assumes lower value. The similar phenomenon was also appeared in [20,37].

The asymptotic behaviors at times close to maturity of the critical exercise prices for the American fractional floating strike lookback put and call options are respectively given by

$$\lim_{t\to T^-} S_t(t,S,\mathbb{M}) = \frac{1}{\mathbb{a}} \min\left(\frac{r}{q+\lambda\sigma_3}, 1\right) \lim_{t\to T^-} \mathbb{M}_t$$

and

$$\lim_{t\to T^-} S_t(t,S,\mathbb{m}) = \frac{1}{\mathbb{b}} \max\left(\frac{r}{q+\lambda\sigma_3}, 1\right) \lim_{t\to T^-} \mathbb{m}_t.$$

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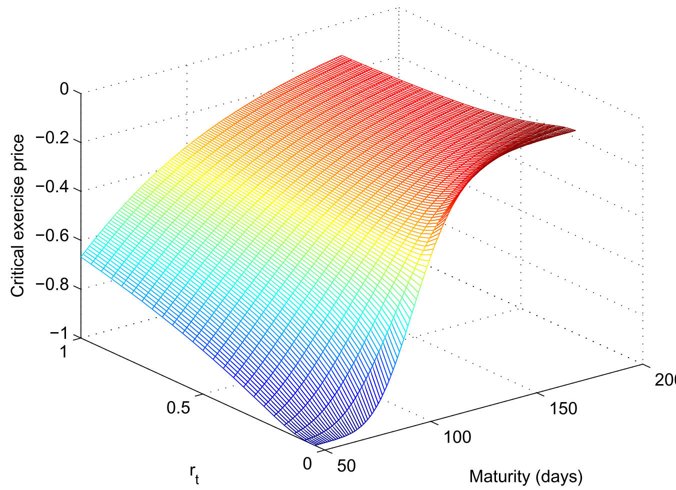

FIGURE 3. Critical exercise price surfaces generated with MJD-fBm.

Hence the strike prices of the American fractional lookback put and call options are set to be $\mathbb{M}_0^T$ and $\mathbb{m}_0^T$, respectively. The usual argument and proof of analysis of limiting behaviors can be applied in a similar manner like literature [18,20,37].

In what follows, the critical exercise prices for the American fractional floating strike lookback put options be displayed by using MJD-fBm for different parameters. We consider the critical exercise prices of our MJD-fBm for various Hurst parameters H and then investigate the critical exercise prices for different jump parameters. Figure 2 shows critical exercise prices of the American fractional floating strike lookback put option against its parameters, H, λ, μ_j, σ_j. The default parameters are $r = 0.0250$, $q = 0.0320$, $\sigma_1 = 0.1073$, $K = 100$, $H = 0.76$, $J = 0.35$, $t = 0$, $T = 2$(Year), $\lambda = 7.88$, $\mu_j = -0.0721$ $\sigma_j = 0.19$, $\sigma_2 = 0.0215$ and $\sigma_3 = 0.07202$. Figure 2 shows that critical exercise prices is a decreasing function of H, λ, μ_j and σ_j. We observed from these figures, that critical exercise price is monotonically decreasing with various Hurst parameters H and the parameters σ_j. It is a convex function of Hurst parameters H and the parameters σ_j. When the jump intensity λ becomes larger, the critical exercise price of lookback option decreases faster, this phenomenon is consistent with empirical evidence.

We compare theoretical exercise prices of some assumptive options among the following models: the pure mixed fBm (hereafter pmfBm), the jump-diffusion Brownian motion (hereafter jdBm), and our MJD-fBm. This exercise consists of some simulations of different pricing models with some chosen parameters that will be not based on empirical data. Using an iterative search procedure like the binomial tree methods, we can easily find the exercise prices for every option critical exercise price and plot an exercise prices surface. In Figure 3, we plot the critical exercise price surfaces generated with our MJD-fBm. The chosen parameters are $r = 0.0250$, $q = 0.0320$, $\sigma_1 = 0.1073$, $K = 100$, $H = 0.76$, $J = 0.35$, $t = 0$, $T = 0.5$(Year), $\lambda = 10.22$, $\mu_j = -0.0721$ $\sigma_j = 0.19$, $\sigma_2 = 0.0215$ and $\sigma_3 = 0.07202$. We compare the two exercise prices derived from the pmfBm and jdBm models for out-of-the-money. The simulation parameters are selected as: $r = 0.0250$, $q = 0.0320$, $\sigma_1 = 0.1073$, $K \in [80, 140]$, $H = 0.76$, $J = 0.8$, $t = 0.1$, $T \in [0, 3.5]$(Year), $\lambda = 2.3$, $\mu_j = 0.0721$ $\sigma_j = 0.19$, $\sigma_2 = 0.0215$ and $\sigma_3 = 0.07202$. For the critical exercise price surfaces generated with pmfBm and jdBm models in Figure 4 and Figure 5, respectively. From these figures it is concluded that the MJD-fBm model be better fitted to the smile curve than pmfBm and jdBm models. As

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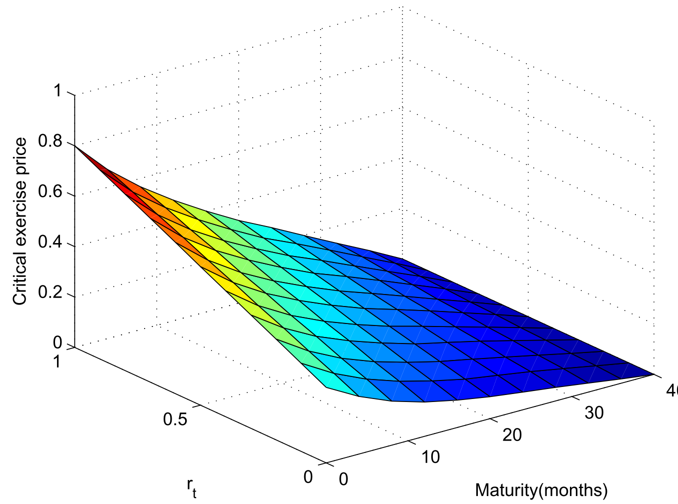

FIGURE 4. Critical exercise price surfaces generated with pmfBm.

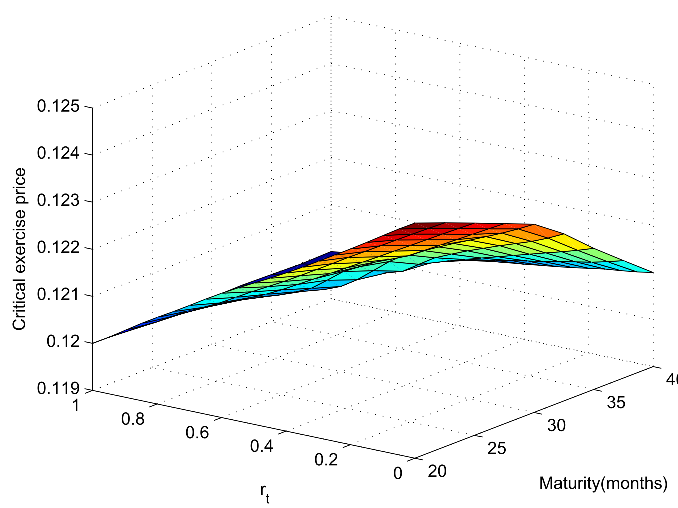

FIGURE 5. Critical exercise price surfaces generated with jdBm.

can be seen, the model indicates that the critical exercise price varies with r_t and maturity, which attempts to explain the curious price movement of a stock (or index) that is sometimes observed during the first few days and last few days leading up to an option's expiration date. The negative correlation between the critical exercise price and r_t of European lookback fixed strike put option is particularly large and is consistent across all sample periods. The critical exercise prices is also negatively and almost perfectively correlated with the maturity(days). This indicates that the non-positive skewness of returns becomes more negative as market volatility rises. Consequently, the critical exercise prices also could serve as an accurate indicator for investor fear. Therefore, we can see that our MJD-fBm model seems reasonable.

5. CONCLUSION

In this paper, we compound the Brownian motion, fBm, and Poisson process by fractional Wick-Itô-Skorohod integral. With an optimal stopping problem and the exercise boundary, the explicit integral representation of early exercise premium and the critical exercise price are estimated by the fundamental solutions of Volterra integral equation. The fundamental solution of stochastic partial differential equations plays an important role in numerical inversion for the put and call case. By solving the free boundary problem and the Black–Scholes–Merton partial differential equations, we characterized asymptotic behaviors of the early exercise boundaries at a time to close to expiration and at infinite time to expiration. We also present and discuss numerical simulations of the critical exercise price.

In the numerical simulations, we further examine the critical exercise prices of our MJD-fBm under various parameters assumptions. We find that critical exercise prices is a decreasing function of H, λ, μ_j and σ_j, and the critical exercise price is monotonically decreasing with various Hurst parameters H and the parameters σ_j. It is a convex function of Hurst parameters H and the parameters σ_j. We compared with the pure mixed fBm, the jump-diffusion Brownian motion and our MJD-fBm, we find that the MJD-fBm model be better fitted to the smile curve than pmfBm and jdBm models. Our model indicates that the critical exercise price varies with r_t and maturity, which attempts to explain the curious price movement of a stock (or index) that is sometimes observed during the first few days and last few days leading up to an option's expiration date. The negative correlation between critical exercise price and r_t of European lookback fixed strike put option is particularly large and is consistent across all sample periods. The critical exercise price is also negatively and almost perfectively correlated with the maturity(days). These results have some reference significance to pricing other European options and exotic options.

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APPENDIX A. PROOF OF LEMMA 2.6

Let $\mathrm{d}P_t$ admits two-point distribution as following:

$$\mathrm{d}P_t = \begin{cases} P\{j_t = \omega_1\} = p, & \text{jumps cannot occur,} \\ P\{j_t = \omega_2\} = 1 - p, & \text{jumps can occur.} \end{cases}$$

Moreover, during the time interval $[t, t + \mathrm{d}t]$, we can write the probability of jumps cannot occur as $\mathbf{Prob}(\omega_1) = 1 - \lambda \mathrm{d}t$, the probability of jumps can occur as $\mathbf{Prob}(\omega_2) = \lambda \mathrm{d}t$. In the case $j_t > 0$, $S_{t+} > S_t$ describes stock price S_t has upward jump at time t. In the case $j_t < 0$, while $S_{t+} < S_t$ describes stock price S_t has downward jump at time t. Hence $j_t > -1$ can ensure the stock price is positive, such that $S_{t+} = S_t(1 + j_t) > 0$.

Let $\Pi_t = V_t - \nabla_t S_t$ is a riskless portfolio and ∇_t is stock shares at time t. In complete financial markets, there are no risk-free arbitrage opportunities. Then

$$\mathbb{E}(\mathrm{d}\Pi_t) = r\Pi_t \mathrm{d}t.$$

No matter the jumps occur or not, by Merton assumptions $\ln(1 + j_t) \sim \mathbb{N}[\ln(1 + \mu_j) - 1/2\sigma_j^2, \sigma_j^2]$ and model (4), the variance σ_j^2 exist surely. Because either $P(X = x) > 0$ for all $x > 0$ or $P(X = x) = 0$ for all $x > 0$. In the first case, every positive point can be hit continuously in X and this phenomena is called creep[2.4]. In the second case, only jumps can occur (almost surely). In time interval $[t, t + \mathrm{d}t]$, the pricing satisfy the following hypothesis:

i) If jumps cannot occur, for the events ω_1, by $V_t = V(S_t, t)$ second order differentiable, hence we can use Itô formula, then

$$\begin{aligned} \mathrm{d}\Pi_t(\omega_1) &= \mathrm{d}V_t - \nabla_t \mathrm{d}S_t \\ &= \left[\frac{\partial V}{\partial t} + (H\sigma_1^2 t^{2H-1} + \frac{1}{2}\sigma_2^2 + \frac{1}{2}\lambda\sigma_3^2)S^2 \frac{\partial^2 V}{\partial S^2}\right]_{(S_t,t)} \mathrm{d}t + \left(\frac{\partial V}{\partial S} - \nabla\right)_{(S_t,t)} \mathrm{d}S_t. \end{aligned}$$

ii) If jumps can occur, for the events ω_2, we have

$$\mathrm{d}\Pi_t(\omega_2) = V(S_{t+}) - V(S_t, t) - \nabla_t(S_{t+} - S_t) = V[(1 + j_t)S_t, t] - V(S_t, t) - \nabla_t j_t S_t.$$

Then

$$\begin{aligned} \mathbb{E}(\mathrm{d}\Pi_t) &= r\Pi_t \mathrm{d}t = (1 - \lambda \mathrm{d}t)[\mathrm{d}\Pi_t(\omega_1)] + \lambda \mathrm{d}t[\mathrm{d}\Pi_t(\omega_2)] \\ &= (1 - \lambda \mathrm{d}t)\left\{\left[\frac{\partial V}{\partial t} + (H\sigma_1^2 t^{2H-1} + \frac{1}{2}\sigma_2^2 + \frac{1}{2}\lambda\sigma_3^2)S^2 \frac{\partial^2 V}{\partial S^2}\right]_{(S_t,t)} \mathrm{d}t \right. \\ &\quad \left. + \left(\frac{\partial V}{\partial S} - \nabla\right)_{(S_t,t)} \mathrm{d}S_t\right\} + \lambda \mathrm{d}t\{V[(1 + j_t)S_t, t] - V(S_t, t) - \nabla_t j_t S_t\}. \end{aligned}$$

Let $\nabla_t = \partial V/\partial S|_{(S_t,t)}$, take expectations for j_t on both side of the above equation, and cancel items of $\mathrm{d}t^2$, then we have a parabolic partial differential equation as following

$$\begin{aligned} &\frac{\partial V}{\partial t} + \left(H\sigma_1^2 t^{2H-1} + \frac{1}{2}\sigma_2^2 + \frac{1}{2}\lambda\sigma_3^2\right) S^2 \frac{\partial^2 V}{\partial S^2} + (r - \lambda\sigma_3)S\frac{\partial V}{\partial S} \\ &\quad + \lambda\mathbb{E}[V(S(1 + j_t), t) - V(S, t)] - (r + \lambda)V = 0, \end{aligned}$$

then under the environment of MJD-fBm, lookback option pricing model can be expressed as a parabolic integral equation which containing expectation.

Notice that $V_t = V(S_t, t)$ is a binary differential function, since we can use Taylor expansion as follows

$$\mathrm{d}V_t = \frac{\partial V}{\partial t}\mathrm{d}t + \frac{\partial V}{\partial S}\mathrm{d}S_t + \frac{1}{2}\frac{\partial^2 V}{\partial S^2}(\mathrm{d}S_t)^2 + o(\mathrm{d}t\mathrm{d}S_t), \tag{A.1}$$

by approximate that

$$(\mathrm{d}B_t^H)^2 \approx \mathsf{Var}(\mathrm{d}B_t^H) = \mathbb{E}(\mathrm{d}B_t^H)^2 = \mathrm{d}t^{2H} = 2Ht^{2H-1}\mathrm{d}t, (\mathrm{d}B_t)^2 = \mathrm{d}t,$$

and

$$(\mathrm{d}P_t, \mathrm{d}P_t) = \mathrm{d}\langle P, P\rangle(t) = (\mathrm{d}P_t)^2 = \lambda\mathrm{d}t,$$

we have

$$\begin{aligned}
(\mathrm{d}S_t)^2 &= [\mu S_t\mathrm{d}t + \sigma_1 S_t\mathrm{d}B_t^H + \sigma_2 S_t\mathrm{d}B_t + \sigma_3 S_t\mathrm{d}P_t]^2 \\
&= \mu^2 S_t^2(\mathrm{d}t)^2 + \sigma_1^2 S_t^2(\mathrm{d}B_t^H)^2 + \sigma_2^2 S_t^2(\mathrm{d}B_t)^2 + \sigma_3^2 S_t^2(\mathrm{d}P_t)^2 + 2[\mu S_t\sigma_1 S_t\mathrm{d}B_t^H\mathrm{d}t \\
&\quad + \mu S_t\sigma_2 S_t\mathrm{d}B_t\mathrm{d}t + \mu S_t\sigma_3 S_t\mathrm{d}t\mathrm{d}P_t + \sigma_1 S_t\sigma_2 S_t\mathrm{d}B_t^H\mathrm{d}B_t \\
&\quad + \sigma_1 S_t\sigma_3 S_t\mathrm{d}B_t^H\mathrm{d}P_t + \sigma_2 S_t\sigma_3 S_t\mathrm{d}B_t\mathrm{d}P_t] \\
&= 2H\sigma_1^2 S_t^2 t^{2H-1}\mathrm{d}t + \sigma_2^2 S_t^2\mathrm{d}t + \lambda\sigma_3^2 S_t^2\mathrm{d}t + o(\mathrm{d}t\mathrm{d}S_t\mathrm{d}P_t).
\end{aligned} \tag{A.2}$$

Substituting (4) and (A.2) into (A.1), then

$$\begin{aligned}
\mathrm{d}V_t &= \frac{\partial V}{\partial t}\mathrm{d}t + \frac{\partial V}{\partial S}[\mu S_t\mathrm{d}t + \sigma_1 S_t\mathrm{d}B_t^H + \sigma_2 S_t\mathrm{d}B_t + \sigma_3 S_t\mathrm{d}P_t] \\
&\quad + \frac{1}{2}\frac{\partial^2 V}{\partial S^2}[2H\sigma_1^2 S_t^2 t^{2H-1}\mathrm{d}t + \sigma_2^2 S_t^2\mathrm{d}t + \lambda\sigma_3^2 S_t^2\mathrm{d}t] \\
&= \left[\frac{\partial V}{\partial t} + \mu S_t\frac{\partial V}{\partial S} + \left(H\sigma_1^2 S_t^2 t^{2H-1} + \frac{1}{2}\sigma_2^2 S_t^2 + \frac{1}{2}\lambda\sigma_3^2 S_t^2\right)\frac{\partial^2 V}{\partial S^2}\right]\mathrm{d}t + \sigma_1 S_t\frac{\partial V}{\partial S}\mathrm{d}B_t^H \\
&\quad + \sigma_2 S_t\frac{\partial V}{\partial S}\mathrm{d}B_t + \sigma_3 S_t\frac{\partial V}{\partial S}\mathrm{d}P_t + \lambda\mathbb{E}[V(S(1+j_t), t) - V(S, t)]\mathrm{d}t.
\end{aligned}$$

APPENDIX B. PROOF OF LEMMA 3.1.2

Let $x = \ln\dfrac{S\prod_{i=1}^n(1+j_{t_i})}{\xi} \in \mathbb{R}$, $\mathfrak{W} = V\cdot\exp\{-r(T-t)\}$, then solution problem of (10) and (11) is equivalent to the following solution problem

$$\begin{cases}
\dfrac{\partial\mathfrak{W}}{\partial t} - (H\sigma_1^2 t^{2H-1} + \dfrac{1}{2}\sigma_2^2 + \dfrac{1}{2}\lambda\sigma_3^2)\dfrac{\partial^2\mathfrak{W}}{\partial x^2} - [(r - q - \lambda\sigma_3) \\
-(H\sigma_1^2 t^{2H-1} + \dfrac{1}{2}\sigma_2^2 + \dfrac{1}{2}\lambda\sigma_3^2)]\cdot\dfrac{\partial\mathfrak{W}}{\partial x} - (r+\lambda)\mathfrak{W} = 0, & \text{(B.1)} \\
\mathfrak{W}(x, T) = \dfrac{\exp\{-r(T-t)\}}{\xi}\delta(x), & \text{(B.2)}
\end{cases}$$

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Let $\mathfrak{W} = \mathfrak{V} \cdot \exp\{\alpha(T,\, t) + \beta(T,\, t)x\}$, where

$$\alpha(T,t) = -r(T-t) - \frac{[(r-q-\lambda\sigma_3)(T-t) - \sigma_1^2(T^{2H}-t^{2H}) - \frac{1}{2}(\sigma_2^2+\lambda\sigma_3^2)(T-t)]^2}{2\sigma_1^2(T^{2H}-t^{2H}) + \sigma_2^2(T-t) + \lambda\sigma_3^2(T-t)}, \quad \textbf{(B.3)}$$

$$\beta(T,t) = \frac{\sum_{n=0}^{\infty} P\{j_t = n\}\mathcal{E}_n}{\xi\sqrt{2\pi[2\sigma_1^2(T^{2H}-t^{2H}) + \sigma_2^2(T-t) + \lambda\sigma_3^2(T-t)]}}$$

$$= \frac{\sum_{n=0}^{\infty}\{\lambda^n(T-t)^n \exp[-\lambda(T-t)](n!)^{-1}\mathcal{E}_n\}}{\xi\sqrt{2\pi[2\sigma_1^2(T^{2H}-t^{2H}) + \sigma_2^2(T-t) + \lambda\sigma_3^2(T-t)]}}, \quad \textbf{(B.4)}$$

here Poisson distribution with intensity $\lambda(T-t)$. Then by (B.1) and (B.2) we get

$$\begin{cases} \dfrac{\partial\mathfrak{V}}{\partial t} - \left(H\sigma_1^2 t^{2H-1} + \dfrac{1}{2}\sigma_2^2 + \dfrac{1}{2}\lambda\sigma_3^2\right)\dfrac{\partial^2\mathfrak{V}}{\partial x^2} \\ \quad - \left[(r-q-\lambda\sigma_3) - \left(H\sigma_1^2 t^{2H-1} + \dfrac{1}{2}\sigma_2^2 + \dfrac{1}{2}\lambda\sigma_3^2\right)\right] \cdot \dfrac{\partial\mathfrak{V}}{\partial x} - (r+\lambda)\mathfrak{V} = 0, & \textbf{(B.5)} \\ \mathfrak{V}(x,0) = \dfrac{\exp(-\beta x)}{\xi}\delta(x) = \dfrac{1}{\xi}\delta(x). & \textbf{(B.6)} \end{cases}$$

The solution of stochastic parabolic partial differential equations B.5 and B.6 can be written as

$$\mathfrak{V}(x,\xi;t,T) = \frac{1}{\xi\sqrt{2\pi[2\sigma_1^2(T^{2H}-t^{2H}) + \sigma_2^2(T-t) + \lambda\sigma_3^2(T-t)]}} \cdot \exp\left\{-\frac{x^2}{2\sigma_1^2(T^{2H}-t^{2H}) + \sigma_2^2(T-t) + \lambda\sigma_3^2(T-t)}\right\}. \quad \textbf{(B.7)}$$

Combine B.7 with B.3 and B.4 we have

$$\mathfrak{W}(x,\xi;t,T) = \frac{\sum_{n=0}^{\infty}\{\lambda^n(T-t)^n \exp[-\lambda(T-t)](n!)^{-1}\mathcal{E}_n\}}{\xi\sqrt{2\pi[2\sigma_1^2(T^{2H}-t^{2H}) + \sigma_2^2(T-t) + \lambda\sigma_3^2(T-t)]}} \cdot \exp\left\{[-r(T-t)] - \frac{1}{2\sigma_1^2(T^{2H}-t^{2H}) + \sigma_2^2(T-t) + \lambda\sigma_3^2(T-t)} \times [x + (r-q-\lambda\sigma_3)(T-t) - \sigma_1^2(T^{2H}-t^{2H}) - \frac{1}{2}(\sigma_2^2+\lambda\sigma_3^2)(T-t)]^2\right\}. \quad \textbf{(B.8)}$$

Substitute original variables $(S,\, t)$ in B.8, we obtain

$$\mathfrak{G}(S,t;\xi,T) = \sum_{n=0}^{\infty}\left\{\frac{\lambda^n(T-t)^n \exp[-\lambda(T-t)]}{n!}\mathcal{E}_n\right\} \cdot \frac{\exp[-r(T-t)]}{\xi\sqrt{2\pi[2\sigma_1^2(T^{2H}-t^{2H}) + \sigma_2^2(T-t) + \lambda\sigma_3^2(T-t)]}} \cdot \exp\left\{-\frac{1}{2\sigma_1^2(T^{2H}-t^{2H}) + \sigma_2^2(T-t) + \lambda\sigma_3^2(T-t)}\left[\ln\frac{S\prod_{i=1}^{n}(1+j_{t_i})}{\xi} \cdot + (r-q-\lambda\sigma_3)(T-t) - \sigma_1^2(T^{2H}-t^{2H})\frac{1}{2}(\sigma_2^2+\lambda\sigma_3^2)(T-t)\right]^2\right\}.$$

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APPENDIX C. PROOF OF THEOREM 3.1.3

For any $\epsilon > 0$, considering integral as follows

$$
\begin{aligned}
0 &= \int_0^\infty \mathrm{d}x \int_{t+\epsilon}^{\eta-\epsilon} [\mathfrak{G}^*(x,y;S,t)\cdot\mathfrak{G}(x,y;\xi,\eta) - \mathfrak{G}(x,y;\xi,\eta)\cdot\mathfrak{G}^*(x,y;S,t)]\mathrm{d}y \\
&= \int_0^\infty \mathrm{d}x \int_{t+\epsilon}^{\eta-\epsilon} \left\{ \frac{\partial(\mathfrak{G}^*\mathfrak{G})}{\partial y} + \left(H\sigma_1^2 t^{2H-1} + \frac{1}{2}\sigma_2^2 + \frac{1}{2}\lambda\sigma_3^2\right)\frac{\partial}{\partial x}\left(x^2\mathfrak{G}^*\frac{\partial \mathfrak{G}}{\partial x}\right)\right. \\
&\quad + \lambda\mathbb{E}[v(S(1+j_t),t) - v(S,t)] - \left(H\sigma_1^2 t^{2H-1} + \frac{1}{2}\sigma_2^2 + \frac{1}{2}\lambda\sigma_3^2\right)\frac{\partial}{\partial x}\left[\mathfrak{G}\frac{\partial}{\partial x}(x^2\mathfrak{G}^*)\right] \\
&\quad \left. + (r-q-\lambda\sigma_3)\frac{\partial}{\partial x}(x\mathfrak{G}\mathfrak{G}^*) - \lambda\mathbb{E}[v(S(1+j_t),t) - v(S,t)]\right\}\mathrm{d}y.
\end{aligned}
$$

When $x \to 0,\ \infty$, we have

$$
x^2\mathfrak{G}^*\frac{\partial\mathfrak{G}}{\partial x} \to 0, \quad \mathfrak{G}\frac{\partial}{\partial x}(x^2\mathfrak{G}^*) \to 0, \quad x\mathfrak{G}\mathfrak{G}^* \to 0,
$$

since

$$
\int_0^\infty \mathfrak{G}^*(x,\eta-\epsilon;S,t)\cdot\mathfrak{G}(x,\eta-\epsilon;\xi,\eta)\mathrm{d}x = \int_0^\infty \mathfrak{G}^*(x,t+\epsilon;S,t)\cdot\mathfrak{G}(x,t+\epsilon;\xi,\eta)\mathrm{d}x.
$$

Let $\epsilon \to 0$, by conditions (9) and (11) we get

$$
\int_0^\infty \mathfrak{G}^*(x,\eta;S,t)\cdot\delta(x-\xi)\mathrm{d}x = \int_0^\infty \delta(x-S)\cdot\mathfrak{G}(x,t;\eta,\xi)\mathrm{d}x.
$$

Then

$$
\mathfrak{G}(S,t;\xi,\eta) = \mathfrak{G}^*(\xi,\eta;S,t).
$$

APPENDIX D. PROOF OF THEOREM 3.2.1

Suppose $V(t,\ S,\ \mathbb{M})$ satisfies an order continuous differentiable on domain Σ: $\Sigma = \{0 \le S \le \mathbb{M} < \infty,\ 0 \le t < T\}$, and $V(t,\ S,\ \mathbb{M})$ exist piecewise continuous of the second order derivative, then

$$
-\mathfrak{L}V(t,S,\mathbb{M}) = \begin{cases} 0, & (t,S,\mathbb{M}) \in \Sigma_1, \\ \mathbb{b}\mathbb{M}r - (q+\lambda\sigma_3)S, & (t,S,\mathbb{M}) \in \Sigma_2, \end{cases} \tag{D.1}
$$

where $\mathfrak{L}$ is the Black-Scholes operator.

Multiplying both sides of the equation (D.1) to $\mathfrak{G}^*(\xi,\ \eta;S,\ t)$ and quadrature on domain $\{0 \le \xi \le \mathbb{M} < \infty,\ t+\epsilon \le \eta \le T\}$, For $\Sigma_2 = \{0 \le \xi \le \mathbb{M} \le S_\eta,\ 0 \le \eta < T\}$, and S_η is continuous monotonous of the asset price, then

$$
\begin{aligned}
&\int_{t+\epsilon}^T \mathrm{d}\eta \int_0^{S_\eta} [\mathbb{b}\mathbb{M}r - (q+\lambda\sigma_3)\xi]\cdot\mathfrak{G}^*(\xi,\eta;S,t)\mathrm{d}\xi \\
&\quad = -\int_{t+\epsilon}^T \mathrm{d}\eta \int_0^\infty \mathfrak{G}^*(\xi,\eta;S,t)\mathfrak{L}V\mathrm{d}\xi
\end{aligned}
$$

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$$= -\int_{t+\epsilon}^{T} \mathrm{d}\eta \int_0^\infty [\mathfrak{G}^*(\xi,\eta;S,t)\mathfrak{L}V(\eta,\xi,\mathbb{M}) - V(\eta,\xi,\mathbb{M})\mathfrak{L}^*\mathfrak{G}^*(\xi,\eta;S,t)]\mathrm{d}\xi$$

$$= -\int_{t+\epsilon}^{T} \mathrm{d}\eta \int_0^\infty \left\{ \frac{\partial(\mathfrak{G}^* V)}{\partial \eta} + \left(H\sigma_1^2 t^{2H-1} + \frac{1}{2}\sigma_2^2 + \frac{1}{2}\lambda\sigma_3^2 \right) \frac{\partial}{\partial \xi}\left(\xi^2 \mathfrak{G}^* \frac{\partial V}{\partial \xi} \right) \right.$$

$$+ \lambda\mathbb{E}[V(t,S(1+j_t),\mathbb{M}) - V(t,S,\mathbb{M})] - \left(H\sigma_1^2 t^{2H-1} + \frac{1}{2}\sigma_2^2 + \frac{1}{2}\lambda\sigma_3^2 \right) \frac{\partial}{\partial \xi}$$

$$\left. \times \left[V\frac{\partial}{\partial \xi}\cdot(\xi^2\mathfrak{G}^*) \right] + (r-q-\lambda\sigma_3)\frac{\partial}{\partial \xi}(\xi V \mathfrak{G}^*) - \lambda\mathbb{E}[V(t,S(1+j_t),\mathbb{M}) - V(t,S,\mathbb{M})] \right\} \mathrm{d}\xi,$$

where $\mathfrak{L}^*$ is the adjoint Black–Scholes operator, and when $\xi \to 0,\ \infty$, we have

$$\xi^2\mathfrak{G}^*\frac{\partial V}{\partial \xi} \to 0, \quad V\frac{\partial}{\partial \xi}(\xi^2\mathfrak{G}^*) \to 0, \quad \xi V\mathfrak{G}^* \to 0,$$

hence

$$\int_0^\infty \mathfrak{G}^*(\xi,t+\epsilon;S,t)V(t+\epsilon,\xi,\mathbb{M})\mathrm{d}\xi$$
$$= \int_0^\infty \mathfrak{G}^*(\xi,T;S,t)V(T,\xi,\mathbb{M})\mathrm{d}\xi + \int_{t+\epsilon}^{T}\mathrm{d}\eta\int_0^{S_\eta}[\mathbb{bM}r - (q+\lambda\sigma_3)\xi]\mathfrak{G}^*(\xi,\eta;S,t)\mathrm{d}\xi.$$

Let $\epsilon \to 0$, by (11) and Corollary 3.1.4, we obtain

$$V(t,S,\mathbb{M}) = \int_0^\infty \mathfrak{G}(S,t;\xi,T)(\mathbb{bM}-\xi)^+\mathrm{d}\xi + \int_t^T \mathrm{d}\eta \int_0^{S_\eta}[\mathbb{bM}r - (q+\lambda\sigma_3)\xi]\mathfrak{G}(S,t;\xi,\eta)\mathrm{d}\xi$$
$$= V_E(t,S,\mathbb{M}) + e(t,S,\mathbb{M}).$$

Then the prove is finished.

APPENDIX E. PROOF OF COROLLARY 3.2.2

By (28) and (B.6), $e(S,\,t)$ can be represented by

$$e(t,S,\mathbb{M}) = \int_t^T \mathrm{d}\eta \int_0^{S_\eta}[\mathbb{bM}r - (q+\lambda\sigma_3)\xi]\cdot\sum_{n=0}^{\infty}\left\{ \frac{\lambda^n(T-t)^n\exp[-\lambda(T-t)]}{n!}\mathcal{E}_n \right\}$$
$$\cdot\frac{\exp\{-r(T-t)\}}{\xi\sqrt{2\pi[2\sigma_1^2(T^{2H}-t^{2H})+\sigma_2^2(T-t)+\lambda\sigma_3^2(T-t)]}}$$
$$\cdot\exp\left\{ -\frac{1}{2\sigma_1^2(T^{2H}-t^{2H})+\sigma_2^2(T-t)+\lambda\sigma_3^2(T-t)} \right.$$
$$\cdot\left[\ln\frac{S\prod_{i=1}^n(1+j_{t_i})}{\xi} + (r-q-\lambda\sigma_3)(T-t) - \sigma_1^2(T^{2H}-t^{2H}) \right.$$
$$\left.\left. -\frac{1}{2}(\sigma_2^2+\lambda\sigma_3^2)(T-t) \right]^2 \right\}\mathrm{d}\xi. \qquad \textbf{(E.1)}$$

Let

$$x^* = \frac{1}{\sqrt{2\sigma_1^2(\eta^{2H} - t^{2H}) + \sigma_2^2(\eta - t) + \lambda\sigma_3^2(\eta - t)}} \cdot \left[\ln \frac{S\prod_{i=1}^n (1 + j_{t_i})}{\xi} + (r - q - \lambda\sigma_3)(\eta - t) - \sigma_1^2(\eta^{2H} - t^{2H}) - \frac{1}{2}(\sigma_2^2 + \lambda\sigma_3^2)(\eta - t)\right],$$

$$\mathrm{d}x^* = -\frac{\mathrm{d}\xi}{\xi\sqrt{2\sigma_1^2(T^{2H} - t^{2H}) + \sigma_2^2(T - t) + \lambda\sigma_3^2(T - t)}},$$

then

$$\begin{aligned}
&e(t, S, \mathbb{M}) \\
&= \frac{\mathbb{bM}r}{\sqrt{2\pi}} \int_t^T \exp\{-r(\eta - t)\} \sum_{n=0}^{\infty} \left\{\frac{\lambda^n(\eta - t)^n \exp[-\lambda(\eta - t)]}{n!} \mathcal{E}_n\right\} \mathrm{d}\eta \\
&\cdot \int_{\frac{\ln \frac{S\prod_{i=1}^n (1+j_{t_i})}{S_\eta} + (r-q-\lambda\sigma_3)(\eta-t) - \sigma_1^2(\eta^{2H}-t^{2H}) - \frac{1}{2}(\sigma_2^2+\lambda\sigma_3^2)(\eta-t)}{\sqrt{2\sigma_1^2(\eta^{2H}-t^{2H})+\sigma_2^2(\eta-t)+\lambda\sigma_3^2(\eta-t)}}}^{\infty} \exp\{-x^2\}\mathrm{d}x \\
&- \frac{(q + \lambda\sigma_3)S}{\sqrt{2\pi}} \int_t^T \exp\{-(q + \lambda\sigma_3)(\eta - t)\} \sum_{n=0}^{\infty} \left\{\frac{\lambda^n(\eta - t)^n \exp[-\lambda(\eta - t)]}{n!} \mathcal{E}_n\right\} \mathrm{d}\eta \\
&\cdot \int_{\frac{\ln \frac{S\prod_{i=1}^n (1+j_{t_i})}{S_\eta} + (r-q-\lambda\sigma_3)(\eta-t) - \sigma_1^2(\eta^{2H}-t^{2H}) - \frac{1}{2}(\sigma_2^2+\lambda\sigma_3^2)(\eta-t)}{\sqrt{2\sigma_1^2(\eta^{2H}-t^{2H})+\sigma_2^2(\eta-t)+\lambda\sigma_3^2(\eta-t)}}}^{\infty} \\
&\times \exp\left\{-\frac{x^2}{2} + [\sqrt{2\sigma_1^2(\eta^{2H} - t^{2H}) + \sigma_2^2(\eta - t) + \lambda\sigma_3^2(\eta - t)}]x\right\} \mathrm{d}x \\
&= \mathbb{bM}r \int_t^T \exp\{-r(\eta - t)\} \sum_{n=0}^{\infty} \left\{\frac{\lambda^n(\eta - t)^n \exp[-\lambda(\eta - t)]}{n!} \mathcal{E}_n\right\} \\
&\cdot \left\{1 - \mathbb{N}\left[\frac{\ln \frac{S\prod_{i=1}^n (1+j_{t_i})}{S_\eta} + (r - q - \lambda\sigma_3)(\eta - t) - \sigma_1^2(\eta^{2H} - t^{2H}) - \frac{1}{2}(\sigma_2^2 + \lambda\sigma_3^2)(\eta - t)}{2\sigma_1^2(\eta^{2H} - t^{2H}) + \sigma_2^2(\eta - t) + \lambda\sigma_3^2(\eta - t)}\right]\right\} \mathrm{d}\eta \\
&- (q + \lambda\sigma_3)S \int_t^T \exp\{-(q + \lambda\sigma_3)(\eta - t)\} \sum_{n=0}^{\infty} \left\{\frac{\lambda^n(\eta - t)^n \exp[-\lambda(\eta - t)]}{n!} \mathcal{E}_n\right\} \mathrm{d}\eta \\
&\cdot \int_{\frac{\ln \frac{S\prod_{i=1}^n (1+j_{t_i})}{S_\eta} + (r-q-\lambda\sigma_3)(\eta-t) - \sigma_1^2(\eta^{2H}-t^{2H}) - \frac{1}{2}(\sigma_2^2+\lambda\sigma_3^2)(\eta-t)}{\sqrt{2\sigma_1^2(\eta^{2H}-t^{2H})+\sigma_2^2(\eta-t)+\lambda\sigma_3^2(\eta-t)}}}^{\infty} \\
&\times \exp\left\{ -\frac{1}{2} \cdot \left(x + \sqrt{2\sigma_1^2(\eta^{2H} - t^{2H}) + \sigma_2^2(\eta - t) + \lambda\sigma_3^2(\eta - t)}\right)^2 \right. \\
&\left. + (r - q - \lambda\sigma_3)(\eta - t) + \sigma_1^2(\eta^{2H} - t^{2H}) + \frac{1}{2}(\sigma_2^2 + \lambda\sigma_3^2)(\eta - t)\right\} \mathrm{d}x \\
&= \mathbb{bM}r \int_t^T \exp\{-r(\eta - t)\} \sum_{n=0}^{\infty} \left\{\frac{\lambda^n(\eta - t)^n \exp[-\lambda(\eta - t)]}{n!} \mathcal{E}_n\right\} \\
&\cdot \left\{1 - \mathbb{N}\left[\frac{\ln \frac{S\prod_{i=1}^n (1+j_{t_i})}{S_\eta} + (r - q - \lambda\sigma_3)(\eta - t) - \sigma_1^2(\eta^{2H} - t^{2H}) - \frac{1}{2}(\sigma_2^2 + \lambda\sigma_3^2)(\eta - t)}{2\sigma_1^2(\eta^{2H} - t^{2H}) + \sigma_2^2(\eta - t) + \lambda\sigma_3^2(\eta - t)}\right]\right\} \mathrm{d}\eta
\end{aligned}$$

$$
\begin{aligned}
&-(q+\lambda\sigma_3)S\int_t^T \exp\{-(q+\lambda\sigma_3)(\eta-t)\}\sum_{n=0}^{\infty}\left\{\frac{\lambda^n(\eta-t)^n\exp[-\lambda(\eta-t)]}{n!}\mathcal{E}_n\right\}\\
&\cdot\left\{1-\mathbb{N}\left[\frac{\ln\frac{S\prod_{i=1}^{n}(1+j_{t_i})}{S_\eta}+(r-q-\lambda\sigma_3)(\eta-t)+\sigma_1^2(\eta^{2H}-t^{2H})+\frac{1}{2}(\sigma_2^2+\lambda\sigma_3^2)(\eta-t)}{2\sigma_1^2(\eta^{2H}-t^{2H})+\sigma_2^2(\eta-t)+\lambda\sigma_3^2(\eta-t)}\right]\right\}\mathrm{d}\eta.
\end{aligned}
\tag{E.2}
$$

Substituting (E.2) into (24), we have

$$V(t,S_t,\mathbb{M})=\mathbb{b}\mathbb{M}-S_t,$$

then we obtain

$$\mathbb{b}\mathbb{M}-S_t=V_E(t,S_t,\mathbb{M})+e(t,S_t,\mathbb{M}),$$

such that

$$
\begin{aligned}
S_t=\;&\mathbb{b}\mathbb{M}+S\exp\{-(q+\lambda\sigma_3)(T-t)\}\\
&\cdot\mathbb{N}\Bigg\{-\left[-\ln\frac{S\prod_{i=1}^{n}(1+j_{t_i})}{\mathbb{b}\mathbb{M}}+(r-q-\lambda\sigma_3)(T-t)+\sigma_1^2(T^{2H}-t^{2H})\right.\\
&\left.+\frac{1}{2}(\sigma_2^2+\sigma_3^2)(T-t)\right]\cdot\left[\sqrt{2\sigma_1^2(T^{2H}-t^{2H})+\sigma_2^2(T-t)+\lambda\sigma_3^2(T-t)}\right]^{-1}\Bigg\}\\
&-\mathbb{b}\mathbb{M}\exp\{-r(T-t)\}\sum_{n=0}^{\infty}\left\{\frac{\lambda^n(T-t)^n\exp[-\lambda(T-t)]}{n!}\mathcal{E}_n\right\}\\
&\cdot\mathbb{N}\Bigg\{-\left[-\ln\frac{S\prod_{i=1}^{n}(1+j_{t_i})}{\mathbb{b}\mathbb{M}}+(r-q-\lambda\sigma_3)(T-t)-\sigma_1^2(T^{2H}-t^{2H})\right.\\
&\left.-\frac{1}{2}(\sigma_2^2+\lambda\sigma_3^2)(T-t)\right]\cdot\left[\sqrt{2\sigma_1^2(T^{2H}-t^{2H})+\sigma_2^2(T-t)+\lambda\sigma_3^2(T-t)}\right]^{-1}\Bigg\}\\
&-\mathbb{b}\mathbb{M}r\int_t^T\exp\{-r(\eta-t)\}\sum_{n=0}^{\infty}\left\{\frac{\lambda^n(T-t)^n\exp[-\lambda(T-t)]}{n!}\mathcal{E}_n\right\}\\
&\cdot\left\{1-\mathbb{N}\left[\left(\ln\frac{S\prod_{i=1}^{n}(1+j_{t_i})}{S_\eta}+(r-q-\lambda\sigma_3)(\eta-t)-\sigma_1^2(\eta^{2H}-t^{2H})\right.\right.\right.\\
&\left.\left.\left.-\frac{1}{2}(\sigma_2^2+\lambda\sigma_3^2)(\eta-t)\right)\cdot\left(\sqrt{2\sigma_1^2(\eta^{2H}-t^{2H})+\sigma_2^2(\eta-t)+\lambda\sigma_3^2(\eta-t)}\right)^{-1}\right]\right\}\mathrm{d}\eta\\
&+(q+\lambda\sigma_3)S\int_t^T\exp\{-(q+\lambda\sigma_3)(\eta-t)\}\sum_{n=0}^{\infty}\{\frac{\lambda^n(T-t)^n\exp[-\lambda(T-t)]}{n!}\mathcal{E}_n\}\\
&\cdot\left\{1-\mathbb{N}\left[\left(\ln\frac{S\prod_{i=1}^{n}(1+j_{t_i})}{S_\eta}+(r-q-\lambda\sigma_3)(\eta-t)+\sigma_1^2(\eta^{2H}-t^{2H})\right.\right.\right.\\
&\left.\left.\left.+\frac{1}{2}(\sigma_2^2+\lambda\sigma_3^2)(\eta-t)\right)\cdot(\sqrt{2\sigma_1^2(\eta^{2H}-t^{2H})+\sigma_2^2(\eta-t)+\lambda\sigma_3^2(\eta-t)})^{-1}\right]\right\}\mathrm{d}\eta.
\end{aligned}
$$

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