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2015 NORTH AMERICAN ANNUAL MEETING OF THE ASSOCIATION FOR SYMBOLIC LOGIC

University of Illinois at Urbana-Champaign
Urbana, IL, USA

March 25–28, 2015

The 2015 North American Annual Meeting of the Association for Symbolic Logic was held at the University of Illinois in Urbana-Champaign, March 25–28, 2015. There were seven plenary talks, two tutorials, six special sessions, and two sessions of contributed talks, as well as the 26th annual Gödel lecture. A reception was held on the evening of Thursday March 26, 2015. A unique feature of this meeting was the panel discussion on academic freedom.

The program committee consisted of D. Haskell (Chair), P. Hieronymi, A. Montalban, A. Razborov, D. Sinapova, and H. Towsner. The local organizing committee consisted of A. Arana (Chair), L. van den Dries, P. Hieronymi, and S. Solecki. Many thanks go to the graduate students at the University of Illinois for their active participation in the organization.

There were 109 registered participants at the meeting, including 46 graduate students. Generous financial support was provided by the Association for Symbolic Logic, the National Science Foundation, as well as the Departments of Mathematics and Philosophy at UIUC and individual logicians at UIUC.

The Gödel Lecture was given by Alex Wilkie: "Complex continuations of functions definable in $\mathbb{R}_{an,exp}$ with a diophantine application".

The plenary addresses at the meeting were:

Verónica Becker (Universidad de Buenos Aires), *Constructing normal numbers*.

David Fernández-Duque (Instituto Tecnológico Autónomo de México), *Arithmetic reflection principles and consistency strength*.

Rami Grossberg (CMU), *Classification theory for abstract elementary classes*.

Yuri Gurevich (Microsoft Research), *Logic in computer science and software engineering*.

Neil Immerman (UMass Amherst), *Dynamic reasoning*.

Russell Miller (CUNY), *Functors and effective interpretations in model theory*.

Anush Tserunyan (UIUC), *Finite generating partitions for continuous actions of countable groups*.

There were two tutorials, with the novel feature that the second hour was held in parallel, given by:

James Cummings (CMU), *Compactness*.

Chris Miller (Ohio State), *Tameness and metric dimensions in expansions of the real field*.

The panel discussion on the topic: *What should the ASL do to protect academic freedom? Should the ASL have cancelled this conference?* was prompted by recent events at the University of Illinois, Urbana-Champaign. The panelists were Matthew Finkin, Deirdre Haskell, and Alisdair Urquhart.

The program included six special sessions (with organizers in parentheses): Computability (Uri Andrews and Steffen Lempp), Constructive mathematics (Douglas Bridges and Vladik Kreinovich), Model theory of ordered structures (Kobi Peterzil and Sergei Starchenko),

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Parametrized complexity (Rod Downey and Serge Gaspers), Philosophy of mathematics (Andrew Arana and Audrey Yap), and Set theory (Slawomir Solecki). There were a total of 32 special session speakers.

There were 12 contributed talks delivered at the meeting, and 3 additional abstracts presented by title.

Abstracts of the invited talks and the contributed talks (given in person or by title) by members of the Association for Symbolic Logic follow.

For the Program Committee
DEIRDRE HASKELL

Abstract of the 26th Annual Gödel Lecture

▶ ALEX WILKIE, *Complex continuations of functions definable in $\mathbb{R}_{an,exp}$ with a diophantine application*.
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Diophantine properties of subsets of $\mathbb{R}^n$ definable in an o-minimal expansion of the ordered field of real numbers have been much studied over the last few years and several applications to purely number theoretic problems have been made. One line of inquiry attempts to characterise the set of definable functions $f\colon \mathbb{R} \to \mathbb{R}$ having the property that $f(\mathbb{N}) \subseteq \mathbb{N}$. For example, a result of Thomas, Jones and myself shows that if the structure under consideration is $\mathbb{R}_{exp}$ (the real field expanded by the exponential function) and if, for all positive r, $f(x)$ eventually grows more slowly than $exp(x^r)$, then f is necessarily a polynomial with rational coefficients. In this talk I shall improve this result in two directions. Firstly, I take the structure to be $\mathbb{R}_{an,exp}$ (the expansion of $\mathbb{R}_{exp}$ by all real analytic functions defined on compact balls in $\mathbb{R}^n$) and secondly, I allow the growth rate to be $x^N \cdot 2^x$ for arbitrary (fixed) N. The conclusion is that $f(x) = p(x) \cdot 2^x + q(x)$ for sufficiently large x, where p and q are polynomials with rational coefficients.

I should mention that over ninety years ago Pólya established the same result for entire functions $f\colon \mathbb{C} \to \mathbb{C}$ and that in 2007 Langley weakened this assumption to f being regular in a right half-plane of $\mathbb{C}$. I follow Langley's method, but first we must consider which $\mathbb{R}_{an,exp}$-definable functions actually have complex continuations to a right half-plane and, as it turns out, which of them have a definable such continuation.

Abstracts of invited tutorials

▶ JAMES CUMMINGS, *Compactness*.
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The "hard core" of many set-theoretic problems often turns out to involve some form of compactness (broadly construed). In these lectures we will aim to explain why this is so, give many examples, and discuss some recent developments.

▶ CHRIS MILLER, *Tameness and metric dimensions in expansions of the real field*.
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It is long known that any expansion, $\mathfrak{R}$, of the field of real numbers that defines $\mathbb{N}$ (the set of all natural numbers) also defines every Borel subset of each $\mathbb{R}^n$, hence also every projective (in the sense of descriptive set theory) subset of each $\mathbb{R}^n$. Thus, one can easily ask questions

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about the definable sets of $\mathfrak{R}$ that can turn out to be independent of ZFC (e.g., whether every definable set is Lebesgue measurable). This leads naturally to wondering what can be said about its definable sets if $\mathfrak{R}$ does not define $\mathbb{N}$. Philipp Hieronymi and I have recently obtained a result that can be stated loosely as: $\mathfrak{R}$ *avoids defining* $\mathbb{N}$ *if and only if all metric dimensions commonly encountered in geometric measure theory, fractal geometry and analysis on metric spaces coincide with topological dimension on all images of closed definable sets under definable continuous maps*. In the first of two hours, I will make this statement precise (assuming essentially no knowledge of dimension theory), explain its significance, and give some easy (yet striking) corollaries and applications. In the second hour, I will explain some crucial (yet remarkably simple) ideas of the proof. Both hours will be aimed at a general ASL audience.

Abstracts of invited plenary lectures

▸ VERÓNICA BECHER, *Constructing normal numbers*.
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Flip a coin a large number of times and roughly half of the flips will come up heads and half will come up tails. Normality makes similar assertions about the sequences of digits in the expansions of a real number. For b an integer greater than or equal to 2, a real number x is simply normal to base b if every digit d in $\{0, 1, \dots, b-1\}$ occurs in the base b expansion of x with asymptotic frequency $1/b$; a real number x is normal to base b if it is simply normal to all powers of b; and a real number x is absolutely normal if it is simply normal to all integer bases greater than or equal to 2.

More than one hundred years ago E. Borel showed that almost all real numbers are absolutely normal, and he asked for one example. He would have liked some fundamental mathematical constant such as π or e, but this remains as the most famous open problem on normality. As for other examples, there have been several constructions of normal numbers since Borel's time, with varying levels of effectivity (computability). I will summarize the latest results, including our constructions of numbers normal to selected bases, a fast algorithm to compute an absolutely normal number which runs in nearly quadratic time, and an algorithm to compute an absolutely normal Liouville number.

This is joint work with Theodore Slaman and Pablo Heiber.

▸ DAVID FERNÁNDEZ-DUQUE, *Arithmetic reflection principles and consistency strength*.
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If T is an arithmetic theory and Λ an ordinal notation system, we can define the *Turing–Feferman Progression of* T *along* Λ by transfinitely iterating consistency assertions about T [5]; to be precise, $T_0 = T$, $T_{\lambda+1} = T_\lambda + \texttt{Cons}(T_\lambda)$, and, for λ a limit, $T_\lambda = \bigcup_{\eta<\lambda} T_\eta$. If T is sound, then Gödel's Second Incompleteness Theorem tells us that $\langle T_\lambda \rangle_{\lambda<\Lambda}$ is a strictly increasing sequence of theories.

This construction can be used as a tool to measure the *consistency strength* of a theory T or, more formally, its Π^0_1-*ordinal*. Suppose that we have a second theory, U, usually weaker than T. Then, the Π^0_1-ordinal of T relative to U and Λ is the supremum of all λ such that $T \vdash \texttt{Cons}(U_\lambda)$. Omitting the dependency on U and Λ, we will denote this ordinal simply by $\|T\|_{\Pi^0_1}$.

A recent approach by Beklemishev uses modal logic and reflection principles to compute $\|T\|_{\Pi^0_1}$ [1]. If we use $\Box_T \phi$ to denote a natural formalization of *The formula* ϕ *is provable in* T, then *uniform reflection over* T is the schema

$$\texttt{RFN}(T) = \forall x\big(\Box_T \phi(\bar{x}) \to \phi(x)\big).$$

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Reflection principles are philosophically appealing, since it can be argued that, once one has accepted T, one should also accept reflection over T; in this sense, $T + \mathtt{RFN}(T)$ gives a natural extension of T. Moreover, this extension is sometimes also mathematically natural [4]; for example, if we let EA (Elementary Arithmetic) be the subsystem of PA with exponentiation where induction may only be applied to Δ^0_0 formulas, then $\mathrm{PA} \equiv \mathrm{EA} + \mathtt{RFN}(\mathrm{EA})$. Beklemishev has used this representation in order to show that $\|\mathrm{PA}\|_{\Pi^0_1} = \varepsilon_0$, and also to provide a consistency proof of PA using only finitary means plus one instance of transfinite induction.

In this presentation we will discuss recent results, obtained in joint work with Andrés Cordón-Franco, Félix Lara-Martín, and Joost J. Joosten [2] that may lead to similar analyses and consistency proofs of theories of second-order arithmetic. In particular, we propose two strong reflection principles which are equivalent to the theory ATR_0 of *Arithmetic Transfinite Recursion* and $(\mathbf{\Pi}^1_1\text{-}\mathtt{CA})_0$ of $\mathbf{\Pi}^1_1$*-comprehension*, respectively. Both principles assert that ACA_0 (or perhaps a weaker theory, such as RCA_0) does not prove false Π^1_2-sentences, even after transfinitely iterating the ω-rule [3]. We then detail how principles of this form might be used to obtain a Π^0_1 analysis of ATR_0, $(\mathbf{\Pi}^1_1\text{-}\mathtt{CA})_0$, or related theories.

[1] L. D. Beklemishev, *Provability algebras and proof-theoretic ordinals, I*. ***Annals of Pure and Applied Logic***, vol. 128 (2004), pp. 103–124.

[2] A. Cordón-Franco, D. Fernández-Duque, J. J. Joosten, and F. Lara-Martín, *Predicativity through transfinite reflection*, `arXiv`, (2014).

[3] D. Fernández-Duque and J. J. Joosten, *The omega-rule interpretation of transfinite provability logic*, submitted, `arXiv:1205.2036 [math.LO]`, 2013).

[4] G. Kreisel and A. Lévy, *Reflection principles and their use for establishing the complexity of axiomatic systems*. ***Zeitschrift für mathematische Logik und Grundlagen der Mathematik***, vol. 14 (1968), pp. 97–142.

[5] A. Turing, *Systems of logics based on ordinals*. ***Proceedings of the London Mathematical Society***, vol. 45 (1939), pp. 161–228.

► RAMI GROSSBERG, *Classification Theory for Abstract Elementary Classes*.
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Stability theory or what was later renamed as *Classification Theory* was mainly developed by Saharon Shelah in the seventies. It is a conceptually rich theory, much of it was motivated by answering rather strange questions involving cardinal numbers: "what is the first λ such that an uncountable first-order stable theory T is stable in λ?" and the much harder "what is the asymptotic behavior of the number of pairwise nonisomorphic models of T of cardinality $\aleph_\alpha$ as a function of α?"

Only 20 years after much of the theory was discovered, applications to hard problems of geometry and number theory started to show up, involving methods and arguments much deeper than what was used by the previous generation of applied model theorists.

After completing much of his theory and answering some of the outstanding questions concerning models of first-order theories, Shelah introduced the category-theoretic notion of *Abstract Elementary Class* (*AEC*) which is a powerful generalization of $L_{\omega_1,\omega}$. Shelah expects that such a theory will have greater effect on classical problems of mathematics, due to the greater expressive power of the logics involved. However so far this has not materialized. Shelah proposed around 1977 a fundamental categoricity conjecture to serve as the test-question for the understanding of AECs. This started in 1977 what he named *Classification Theory for AECs*. Shelah published a two volume book in 2009 on the subject. I estimate that Shelah is responsible as an author or coauthor for publishing more than 2,500 pages of original work on this field.

I will focus on describing several recent developments along the main lines of Shelah's program, due to former and present students of mine among them: VanDieren, Kolesnikov, Boney, and Vasey.

► YURI GUREVICH, *Logic in computer science and software engineering*.
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Eugene P. Wigner and Richard W. Hamming wrote about the "unreasonable effectiveness" of mathematics in the natural sciences [4, 2]. This inspired Joseph Y. Halpern, Robert Harper, Neil Immerman, Phokion G. Kolaitis, Moshe Y. Vardi, and Victor Vianu to write about the "unusual effectiveness" of logic in computer science [1]. Moshe Vardi gives talks about a "logic revolution" in computer science and, to an extent, software engineering [3].

We are less jubilant about the role that logic and logicians have been playing in science and engineering. Logic had the greatest prestige in the first part of the 20th century. Its role could be bigger nowadays, especially in software engineering. Software engineers use formal logic day in and day out, even though many of them do not realize that. They studied calculus but rarely, if ever, use it. They did not study formal logic but use it daily. We will try to illustrate why formal logic is so relevant for software engineers, and why it is hard for them to pick it up.

[1] Joseph Y. Halpern, Robert Harper, Neil Immerman, Phokion G. Kolaitis, Moshe Y. Vardi, and Victor Vianu, *On the unusual effectiveness of logic in computer science*, this Bulletin, vol. 7 (2001), no. 2, pp. 213–236.

[2] Richard W. Hamming, *The unreasonable effectiveness of mathematics*. ***American Mathematical Monthly***, vol. 87 (1980), pp. 81–90.

[3] Moshe Vardi, *A logical revolution*, A lecture at Microsoft Research, Redmond, August 2013, www.cs.rice.edu/~vardi/comp409/logicrevo13.pdf.

[4] Eugene P. Wigner, *The unreasonable effectiveness of mathematics in the natural sciences*. ***Communications on Pure and Applied Mathematics***, vol. 13 (1960), pp. 1–14.

► NEIL IMMERMAN, *Dynamic reasoning*.
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Reasoning about reachability—can we get to b from a by following a sequence of pointers—is crucial for proving that programs meet their specifications. However reasoning about reachability in general is undecidable. A related problem concerns reachability in dynamic graphs (a dynamic graph is a graph in which edges are added or deleted over time). It would be useful to revise reachabilty relations with minimal recomputation, and indeed, in undirected graphs and functional graphs, this can be done efficiently.

The problems of reasoning about reachability in programs and computing reachability in dynamic graphs are connected. For some graphs we can keep track of reachability after each small change in a quantifier-free language. This leads to an automatic way to check the correctness of programs whose data structures are appropriately simple. In this talk, I discuss progress and open questions concerning where such simple dynamic reasoning is possible and where it is not.

► RUSSELL MILLER, *Functors and effective interpretations in model theory*.
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We present and connect several recent results in computable model theory in which one class of countable structures is coded into another class. These include a coding of countable graphs into fields, by Park, Poonen, Schoutens, Shlapentokh, and the speaker, which is quantifiably more effective than the well-known Friedman–Stanley coding and which shows that fields have the same completeness properties as graphs in regard to Turing degree spectra and computable categoricity. Additionally, we will describe Ocasio's version of Marker's coding of linear orders into real closed fields, followed by a recent coding of graphs into differentially closed fields using the ENI-DOP for the theory $\mathbf{DCF}_0$. Both of these are seen to be somewhat less effective, although they are still Turing-computable embeddings, in the sense defined by Knight and her many co-authors.

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To a model theorist, the map from graphs into fields will be readily recognizable as an interpretation, using existential formulas. The other two maps can also be seen as interpretations, but require the use of computable infinitary $\exists\forall$ formulas. Montalbán's definition of *effective interpretation* is relevant here. On the other hand, the map from graphs into fields can be viewed as a kind of functor, since it codes each graph into a field in such a way that the isomorphisms between any two graphs give rise (in an effective way) to all isomorphisms between the corresponding fields. We will state the natural definition of *computable functor* which arose, along with its generalization to the other two maps. Finally, we will describe work by Harrison-Trainor, Melnikov, Montalbán, and the author in which these two notions (along with the longstanding notion of Σ-definability, used mostly in Novosibirsk) are all shown to be equivalent.

▸ ANUSH TSERUNYAN, *Finite generating partitions for continuous actions of countable groups*.
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Let a countable group G act continuously on a Polish space X. A countable Borel partition $\mathcal{P}$ of X is called a *generator* if the set of its translates $\{gP : g \in G, P \in \mathcal{P}\}$ generates the Borel σ-algebra of X. For $G = \mathbb{Z}$, the Kolmogorov–Sinai theorem gives a measure-theoretic obstruction to the existence of finite generators: they do not exist in the presence of an invariant probability measure with infinite entropy. It was asked by Benjamin Weiss in the late 80s whether the nonexistence of any invariant probability measure guarantees the existence of a finite generator. We show that the answer is positive for an arbitrary countable group G and σ-compact X (in particular, for locally compact X). We also show that finite generators always exist for aperiodic actions in the context of Baire category (allowing ourselves to disregard a meager set), thus answering a question of Alexander Kechris from the mid-90s.

Abstracts of invited talks in the Special Session on Computability

▸ WESLEY CALVERT AND JOHANNA N.Y. FRANKLIN, *UD-randomness and the Kurtz random reals*.
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UD-randomness is an extremely weak randomness notion. For instance, there are UD-random reals that are not Kurtz random. We will discuss some recent work on the relationship between UD-randomness and two disjoint subclasses of the Kurtz random reals: the weakly 1-generic and Schnorr random reals. Avigad has proven that all Schnorr random reals are UD-random. Here, we will show that there is a Kurtz random real that is neither UD-random nor weakly 1-generic.

▸ FRANÇOIS G. DORAIS, *Nonstandard models and nonuniformity in recursion theory*.
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We will illustrate the use of nonstandard models of subsystems of second-order arithmetic for the analysis of nonuniform results in recursion theory through the analysis of a classical result on n-bounded diagonally nonrecursive functions (DNR_n functions, for short).

An elegant argument due to Friedberg (see [2, Theorem 5]) shows that every DNR_{n^2} function computes a DNR_n function. However, Friedberg's proof is nonuniform since it requires

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deciding a Σ^0_2 statement about the DNR_{n^2} function in order to compute the DNR_n function. Jockusch [2] showed that this nonuniformity is essential: there is no uniform reduction procedure to compute a DNR_n function from a DNR_{n+1} function. Recently, Hirst, Shafer and the author [1] showed that the Σ^0_2 decision in Friedberg's argument cannot be replaced by a Δ^0_2 decision by constructing a nonstandard model of RCA_0 with Δ^0_2-induction where there is a DNR_n function for some n but there are no DNR_2 functions.

In this talk, we will show that Friedberg's proof is optimal in the sense that if $f(n)$ is a primitive recursive function such that RCA_0 proves that every $\mathrm{DNR}_{f(n)}$ function computes a DNR_n function then $f(n)$ must have polynomial growth rate. This will be accomplished by forcing over nonstandard models of RCA_0. If time permits, we will also discuss related joint work with Seth Harris on graph colorings.

[1] F. G. Dorais, J. L. Hirst, and P. Shafer, *Comparing the strength of diagonally nonrecursive functions in the absence of Σ^0_2 induction*, submitted.

[2] C. G. Jockusch, Jr. *Degrees of functions with no fixed points*, ***Logic, Methodology and Philosophy of Science. VIII. (Proceedings of the Eighth International Congress held at Moscow State University, Moscow, August 17–22, 1987)*** (J. E. Fenstad, I. T. Frolov, and R. Hilpinen, editors), North-Holland, Amsterdam, 1989, pp. 191–201.

► MUSHFEQ KHAN, *Mass problems and recursively-bounded DNR functions*.
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If h is a nondecreasing, recursive, and unbounded function, let DNR_h be the class of DNR functions that are bounded pointwise by h. What mass problems are weakly reducible to DNR_h? The motivation for this question came partly from algorithmic randomness, where an analysis of DNR_h functions for a specific h was used to show that it is not always possible to compute a (ML) random sequence from one of positive information density (as measured by effective Hausdorff dimension).

However, we need not limit ourselves to mass problems that arise in randomness. For any problem that is weakly reducible to PA (the class of functions of PA degree), it is natural to ask if it is reducible to DNR_h for some h. I will discuss some of the interesting things that we know and the questions that remain. Answers to these questions may have consequences not just for algorithmic randomness, but also for the theory of the Turing degrees.

► JACK H. LUTZ, *Mutual dimension and random sequences*.
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Recent results relating mutual dimension to algorithmic randomness of sequences will be presented.

This is joint work with Adam Case.

► STEPHEN G. SIMPSON, *Aspects of the Muchnik lattice*.
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Let P and Q be sets of reals. Intuitively we may view a set of reals as a "problem," namely, the problem of "finding" some real in the set. Accordingly, we say that P is *Muchnik reducible* to Q if for all $y \in Q$ there exists $x \in P$ such that x is Turing reducible to y. The *Muchnik degree* of P is the equivalence class of P under mutual Muchnik reducibility. Let $\mathcal{D}_\mathrm{w}$ be the lattice of all Muchnik degrees, and let $\mathcal{E}_\mathrm{w}$ be the sublattice consisting of the Muchnik degrees of nonempty, effectively closed (i.e., Π^0_1) sets of reals. It is well known that $\mathcal{D}_\mathrm{w}$ is the natural completion of the upper semilattice $\mathcal{D}_\mathrm{T}$ of Turing degrees. Similarly, $\mathcal{E}_\mathrm{w}$ is a natural extension of the upper semilattice $\mathcal{E}_\mathrm{T}$ of recursively enumerable Turing degrees. In a recent paper by Sankha S. Basu and the speaker, we show that the category of sheaves of sets over $\mathcal{D}_\mathrm{w}$ is an interesting model of intuitionistic higher-order logic. We call this model the *Muchnik topos*. In recent work by Stephen E. Binns, Richard A. Shore and the speaker, we show that $\mathcal{E}_\mathrm{w}$ is

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dense, i.e., for all $\mathbf{p}, \mathbf{q} \in \mathcal{E}_w$ such that $\mathbf{p} < \mathbf{q}$ there exists $\mathbf{r} \in \mathcal{E}_w$ such that $\mathbf{p} < \mathbf{r} < \mathbf{q}$. We now sketch the proof of this latter result. The proof involves some hyperarithmetical theory.

► LINDA BROWN WESTRICK, *Comparing subshifts in Z and Z^2*.
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A subshift is a closed, shift-invariant subset of Cantor space. Also known as symbolic dynamical systems, subshifts were originally used to discretize information from continuous dynamical systems. The simplest kind of subshift is a shift of finite type (SFT), obtained by forbidding finitely many words (or in the two-dimensional case, finitely many $n \times n$ blocks) from occurring anywhere in an element of the subshift. SFTs have been extensively studied in both the one and two-dimensional cases, but the results have divergent characteristics. This difference often stems from the fact that two-dimensional SFTs can embed a Turing machine, while one-dimensional SFTs cannot. The distinction raises the question of whether some more computationally-powerful class of one-dimensional subshifts, such as Π^0_1 subshifts, could provide a better analogy to two-dimensional SFTs. For example, Hochman (2009), Durand, Romashchenko and Shen (2012), and Aubrun and Sablik (2013) have described ways to embed one-dimensional Π^0_1 shifts into higher-dimensional SFTs. However, Π^0_1 subshifts can have more complicated effective dimension spectra than two-dimensional SFTs. I will discuss the search for a nice one-dimensional analogue to two-dimensional SFTs, considering in particular the effective dimension of individual elements of the subshift.

Abstracts of invited talks in the Special Session on Constructive Mathematics

► SANKHA S. BASU AND STEPHEN G. SIMPSON, *Mass problems and intuitionistic higher-order logic*.
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We study a model of intuitionistic higher-order logic which we call *the Muchnik topos*. The Muchnik topos may be defined briefly as the category of sheaves of sets over the topological space consisting of the Turing degrees, where the Turing cones form a base for the topology. We note that our Muchnik topos interpretation of intuitionistic mathematics is an extension of the well known Kolmogorov/Muchnik interpretation of intuitionistic propositional calculus via Muchnik degrees, i.e., mass problems under weak reducibility. We introduce a new sheaf representation of the intuitionistic real numbers, *the Muchnik reals*, which are different from the Cauchy reals and the Dedekind reals. Within the Muchnik topos we obtain a *choice principle* $(\forall x\, \exists y\, A(x, y)) \Rightarrow \exists w\, \forall x\, A(x, wx)$ and a *bounding principle* $(\forall x\, \exists y\, A(x, y)) \Rightarrow \exists z\, \forall x\, \exists y\, (y \leq_T (x, z) \wedge A(x, y))$ where x, y, z range over Muchnik reals, w ranges over functions from Muchnik reals to Muchnik reals, and $A(x, y)$ is a formula not containing w or z.

► ULRIK BUCHHOLTZ, *Primitive recursive homotopy type theory*.
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Homotopy type theory (HoTT) refers to the study of homotopical interpretations [1] of Martin-Löf's constructive type theory (MLTT) and various principles motivated by such interpretations.

Voevodsky's *Univalent Foundations* program [4] aims to leverage HoTT to provide a foundation of mathematics that is suitable for implementation in proof assistants, particularly those already based on MLTT.

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The main principles added to MLTT are the *Univalence Axiom* (UA) [3] and *higher inductive types* (HITs) which can represent constructions such as finite cell complexes, truncations, bracket types, and quotients by equivalence relations.

Primitive Recursive Arithmetic (PRA) is an important subsystem of first-order *Heyting arithmetic* (HA), which in turn naturally embeds into a fragment of MLTT. PRA suffices to prove many syntactical or metamathematical results, e.g., from proof theory. Herbelin and Patey have presented a *Calculus of Primitive Recursive Constructions* (CPRC) [2] as an enriched version of PRA presented as a constructive type theory.

Working with a weak framework often allows us to get a sharper of view of concepts compared to working in a stronger setting. Here, I shall discuss what happens to HoTT when pushed down to a weak setting. I shall present a version of UA and a class of HITs that can be added to a type-theory similar to CPRC while remaining conservative over PRA.

[1] Stewe Awodey and Michael A. Warren, *Homotopy theoretic models of identity types*. ***Mathematical Proceedings of the Cambridge Philosophical Society***, vol. 146 (2009), no. 1, pp. 45–55.

[2] Hugo Herbelin and Ludovic Patey, *A Calculus of Primitive Recursive Constructions*, (Abstract #41). Contributed talk presented at the Types 2014 meeting, May 12–15, 2014, Paris, France.

[3] Chris Kapulkin, Peter LeFanu Lumsdaine, and Vladimir Voevodsky, *The simplicial model of Univalent Foundations*, preprint on `arXiv`:
`http://arxiv.org/abs/1211.2851`, 2014.

[4] Vladimir Voevodsky, *Univalent foundations project*, Modified version of an NSF grant application, `http://www.math.ias.edu/~vladimir/Site3/Univalent_Foundations_-files/univalent_foundations_project.pdf`, 2010.

► HANNES DIENER, *Continuity is overrated* (... *sometimes*).
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The foundations of constructive mathematics are closely linked to continuity, in particular continuity of functions of type $\mathbb{R} \to \mathbb{R}$. A, for the uninitiated, bizarre feature of many constructive varieties of mathematics is that often all such functions are continuous. In some varieties such as Brouwer's intuitionism this comes built in as an axiom (with philosophical backing), whereas in others, such as recursive models it is a proven consequence. The situation is a bit more subtle in Bishop style constructive mathematics (BISH). Since Bishop did not want to break compatibility to either classical mathematics or Brouwer's intuitionism, in Bishop's system it is neither provable nor disprovable that all functions $\mathbb{R} \to \mathbb{R}$ are continuous; he simply assumed that all functions he worked with were continuous. One might assume that it is not possible to improve much on this situation. However, there is a group of results, provable in BISH, that do not make any continuity assumption on the function. Among them, for example, Ishihara's second trick:

> Let $f\colon \mathbb{R} \to \mathbb{R}$ be a strongly extensional mapping, and let $(x_n)_{n\geqslant 1}$ be a sequence in $\mathbb{R}$ converging to a limit x. Then for all positive numbers $\alpha < \beta$, either $|f(x_n) - f(x)| < \beta$ eventually or $|f(x_n) - f(x)| > \alpha$ infinitely often.

Notice that this is almost saying that we can *decide* sequential continuity; which is, of course, not surprising if all functions are continuous. It is, however, surprising that we can make this decision a priori without knowing whether all functions are continuous or not.

In this talk we are going to present a variety of similar results, showing that sometimes continuity is overrated.

► HANNES DIENER AND ROBERT LUBARSKY, *Weakenings of Cauchy convergence*.
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Weakenings of the notion of a Cauchy sequence have been considered by various researchers: Tao [4] (albeit from a classical perspective), Berger, Bridges and Palmgren [1], Richman [3], the current authors [2]. We investigate these and other related principles, with an eye toward determining the valid implications among them, and, for those implications that do not hold, just what additional principle is needed for the implication. For instance, Tao [4] showed classically that every metastable sequence is Cauchy; we show that that theorem is equivalent to LPO.

[1] Josef Berger, Douglas Bridges, and Eric Palmgren, *Double sequences, almost cauchyness and BD-N*. ***Logic Journal of Interest Group in Pure and Applied Logics***, vol. 20 (2012), no. 1, pp. 349–354.

[2] Robert Lubarsky and Hannes Diener, *Principles weaker than BD-N*. ***The Journal of Symbolic Logic***, vol. 78 (2013), no. 3, pp. 873–885.

[3] Fred Richman, private communication.

[4] Terrence Tao, *Soft analysis, hard analysis, and the finite convergence principle*, http://terrytao.wordpress.com/2007/05/23/soft-analysis-hard-analysis-and-the-finite-convergence-principle/, May 2007.

► VLADIK KREINOVICH AND OLGA KOSHELEVA, *How physics can influence what is computable: taking into account that we process physical data and that we can use nonstandard physical phenomena to process this data*.

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It is well known that many computational problems—e.g., computing the root of a computable function—are, in general, not algorithmically solvable. Since we mostly process physical data, it makes sense to restrict ourselves to processing *physical* inputs. Physicists believe that events with very small probability cannot occur—this is a standard explanation of why, e.g., in spite of a stochastic molecular movements, a kettle on a cold stove cannot start boiling by itself. We show under that a proper formalization of this idea, if we restrict computations to situations which are "not-abnormal" in this sense, then problems which are noncomputable in the general case—like comparing real numbers or finding the roots of a computable function—become computable [2]. Moreover, under the usual physicists' belief that no physical theory is perfect, the use of physical phenomena potentially allows to make the corresponding computations feasible for almost all inputs [1].

[1] O. Kosheleva, M. Zakharevich, and V. Kreinovich, *If many physicists are right and no physical theory is perfect, then by using physical observations, we can feasibly solve almost all instances of each NP-complete problem*. ***Mathematical Structures and Modeling***, vol. 31 (2014), pp. 4–17.

[2] V. Kreinovich, *Negative results of computable analysis disappear if we restrict ourselves to random (or, more generally, typical) inputs*. ***Mathematical Structures and Modeling***, vol. 25 (2012), pp. 100–103.

► BAS SPITTERS, *Formalizing mathematics in the univalent foundations*.

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We have been formalizing mathematics in type theory for many years. Although great progress has been made, there were still a number of issues that are hard to tackle, especially when leaving the realm of discrete mathematics. We mention a lack of quotient types and the absence of a clear mathematical semantics for the Coq type theory.

HoTT promises a solution to these challenges. I will discuss how to develop some mathematics in the univalent foundations and report on the design of our Coq library.

Basic familiarity with type theory and some categorical semantics will be assumed. For more information see: http://homotopytypetheory.org/,

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https://github.com/HoTT/HoTT/, and our book:
http://homotopytypetheory.org/book/

Abstracts of invited talks in the Special Session on Model Theory of Ordered Structures

▸ SAUGATA BASU, *From combinatorial complexity to triangulations of monotone families*.
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I will explain how to extend the combinatorial parts of certain well known bounds on the topology (the Betti numbers) of semi-algebraic sets to the general o-minimal setting and mention some applications of such bounds in discrete geometry. If time permits, in the second part of the talk I will explain a result of Gabrielov and Vorobjov which reduces the problem of bounding the topology of arbitrary definable sets to that of compact ones, and show how it leads to the problem of proving the existence of triangulations compatible with monotone definable families. I will mention some partial results in this direction.

The last part of the talk is joint work with A. Gabrielov and N. Vorobjov.

▸ WILL JOHNSON, *Interpretable sets in o-minimal structures*.
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It is known that not all o-minimal structures have elimination of imaginaries. Definable quotients cannot always be eliminated. Peterzil asked whether this became true after naming additional parameters. We will discuss an example of this failing to hold: an o-minimal structure with a definable quotient that cannot be eliminated even after naming parameters. We'll then discuss what can be said about interpretable sets in o-minimal structures. For example, Kamenkovich and Peterzil have shown that interpretable sets still admit well-defined Euler characteristics. We'll discuss a sense in which interpretable sets look "locally" like manifolds.

▸ TOBIAS KAISER AND PATRICK SPEISSEGGER, *Holomorphic extensions of functions definable in* $\mathbb{R}_{an,\exp}$.
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We are interested in holomorphic extensions of one-variable functions definable in the expansion of the real field by all restricted analytic functions and the exponential function. I will try to precisely state a theorem about such extensions, which in a certain sense is optimal.

▸ JANA MAŘÍKOVÁ AND ERIK WALSBERG, *Measures and definable metric spaces in o-minimal fields*.
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We extend the results in [1] and give an application to definable metric spaces in o-minimal structures.

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This is joint work in progress with Erik Walsberg.

[1] J. Maříková and M. Shiota, *Measuring definable sets in o-minimal fields*. ***Israel Journal of Mathematics***, to appear.

▸ ATHIPAT THAMRONGTHANYALAK, *Continuous definable Skolem functions in d-minimal structures*.
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It is well-known that o-minimal structures admit definable Skolem functions which are piecewise continuous. However, we cannot always find continuous definable Skolem functions for all definable families. In 2013, M. Aschenbrenneer and I gave a sufficient condition on definable families that guarantees the existence of continuous definable Skolem functions. In this talk, we will discuss the similar problem in d-minimal expansions of the real field. An analog of our result will be proved in this context.

Abstracts of invited talks in the Special Session on Parametrized Complexity

▸ CHANDRA CHEKURI, *Recent progress in structure of large treewidth graphs and some applications*.
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The seminal work of Robertson and Seymour on graph minors developed and utilized several important properties of tree decompositions and treewidth. Treewidth has since become a fundamental tool for structural and algorithmic results on graphs. One of the key results of Robertson and Seymour is the Excluded Grid Theorem which states that there is an integer valued function f such that every graph G with treewidth at least $f(k)$ contains a $k \times k$ grid as a minor.

In this talk we will discuss some recent developments on the structure of graphs with large treewidth. In particular, Julia Chuzhoy and the author showed that f can be chosen to be a polynomial, improving previous bounds that were at least exponential. We will discuss this and related structural results and some applications to fixed parameter tractability and Erdos–Posa type theorems.

▸ SERGE GASPERS, *Backdoors to satisfaction*.
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A backdoor set is a set of variables of a propositional formula such that fixing the truth values of the variables in the backdoor set moves the formula into some class where the Satisfiability problem is polynomial-time decidable. If we know a small backdoor set we can reduce the question of whether the given formula is satisfiable to the same question for one or several easy formulas that belong to the tractable class under consideration. In this talk I will review parameterized complexity results for problems that arise in the context of backdoor sets, such as the problem of finding a backdoor set of size at most k, parameterized by k. In particular, I will outline backdoors to combinations of base classes and a recent FPT-approximation algorithm finding a backdoor to the class of formulas with bounded incidence treewidth.

[1] Serge Gaspers, Neeldhara Misra, Sebastian Ordyniak, Stefan Szeider, and Stanislav Zivny, *Backdoors into heterogeneous classes of SAT and CSP*, ***Proceedings of the 28th AAAI Conference on Artificial Intelligence (AAAI 2014), (Québec City, Québec, Canada)*** (Carla E. Brodley and Peter Stone, editors), AAAI Press, Menlo park, CA, 2014, pp. 2652–2658.

[2] Serge Gaspers and Stefan Szeider, *Strong backdoors to bounded treewidth SAT*, ***Proceedings of the 54th Annual IEEE Symposium on Foundations of Computer Science (FOCS 2013), (Berkeley, California, USA)***, IEEE Computer Society, 2013, pp. 489–498.

[3] ———, *Backdoors to satisfaction*, ***The Multivariate Algorithmic Revolution and Beyond: Essays Dedicated to Michael R. Fellows on the Occasion of his 60th Birthday*** (Hans L. Bodlaender, Rodney G. Downey, Fedor V. Fomin, Dániel Marx, editors), Springer, Berlin, Lecture Notes in Computer Science, vol. 7370, 2012, pp. 287–317.

[4] Ryan Williams, Carla P. Gomes, and Bart Selman, *Backdoors To typical case complexity*, ***Proceedings of the 18th International Joint Conference on Artificial Intelligence (IJCAI 2003), (Acapulco, Mexico)*** (Georg Gottlob and Toby Walsh, editors), Morgan Kaufmann, 2003, pp. 1173–1178.

▸ IYAD KANJ, *When is weighted satisfiability in FPT?*
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We consider the weighted monotone and antimonotone satisfiability problems on normalized circuits of depth at most $t \geq 2$, abbreviated WSAT$^+[t]$ and WSAT$^-[t]$, respectively. These problems model the weighted satisfiability of monotone and antimonotone propositional formulas (including weighted monotone/antimonotone CNF-SAT) in a natural way, and serve as the canonical problems in the definition of the parameterized complexity hierarchy. In particular, WSAT$^+[t]$ $(t \geq 2)$ is W$[t]$-complete for even t and W$[t-1]$-complete for odd t, and WSAT$^-[t]$ $(t \geq 2)$ is W$[t]$-complete for odd t and W$[t-1]$-complete for even t. Moreover, several well-studied problems, including important graph problems, can be modeled as WSAT $^+[t]$ and WSAT$^-[t]$ problems in a straightforward manner. We study the parameterized complexity of WSAT$^-[t]$ and WSAT$^+[t]$ with respect to the genus of the circuit. For WSAT$^-[t]$, we give a fixed-parameter tractable (FPT) algorithm when the genus of the circuit is $n^{o(1)}$, where n is the number of the variables in the circuit. For WSAT$^+[2]$ (i.e., weighted monotone CNF-SAT), which is W[2]-complete, we also give FPT-algorithms when the genus is $n^{o(1)}$. For WSAT$^+[t]$ where $t \geq 3$, we give FPT-algorithms when the genus is $o(\log^{2/3}(n))$. We also show that both WSAT$^-[t]$ and WSAT$^+[t]$ on circuits of genus $n^{\Omega(1)}$ have the same W-hardness as the general WSAT$^+[t]$ and WSAT$^-[t]$ problem (i.e., with no restriction on the genus), thus drawing a precise map of the parameterized complexity of WSAT$^-[t]$ and of WSAT$^+[2]$ with respect to the genus of the underlying circuit. As a byproduct of our results, we obtain, via standard parameterized reductions, tight results on the parameterized complexity of several problems with respect to the genus of the underlying graph.

This is joint work with Dimitrios Thilikos and Ge Xia.

▸ AMER MOUAWAD, *The complexity of CSP reconfiguration*.
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Given an n-vertex graph G and two vertices s and t in G, determining whether there exists a path and computing the length of the shortest path between s and t are two of the most fundamental graph problems. In the classical battle of P versus NP or "easy" versus "hard", both of these problems are on the easy side. That is, they can be solved in $poly(n)$ time, where $poly$ is any polynomial function. But what if our input consisted of a 2^n-vertex graph? Of course, we can no longer assume G to be part of the input, as reading the input alone requires more than $poly(n)$ time. Instead, we are given an oracle encoded using $poly(n)$ bits and that can, in constant or $poly(n)$ time, answer queries of the form "is u a vertex in G" or "is there an edge between u and v?". Given such an oracle and two vertices of the 2^n-vertex graph, can we still determine if there is a path or compute the length of the shortest path between s and t in $poly(n)$ time? This seemingly artificial question is in fact quite natural and appears in many practical and theoretical problems. In particular, this is exactly the types of questions asked under the reconfiguration framework. Under the reconfiguration framework, instead

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of finding a feasible solution to some instance $\mathcal{I}$ of a search problem $\mathcal{Q}$, we are interested in structural and algorithmic questions related to the solution space of $\mathcal{Q}$. For example, a reconfiguration variant of the k-COLORING problem asks, given two k-colorings α and β of a graph G and an integer ℓ, can α be modified into β by recoloring vertices one at a time, while maintaining a k-coloring throughout and using at most ℓ such recoloring steps? This problem is weakly PSPACE-hard for every constant $k \geq 4$. We show that the problem is W[1]-hard (but in XP) when parameterized only by ℓ. On the positive side, we show that the problem is fixed-parameter tractable when parameterized by $k + \ell$. In fact, we show that the more general problem of length-bounded reconfiguration of constraint satisfaction problems (CSPs) is fixed-parameter tractable parameterized by $k + \ell + r$, where r is the maximum constraint arity and k is the maximum domain size. We show that for parameter ℓ, the latter problem is W[2]-hard, even for $k = 2$. Finally, if p denotes the number of variables with different values in the two given assignments, we show that the problem is W[2]-hard when parameterized by $\ell - p$, even for $k = 2$ and $r = 3$.

This is joint work with Paul Bonsma, Naomi Nishimura, Vinayak Pathak, and Venkatesh Raman.

▶ SRIDHARAN RAMANUJAN, *A parameterized complexity dichotomy for Boolean QCSP Deletion*.
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In the BOOLEAN QCSP DELETION problem, the input is a *totally quantified* CSP instance over a fixed boolean constraint language Γ and an integer k and the objective is to test if there is a set of at most k constraints whose deletion makes the resulting quantified CSP instance true. This problem has the power to model several fundamental and well-studied problems in parameterized complexity. A prime example is the 2-SAT DELETION problem which is the special case of BOOLEAN QCSP DELETION where Γ is 2-cnf or bijunctive, and the quantification is purely existential. The 2-SAT DELETION problem itself has played a central role in the evolution of FPT algorithms, both as an important subroutine in several algorithms and as a source of new techniques.

In this work, we study the BOOLEAN QCSP DELETION problem over *Schaefer languages* and show that the parameterized complexity of this problem is completely characterized by the choice of a Schaefer language Γ and the arity of the relations in Γ.

As a corollary of the positive side of the dichotomy, we obtain the fixed parameter tractability of the QUANTIFIED H-COLORING DELETION problem, implying an FPT-version of Hell and Nesetril's dichotomy for the H-coloring problem. Furthermore, the fixed parameter tractability of several important problems in parameterized complexity follows as a direct corollary of our theorem, including ODD CYCLE TRANSVERSAL [3], DELETION RHORN-BACKDOOR SET DETECTION, 2-SAT DELETION [2], and DIRECTED MULTIWAY CUT [1].

[1] RAJESH CHITNIS, MOHAMMADTAGHI HAJIYAGHAYI, AND DÁNIEL MARX, *Fixed parameter tractability of Directed Multiway Cut parameterized by the size of the cutset*. ***SIAM Journal on Computing***, vol. 42 (2013), no. 4, pp. 1674–1696.

[2] IGOR RAZGON AND BARRY O'SULLIVAN, *Almost 2-SAT is fixed parameter tractable*. ***Journal of Computer and System Sciences***, vol. 75 (2009), no. 8, pp. 435–450.

[3] BRUCE REED, KALEIGH SMITH, AND ADRIAN VETTA, *Finding odd cycle transversals*. ***Operations Research***, vol. 32 (2004), no. 4, pp. 299–301.

Abstracts of invited talks in the Special Session on Philosophy of Mathematics

▶ JOHN T. BALDWIN, *Model theory and set theory*.
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At least since Skolem's formulation of his paradox set theory and model theory have been intertwined. In contrast to Skolem, we investigate the methodological role of set theory in model theory. We will address several questions. What is the role of axiomatic set theory in model theory? How does this depend on whether the object of study is a logic or a theory? What is the role of combinatorial set theory in model theory? In particular, what is the role of indiscernibles in model theory? What is the role of cardinality in model theory. How do properties of largish cardinals affect countable models? In this talk we will outline some of these issues and discuss some competing views.

▸ KATALIN BIMBÓ AND J. MICHAEL DUNN, *On the decidability of classical linear logic*.
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Classical linear logic was introduced in [4]. We prove that LL, the propositional fragment of this logic is decidable. We provide two conceptually different proofs (for the sake of comparison with [6], where undecidability is claimed).

The relevance logic LR ("lattice-R") was introduced and proved decidable in [7]. Our first proof relies on a translation between LR (with constants) and LL.

LL has a simple sequent calculus formulation, which is amenable to an application of the same proof technique that had been used to show a range of relevance and modal logics decidable. (See for example, [5], [7], [3], [2], and [1].) A core idea is to allow a limited amount of contraction in connective rules in lieu of explicit contraction rules. We formulate a sequent calculus for LL without the $(\,!W\vdash)$ and $(\vdash ?W)$ rules, for which we prove the cut theorem and Curry's lemma. Then, we show—using a version of Kripke's lemma and König's lemma—that a finite proof-search tree can be constructed for a given formula.

[1] Katalin Bimbó, ***Proof Theory: Sequent Calculi and Related Formalisms***, Discrete Mathematics and its Applications, CRC Press, Boca Raton, FL, 2014.

[2] Katalin Bimbó and J. Michael Dunn, *On the decidability of implicational ticket entailment*. ***The Journal of Symbolic Logic***, vol. 78 (2013), pp. 214–236.

[3] J. Michael Dunn, *Relevance logic and entailment*. ***Handbook of Philosophical Logic*** (D. Gabbay and F. Guenthner, editors), vol. 3, 1st ed., D. Reidel, Dordrecht, 1986, pp. 117–224.

[4] Jean-Yves Girard, *Linear logic*. ***Theoretical Computer Science***, vol. 50 (1987), pp. 1–102.

[5] Saul A. Kripke, *The problem of entailment, (abstract)*. ***The Journal of Symbolic Logic***, vol. 24 (1959), p. 324.

[6] Patrick Lincoln, John Mitchell, Andre Scedrov, and Natarajan Shankar, *Decision problems for linear logic*. ***Annals of Pure and Applied Logic***, vol. 56 (1992), pp. 239–311.

[7] Robert K. Meyer, ***Topics in Modal and Many-valued Logic***, PhD thesis, University of Pittsburgh, Ann Arbor (UMI), 1966.

▸ MICHAEL ERNST, *What graphs could not be*.
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Categorial structuralism suggests that categories are the most effective, and potentially the most correct, method for describing mathematical structures. I will show that categorial structuralism has problems similar to the Benacerraf-style difficulties that originally motivated the structuralist view in the philosophy of mathematics. This will be done by looking at the categorial treatment of graphs.

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► COLIN MCLARTY, *How Alexander Grothendieck (1928–2014) simplified number theory, and why*.
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Philosophers can learn much about simplicity, structure, historical continuity, and rigor in mathematics from the extremely productive state of number theory since the "Grothendieck revolution" (a term used by David Mumford and Barry Mazur). This talk will emphasize how "etale cohomology" simplifies insights from Bernhard Riemann's 1851 dissertation on complex analysis and makes them work in number theory. Riemann and Kronecker, among others, both believed things like this should be possible and should be done.

► REBECCA MORRIS, *Understanding proof steps*.
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When reading a proof, we not only want to be able to check its correctness, but desire to *understand* it. Philosophers of mathematical practice have started to analyze what it means to understand a proof, for example see [1]. In this talk, I will consider the different senses in which a reader may understand a particular *step* in a proof, the associated benefits of such understanding, and how it may be promoted or stifled. To do this, I will examine case studies from number theory, comparing different proofs of the same theorem. Using these case studies, I identify two types of understanding associated with proof steps: (i) understanding where the proof step comes from; (ii) understanding how the proof step contributes to the overall argument. I suggest that these two kinds of understanding work together to allow the reader to reuse the proof step effectively in other contexts, and thus have a practical benefit. I then argue that the choice of notation, as well as the structure of the proof, can foster, or suppress, such understanding.

[1] Jeremy Avigad, *Understanding proofs*, ***The Philosophy of Mathematical Practice*** (Paolo Mancosu, editors), Oxford University Press, 2008, pp. 317–353.

Abstracts of invited talks in the Special Session on Set Theory

► MARTINO LUPINI, *Noncommutative analogs of the Gurarij Banach space*.
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Working in the framework of Fraïssé theory for metric structures developed by Ben Yaacov, we show that finite-dimensional 1-exact operator spaces form a Fraïssé class. We identify the limit as the (noncommutative) Gurarij operator space introduced by Oikhberg. As a consequence we obtain that such a space is unique up to complete isometry, homogeneous, and universal for separable 1-exact operator spaces. We also show that finite-dimensional 1-exact operator systems form a Fraïssé class. The corresponding limit is a nuclear operator system that contains unitally completely isometrically every separable 1-exact operator system. It is moreover universal in the sense of Kirchberg and Wasserman, i.e., the canonical *-homomorphism from the universal C^*-algebra to the C^*-envelope is a *-isomorphism.

► KONSTANTIN SLUTSKY, *Orbit equivalences of Borel flows*.
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We shall give an overview of different variants of orbit equivalence for multidimensional Borel flows, that is Borel actions of $\mathbb{R}^n$. Of main importance for us will be the concept of Lebesgue orbit equivalence. We say that two free flows are Lebesgue orbit equivalent if there exists a Borel bijection between their phase spaces sending orbits onto orbits which preserves Lebesgue measure within each orbit. The talk will cover old and new classification results up to this notion of equivalence in ergodic theory and Borel dynamics.

▸ ROBIN TUCKER-DROB, *Planarity and treeability of groups and equivalence relations*.
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I will discuss recent joint work with C. Conley, D. Gaboriau, and A. Marks, in which we show that any free Borel action of a group with a planar Cayley graph gives rise to an equivalence relation which is measure treeable. This provides the first examples of nonamenable groups with one end which are strongly treeable.

▸ SPENCER UNGER, *The tree property at successive regular cardinals*.
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We explore recent progress towards a positive solution to a question of Magidor: "Is it consistent that every regular cardinal greater than $\aleph_1$ has the tree property?" We present two main advances. First, in joint work with Dima Sinapova we obtained partial progress towards a positive answer to a question of Woodin: "Is it consistent that the singular cardinals hypothesis fails at $\aleph_\omega$ and the tree property holds at $\aleph_{\omega+1}$?". Second, we have extended the longest known initial segment of regular cardinals on which the tree property can hold.

▸ TREVOR M. WILSON, *Covering properties of derived models*.
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After forcing with the Levy collapse at a limit λ of Woodin cardinals, one obtains a model $L(\mathbb{R}^*)$ satisfying AD, the Axiom of Determinacy. (The set $\mathbb{R}^*$ consists of all reals added by proper initial segments of the forcing.) In fact the proof shows that $L(\mathbb{R}^*)$ satisfies AD^+. More generally, one may consider the derived model $D(V, \lambda)$, which is the maximal submodel of the symmetric model $V(\mathbb{R}^*)$ that satisfies AD^+ and contains all ordinals and all reals in $\mathbb{R}^*$. This model may properly extend $L(\mathbb{R}^*)$. The existence of such a maximal model follows from Woodin's derived model theorem [1, Thm. 31].

We outline some arguments showing that under certain circumstances the derived model at a limit λ of Woodin cardinals is "close to V" in terms of its ordinal Θ, the successor of the continuum in the sense of surjections. Namely, we show that if λ is inaccessible then the cofinality of $\Theta^{D(V,\lambda)}$ is at least λ, and if in addition λ is weakly compact then either $\Theta^{D(V,\lambda)} = \lambda^+$ or $D(V, \lambda)$ satisfies "every set of reals is Suslin." We discuss a question that remains in the case that λ is singular.

[1] W. Hugh Woodin, *Suitable extender models I*. ***Journal of Mathematical Logic***, vol. 10 (2010), no. 1,2, pp. 101–339.

Abstracts of contributed talks

▸ WESLEY CALVERT, *PAC learning, VC dimension, and the arithmetic hierarchy*.
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We compute that the index set of PAC-learnable concept classes is m-complete Σ^0_3 within the set of indices for all concept classes of a reasonable form. All concept classes considered

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are computable enumerations of computable Π^0_1 classes, in a sense made precise here. This family of concept classes is sufficient to cover all standard examples, and also has the property that PAC learnability is equivalent to finite VC dimension.

► GABRIEL CONANT, *Model theory and combinatorics of homogeneous metric spaces*.
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Given a countable, totally and positively ordered commutative structure $\mathcal{R} = (R, \oplus, \leq, 0)$, there is a natural notion of an $\mathcal{R}$-metric space. In this talk, we assume the existence of a countable, universal, and homogeneous $\mathcal{R}$-Urysohn space, denoted $\mathcal{U}_\mathcal{R}$. By a generalization of results due to Delhommé, Laflamme, Pouzet, and Sauer, this is equivalent to the assumption that $\oplus$ is associative. Let $T_\mathcal{R}$ denote the complete theory of $\mathcal{U}_\mathcal{R}$ in a relational language with distance predicates $d(x, y) \leq r$, for $r \in R$. We construct a nonstandard extension $\mathcal{R}^* = (R^*, \oplus^*, \leq^*, 0)$ such that any model of $T_\mathcal{R}$ is an $\mathcal{R}^*$-metric space and, moreover, quantifier elimination in $T_\mathcal{R}$ is equivalent to a natural continuity property in $\mathcal{R}^*$. We also show that stability, simplicity, and Shelah's n-strong order property for $T_\mathcal{R}$ are characterized by the archimedean complexity of $\mathcal{R}$. Finally, we focus on the case when R is a finite subset of $\mathbb{N}$ and $r \oplus s = \max\{x \in R : x \leq r + s\}$. Building on work of Nguyen Van Thé, we discuss some results on the combinatorial and enumerative behavior of $\mathcal{R}$, as well as conjectures and connections in other areas of additive combinatorics.

► MATTHEW CORDES, MOON DUCHIN, YEN DUONG, MENG-CHE HO, and ANDREW P. SÁNCHEZ *Random nilpotent groups*.
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We study random nilpotent groups in the well-established style of random groups. Whereas random groups are quotients of the free group by a random set of relators, random nilpotent groups are quotients of a free nilpotent group $N_{p,m}$ by a similarly chosen set of relators.

We establish results about the distribution of rank and step for random nilpotent groups. We show that a random nilpotent group is almost never abelian but not cyclic. We also describe how to lift results about random nilpotent groups to obtain information about standard random groups. A random nilpotent group is trivial if and only if the corresponding random group is perfect, i.e., is equal to its commutator subgroup. Considering adding relators one by one in a stochastic process, we study the threshold number of relators required.

► BARBARA CSIMA AND MATTHEW HARRISON-TRAINOR, *Computable structures relative to a cone*.
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Computable structures can often be constructed to exhibit pathological behaviour. Structures which naturally occur in mathematics are often more nicely behaved and do not have such pathological behaviour. One can study the properties of natural structures by relativizing to a cone of Turing degrees; if a natural structure satisfies some property P on a cone,

we would expect it to satisfy the unrelativized property P. Some examples of such properties involve degree spectra of relations and degrees of categoricity.

▸ PHILIP EHRLICH AND ELLIOT KAPLAN, *Number systems with simplicity hierarchies: A generalization of Conway's theory of surreal numbers II*.
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In [1], the first author brought to the fore the algebraico-tree-theoretic simplicity hierarchical structure of J. H. Conway's ordered field **No** of surreal numbers and employed it to provide, among other things, necessary and sufficient conditions for an ordered field to be isomorphic to an initial subfield of **No**, i.e., a subfield of **No** that is an initial subtree of **No**. This led to the result that every real-closed ordered field is isomorphic to an initial subfield of **No**. In this paper the authors establish analogous results for domains, groups, and semigroups.

[1] Philip Ehrlich, *Number systems with simplicity hierarchies: A generalization of conway's theory of surreal numbers*. ***The Journal of Symbolic Logic***, vol. 66 (2001), no. 3, pp. 1231–1258.

▸ TIGRAN HAKOBYAN AND MINH CHIEU TRAN, *Algebraically closed fields with a generic multiplicative character*.
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We study the class $\mathcal{C}$ of two sorted structures $(F, K; \chi)$, where F and K are algebraically closed fields, K has characteristic 0, and $\chi\colon F \to K$ is a generic multiplicative character which means χ is injective, multiplication preserving, and takes multiplicatively independent elements to algebraically independent elements over $\mathbb{Q}$. The examples of main interest are when $F = \mathbb{F}_p^{ac}$ and $K = \mathbb{Q}^{ac}$. We prove the following: $\mathcal{C}$ is an elementary class with a natural axiomatization denoted by ACFC; for p prime or $p = 0$, $\mathrm{ACFC}_p = \mathrm{ACFC} \cup \{\mathrm{char}(F) = p\}$ has a categoricity property, is complete, has relative quantifier elimination, is ω-stable, has all models algebraically bounded, and has all definable functions piecewise closed-graph in a natural topology.

▸ ALEX KRUCKMAN, *Amalgamation and the finite model property*.
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A theory T has the *finite model property* if every sentence in T has a finite model. In many examples, such as the theory of the random graph, the finite model property can be established via a probabilistic argument, i.e., a probabalistic construction of finite structures of size n which produces a model of any sentence $\varphi \in T$ with positive probability for large enough n.

It turns out that higher order amalgamation properties for quantifier-free types have a key role in such probabilistic arguments. From this point of view, I will discuss the question of whether instances of the finite model property can always be explained by probabilistic arguments for a natural class of "purely combinatorial" theories.

▸ JOACHIM MUELLER-THEYS, *The positive inexpressibility of consistency*.
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By the occurrence of immanent unsoundness and inconsistency (due to the existence of negative fixed points), $\imath(x)$ (e.g., Gödel's Prov (x)) fails to state consequence (provability), whence $\kappa := \neg\imath(\langle\bot\rangle)$ (e.g., Con $:= \neg\mathrm{Prov}(\#(\bot))$) fails to state consistency ([1], [2]).

Consistency is a metatheoretic property defined negatively, by the absence of contradictions. Gödelian and modal approaches try to represent it positively, by the presence of certain formulæ. We will now show that this is *generally* impossible.

0. Let, at least, $\bot \in L$, $\mathfrak{T} \subseteq \operatorname{Pot} L$ ("theories"), $L \in \mathfrak{T}$. $T \in \mathfrak{T}$ is *consistent* :iff $\bot \notin T$; T is *inconsistent* :iff T is not consistent. L is inconsistent.

Logical systems $\mathfrak{L} = \langle L, \operatorname{seq} \subseteq \operatorname{Pot} L \times L\rangle$ with $\Phi \subseteq \operatorname{seq}\Phi$ $(:= \{\phi\colon \Phi \operatorname{seq} \phi\})$ can be subsumed by $\mathfrak{T}_{\mathfrak{L}} := \{T : \exists\Phi\, T = \operatorname{seq}\Phi\}$. $L \in \mathfrak{T}_{\mathfrak{L}}$ follows.

$\mathfrak{P} \subseteq \mathfrak{T}$ is a *metatheoretic property*. *Cs* $:= \{T \in \mathfrak{T}\colon T$ consistent$\}$; *Incs* $:= \mathfrak{T} \setminus$Cs. $\mathfrak{P}$ is *L-free* :iff $L \notin \mathfrak{P}$. Cs is L-free.

I. A formula expresses a metatheoretic property if it is present at theories having it and absent from theories having it not.

Definition.

(i) $\phi \in L$ *expresses* $\mathfrak{P}$ iff for every $T \in \mathfrak{T}$: $\phi \in T$ iff $T \in \mathfrak{P}$;
(ii) $\mathfrak{P}$ is *expressible* iff there is ϕ such that ϕ expresses $\mathfrak{P}$;
(iii) $\mathfrak{P}$ is *inexpressible* iff $\mathfrak{P}$ is not expressible.

Proposition. *Incs is expressible.*

II. However, metatheoretic properties, which the language does not have, are inexpressible.

Theorem. *If $\mathfrak{P}$ is L-free, $\mathfrak{P}$ is inexpressible.*

Proof. Assume that ϕ_0 expresses $\mathfrak{P}$. Thence $L \in \mathfrak{P}$.

Corollary. *Cs is inexpressible.*

In particular, formulæ like κ and $\neg\Box\bot$ cannot express consistency.

Note. This is also joint work with Wilfried Buchholz, who found the definiens $\approx$ 2011. Andreas Haltenhoff was stimulating further development.

[1] Joachim Mueller-Theys, *Metalogical extensions—Part II: First-order consequences and Gödel*, ***2014 ASL European Summer Meeting***.

[2] ———, *Immanent inconsistency*, ***2014–15 ASL Winter Meeting*** (***with Joint Mathematics Meeting***).

▶ CAROLINE TERRY, *Finite distance metric spaces: global structure and 0-1 laws*.
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Fix an integer $r \geq 3$. Given an integer n, we define $M_r(n)$ to be the set of metric spaces with underlying set $\{1, \ldots, n\}$ such that the distance between any two points lies in $\{1, \ldots, r\}$. We present results describing the approximate structure of these metric spaces when n is large. We then present that as a consequence of these structural results, when r is even, this family of structures has a first-order labeled 0-1 law in the language consisting of r binary relation symbols, one for each distance in $\{1, \ldots, r\}$. We end by discussing similarities between this example, in the case when r is even, and the family of finite labeled triangle free-graphs. In particular, both families produce two first order theories: the almost sure theory, and the theory of the Fraïssé limit associated to the family. In both examples, these two theories are different. We discuss how the two theories produced by each example differ analogously in their model theoretic complexity.

This is joint work with Dhruv Mubayi.

▶ SEBASTIEN VASEY, *Independence in abstract elementary classes*.
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Independence (or forking) is one of the central notion of modern classification theory. In first-order model theory, it was introduced by Shelah and is one of the main devices of his book. One can ask whether there is such a notion outside of elementary classes. We will focus on abstract elementary classes (AECs), a very general axiomatic framework introduced by Shelah in 1985. In Shelah's book on AECs, the central concept is that of a good frame

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(a local independence notion for types of singletons) and conditions are given for their existence. However, the question of when there is a global independence notion (for types of all lengths) is still left open.

We show how to construct such a notion assuming categoricity (in a high-enough cardinal), amalgamation, tameness (a locality condition for types introduced by Grossberg and VanDieren), and type-shortness (a relative of tameness introduced by Boney). We also introduce a definition of superstability that can replace categoricity in our construction.

Abstracts of talks presented by title

▸ JOHN CORCORAN, *Teaching basic logics*.
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This accessible presentation explains and demonstrates the quantifier-free, Jaskowski-style, natural-deduction logics introduced in [2].

A *basic logic BL* is a “zero-order logic”: more rudimentary than first-order and transcended by first-order as first-order is transcended by second-order. Every BL includes identity, individual constants, and the standard truth-functional connectives. BLs may also have inidentity (diversity or nonidentity), relation symbols, or function symbols. Although they lack variables and quantification, some BLs are genuine underlying logics suitable for naturally representing actual reasoning in rudimentary sciences including “Leibniz Arithmetic”, certain versions of “Aristotelian syllogistic”, and portions of high-school arithmetic and trigonometry.

BLs are not mere logic-schemata like propositional logics, which are incapable of serving as underlying logics of actual sciences. After all, no proposition’s logical form is adequately represented by a propositional constant: all propositions have internal structure [1, §00, §01, and §04]. Unlike propositional logics, under certain interpretations, a BL’s formal deductions can be used by persons to express their deductive reasoning.

Without underlying logics having sentences exhibiting logical forms of genuine propositions, understanding the epistemic significance of natural-deduction formalization is awkward if not impossible.

Some logics in [4]—certain underlying logics of set theories—also lack variables and quantification. Nevertheless, they aren’t BLs; lacking truth-functional connectives, their sentences are exclusively constant equations such as $(1 + 2) = (4 - 1)$. Such logics in [4], and thus BLs, extend what are called *identity logics* in [3]—where sentences are identities or inidentities between individual constants.

[1] Alonzo Church, ***Introduction to Mathematical Logic***, Princeton University Press, Princeton, NJ, 1956.

[2] John Corcoran, *Second-order logic*, ***Logic, Meaning, and Computation: Essays in Memory of Alonzo Church*** (C. Anderson et al., editors), Kluwer, Dordrecht, 1998.

[3] John Corcoran and Stanley Ziewacz, *Identity logics*. ***Notre Dame Journal of Formal Logic***, vol. 20 (1979), pp. 777–784.

[4] Alfred Tarski and Steven Givant, ***Formalization of Set Theory without Variables***, American Mathematical Society, Providence, RI, 1987.

▸ JOHN CORCORAN AND WILLIAM FRANK, *Assumptions: Illative and typographical*.
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The word ‘assumption’ is appropriate in contrasting contexts with contrasting meanings, often in one paper, e.g., [3]. We distinguish three context classes.

In the first—possibly the earliest—people making an assumption believe it and thus necessarily understand it; axioms, postulates, and definitions are sometimes called assumptions [1]. No such context occurs in [3].

In the second—also old—people making an assumption necessarily understand it but often don’t believe it, or may even disbelieve it, e.g., assumptions “for purposes of reasoning”:

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reductio assumptions in indirect proofs and antecedents in conditional proofs. The first such context in [3] is Section 1.1, pages 371f: a conditional proof of an implication between two metalanguage propositions. Such assumptions are called *illative* indicating their role in illation, reasoning productive of knowledge.

In the third—beginning around 1900—'assumption' is used in connection with uninterpreted syntactic strings [2]: there is nothing to understand much less believe. Dozens of such contexts occur in [3]—some in proofs of metatheorems. The first such context in [3] is Section 1.2, pages 372ff, where 'assumption' is used in the relation-verb phrase 'is an assumption in'. Assumptions in this sense are called *typographical* indicating their connection with string-manipulation.

Assumptions from the first context—understood and believed—are excluded from this study. We investigate illative assumptions—understood but possibly disbelieved—and typographical assumptions—uninterpreted character-strings. We question how illative assumptions are modeled, represented, or expressed by typographical assumptions.

[1] John Corcoran, *Sentence, proposition, judgment, statement, and fact*, ***Many Sides of Logic*** (Walter Carnielli et al., editors), College Publications, London, 2009.

[2] John Corcoran, William Frank, and Michael Maloney, *String theory*. ***The Journal of Symbolic Logic***, vol. 39 (1974), pp. 625–637.

[3] John Corcoran and George Weaver, *Logical Consequence in Modal Logic: Natural Deduction in S5*. ***Notre Dame Journal of Formal Logic***, vol. 10 (1969), pp. 370–384.

▸ JOHN CORCORAN AND LEONARDO WEBER, *Two satisfactions in Tarski's truth-definition paper*.

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Tarski used one satisfaction in his truth-definition [2, p. 195] but another in the preceding definitions, axioms, and theorems. His metalanguage used the same relation-verb 'satisfy' for both—creating ambiguity and an appearance of circularity. Surprisingly, Tarski never noted the ambiguity nor did he address the apparent circularity [1].

We use 'satisfy$_1$'for the first and 'satisfy$_2$' for the second.

Thus, 'satisfy$_1$' expresses a semantic *nonlogical* notion: "a given infinite sequence of classes satisfies$_1$ a given sentential function iff [. . .]"; 'satisfy$_1$' expresses a nonhomogeneous two-place *semantic* relation connecting infinite sequences to object-language expressions.

In contrast, 'satisfy$_2$' occurs in various locutions for an arguably *logical* notion. One such locution is '[. . .] satisfies$_2$ the condition: [. . .]'—metalanguage formulas follows the colon. Sometimes some expressions ending with three-word expression 'satisfies$_2$ the condition' seem redundant: deletable without altering meaning.

(1) n is positive iff n satisfies$_2$ the condition: $n > 0$
(2) n is positive iff $n > 0$

Occasionally the prose reads better if the colon is retained and 'satisfies$_2$ the condition' becomes 'is such that'—a logical expression for many logicians including Peano, Russell, and Quine. In such contexts 'satisfies$_2$ the condition' means the same as 'is such that'.

(3) n is positive iff n is such that: $n > 0$

Tarski uses 'satisfies the condition:' several times in [2]: e.g., pages 176, 177, 182, 193, 198, 225, and 275. Building on [1], this paper catalogues and analyses all uses of 'satisfy$_2$' in [2] and considers alternative interpretations and justifications of [2]'s uses of 'satisfy'.

[1] John Corcoran and Leonardo Weber, *Tarski's convention T: condition beta*. ***South American Journal of Logic***, vol. 1 (2014), pp. TBA.

[2] Alfred Tarski, *The concept of truth in formalized languages*, ***Logic, Semantics, Metamathematics, Papers from 1923 to 1938*** (John Corcoran, editor), Hackett, Indianapolis, 1983, pp. 152–278.