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ORIGIN OF FOLIATION IN GLACIERS

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Abstract. Laboratory studies suggest that neither bubbles nor dirt particles migrate rapidly enough in glacier ice to be responsible for the alternating layers of bubbly and clear ice or dirty and clean ice which constitute foliation. We therefore suggest that these variations in bubble or dirt content are inherited from primary inhomogeneities such as may occur in sedimentary stratification in the accumulation region, in crevasse fillings, or during debris entrainment at the base of the glacier: the appearance of these inhomogeneities is later modified by strain during flow to produce foliation. We consider six types of inhomogeneity, or components of foliation, and show that, at the very large total strains expected in glaciers, all are eventually flattened, stretched out, and rotated to form a layered structure roughly perpendicular to the direction of maximum total shortening. Most characteristics of observed foliation can be explained by this hypothesis. For example, in the marginal zones of polar ice sheets the rapid decrease in dip of foliation with depth and with distance up-glacier from the margin can be explained by a model in which the foliation is assumed to be nearly parallel to the base of the glacier some distance from the margin, and is deformed passively with the ice thereafter. However, some observations of cross-cutting foliations may require localized inhomogeneous shear parallel to the "new" foliation.

Résumé. *Origine de la foliation des glaciers.* Des études de laboratoire suggèrent que ni les bulles ni les particules de poussière ne migrent assez rapidement dans la glace de glacier pour être responsables de l'alternance de couches de glace bulleuse et claire ou de glace propre et sale qui constituent la foliation. Nous suggérons donc que ces variations dans la teneur en bulles ou en poussière sont héritées des irrégularités primitives comme il peut s'en produire dans la stratification de la zone d'accumulation par les remplissages de crevasses ou pendant l'entraînement des sédiments à la base des glaciers: l'apparence de ces irrégularités est plus tard modifiée par la déformation durant l'écoulement pour produire la foliation. Nous considérons six types d'irrégularités ou de composantes de la foliation et montrons que, au long de très fortes, déformations totales que l'on doit s'attendre à trouver dans les glaciers, elles sont toutes susceptibles d'être laminées, étirées et retournées jusqu'à former une structure stratifiée grossièrement perpendiculaire à la direction du retrécissement total maximum. La plupart des caractéristiques des foliations observées peuvent être expliquées par ces hypothèses. Par exemple, dans les zones marginales des calottes glaciaires polaires la rapide diminution de l'épaisseur de la foliation avec la profondeur et la distance du sommet du glacier à la bordure peut être expliquée par un modèle dans lequel on fait l'hypothèse que la foliation est presque parallèle à la base du glacier à quelque distance du bord, et se trouve déformée passivement avec la glace par la suite. Cependant, quelques observations de foliations recoupant les autres en croix peuvent plaider en faveur de cisaillements localisés irréguliers parallèles à la "nouvelle" foliation.

Zusammenfassung. *Der Ursprung der Bänderung in Gletschern.* Aus Laborversuchen geht hervor, dass weder Blasen noch Schmutzpartikel sich schnell genug im Gletschereis bewegen, um die Wechselschichtung von blasenreichem und klarem Eis bzw. verschmutztem und reinem Eis bewirken zu können, aus der die Bänderung besteht. Wir nehmen deshalb an, dass diese Schwankungen im Blasen- oder Schmutzgehalt von primären Inhomogenitäten herrühren, wie sie in der sedimentären Lagerung im Akkumulationsgebiet, in Spaltenfüllungen oder bei der Schuttaufnahme am Grunde des Gletschers vorkommen: das Erscheinungsbild dieser Inhomogenitäten wird später durch die Fliessspannungen so verändert, dass die Bänderung entsteht. Wir betrachten 6 Arten von Inhomogenität als Komponenten der Bänderung und zeigen, dass alle unter den sehr hohen Gesamtspannungen, die in Gletschern auftreten, im Laufe der Zeit ausgewalzt, gestreckt und gedreht werden, so dass sie eine geschichtete Struktur annähernd senkrecht zur Richtung der maximalen Gesamtkürzung bilden. Die meisten Eigenschaften beobachteter Bänderungen können mit dieser Hypothese erklärt werden. Zum Beispiel lässt sich die rasche Abnahme des Einfallswinkels der Bänderung mit der Tiefe und mit dem Randabstand gletscheraufwärts am Rande polarer Eisdecken durch ein Modell erklären, in dem die Bänderung annähernd parallel zum Untergrund des Gletschers in einigem Abstand vom Rand angenommen wird, aber mit dem Eis darnach eine passive Deformation erleidet. Einige Beobachtungen von gekreuzten Bänderungen allerdings können es erforderlich machen, eine lokale, inhomogene Scherung parallel zu der "neuen" Bänderung anzunehmen.

Introduction

Most glacier ice displays a planar or layered structure, developed during deformation and defined by variations in bubble or dirt content (Fig. 1) (Allen and others, 1960; Ragan, 1969, among others). Crystal size, texture and orientation may also vary from layer to layer (Kamb, 1959; Hooke, 1973[b], among others) but such changes are usually less obvious.

The origin of this foliation, as it is usually called, is not well understood, and there is even some disagreement as to just what structure the term foliation describes. Ragan (1967, 1969)

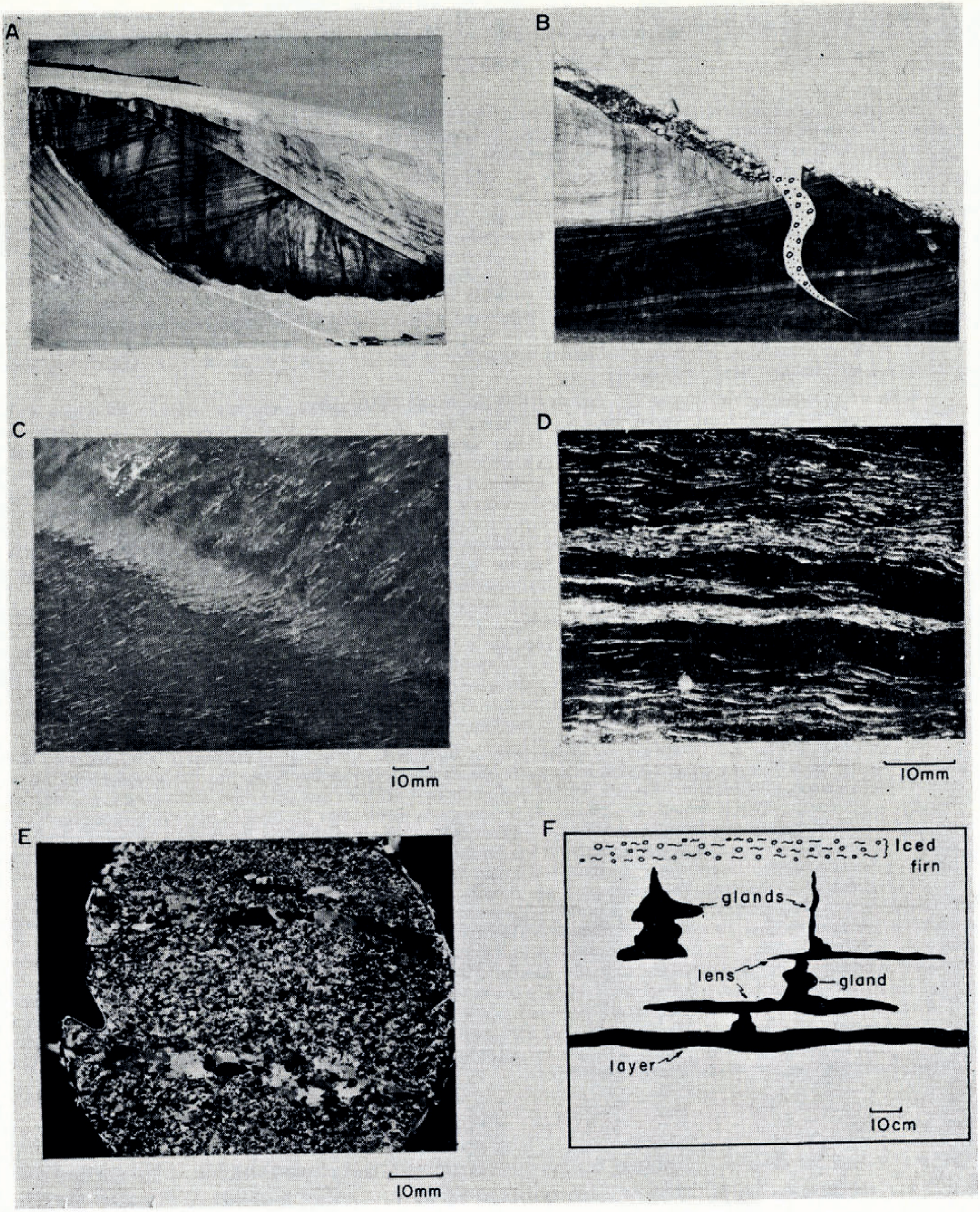

Fig. 1.

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argues that the essential structural element is a planar one, and he considers that the common use of the word in glaciology to include layered elements as well is unfortunate, as planar and layered elements may form in different ways.

However, the terms "layered" and "planar", as applied to ice in particular, are somewhat ambiguous. A layered structure is one made up of tabular bodies of material, distinguished by differences in internal composition or texture or by the nature of the interfaces. Examples in rocks are sedimentary layering and the bands of varying lithology in gneiss. Planar structural elements are surfaces of discontinuity which are characteristically penetrative such as cleavage or schistosity in deformed rocks. It is clear that the boundaries of layers define a planar structure that is usually penetrative on some scale (Turner and Weiss, 1963, p. 23), and it is such a structure that is the most prominent component of foliation in glacial ice (Ragan, 1969; Allen and others, 1960). It thus seems academic to argue whether it is the planar or the layered aspect of foliation in ice which is the more fundamental. It should be added, however, that there are commonly other planar structural elements which are parallel to the structure defined by the boundaries of layers but which have no layered affinities. Examples of such elements are those due to the preferred orientation of inequant grains or bubbles.

Thus we feel that the traditional glaciological use of the term foliation, embracing both planar and layered structures, is perhaps appropriate to describe the structure commonly observed in ice. In the discussion which follows, however, we will treat each identifiable component of foliation separately, and in this way try to avoid the problems of nomenclature evident in the literature.

There are a number of characteristic features that must be accounted for in any satisfactory hypothesis for the origin of foliation:

(1) In valley glaciers, foliation may form longitudinally with steep to vertical dips throughout the length and breadth of the glacier (Meier, 1960; Gunn, 1964), but with the most intense development normally near the margins.

(2) Foliation may also form transversely, with up-glacier dips and an overall spatial arrangement resembling "nested spoons". There may be, in fact, several sets of nested-spoon axes corresponding to separate streams of ice (Untersteiner, 1955; Rutter, 1965), and in this case both longitudinal and transverse components of foliation occur, the zones of longitudinal foliation separating ice streams which each have their own transverse foliation.

(3) In glaciers with predominantly longitudinal foliation there is usually a short section near the snout where transverse attitudes appear (Meier, 1960).

(4) In general it seems that foliation is most strongly developed near the base and towards the margins of glaciers, and is usually parallel to the base or valley sides as these are approached. It may be folded in these locations (Hudleston, 1976).

Fig. 1.

A. *Foliation in an ice cliff at the margin of Barnes Ice Cap. Cliff is approximately parallel to flow lines. Foliation is defined by variations in bubble and dirt content. The foliation dipping up-glacier (left) in the main part of the cliff was derived from sedimentary stratification similar to that dipping down-glacier (right) near the top of the cliff. The stratification was overturned as it was overridden during an advance of the glacier (Hooke, 1973[a]). There is an unconformity between the stratification dipping down-glacier and that dipping up-glacier. An unconformity of similar origin but now overturned is preserved in the lower part of the cliff.*

B. *Foliation defined by variations in dirt content exposed in a cliff at the margin of Barnes Ice Cap. The filled crevasse was exposed in the cliff and has been cut out of the photograph to improve the contrast. Flow is to* left.

C. *Right-dipping foliation band, defined by increase in bubble concentration, is cross-cut by second foliation defined by bubble elongation. Sample is from complexly deformed marginal zone similar to that shown in A.*

D. *Foliation defined by variations in bubble content and by bubble elongation.*

E. *Foliation defined by variations in crystal size and dirt content. Fine-grained ice contains a few per cent dirt which has inhibited crystal growth. Note elongation of crystals in clean (coarse) folia.*

F. *Glands, lenses, and layers formed by percolation of melt water into firn. During deformation such general inhomogeneities may be deformed and flattened to form folia. Based on Benson (1959, figs 5 and 16).*

(5) Foliation is frequently observed to cross-cut primary sedimentary stratification (Meier, 1960; Ragan, 1969), but in some instances may be derived from this stratification by shear parallel to the bedding or transposition of the bedding (Hambrey, 1975, 1976). It is also common for a second foliation to cross-cut an earlier one (Allen and others, 1960; Ragan, 1969).

Migration of bubbles and dirt

Foliation could result from migration of either bubbles or dirt in the ice. Both bubbles and dirt migrate under temperature gradients, but the rates appear to be too low to account for more than minor modification of the boundaries between ice layers of varying characteristics (Shreve, 1967; Stehle, 1967; Römkens, unpublished). Furthermore, temperature gradients in ice are unidirectional, except very near the surface, and thus migration under a temperature gradient cannot explain the intercalation of layers of different types of ice.

There is a possibility of migration of mineral particles or bubbles in ice under a shear gradient, however, as it has been demonstrated that solid particles (Bhattacharji, 1967) and deformable spheres (Goldsmith and Mason, 1969) in a viscous matrix move to regions of low velocity gradient.

Dirt-bearing ice generally deforms less readily than clean ice because the dirt particles tend to inhibit movement of dislocations (Hooke and others, 1972), and also because crystal sizes tend to be smaller (generally <1 mm) in dirty folia than in intervening clean folia (Fig. 1E), and creep rate increases with increasing grain size for grain sizes above 1 mm (Baker, in press). Thus there is a possibility that higher shear strain-rates occur in clean ice than in dirty ice, and that dirt particles consequently migrate toward the dirty ice. Initially random variations in dirt concentration could thus become accentuated to produce a foliation. Inasmuch as dirty ice comprises only a small percentage of the foliated ice observed in glaciers, such migration of dirt particles is not quantitatively significant even if it occurs.

As to the possibility of bubble migration under a shear gradient, bubbly ice appears to be weaker than bubble-free ice and bubble-rich layers in glaciers are often found to deform at higher strain-rates than adjacent layers (Hooke, 1973[b]; Hudleston, 1977). Migration of bubbles would thus tend to be out of the bubble-rich layers, weakening the layered appearance rather than enhancing it.

We are thus led to the conclusion that most of the foliation observed in glacier ice probably results from deformation of pre-existing primary inhomogeneities of one type or another. In the remainder of this paper we identify a number of these inhomogeneities, or components, and consider, separately, how each of them behaves during flow of the ice.

Components of foliation and their behavior during flow

We distinguish between large- and small-scale components of foliation in this section. It is important to remember, however, that several components of both categories commonly occur together and are mutually sub-parallel, defining a single foliation (Ragan, 1969; Hashimoto and others, 1966).

Large-scale components

The three large-scale components of foliation which we consider are sedimentary stratification marked by variations in bubble content between layers and often by bands of fine dirt on old ablation surfaces (Fig. 1A) (Hambrey, 1975, 1976), general inhomogeneities such as ice glands or lenses resulting from percolation (Fig. 1F), and fractures which later become filled with snow or clear vein ice (Fig. 1B) (Allen and others, 1960; Hambrey, 1975). During flow

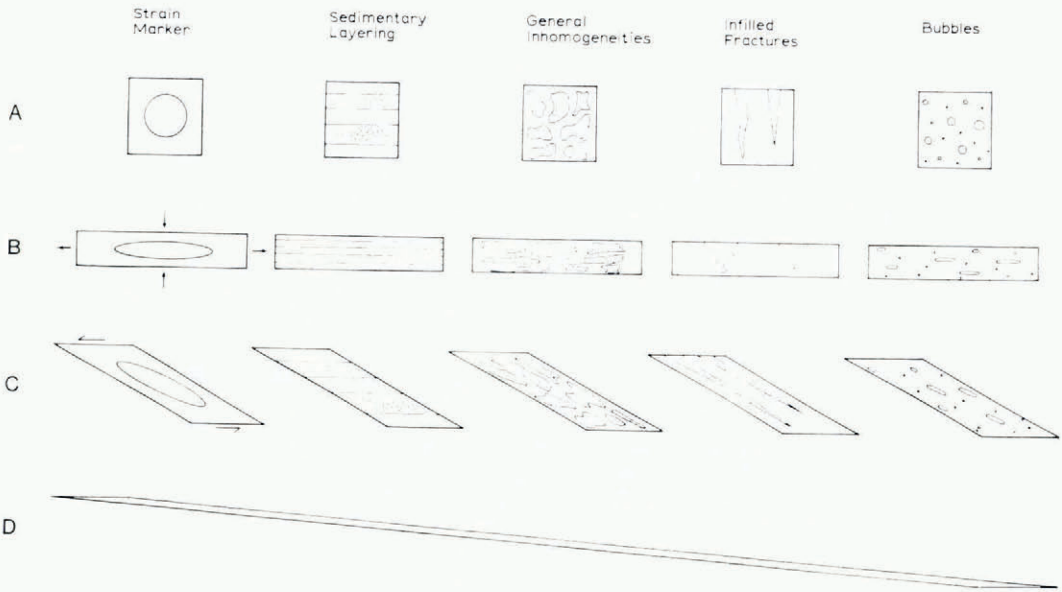

Fig. 2. Behavior of various components of foliation as a result of homogeneous strain.

A. Initial configuration of components. Square and circle in left-hand diagram do not represent specific features, but are for reference only.
B. Components after pure shear with strain-ellipse axial ratio of 6.5.
C. Components after simple shear with strain-ellipse axial ratio of 6.5.
D. Square in upper left diagram after simple shear of 12.2. Strain-ellipse axial ratio 150.

The appearance of the bubbles after straining is schematic, based on observations on natural and experimentally deformed ice, and theoretical considerations (see text). In nature the components will be affected by a complex strain history that may involve both pure and simple shear (Fig. 3A).

of the ice these components become deformed as illustrated schematically in Figure 2. In analyzing this deformation we consider only simple, essentially two-dimensional flow regimes such as are found in ice caps or ice sheets (Fig. 3), or along axial flow lines in valley glaciers. We will restrict ourselves to glaciers under steady-state flow conditions, as departures from a steady state can cause complications such as the formation of folds (Hudleston, 1976). The reader should refer to Figure 2 throughout this discussion.

It is not generally accepted that foliation may be derived from primary sedimentary stratification, although this clearly has occurred in some instances (Fig. 1A) (Hambrey, 1975, 1976). Under normal flow conditions stratification will be passively deformed so that it first tends to become normal to the direction of maximum total shortening (Ramsay, 1967, p. 334). Later, deeper in the glacier it will also tend towards parallelism with the flow lines (Fig. 3) because, under simple shear, which predominates near the base of a glacier, material lines become rotated into this orientation at very high shear strains (Fig. 2D) (Ramsay, 1967, p. 87). Small departures of basal flow from simple shear do not effect the general nature of this conclusion.

General inhomogeneities will also tend to become "flattened" during flow and will produce a planar structure reflecting the total deformation undergone by the ice. This planar structure will ultimately be perpendicular to the direction of maximum total shortening. If the initial inhomogeneities are roughly equant, a necessary condition for formation of planar structures by this process is that the intermediate principal longitudinal strain be either zero or extensional. In the case of an ice cap or ice sheet this will generally be true, as there is commonly extensional strain parallel to the margin.

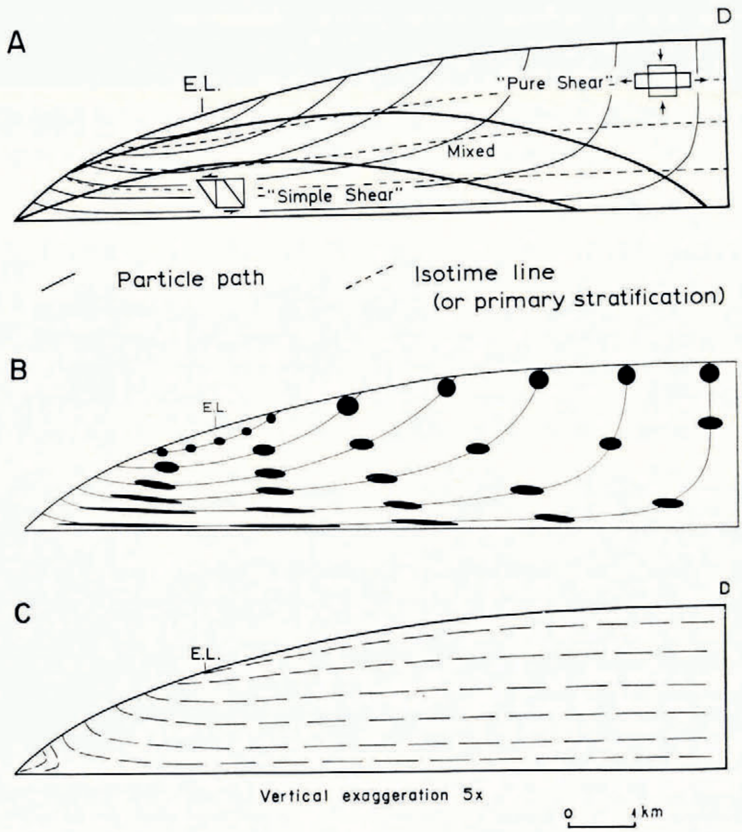

Fig. 3.

A. *Schematic flow pattern in an ice sheet. D = divide; E.L. = equilibrium line. The strain fields delineated are for small increments of deformation only, and transverse strains are ignored. The horizontal component of the "pure shear" field will vary from extensional above the equilibrium line to compressive below.*

B. *Schematic illustration of cumulative strain at selected points along particle paths shown in Figure 3A. Initial circles become deformed into ellipses during flow.*

C. *Schematic sketch of foliation in south dome of Barnes Ice Cap. Pattern based on observations at surface, in a tunnel, and in several bore holes.*

Snow- or ice-filled fractures will behave in a similar passive manner during deformation. However, the appearance of such fractures will depend on where in the glacier the fracturing occurs and on its orientation. For example, if fractures form in a vertical transverse orientation in the upper part of the accumulation zone, they will initially be flattened in the zone of "pure shear" near the glacier surface, but will later be rotated and thinned in the zone of "simple shear" near the base (Figs 2, 3). However such fractures forming near or below the equilibrium line would be simply rotated and thinned as they passed down-glacier. Allen and others (1960) have suggested that closed transverse crevasses formed during passage of ice through the ice fall on the Blue Glacier were deformed in this way, and that this is the origin of the nested-spoon foliation seen below the ice fall. Hambrey (1976) also identified structures of this type on Charles Rabots Bre, Norway.

From the above discussion it is apparent that under the conditions met near the base of an ice sheet and towards its margin (Figs 2 and 3), all of these components tend to form a composite structure of mutually parallel planar and layered elements that should, under steady-

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state conditions, be almost parallel to the flow lines and nearly perpendicular to the direction of maximum finite shortening. Where total deformation is not so great, some divergence between the preferred orientation of the components might be expected.

Small-scale components

There are three small-scale features of foliated ice which are also important to consider. These components are non-spherical bubbles which may show a preferred orientation that constitutes a planar fabric (Fig. 1D), elongate crystals which may also develop such a preferred orientation (Fig. 1E) (Kamb, 1959; Ragan, 1969), and variations in dirt concentration (Fig. 1B and E).

Bubbles in unstrained glacier ice usually have sub-spherical initial shapes (Gow, 1968; Hooke, 1973[a], p. 3940). During deformation these bubbles become deformed into shapes which reflect some cumulative aspect of the deformation (Kamb, 1972; Hudleston, 1977). As a first approximation they might be considered as passive markers of deformation, in which case their appearance would reflect the shape and orientation of the strain ellipsoid. In a more rigorous analysis, the bubbles may be considered as initially-spherical inclusions of zero viscosity in a non-linearly viscous host. As such, under pure shear they reflect the orientation of the strain ellipsoid, but over-estimate its shape; under simple shear they reflect neither the shape nor the orientation of the strain ellipsoid, but will over-estimate the former and be inclined at greater angles to the shear direction than the latter (Gay, 1968). At large strains, however, when the bubbles are highly elongate, their rate of further elongation and rotation will become that of the surrounding ice and thus that of a passive marker (Smith, 1975). Under simple shear of large magnitude the bubbles will tend towards parallelism with the shear plane.

Bubble behavior is complicated by the fact that surface-tension forces tend to restore the shape to spherical (Hudleston, 1977). This is more effective for the smallest bubbles, which seem to maintain a spherical form in even the most strongly deformed ice. The white Pleistocene ice at the base of the Barnes Ice Cap is an example (Hooke, 1976). Another complicating factor is the observation that bubbles are usually more cylindrical than ellipsoidal when deformed, probably again due to effects of surface tension. The rate of restoration of elongate bubbles to a spherical shape is likely to be related more to strain-rate than to total strain. Thus bubbles probably do not reflect the total deformational history of the ice, but record instead, some limited recent part of that deformation (Hudleston, 1977). It is not uncommon, for example, to see bubbles in well-foliated ice that are near a boundary between bubbly and clear ice but that are not parallel to that boundary (Fig. 1C). This would be expected in a simple shear deformation in which the banding reflects the total strain, and has become nearly parallel to the shear plane, whereas the bubbles, due to recovery, only reflect some small increment of simple shear.

It is unclear just how the shapes of individual crystals should be related to deformation. In most polar glacial ice, recovery processes and recrystallization maintain nearly equi-dimensional crystal shapes, even where deformation rates are very high. Where crystals do show elongate shapes with preferred orientations, they appear to be extended parallel to the direction of maximum longitudinal strain-rate $\dot{\epsilon}_1$ or to reflect some slightly greater increment of strain (Hudleston, 1977; Kamb, 1972).

Dirt can become incorporated into the ice either at the upper surface through wind action or rock falls, or at the lower or lateral surfaces through erosion. The actual mechanics of the erosion process have been disputed recently. Goldthwait (1951, p. 570–71) refers to dirt-bearing folia as "shear planes", and Bishop (1957, p. 17) suggested that these ". . . shears project down to the glacier floor where they pick up . . . 'old' ground moraine". This concept has unfortunately been widely accepted among glacial geologists (e.g. Souchez, 1967;

Hambrey, 1976, p. 52).* None of the authors who have suggested this mechanism seem to have actually observed it in action, however, and Weertman (1961) and Hooke (1968) have presented cogent reasons for doubting that it is mechanically sound. An alternative mechanism of entrainment, involving refreezing of melt water at the bed, has been advanced by Weertman (1961). Kamb and LaChapelle (1964) and Boulton (1970) have observed dirt in basal ice which formed during regelation around protrusions in the bed. While this source of melt water is slightly different from that originally envisaged by Weertman, the process of entrainment is the same.

Dirt incorporated into the ice at the upper surface will normally form layers of dirty ice which parallel sedimentary stratification. Deformation of such dirty layers is thus similar to deformation of the stratification. Debris incorporated along the margins or at the base of a glacier will also probably have a layered appearance (Boulton, 1970) with lenses of dirty ice alternating with layers of clean ice (Fig. 1B and E). The concentration of dirt in such lenses is normally only a few per cent by weight (Hooke, 1970, p. 306). Because foliation near the sides and bottom of a glacier is usually nearly parallel to these boundaries, lenses of dirty ice originating at the boundaries will initially be sub-parallel to the foliation. These lenses subsequently deform in a manner similar to the deformation of other inhomogeneities in the ice (Fig. 2).

Cross-cutting relations and inhomogeneous deformation

Cross-cutting of planar elements is common in glaciers. Crevasse fillings, for example, initially cut across sedimentary stratification or foliation derived therefrom (Fig. 1B). After simple shear of large magnitude, however, these two components will be nearly parallel to one another (Fig. 2). Despite large strains, the cross-cutting character must persist if bubble or dirt migration has not occurred. This is not to say, of course, that the cross-cutting relation will be readily visible after such large strains.

The most detailed discussion of such relations that we have seen is that by Ragan (1969) who measured two or more non-parallel foliations at a number of places during a study of a small area below an ice fall on the Gulkana Glacier, Alaska. Ragan (1969, p. 654) lists several elements of the foliation he observed. Alternating layers of bubbly and clear ice were the most common element, but fracture cleavage, flattened bubbles, sub-parallel crystal boundaries, and Tyndall flowers were also seen. Ragan does not describe in detail the types of cross-cutting relations observed, but his figures 4 and 5 illustrate a foliation defined by alternating layers of bubbly and clear ice cross-cut by a foliation defined by fractures or crevasse fillings. Such relations can, of course, form in a manner similar to that discussed above.

Another way in which cross-cutting foliations develop is by local inhomogeneous deformation in the form of planar zones of high strain separated by zones of less deformed ice. The most likely form of this inhomogeneous strain would be a pattern of discrete shear zones such as Ragan (1969, fig. 6), describes and we have observed in the Barnes Ice Cap (Hudleston, 1977). As Ragan notes, these may develop in response to some form of creep instability (Orowan, 1965). The fabric and texture of the ice could change within such shear zones and thus produce a foliation parallel to a direction of high shear strain-rate, quite independent of any earlier structures related to total strain. Bubbles within the shear zones, for example, can differ in shape and orientation from those outside, and may be reduced in number and increased in size by coalescence (Weertman, 1968[a]). Crystal texture may change by re-crystallization with accompanying increase or decrease in grain size (Hudleston, 1977).

* Hambrey (1976, p. 58) cites several authors whom he says have "suggested" such a mechanism. Two of the authors thus referenced are misquoted; Swinzow (1962, p. 228) says that the process of entrainment is "separate" from the processes discussed in his paper, and Weertman (1968[b], p. 164) simply argues that the shear mechanism could not occur in regions of extending flow.

It seems likely that such shear zones will remain active as long as they remain parallel to a direction of high shear stress, and it is clear that this must be long enough for an appreciable finite shear strain to develop. A new cross-cutting foliation may form once the first set of shear zones are sufficiently rotated that the shearing stress in another plane can initiate a new set.

The importance of such inhomogeneous deformation in the development of foliation is difficult to assess. Its effects will be most important in areas where the directions of maximum shear stress are not parallel to the components of foliation already present. In the region of nearly simple shear at the base of a glacier, on the other hand, only one foliation is likely to develop, and it will be parallel to the base. This is the direction of maximum shear stress and is *also*, after very large strains, within a small fraction of a degree of the direction of maximum finite extension. Thus, in this case, there is almost no detectable difference in result from a hypothesis of foliation developing parallel to the direction of maximum shear stress or strain-rate and one of foliation forming perpendicular to the direction of maximum total shortening. Furthermore, cross-cutting foliations are not possible at the base even if inhomogeneous shear does occur, because the horizontal direction of maximum shear strain-rate is acting on a non-rotating material plane. The near-vertical conjugate direction of maximum shear strain-rate acts on material planes that rotate too rapidly for any finite structure to appear in that direction.

Foliation in the margin of a polar ice sheet

The hypothesis we have presented is that foliation is derived from primary inhomogeneities which have become highly deformed during flow. To illustrate this hypothesis we now examine the foliation near the margin of the south dome of the Barnes Ice Cap. This foliation is visualized as resulting from passive deformation primarily of sedimentary stratification inherited from the accumulation region. Specifically excluded from the discussion are down-glacier-dipping layered structures very near the margin which have developed during over-riding of superimposed ice formed at the margin (Fig. 1A) (Hooke, 1973[a]).

The foliation pattern, as revealed in several bore holes, a tunnel, and surface exposures, is shown in Figure 3C. Rigorous analysis of the change in attitude of the foliation as it passes from the surface in the accumulation area to a position near the bed and some distance down-glacier from the equilibrium line has not been undertaken, and would be of limited value in any case because it is clear from Figure 3 that the foliation remains virtually horizontal throughout this region. However, near the margin the change in dip of the foliation is substantial, and we now show that this is consistent with our hypothesis.

To evaluate the deformation involved in the marginal zone, consider a plane which is roughly parallel to the bed and which is contained in an element of ice situated a distance ξ_0 from the margin and height η_0 above the bed. This problem was studied previously (Hooke, 1973[b]) but an error was made in that analysis and one purpose of this part of the present paper is to correct that error.

We assume (1) that the ice-sheet profile is parabolic so $h = (cx)^{\frac{1}{2}}$ where x is the distance from the margin, h is the glacier thickness, and c is a constant which depends on the strength or viscosity of the ice (Nye, 1951, p. 571), (2) the ablation rate $\dot{A}_b$ is constant over the ablation area, (3) the horizontal velocity is independent of depth, (4) the mass budget is balanced, and (5) ice is incompressible. It can then be shown (Hooke, 1973[b]) that the path followed by this element of ice is given by:

$$\xi = (\xi_0^{\frac{1}{2}} - \tfrac{1}{2}\alpha t)^2, \tag{1}$$

$$\eta = \eta_0(\xi_0/\xi)^{\frac{1}{2}}, \tag{2}$$

where η is the height of the element above the bed after it has traveled a distance $\xi_0 - \xi$ from its initial position, t is the time required for this movement, and $\alpha = \dot{A}_b c^{-\frac{1}{2}}$. A plot of η versus

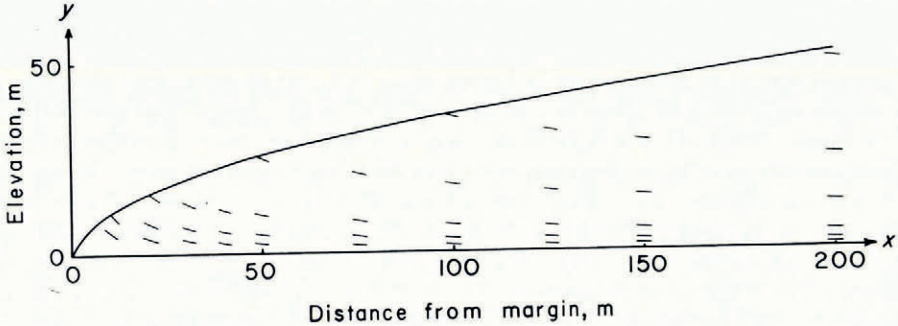

Fig. 4. Foliation attitudes in the margin of an idealized, perfectly plastic glacier with parabolic surface profile, $h = (22.6x)^{\frac{1}{2}}$ (Nye, 1951). Foliation is assumed to have had an up-glacier dip $\frac{1}{4}°$ steeper than the flow line at a point $\xi_0 = 1\,250$ m from the margin. An element of ice containing the foliation plane then followed the path defined by Equation (2), and the dip of the foliation plane at various points along this path was determined from Equation (7). The six paths shown are for six different values of η_0, the height of the element of ice at $\xi = \xi_0$. The values of η_0 used were 0.5, 1.0, 2, 5, 10, and 20 m.

ξ defines the path of the element, and by setting $x = \eta_0(\xi_0/c)^{\frac{1}{2}}$, the x coordinate of the point where the element reaches the glacier surface is obtained. The path is observed to be hyperbolic with a rapid increase in slope near the margin (Fig. 4).

The deformation of a plane in this element of ice is given by

$$\frac{\partial \zeta}{\partial t} = \frac{\partial v}{\partial \xi} + 2\zeta \frac{\partial v}{\partial \eta} - u \frac{\partial \zeta}{\partial \xi}, \tag{3}$$

(Shreve and Sharp, 1970, equation (7)) where ζ is the tangent of the angle of dip of the plane, and u and v are the horizontal and vertical components of the velocity respectively. (In the earlier treatment the third term on the right was erroneously omitted. Consequently the solution predicted that two points originally on a flow line would *not* remain on this flow line. It was the realization of this inconsistency that provoked the additional study of the problem presented herein).

To solve this equation we make use of the equations for u and v derived in the earlier paper:

$$u = -\alpha x^{\frac{1}{2}}, \tag{4}$$

$$v = \tfrac{1}{2}\alpha x^{-\frac{1}{2}} y, \tag{5}$$

$\partial v/\partial \xi$ and $\partial v/\partial \eta$ are obtained from Equation (5) making use of Equation (2). The term on the left in Equation (3) is evaluated by using the chain rule

$$\frac{d\zeta}{dt} = \frac{d\zeta}{d\xi}\frac{d\xi}{dt},$$

and obtaining $d\xi/dt$ from Equation (1). The resulting differential equation is

$$\frac{d\zeta}{d\xi} + \frac{\zeta}{\xi} - \frac{\eta_0 \xi_0^{\frac{1}{2}}}{4\xi^{\frac{3}{2}}} = 0, \tag{6}$$

which has the solution

$$\zeta = -\frac{1}{\xi}\left[\frac{\eta_0}{2}\left(\frac{\xi_0}{\xi}\right)^{\frac{1}{2}} - \xi_0\zeta_0 - \frac{\eta_0}{2}\right], \tag{7}$$

where ζ_0 is the attitude of the plane at (η_0, ξ_0). The minus sign is consistent with the coordinate axes used (Fig. 4), and indicates an up-glacier dip.

It can be readily verified that this solution satisfies Equation (6). To show that it also satisfies the condition that two points on a flow line remain on that flow line, consider the case where $\zeta = \mathrm{d}\eta/\mathrm{d}\xi = -\frac{1}{2}\eta_0\xi_0^{\frac{1}{2}}\xi^{-\frac{3}{2}}$. At (η_0, ξ_0), $\zeta_0 = -\eta_0/2\xi_0$. Substituting this into Equation (7) we obtain the attitude of the foliation plane at some later time

$$\zeta = -\frac{\eta_0\xi_0^{\frac{1}{2}}}{2\xi^{\frac{3}{2}}}.$$

Thus ζ remains equal to $\mathrm{d}\eta/\mathrm{d}\xi$ for all time.

Dips of foliation planes at different positions in the ice are plotted in Figure 4. ξ_0 was arbitrarily taken as 1 250 m, and ζ_0 as $|\mathrm{d}\eta/\mathrm{d}\xi|_{\xi_0} - 0.004\,4$ ($0.004\,4 = \tan\frac{1}{4}°$). Thus the initial up-glacier dip of the foliation plane is $\frac{1}{4}°$ greater than the dip of the flow line. ζ and η were then calculated from Equations (2) and (7) for different values of η_0 and ξ. Note that the rapid increase in dip of foliation near the glacier surface which was observed in our ice tunnel (Hooke, 1973[a]) is clearly present in the model, even though $|\zeta_0|$ is only slightly greater than $|\mathrm{d}\eta/\mathrm{d}\xi|$ at ξ_0. ζ_0 is also clearly greater than the dip of the flow lines at the surface. This is consistent with observations on the Barnes Ice Cap (Hooke, 1973[a], [b]) and re-emphasizes the fact that foliation planes are not necessarily parallel to flow lines in the ice.

It is pertinent to note that if $|\zeta_0|$ is less than $|\mathrm{d}\eta/\mathrm{d}\xi|$ at ξ_0, dips of foliation planes will be less than dips of flow lines at the surface. That this is, in fact, contrary to observation supports the interpretation that the foliation developed normal to the axis of maximum total shortening which can never actually become perpendicular to the bed or the flow line, but must always be inclined slightly up-glacier. However it should be noted that changes in mass balance can change flow lines so that if flow lines were parallel to foliation at one time, as required by Ragan's (1969) hypothesis, they might not remain so. Hudleston (1976) has shown that such changes in mass balance can result in $|\zeta_0| < |\mathrm{d}\eta/\mathrm{d}\xi|$ locally, and that this can result in folding.

In the earlier paper it was shown that models which assume either a linear decrease in horizontal velocity with depth or a linear increase in ice thickness with distance from the margin both lead to $v = 0$, so Equation (3) becomes

$$\frac{\partial \zeta}{\partial t} = -u\frac{\partial \zeta}{\partial \xi}, \tag{8}$$

which has the solution $\zeta = \zeta_0$, so ζ is independent of ξ and t. As ice-sheet margins generally have some curvature, though not as much as implied by the parabolic profile, and as velocities generally do not decrease linearly with depth, the true solution to the problem probably lies between the solutions represented by Equations (6) and (8). As the initial dip is not known in any case, the effect of this error cannot be evaluated.

In the earlier paper it was also shown that present strain-rates ($\mathrm{d}v/\mathrm{d}\eta$ and $\mathrm{d}v/\mathrm{d}\xi$, Equation (3)) in the margin of the Barnes Ice Cap are too low to explain the observed change in attitude of the foliation between two bore holes 81.5 m apart along a flow line, assuming steady state, and it was suggested that the recent negative mass balance of the glacier has resulted in thinning with a consequent decrease in $\mathrm{d}v/\mathrm{d}\eta$ and more importantly in $\mathrm{d}v/\mathrm{d}\xi$. Since the earlier paper was published, revised estimates of the velocity derivatives have been made based on an additional bore hole which indicated that the ice was 3.4 m thinner than expected in the vicinity of the original up-glacier hole. From cores and surface measurements it is known that $\tan^{-1}\zeta_0 \approx 5°$ and that the increase in dip of foliation between these two holes is about 30°. Following the procedure used previously, but using the new estimates of the velocity derivatives and the revised differential equation, the predicted increase in dip is about 14°. Thus the discrepancy still exists, but it is substantially less than in the original calculation.

Allen and others (1960) made a comparable calculation of the magnitude of the change in dip expected as transverse foliation, formed at the base of an ice fall, is carried down-glacier with up-glacier dips decreasing progressively toward the snout. They found that the calculated change in dip was in rough agreement with the observed change.* Both their calculation and that outlined here demonstrate that the magnitudes of the changes in attitude of the foliation in the two examples studied are consistent with our hypothesis that foliation results from passive deformation of pre-existing inhomogeneities. Neither calculation, however, is precise enough to rule out other possible mechanism for development of foliation. The calculations are thus necessary but not sufficient evidence for our thesis.

Concluding statement

The final test of our hypothesis is to ask whether it can explain observed characteristics of foliation. Several examples will be presented to demonstrate that it can explain many of these characteristics, but as with the calculations discussed above it must be recognized that such examples are necessary but not sufficient proofs of the validity of the hypothesis.

Most of the foliation parallel to the bed and valley sides is interpreted as being derived from sedimentary stratification in the accumulation region. The reasoning involved is obvious for ice caps and ice sheets, but may not be as apparent for valley glaciers. Longitudinal (extending) strain-rates in the accumulation regions of valley glaciers are highest near the centers of the ice streams, where the ice is thickest (Raymond, 1971, p. 70). This effect may be accentuated if basal sliding occurs, as Raymond showed that sliding rates were greatest where the ice was thickest. A consequence of this higher rate of extension is that the downward velocity perpendicular to the glacier surface will also be highest near the center of the valley. Thus sedimentary layers which are initially parallel to the surface are gradually warped downward into parallelism with the bed and valley sides. In addition, many cirques have a bowl-shaped character which gives the sedimentary layers a primary synclinal form. In these cases the primary form is simply accentuated by the flow. The reasonableness of this suggestion will be more apparent if it is recalled that most valley glaciers are several times as wide as they are deep. Even fjords one or two thousand meters deep are typically several kilometers wide.

Where two ice streams meet, the zones of foliation which were parallel to either side of the rock divider above the junction will be juxtaposed against one another. The frequent observation that zones of longitudinal foliation seem to extend back to such junctions (Allen and others, 1960; Untersteiner, 1955, p. 504) supports this mode of origin for such zones. Because one or both ice streams are likely to be turned or deflected from their original course at such junctions, transverse compression will occur there and any resulting vertical extension will serve to increase the steepness of the dip of foliation in the longitudinal zone. Longitudinal foliation throughout a glacier may be caused in this way by the confluence of many streams of ice.

Junctions of ice streams down-glacier from a bedrock high which does not reach the glacier surface will also be reflected in the foliation pattern at depth. The effects of such highs may not appear until the ice has moved a considerable distance down-valley and much of the overlying ice has been lost through ablation. Such a process could account for some of the complications in the foliation pattern in the lower Blue Glacier (Allen and others, 1960, p. 607–08) where adjacent foliations "intersect" at high angles. (It is perhaps significant that Allen and others specifically avoided saying that these foliations cross-cut one another.)

* Some parts of Allen and others' calculation, as published, are incorrect. Their equation (3), for example, is not dimensionally homogeneous and a point which they say is 500 m up-glacier from the snout is given an x-coordinate which falls some distance below the terminus. Furthermore, some details of the calculations are not discussed in sufficient detail to allow the reader to reproduce their numerical results. However, we have repeated their calculation using some of our own assumptions and reach the same conclusion that they reached.

A major source of transverse foliation seems to be in and below ice falls where crevasse fillings are expected (Allen and others, 1960; Ragan, 1969, among others). Crevasses of varying width and depth may occur elsewhere on a glacier, of course, and once they are closed, whether filled or not, a recognizable band or layer is likely to remain. Near valley sides such components are rapidly rotated into parallelism with the valley side, and though actually cross-cutting the earlier foliation which was parallel to the boundary prior to opening of the crevasse, they are so nearly parallel to this earlier foliation that the cross-cutting relations are usually obscured. However near the center of the glacier such transverse attitudes and cross cutting relations are normally more obvious (Allen and others, 1960; Ragan, 1969).

The commonly mentioned increase in intensity of foliation near the base or valley sides can also be explained by our hypothesis, as these are the areas of highest total shear strain. Thus inhomogeneities in these locations will have undergone greater finite compression than elsewhere, and layers of ice of varying composition will be thinner and closer together, and hence more apparent. Entrainment of dirt along these boundaries also serves to accentuate the foliation in some cases.

In conclusion, we recognize that most of the basic ideas in this paper are not new, but believe that this synthesis will provide a useful framework for further investigation. The critical assumptions we make are that variations in bubble or dirt content cannot arise through migration of either during flow, and that inhomogeneous deformation of a type that might produce such inhomogeneities is quantitatively inadequate to account for more than a small fraction of the foliation observed. Thus these important inhomogeneities must be primary. Failure to come to grips with the origin of these inhomogeneities is one of the major shortcomings of several discussions of foliation, including Ragan's (1969) thought-provoking but sometimes unnecessarily critical paper, and Hambrey's (1976) study of Charles Rabots Bre, Norway. Recognizing the necessity of accounting for these inhomogeneities brings the problem of the origin of foliation into clearer focus.

Quantitative verification of our hypothesis will be difficult, as we have already shown, because it is probably impossible to make measurements of strain-rate which are sufficiently complete in space and time to calculate total strain accurately. But by the same token the hypothesis will be difficult to refute because of the difficulty in visualizing, even qualitatively, the complex character of the total strain. For example, in Figure 2D we show the effects of a shear strain of $\gamma = 12$. Visualize, if you can, the geometry of some of these components after homogeneous strains which are an order of magnitude larger. Such strains are not uncommon in glaciers.

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