J. Plasma Physics (2013), vol. 79, part 3, pp. 305–310. © Cambridge University Press 2012
doi:10.1017/S0022377812000980

Phase shifts of magneto-acoustic solitons in spin-1/2 fermionic quantum plasma during head-on collision

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(Received 4 July 2012; revised 14 October 2012; accepted 15 October 2012; first published online 26 November 2012)

Abstract. The head-on collision between two magneto-acoustic solitons in spin-1/2 fermionic quantum plasma is studied in the framework of the model proposed by Marklund et al. (Marklund, M., Eliasson, B. and Shukla, P. K. 2007 *Phys. Rev.* E. **76**, 067401). The extended Poincare–Lighthill–Kuo method is used to obtain the phase shifts and the trajectories during the head-on collision of two solitons. The effect of the Zeeman energy for different speeds of the waves, the effect of the total mass density of the *charged* plasma particles for different strengths of magnetic field, the effect of the speed of the wave for different values of the Zeeman energy, and that of the ratio of the sound speed to Alfven speed for different values of Zeeman energ on the phase shift are studied. It is observed that the phase shifts are significantly affected in all the cases. The most interesting observation of this paper is that the phase shifts increase as well as decrease, and also they may be positive as well as negative depending upon the domain of the chosen parameters.

1. Introduction

After the pioneering work on quantum plasma by Haas (2005), Garcia et al. (2005), and other workers (Haas et al. 2000; Manfredi and Haas 2001; Chatterjee et al. 2009), there has been a surge of interest in the physics of quantum plasma. Quantum plasma is of considerable interest for its relevance in describing quantum effects in ultra small electronic devices (Markowich et al. 1990), dense astrophysical plasma system (Jung 2001), and laser-produced plasmas (Marklund and Shukla 2006; Glenzer et al. 2007). Other applications are in ultra cold plasma (Killian 2006), biophotonics (Barnes et al. 2003), spintronics (Wolf et al. 2001), and plasmonics (Atwater 2007). On the other hand, superdense quantum plasmas are present in the interior of massive white dwarfs, interior of Jupitors, magnetars (Beskin et al. 1993; Harding and Lai 2006; Fortney et al. 2009), etc. Ion-acoustic waves in quantum plasma have been studied by a number of authors (Haas et al. 2003; Misra and Bhowmik 2007; Moslem et al. 2007; Roy et al. 2008; Masood et al. 2009; Roy and Chatterjee 2011). In quantum plasma the electronic equilibrium is described by the Fermi Dirac distribution rather than the Maxwellian–Boltzmann distribution as is done in the classical plasma. Now it is well known that in dense plasmas, degenerate electrons follow the Fermi–Dirac pressure law, and the quantum force is connected with the Bohm de Broglie potential, as a product of which the waves disperse at nanoscales (Gardner and Ringhofer 1996). Not only that, the effects of the electron spin appear itself in terms of a magnetic dipole force, as well as spin precession, which can be obtained by transforming the Pauli equation to fluid-like variables (Oraevsky and Semikoz 2003; Brodin and Marklund 2007; Marklund and Brodin 2007). Hence, the dynamics of electrons in Fermi degenerate plasmas will be intervened not only by the Lorentz force, but also by the effects of quantum statistical pressure, the Bohm force, as well as the intrinsic spin of electrons.

Dynamics of the spin-1/2 quantum plasma was introduced by Marklund and Brodin (2007) in the nonrelativistic framework. Brodin and Marklund (2007) showed that the spin properties of electrons and positrons may lead to interesting collective effects in quantum magneto plasma *by following the Pauli equation*. Marklund et al. (2007) showed the existence of magneto solitons in a fermionic quantum plasma. In their work they found that if one neglects the magnetic diffusivity, the magnetic field satisfies an equation identical to the equation of continuity and hence one can take the magnetic field linearly proportional to the density of plasma fluid by applying the simplification of the governing equations. Later Misra and Ghosh (2008) obtained the spin magnetosonic shock like waves in quantum plasma, where they took account of magnetic diffusivity, using the reductive perturbation technique. Very recently, linear and nonlinear compressional magnetosonic waves in magnetized degenerate spin-1/2

Fermi plasmas are also investigated (Mushtaq and Vladimirov 2011). Relativistic corrections to the Pauli Hamiltonian in the context of the scalar kinetic theory for spin-1/2 quantum plasmas were also established (Asenjo et al. 2012).

A parallel development in solitonic studies is the head-on collision effects. It is known that one of the interesting properties of solitons is their asymptotic preservation of form when they undergo a head-on collision (i.e. $\theta = \pi$, θ being the angle between two propagation directions of two solitons). This fact was first observed by Zabusky and Kruskal (1995). Recently, extensive investigations have been made by several authors (Han et al. 2008; El-Shamy et al. 2009, 2010; Akbari-Moghanjoughi 2010; Chatterjee et al. 2010; Chatterjee and Ghosh 2011) to study the head-on collision between two ion-acoustic solitary waves (IASWs) with the help of the extended Poincare–Lighthill–Kuo (PLK) method in different classical plasma models, e.g, head-on collision of IASWs in a weakly relativistic electron–positron–ion (e–p–i) plasma by Han et al. (2008) and the head-on collision of IASWs in e–p–i plasma with superthermal electrons and positrons by Chatterjee and Ghosh (2011). In all these investigations the researchers observed that bidirectional solitary waves are propagated and hence head-on collision occurs. They also found the phase shifts and the trajectories of the two solitary waves after collision, which are the characteristics of the collision. It is noteworthy that a few investigations had already been made about the head-on collision phenomena in quantum plasma (El-Labany et al. 2010; Chatterjee et al. 2011; Ning et al. 2011; Xu et al. 2011). For the first time El-Labany et al. (2010) investigated the head-on collision between two quantum IASWs in a dense e–p–i plasma to discuss the effects of both quantum diffraction corrections and the Fermi temperature ratio of positrons to electrons on the phase shifts. The head-on collision between two IASWs with arbitrary colliding angle in an unmagnetized ultracold quantum threecomponent e–p–i plasma had been investigated by Xu et al. (2011). The propagation and interaction of IASWs in quantum e–p–i plasma had been investigated by Ning et al. (2011). Head-on collision of dust-ion-acoustic soliton in quantum pair ion plasma has also been studied by Chartterjee et al. (2011). However, head-on collision of magnetosonic solitons in a fermionic quantum plasma, taking into account spin effects, has not been studied so far. On the above background, the present study has been undertaken on the head-on collision of magnetosonic solitons in the framework of the model proposed by Marklund et al. (2007). We expect that the present results described here could be useful in strongly magnetized astrophysical plasmas like pulsars, magnetars, etc.

The bauplan of the work is as follows: In Sec. 2 we present basic equations and derive the two-sided KdV equations and the phase shifts. Section 3 is kept for discussion and conclusion.

2. Basic equations and derivation of KdV equations and phase shifts

Let us consider the governing equations for the quantum plasma in which the electron spin-1/2 effects are included. The total mass density, the center-of-mass fluid flow velocity, and the current density are defined respectively as $\rho = (m_e\, n_e + m_i\, n_i)$, $V = (m_e\, n_e\, v_e + m_i\, n_i\, v_i)/\rho$, and $j = (-e\, n_e\, v_e + e\, n_i\, v_i)$. Here m_e, n_e, and v_e are respectively the mass, number density, and fluid velocity of electron, and m_i, n_i, and v_i are respectively the mass, number density, and fluid velocity of ion, and e is the magnitude of the electron charge. Assuming the quasi-neutrality condition ($n_e = n_i$), and taking the magnetic field along the z-axis so that $\mathbf{B} = B(x,t)\vec{z}$ and taking the velocity $\mathbf{V} = V(x,t)\vec{x}$ and the density as $\rho(x,t)$, we get the following system of normalized basic equations (Moslem et al. 2007; Roy et al. 2008) as

$$\frac{\partial b}{\partial t} + \frac{\partial (bv)}{\partial x} = 0, \tag{1}$$

$$\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial x} = -\frac{\partial b}{\partial x} - c_s^2\frac{\partial(\ln b)}{\partial x} + \frac{2\omega_{pe}^2}{\omega_c\,|\omega_{ce}|}\frac{\partial}{\partial x}\left(\frac{1}{\sqrt{b}}\frac{\partial^2\sqrt{b}}{\partial x^2}\right) + v_B^2\frac{\partial}{\partial x}[\ln(\cosh(z_e b)) + z_e b(\tanh(z_e b))], \tag{2}$$

where $\rho = \rho_0 b$ with $b = B/B_0$, $c_s = C_s/C_A$, $v = V/C_A$, $C_A = [\frac{B_0^2}{\mu_0\rho_0}]^{\frac{1}{2}}$ is the Alfven speed, $C_s = [\frac{K_B(T_e+T_i)}{m_i}]^{1/2}$ is the sound speed, $\omega_{pj} = [\frac{n_{j0}e^2}{\epsilon_0 m_i}]^{1/2}$, $j = e, i$, are respectively the plasma frequencies for electron and ion, $\omega_c = \frac{2m_e c^2}{h}$ is the compton frequency, $z_e = \frac{\mu_B B_0}{K_B T_e}$ is the temperature-normalized Zeeman energy, $v_B^2 = \frac{K_B T_e}{m_i C_A^2} = \frac{\mu_B B_0}{z_e m_i C_A^2}$, where μ_0 is the permeability of the vacuum, B_0 is the magnetic field strength, ρ_0 is the total mass density of the charged plasma particles, T_i and T_e are ion and electron temperatures, K_B is the Boltzmann constant, and μ_B is the magnitude of Bohr magneton. Moreover, it is to be noted that the normalized variables are taken as $t \to \omega_{ci}t$, $x \to \frac{\omega_{ci}x}{C_A}$. In deriving (1) and (2) the magnetic resistivity is neglected.

Now we assume that the two solitons α and β are asymptotically far apart in the initial state and travel toward each other. After some time they interact, collide, and then depart. We also assume that the solitons have small amplitudes of the order ϵ (where ϵ is the small parameter characterizing the strength of nonlinearity) and the interaction between the two solitons is weak. Hence, we expect that the collision will be quasi-elastic, and hence it will only cause shifts of post-collision trajectories (phase shifts). In order to analyze the effects of collision, we employ an extended PLK method. According to this method, the dependent variables are expanded as

$$b = 1 + \epsilon b_1 + \epsilon^{3/2} b_2 + \epsilon^2 b_3 + \cdots, \tag{3}$$

$$v = \epsilon v_1 + \epsilon^{3/2} v_2 + \epsilon^2 v_3 + \cdots. \tag{4}$$

The independent variables are given by

$$\xi = \epsilon^{1/2}(x - v_p t) + \epsilon^{3/2} P_0(\eta, \tau) + \epsilon^{5/2} P_1(\xi, \eta, \tau) + \cdots, \tag{5}$$

$$\eta = \epsilon^{1/2}(x + v_p t) + \epsilon^{3/2} Q_0(\xi, \tau) + \epsilon^{5/2} Q_1(\xi, \eta, \tau) + \cdots, \tag{6}$$

$$\tau = \varepsilon^3 t, \tag{7}$$

where ξ and η denote the trajectories of two solitons travelling toward each other and v_p is the unknown phase speed normalised by C_A. The variables of $P_0(\eta, \tau)$ and $Q_0(\xi, \tau)$ are also to be determined. These types of strained co-ordinates were used in perturbation methods in fluid mechanics (Van-Dype 1975).

Substituting (3)–(7) into (1)–(2) and equating the quantities with equal power of ϵ, we obtain a set of coupled equations in different orders of ϵ. To the leading order, we have

$$v_p \left(-\frac{\partial}{\partial \xi} + \frac{\partial}{\partial \eta} \right) b_1 + \left(\frac{\partial}{\partial \xi} + \frac{\partial}{\partial \eta} \right) v_1 = 0, \tag{8}$$

$$v_p \left[-\frac{\partial}{\partial \xi} + \frac{\partial}{\partial \eta} \right] v_1 + [1 + c_s^2 - z_e v_B^2 (2 \tanh z_e + z_e sech^2 z_e)] \times \left[\frac{\partial}{\partial \xi} + \frac{\partial}{\partial \eta} \right] b_1 = 0. \tag{9}$$

Solving the above two equations we get

$$b_1 = b_{11}(\xi, \tau) + b_{12}(\eta, \tau), \tag{10}$$

$$v_1 = v_p [b_{11}(\xi, \tau) - b_{12}(\eta, \tau)], \tag{11}$$

and with the solvability condition (i.e., the condition to obtain a uniquely defined v_1 from (11) when b_1 is given by (10)), the phase velocity is also obtained as

$$v_p = \sqrt{1 + c_s^2 - z_e v_B^2 (2 \tanh z_e + z_e sech^2 z_e)}. \tag{12}$$

The unknown functions b_{11} and b_{12} will be determined from the next orders. Relations (10) and (11) imply that, at the leading order, we have two waves, one of which is $b_{11}(\xi, \tau)$ travelling in the forward direction, and the other is $b_{12}(\eta, \tau)$ travelling in the backward direction. At the next order, we have two similar equations as in the leading order. Hence, the solutions are given by

$$b_2 = b_{21}(\xi, \tau) + b_{22}(\eta, \tau), \tag{13}$$

$$v_2 = v_p [b_{21}(\xi, \tau) - b_{22}(\eta, \tau)]. \tag{14}$$

At the next higher order, we obtain the following set of equations

$$-2 \frac{\partial^2 v_3}{\partial \xi \partial \eta} = \frac{\partial}{\partial \xi} \left(\frac{\partial b_{11}}{\partial \tau} + A_1 b_{11} \frac{\partial b_{11}}{\partial \xi} + B_1 \frac{\partial^3 b_{11}}{\partial \xi^3} \right) + \frac{\partial}{\partial \eta} \left(\frac{\partial b_{12}}{\partial \tau} - A_1 b_{12} \frac{\partial b_{12}}{\partial \eta} - B_1 \frac{\partial^3 b_{12}}{\partial \eta^3} \right) + \left(C_1 \frac{\partial P_0}{\partial \eta} - D_1 b_{12} \right) \frac{\partial^2 b_{11}}{\partial \xi^2} - \left(C_1 \frac{\partial Q_0}{\partial \xi} - D_1 b_{11} \right) \frac{\partial^2 b_{12}}{\partial \eta^2}. \tag{15}$$

Integrating the above equation with respect to the variables ξ and η yields

$$-2 v_3 = \int \left(\frac{\partial b_{11}}{\partial \tau} + A_1 b_{11} \frac{\partial b_{11}}{\partial \xi} + B_1 \frac{\partial^3 b_{11}}{\partial \xi^3} \right) d\eta + \int \left(\frac{\partial b_{12}}{\partial \tau} - A_1 b_{12} \frac{\partial b_{12}}{\partial \eta} - B_1 \frac{\partial^3 b_{12}}{\partial \eta^3} \right) d\xi + \int \int \left(C_1 \frac{\partial P_0}{\partial \eta} - D_1 b_{12} \right) \frac{\partial^2 b_{11}}{\partial \xi^2} d\xi d\eta - \int \int \left(C_1 \frac{\partial Q_0}{\partial \xi} - D_1 b_{11} \right) \frac{\partial^2 b_{12}}{\partial \eta^2} d\xi d\eta, \tag{16}$$

where

$$A_1 = \frac{3 v_p^2 - c_s^2 - z_e^2 v_B^2 sech^2 z_e (3 - 2 z_e \tanh z_e)}{2 v_p},$$

$$B_1 = -\frac{\omega_{pe}^2}{2 |\omega_{ce}| v_p \omega_c}, C_1 = 2 v_p,$$

$$D_1 = \frac{v_p^2 + c_s^2 + z_e^2 v_B^2 sech^2 z_e (3 - 2 z_e \tanh z_e)}{2 v_p}.$$

Now the question arises, how to extract the equations satisfied by b_{11}, b_{12}, P_0, and Q_0 from (16). The clue lies in the fact that we should not allow any secular terms. Note that the integrand of the first integral on the right-hand side of (16) depends only on ξ and τ and therefore if the integrand is not identically equal to zero, then the integral will be proportional to η, which will give rise to secular terms. Hence, we must have the integrand to be identically zero. The same argument holds for the second integral. Hence, we obtain the following equations for b_{11} and b_{12}:

$$\frac{\partial b_{11}}{\partial \tau} + A_1 b_{11} \frac{\partial b_{11}}{\partial \xi} + B_1 \frac{\partial^3 b_{11}}{\partial \xi^3} = 0, \tag{17}$$

$$\frac{\partial b_{12}}{\partial \tau} - A_1 b_{12} \frac{\partial b_{12}}{\partial \eta} - B_1 \frac{\partial^3 b_{12}}{\partial \eta^3} = 0. \tag{18}$$

However, the same argument will not hold for the third and the fourth integrals of (16). But while they may not be secular at this order, they will be secular in the higher order (Su and Mirie 1980; Jaffery and Kawahawa 1982). Hence, we must have

$$C_1 \frac{\partial P_0}{\partial \eta} = D_1 b_{12}, \tag{19}$$

$$C_1 \frac{\partial Q_0}{\partial \xi} = D_1 b_{11}. \tag{20}$$

Equations (17) and (18) are the two side-travelling wave KdV equations in the reference frames of ξ and η

respectively. Their special solutions are

$$b_{11} = b_A sech^2\left[\left(\frac{A_1 b_A}{12B_1}\right)^{1/2}\left(\xi - \frac{1}{3}A_1 b_A \tau\right)\right], \quad (21)$$

$$b_{12} = b_B sech^2\left[\left(\frac{A_1 b_B}{12B_1}\right)^{1/2}\left(\eta + \frac{1}{3}A_1 b_B \tau\right)\right], \quad (22)$$

where b_A and b_B are the amplitudes of the two solitons in their initial positions. The leading phase changes due to the collision can be calculated from (19) and (20) and are given by

$$P_0(\eta,\tau) = \frac{D_1}{C_1}\left(\frac{12B_1 b_B}{A_1}\right)^{1/2}\left[\tanh\left(\frac{A_1 b_B}{12B_1}\right)^{1/2} \times \left(\eta + \frac{1}{3}A_1 b_B \tau\right) + 1\right], \quad (23)$$

$$Q_0(\xi,\tau) = \frac{D_1}{C_1}\left(\frac{12B_1 b_A}{A_1}\right)^{1/2}\left[\tanh\left(\frac{A_1 b_A}{12B_1}\right)^{1/2} \times \left(\xi - \frac{1}{3}A_1 b_A \tau\right) - 1\right]. \quad (24)$$

Therefore, up to O(ϵ^2), the trajectories of the two solitary waves for head-on interactions are

$$\xi = \epsilon(x - v_p t) + \epsilon^2\frac{D_1}{C_1}\left(\frac{12B_1 b_B}{A_1}\right)^{1/2}\left[\tanh\left(\frac{A_1 b_B}{12B_1}\right)^{1/2} \times \left(\eta + \frac{1}{3}A_1 b_B \tau\right) + 1\right] + \cdots, \quad (25)$$

$$\eta = \epsilon(x + v_p t) + \epsilon^2\frac{D_1}{C_1}\left(\frac{12B_1 b_A}{A_1}\right)^{1/2}\left[\tanh\left(\frac{A_1 b_A}{12B_1}\right)^{1/2} \times \left(\xi - \frac{1}{3}A_1 b_A \tau\right) - 1\right] + \cdots. \quad (26)$$

To obtain the phase shifts after a head-on collision of the two solitons, we assume that solitons α and β are asymptotically far from each other at the initial time ($t = -\infty$), i.e., soliton α is at $\xi = 0$, $\eta = -\infty$ and soliton β is at $\eta = 0$, $\xi = +\infty$ respectively. After the collision ($t = +\infty$), soliton α is far to the right of soliton β, i.e., soliton α is at $\xi = 0$, $\eta = +\infty$ and soliton β is at $\eta = 0$, $\xi = -\infty$. Using (25) and (26), we obtain the corresponding phase shifts $\triangle P_0$ and $\triangle Q_0$ as follows:

$$\triangle P_0 = -2\epsilon^2\frac{D_1}{C_1}\left(\frac{12B_1 b_B}{A_1}\right)^{1/2}, \quad (27)$$

$$\triangle Q_0 = 2\epsilon^2\frac{D_1}{C_1}\left(\frac{12B_1 b_A}{A_1}\right)^{1/2}. \quad (28)$$

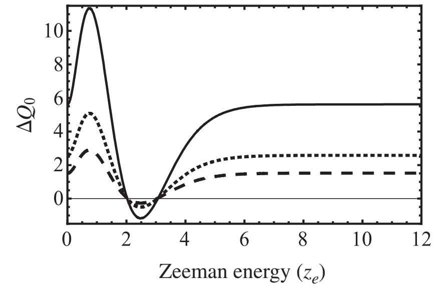

Figure 1. Graphs of the variation of phase shift $\triangle Q_0$ against the Zeeman energy z_e for phase velocities $v_p = 0.01$ (solid line), 0.015 (dotted line), 0.02 (dashed line) when $\epsilon = 0.1$, $b_A = 0.1$, $\frac{\omega_{pe}^2}{\omega_c\,|\omega_{ce}|} = 1$, $v_B = 0.2$, and $c_s = 0.1$.

3. Results and discussion

In this paper, the head-on collision phenomenon of the two magneto-acoustic solitons in a fermionic quantum plasma with the spin effect is studied using the extended version of the PLK method. We know that the KdV-type soliton-like solutions are formed due to the balance between nonlinearity and dispersion in a nonlinear dispersive media. So the condition for the existence of soliton-like solutions are $A_1 \neq 0$ and $B_1 \neq 0$. Since the soliton α is travelling from the left and β is travelling from the right, (27) and (28) imply that each soliton has positive or negative phase shift depending upon the sign of coefficient D. Moreover, it is clear from (27) and (28) that both $\frac{B_1 b_B}{A_1}$ and $\frac{B_1 b_A}{A_1}$ must be positive if both $\triangle P_0$ and $\triangle Q_0$ are real. It follows logically that b_B and b_A must have the same sign, hence both solitons will be either hump types or dip types. Thus, the positive or negative phase shift does not depend on the type of mode (i.e, ion-acoustic, dust-ion-acoustic, dust-acoustic, magneto-acoustic, and electrostatic waves). Several authors (Han et al. 2008; El-Labany et al. 2010) have argued that for IASWs, due to collision, each soliton has a negative phase shift in its travelling direction. Han et al. (2008) demonstrated that due to collision, each IASW has a positive phase shift in its travelling direction. El-Labany et al. (2010) claimed that due to collision, each dust-acoustic solitary wave has a negative phase shift in its travelling direction. However, their own results do not support this hypothesis.

To draw the figures, we consider $\epsilon = 0.1$, $b_A = 0.1$, $\frac{\omega_{pe}^2}{\omega_c\,|\omega_{ce}|} = 1$, and $v_B = 0.2$ in all the cases. Figure 1 represents the variations of the phase shift $\triangle Q_0$ with the Zeeman energy (z_e) for different values of the speed of the waves $v_p = 0.01$ (solid line), 0.015 (dotted line), and 0.02 (dashed line) when $c_s = 0.1$. Here it is interesting to note that the phase shift $\triangle Q_0$ is (i) positive and increasing when $0 < z_e < 0.8$ and $3 < z_e < 7$, (ii) positive and decreasing when $0.8 < z_e < 2$, (iii) negative and decreasing when $2 < z_e < 2.5$, (iv) negative and increasing for $2.5 < z_e < 3$, and (v) positive and constant for $z_e > 7$. Thus, the phase shift is positive as well as negative for different domains of the Zeeman energy, and for a particular value of z_e the magnitude of the

https://doi.org/10.1017/S0022377812000980 Published online by Cambridge University Press

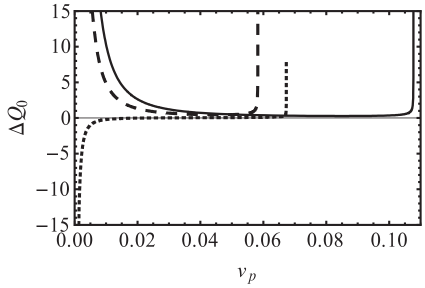

Figure 2. Graphs of the variation of phase shift $\triangle Q_0$ against phase velocity v_p for the Zeeman energy z_e =1 (solid line), 3 (dotted line), and 5 (dashed line) when all other physical parameters are as in Fig. 1.

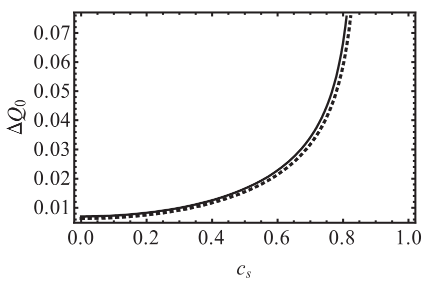

Figure 3. Graphs of the variation of phase shift $\triangle Q_0$ against the ratio of the sound speed to Alfven speed c_s for the Zeeman energy z_e =0.6 (solid line) and 6.5 (dotted line) when $b_A = -0.1$, $v_p = 0.5$, and all other physical parameters are as in Fig. 1.

phase shift increases as the speed of the wave decreases. Qualitatively one can say that the phase shifts become flat when the Zeeman energy is large. Since we consider the case where the Zeeman energy, $z_e \geqslant 1$, so the spin contribution to the soliton dynamics is enhanced and the Zeeman energy plays a crucial role on the phase shifts.

Figure 2 shows the variation of the phase shift $\triangle Q_0$ with the speed of the wave for different values of the Zeeman energy: $z_e = 1$ (solid line), $z_e = 3$ (dotted line), and $z_e = 5$ (dashed line) when $c_s = 0.1$. Figure 2 implies that the phase shift decreases rapidly by taking the positive values, then slowly tends to zero and ultimately it blows up irrespective of the speed of the wave for Zeeman energy, $z_e = 1$ and 5, but for $z_e = 3$ the phase shift increases taking negative values, then slowly becomes zero and ultimately it blows up irrespective of the speed of the wave.

To draw Fig. 3 we consider $b_A = -0.1$ and all other physical parameters as in Fig. 1. Figure 3 shows the variations of the phase shift $\triangle Q_0$ with the ratio of sound speed to Alfven speed c_s for the Zeeman energy, $z_e = 0.6$ (solid line) and $z_e = 6.5$ (dotted line) when $v_p = 0.5$. Figure 3 indicates that the phase shift strictly increases for increasing c_s. It is evident from (12) that there exists a critical value of z_e beyond which v_p will not have real values.

In Fig. 4 we plot the phase shift versus the parameter ρ_0 in the domain $0 < \rho_0 < 3\times10^{17}$ for different strengths of the magnetic field $B_0 = 10^5$T (solid line), $B_0 = 5 \times 10^5$T (dotted line), and $B_0 = 10^7$T (dashed line) when $T_e = 10^6$, $T_i = 10^4$, $m_i = 10^{-10}$ kg, and all other physical parameters are as in Fig. 3. Figure 4 indicates that for $B_0 = 10^5$, initially the phase shift increases but then it decreases, but for $B_0 = 5 \times 10^5$ and 10^7 the phase shift increases slowly. It is clear from (27) and (28) that $\triangle P_0$ will always be of the opposite sign of $\triangle Q_0$. However, magnitude-wise they will have the same behavior.

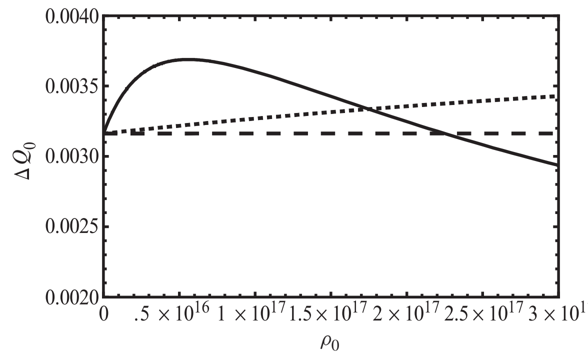

Figure 4. Graphs of the variation of phase shift $\triangle Q_0$ against the total mass density ρ_0 for the strength of the magnetic field $B_0 = 10^5$T (solid line), 5×10^5T (dotted line), and 10^7T (dashed line) when $T_e = 10^6$, $T_i = 10^4$, $m_i = 10^{-10}$kg, and all other physical parameters are as in Fig. 3.

The results obtained here indicate that the effects of the parameters z_e, B_0, c_s, v_p, and ρ_0 play important roles not only on the formation of solitary waves but also on soliton collision. To summarize, the phase shifts, the trajectories, and also the head-on collision of two magnetosonic solitons in a fermionic quantum plasma (taking the spin effects into account by using the extended PLK method) are discussed in this paper. It is found that the nature and the magnitude of the phase shift depend crucially on the chosen parameters.

Acknowledgements

The authors are grateful to the referee for useful and constructive comments and suggestions toward improving this manuscript.

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