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PREDICTING THE PROBABILITY OF A RECESSION WITH NONLINEAR AUTOREGRESSIVE LEADING-INDICATOR MODELS

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We develop nonlinear leading-indicator models for GDP growth, with the interest-rate spread and growth in M2 as leading indicators. Since policy makers typically are interested in whether a recession is imminent, we evaluate these models according to their ability to predict the probability of a recession. Using data for the United States, we find that conditional on the spread, the marginal contribution of M2 growth in predicting recessions is negligible.

Keywords: Event Probabilities, Leading Indicators, Nonlinear Models

1. INTRODUCTION

Forecasting recessions has always been important for policy makers and business planners, and this concern has led to a large time-series literature on macroeconomic forecasts. Much of this literature is based on linear models, but these cannot account for asymmetries that are often associated with business cycles. Typically, recessions involve sharp but short-lived declines in economic activity, but expansions are gradual, and often last longer than recessions. Also, negative shocks to output appear to be less persistent than positive shocks.

The recent time-series literature on business cycles has paid much attention to the modeling of asymmetries in output, and there are now many nonlinear *univariate* models that allow for recessionary and expansionary regimes. Well-known examples include the Markov-switching (MS) model [Hamilton (1989)], the current-depth-of-recession (CDR) model [Beaudry and Koop (1993)], the smooth-transition autoregressive (STAR) model [Teräsvirta (1994)], and the threshold autoregressive (TAR) model [Potter (1995)]. All of these models have provided useful information on the dynamics of business cycles, but their forecast performance has been disappointing.

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One reason why forecast evaluations have not found significant differences between linear and nonlinear models may be that the out-of-sample forecast periods are usually short, and they do not necessarily contain the nonlinear features that the nonlinear model was designed to capture. Thus, the potential advantages of nonlinear specifications are not necessarily realized out of sample. Noting that STAR specifications fit the data very well during recessions, Teräsvirta and Anderson (1992) argued that the forecasting gains from such models might be seen only when one was forecasting over a recessionary period. Tiao and Tsay (1994) and Montgomery et al. (1998) have found evidence that supports this argument.

Most nonlinear univariate models of business cycles have used GDP as the indicator of economic activity, despite the fact that formal tests that GDP is a linear process usually fail to reject linearity. If GDP is a linear process, then the MS, TAR, and STAR models are not even identified. It is, of course, possible that models of GDP conditional on other variables are nonlinear, whereas the marginal model is linear. There is now ample evidence that GDP conditioned on other variables is nonlinear. Granger et al. (1993) rejected the null hypothesis that output was linear in the context of a bivariate model of GDP and a leading indicator, and Anderson and Vahid (1998) found statistically significant evidence of a common nonlinear factor in a trivariate model of GDP, consumption, and investment. Recently, Skalin and Teräsvirta (1999) proposed a STAR-based test of Granger noncausality and applied it to Swedish GDP. These nonlinear causality tests found strong evidence that exports and wages Granger-cause GDP.

The fact that other variables are useful for predicting GDP is very well known, and it has frequently been used in developing leading-indicator models. Stock prices and short-term interest rates have been used as leading indicators since the twentieth century. Further, since Mitchell and Burns (1961) first nominated several variables that they called "leading indicators," the NBER has explicitly funded the development of new leading indicators. Given then that other variables are known to forecast output well, the exercise of comparing the forecasting abilities of linear and nonlinear *univariate* models seems to be merely of academic value. If the goal is to develop good forecasting models for GDP, then multivariate models seem to offer a more appropriate framework.

This paper develops some nonlinear leading-indicator models for output. These models are generalizations of the linear leading-indicator models advocated by Zellner and Hong (1989), Zellner et al. (1991), and Zellner and Min (1999). Some previous attempts to incorporate leading indicators in nonlinear models include the bivariate smooth-transition error-correction model by Granger et al. (1993) and the bivariate MS models by Hamilton and Perez-Quiros (1996) and Ravn and Solas (1999). Chauvet (1998) uses a single-factor MS model as an alternative index of coincident indicators. Recent papers, such as those by Estrella and Mishkin (1998) and Birchenhall et al. (1999), have used leading indicators in probit and logit models to forecast recessions. Other related work includes the nonlinear impulse response analyses by Weise (1999) and Choi (1999), which illustrate various asymmetries in the effects of monetary policy.

Our empirical models emphasize the ability of the interest-rate spread to predict output. Friedman and Kuttner (1998) provide a useful overview of the theoretical and empirical literature on the predictive power of the interest-rate spread. Generally, it is believed that monetary policy affects the short end of the term structure more than the long end, so that changes in the interest-rate spread will lead the output changes induced by monetary policy. Also, credit conditions can affect interest-rate spreads by changing long-term rates more than short-term rates. Thus, interest-rate spreads may reflect an economy's financial climate, and whether or not this climate will be conducive for real growth. Sims (1980) and, more recently, Estrella and Mishkin (1998) and Karunaratne (1999) have found that interest-rate variables are better predictors of real activity than direct measures of money. However, given the large theoretical and empirical literatures that relate output and money, and recent concern that the spread did not predict the 1990 recession, we also include money in some of our models.

We evaluate our models according to their ability to predict recessions. Section 2 explains why we focus on predicting recessions, and it also describes the calculation and evaluation of event forecasts. We discuss our data in Section 3, and then undertake our empirical analysis in Sections 4 and 5. We find that, although univariate nonlinear models cannot predict recessions any better than AR models, the multivariate nonlinear models *do* outperform VARs. We also find that the predictive ability of our models increases significantly as we go from univariate models of output to bivariate models of output and the spread. Using money instead of (or as well as) the spread does not improve forecasts. Section 6 summarizes and concludes.

2. FORECASTING RECESSIONS

Time-series models have always been assessed by their ability to predict out of sample. Thus, the calculation of forecasts and the comparison of forecasts obtained from different models have become quite routine. Two typical examples that study forecasts of detrended output include a comparison of the forecasting ability of AR, MS, and TAR models of GDP by Clements and Krolzig (1998), and a comparison of the forecasting ability of AR, CDR, and STAR models of industrial production by Jansen and Oh (1999).

Comparisons of point forecasts are useful for model selection, but they do not directly address the interests of forecast users. Business-cycle analysts are generally more interested in forecasts of future turning points or predictions of events such as recessions. In this paper, we compare models according to their ability to predict recessions. Previous related work includes that of Neftçi (1982), Diebold and Rudebusch (1989), Zellner et al. (1991), and Fair (1993).

It is difficult to take expectations in nonlinear contexts, and simulation techniques generally are used when forecasting. The relevant issues are discussed by Granger and Teräsvirta (1993), and Koop et al. (1996) provide a closely related discussion on nonlinear impulse response analysis. Clements and Smith (1997) discuss and compare the various Monte Carlo methods that are used to obtain multiperiod forecasts. Fair (1993) bases probability estimates of events on simulations of sequences of multistep conditional forecasting densities, and we use his approach and our leading-indicator models to predict the probability of recession. The main idea behind this approach is that one defines the event of interest as a property of a sequence of predictions, and then classifies each predicted sequence from the Monte Carlo as either having or not having that property. The predicted probability of the event is then based on the proportion of the Monte Carlo sequences that have the property of interest.

2.1. Stochastic Simulations of Event Probabilities

Let y_t be an $\boldsymbol{m}$-vector of endogenous variables, let Y^{t-1} be the history of y_t, that is, $Y^{t-1} = \{y_{t-1}, y_{t-2}, \ldots, y_1\}$, and define the forecasting model f for y_t as

$$y_t = f(Y^{t-1}; \beta) + u_t,$$

where β is a $\boldsymbol{k}$-vector of parameters, $E(u_t) = 0$, and $\mathrm{Var}(u_t) = \Sigma$. The model can be linear or nonlinear in Y^{t-1} and/or β, but we specify it so that the u_t are i.i.d. For the simulations that follow, we assume that the u_t have a multivariate normal distribution but, in principle, one could make other distributional assumptions about u_t without affecting the forecasting procedure.

Given consistent estimates $\hat{\beta}$ and $\hat{\Sigma}$, we construct a trial sequence of forecasts for $y_{t+1}, y_{t+2}, \ldots, y_{t+h}$ conditional on Y^t as follows: We draw a random $\boldsymbol{m}$-vector ε_{t+1} from the distribution $\varepsilon \sim N(0, \hat{\Sigma})$, calculate $\hat{y}_{t+1} = f(Y^t; \hat{\beta}) + \varepsilon_{t+1}$ and form $\hat{Y}^{t+1} = \{\hat{y}_{t+1} \cup Y^t\}$. Then we draw an ε_{t+2} from $\varepsilon \sim N(0, \hat{\Sigma})$, calculate $\hat{y}_{t+2} = f(\hat{Y}^{t+1}; \hat{\beta}) + \varepsilon_{t+2}$ and form $\hat{Y}^{t+2} = \{\hat{y}_{t+2} \cup \hat{Y}^{t+1}\}$. We continue doing this until we have an entire forecast sequence consisting of $\{\hat{y}_{t+1}, \hat{y}_{t+2}, \ldots, \hat{y}_{t+h}\}$. We label this forecast sequence as S_1 (for simulation 1) and then repeat the trial many times to obtain a set of n forecast sequences $S_1, S_2, \ldots S_n$, each of which has been based on independent draws of h observations from the distribution of ε. We use the empirical density of these n trial sequences to approximate the density of $(y_{t+1}, y_{t+2}, \ldots, y_{t+h}|Y^t)$, and to ensure a reasonable approximation in our case, we set $n = 2000$.

Forecasters often use the simulated density of $(y_{t+1}, y_{t+2}, \ldots, y_{t+h} \mid Y^t)$ to obtain one- to h-step-ahead point forecasts (i.e., $\hat{y}_{t+j} \mid Y^t$ for $j = 1, 2 \ldots h$) together with their associated confidence intervals. Here, however, we are interested in forecasting the conditional probability of an *event* rather than a conditional mean, but this simply involves analyzing the trial sequences in a different way. One starts by defining the event of interest as some characteristic of $\{y_{t+1}, y_{t+2}, \ldots, y_{t+h}\}$. Then one simply determines the proportion of the trial sequences that are predicted to have this particular characteristic and uses this as the estimated probability of the event of interest.

In this paper, we are interested in the event of a recession, and this could be defined in many ways. Most definitions of recession involve notions of a slowdown in output growth (and other procyclical variables) over some period of time, but researchers differ in opinions as to what a "slowdown" is, and how long economic activity should decline before we declare a recession. Here, we use Fair's (1993) two definitions of a recession, which are

(a) at least two *consecutive* quarters of negative growth in real GDP over the next five quarters and
(b) at least two quarters of negative growth in real GDP over the next five quarters.

As noted by Fair, the first of these definitions is in common use. The other definition is broader, and allows us to assess how predictions might change as we change the event that we are trying to predict. Our definitions do not necessarily coincide with turning points or even "slowdowns" in growth, and negative values of $y_t, \ldots y_{t-3}$ are not given special treatment, even though one might want to count them as part of an ongoing recession. One could, of course, use other definitions of recession or calculate the probabilities of other sorts of events, all using the same set of Monte Carlo simulations.

All of our estimated models relate to the sample period from 1960:1 to 1996:4. For each observation in this sample, we take lagged observations and our estimated parameters as given, and then we use the simulation process to estimate the probabilities of events A and B. We discuss evaluation in the next section.

At this point, note the similarities and differences between our probability predictions and those of others. First, we define a recession as an observable event. Therefore, we do not need to make an inference about an unobservable state, as is done in Markov-switching models. Moreover, since our definition of recession is directly related to one- to five-period-ahead forecasts of output, an appropriate model for forecasting the probability of recession is one that delivers the one- to five-period-ahead predictive density of output. In this context, binary dependent-variable models ignore available and relevant information. Also, the binary dependent-variable models are problem-specific, and if there is interest in estimating the probabilities associated with other events, then the dependent variable needs to be redefined in each case, and the model needs to be reestimated.

2.2. Evaluation of Probability Forecasts

There are many available procedures for evaluating probability forecasts, and we use the two well-known measures that are described by Diebold and Rudebusch (1989). Defining P_t to be the estimated probability that an event of interest will occur between time $(t + 1)$ and $(t + h)$, and a dummy variable D_t to be equal to 1, if, according to the realized sequence of the relevant $\{y_{t+1}, y_{t+2}, \ldots, y_{t+h}\}$, the event actually occurred, then each of these evaluation methods assesses the accuracy of the forecasts, by comparing the estimated P_t with the observed D_t.

If we use a model f to generate a sequence of T forecasts of the probability of some event, then Brier's (1950) quadratic probability score, defined by

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$$QPS = \frac{1}{T}\sum_{t=1}^{T} 2(P_t - D_t)^2 \qquad (0 < QPS < 2),$$

provides a probability analogue to the usual mean squared error (MSE) criterion. Like the MSE measure, a low QPS implies that forecasts are accurate. The penalties for over- and underprediction are symmetric, but since QPS grows with the square of the prediction error, large mistakes are penalized more heavily.

The other common measure of the accuracy of probability forecasts is given by the log probability score, which is defined by

$$LPS = -\frac{1}{T}\sum_{t=1}^{T}[(1 - D_t)\ln(1 - P_t) + D_t \ln P_t] \qquad (0 < LPS < \infty).$$

Like QPS, this measure will be low when forecasts are accurate, but LPS penalizes large mistakes more heavily than QPS. LPS is the negative of the average log likelihood of the observed sequence of D_t for $t = 1, 2, \dots T$, given the model.

Our forecasts about recessions are not genuine out-of-sample forecasts, but the QPS and LPS criteria differ from the loss functions that are minimized when the parameters are estimated. Although these criteria are not independent of in-sample sum of squared errors, there is little reason to believe that they necessarily improve with the fit of the model. We evaluate our forecasts over the estimation sample so that our analysis relates to all phases of the business cycle. Further, we use nine out-of-sample observations (1997:1–1999:1) to provide conventional MSE of one-step-ahead forecasts for each model that we estimate. These are reported in the Appendices, with the estimated models.

3. DATA

Our empirical analysis is based on data for the United States drawn from the Federal Reserve Data Base (FRED), and it consists of quarterly series on real GDP (seasonally adjusted, in billions of chained 1992 dollars), real M2 (seasonally adjusted M2, in billions of dollars, deflated by the chained GDP implicit price deflator), and an interest-rate spread (the 10-year Treasury Bond rate minus the 3-month Treasury Bill rate, expressed as a percentage). The longest sample period available for all of these series extends from 1959:1 to 1999:1. However, our estimated models relate to an effective sample of 148 observations, which starts in 1960:1 and ends in 1996:4. We left out the early observations to allow for the lagging of variables, and we left out the last nine observations because they would change as a result of the two-sided seasonal adjustment process.

The variables that we use in our models are $y_t(100 \times \Delta \ln \text{real GDP}_t)$ for output growth, $m_t(100 \times \Delta \ln \text{real M2}_t)$ for money growth, and s_t for the interest-rate spread [see Figure 1a–c]. The logarithms of real GDP and real M2 are both I(1), but there is no evidence of a stable long-run relationship between these two series, as is clear from Figure 1d. The interest-rate spread is I(0). Since there is no cointegration

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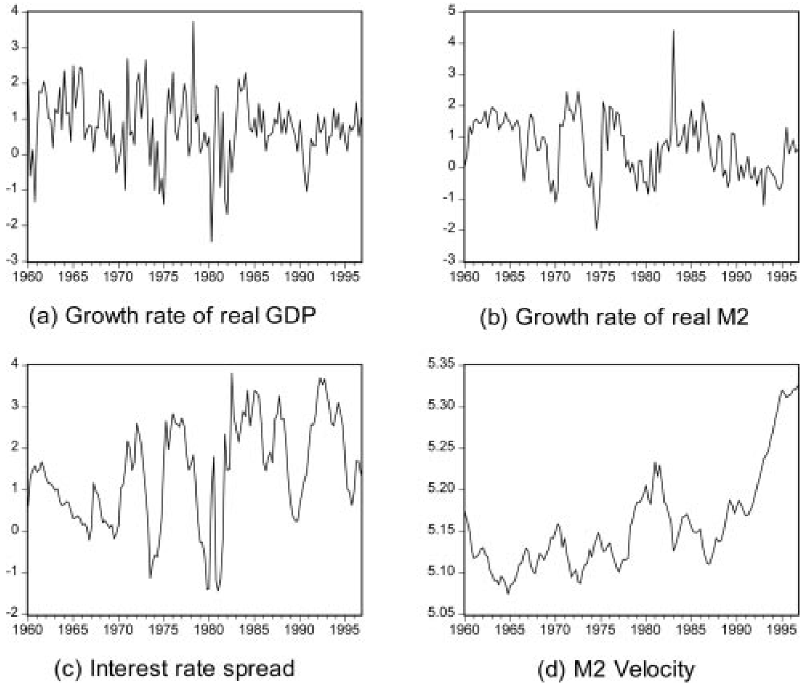

FIGURE 1. Data from FRED for empirical analysis.

between output and money and our models use only first differences of these variables, we refer to the growth rates of real GDP and real M2 as "output" and "money" for the remainder of the paper.

4. UNIVARIATE MODELS OF OUTPUT

Although our goal is to develop a nonlinear leading-indicator model, we begin by investigating how well several univariate models of GDP perform in predicting recessions. We estimate an AR(2), which is the best linear model according to AIC. Table 1, which tests the null hypothesis of linearity against several nonlinear alternatives, shows that there is only weak evidence of an omitted lagged CDR term in the residuals of this model. However, we fit a TAR model, a STAR model, a CDR model, and a Markov-switching model to see how well they can predict recessions relative to the AR(2). The estimated models are reported in Appendix A, and are discussed briefly below.

4.1. TAR Model

Following Potter (1995), we estimated a TAR(5) with $y_{t-2} < 0$ defining the recessionary regime. For this model, and for the others discussed below, we experimented with leaving in insignificant lags or dropping them, but we found that our

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TABLE 1. p-Values of linearity tests

Test	Omitted nonlinear terms	p-Value
Luukkonen et al. (1988)	$y_{t-1}y_{t-2}, y_{t-1}^2, y_{t-2}^2, y_{t-1}^3, y_{t-2}^3$	0.44
Omitted CDR	CDR_{t-1}	0.09
Ramsey (1969) Reset	$\hat{y}_t^2, \hat{y}_t^3$	0.40
Tsay (1989)[a]	NA	0.66

[a]The Tsay test is a test of linearity against a TAR alternative. It is based on the recursive residuals of a regression in which variables are ordered according to the assumed threshold variable. The reported p-value is the smallest we found for thresholds corresponding to $y_{t-1}, \ldots, y_{t-5}$.

final conclusions do not depend on our specification strategy. Our reported results correspond to the most parsimonious specification.

4.2. STAR Model

Using the AR(2) as a baseline and the results of STAR (Luukkonen et al. 1988) specification tests, we started by estimating a logistic STAR(2), with y_{t-2} as the transition variable. Our tests did not support nonlinearity, but the transition variable y_{t-2} had led to the lowest p-value. In this and all subsequent STAR specifications, we restricted the smoothing parameter to lie between 0 and 16 (to avoid difficulties with numerical precision), and we also restricted the centrality parameter to lie between the tenth and the ninetieth percentile of the observed transition variable (to avoid identification problems). Given that the smoothing parameter is rather large, the resulting transition function is close to that of a TAR model with the transition indicator function $1\,(y_{t-2} > -0.55)$.

4.3. CDR Model

This model is the same as that specified by Beaudry and Koop (1993), and relative to the other nonlinear models, it is the only model justified at the 10% significance level, by the tests in Table 1.

4.4. Markov-Switching Model

Our estimated model contains two states with two lags. As in Clements and Krolzig (1998), we found that the third and fourth lags in Hamilton's original four-lag specification were insignificant.

The event-forecast performance of our univariate models is illustrated in Figure 2A and 2B, which compare the predicted probabilities of events A and B, respectively, with observed indicators for these events. We see that none of the univariate models provide clear signals of recessions, and that their probability predictions are not much better than an unconditional model (i.e., a model that always predicts a constant probability for an event, equal to the observed frequency of that event).

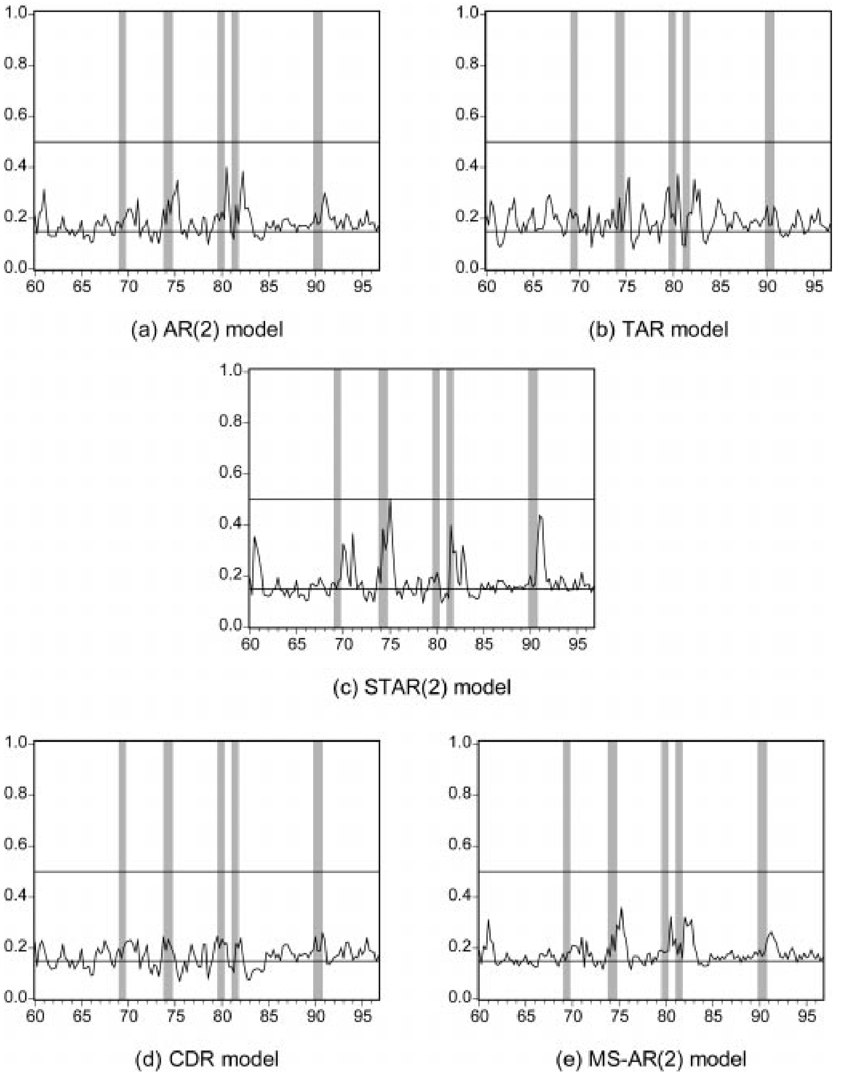

FIGURE 2A. Performance of univariate models in predicting event A. The shaded areas indicate the times that the event occurs. The unconditional probability of event A is 0.149.

This observation is reinforced by the summary statistics in Table 2. There is very little difference between the accuracy of these probability predictions, although the CDR model appears to be marginally better than the rest. Overall, our results are similar to those in the literature that relate to the accuracy of multistep-ahead point forecasts of univariate models of output.

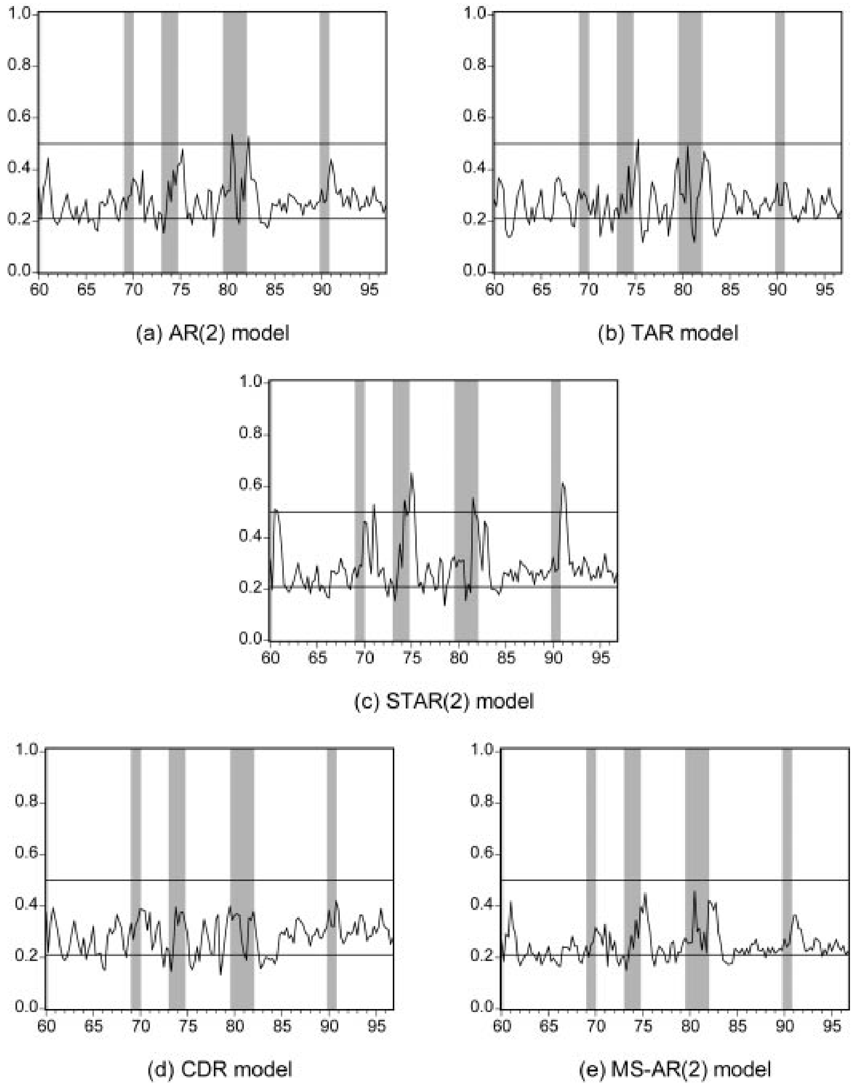

FIGURE 2B. Performance of univariate models in predicting event B. The shaded areas indicate the times that the event occurs. The unconditional probability of event B is 0.209.

5. MULTIVARIATE LEADING-INDICATOR MODELS

There is a large historical literature that documents evidence that financial variables Granger-cause output. M2 traditionally has been recognized as a particularly reliable indicator of economic conditions. Recently, however, with the introduction of new financial instruments and the increasing importance of credit markets, the value of M2 as a leading indicator has been questioned. For example, Stock and

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Table 2. Performance of univariate models in predicting recessions

Model	Event A		Event B	
	QPS	LPS	QPS	LPS
Constant	0.253	0.420	0.331	0.513
AR(2)	0.249	0.410	0.323	0.503
TAR	0.256	0.423	0.326	0.505
STAR(2)	0.242	0.399	0.323	0.515
CDR	0.237	0.392	0.318	0.498
MS-AR(2)	0.250	0.412	0.322	0.500

Watson (1989) decided against including M2 in their index of leading indicators. In 1993, the Federal Reserve Chairman, Alan Greenspan, informed Congress that M2 had been "downgraded as a reliable indicator of financial conditions in the economy" and that "the historical relationships between money and income had broken down" [see Ragan and Trehan (1998)].

The spread between long-term and short-term interest rates is now becoming popular as a leading indicator for output. For example, Estrella and Mishkin (1998), through a series of probit regressions, found that interest-rate spread provides better predictions of the NBER recessions than a large list of other candidate leading indicators. Karunaratne (1999) confirms the power of interest-rate spread as the best single leading indicator for Australian output.

We therefore begin by developing a possibly nonlinear bivariate leading-indicator model of output and the spread, and we then pose the question of whether using money instead of or as well as the spread in this model can significantly improve the prediction of recessions.

5.1. Output–Spread Models

Our starting point is a VAR for output and the spread. Although AIC selects a lag order of 2, residual analysis on the VAR(2) shows that a third lag of spread is needed in the spread equation to eliminate the serial correlation in the errors of that equation. Since the validity of our nonlinearity tests depends on the absence of serial dependence in the errors of the linear model, we start from a VAR(3).

After the omission of insignificant lags, the VAR(3) forms the benchmark for our bivariate analysis. We refer to this model as a linear autoregressive leading-indicator (ARLI) model, following Zellner and Hong's (1989) terminology. We reduce the VAR(3) once, equation by equation, using OLS, and then again using iterative SUR estimation. We end up with the same final specification, but of course with slightly different parameter estimates. The estimated models are reported under ARLI-OLS and ARLI-SYS in Appendix B, and their event-forecast performance is illustrated in Figures 3A(a), 3A(b), 3B(a), and 3B(b). We report the probability scores in the first two lines of Table 4.

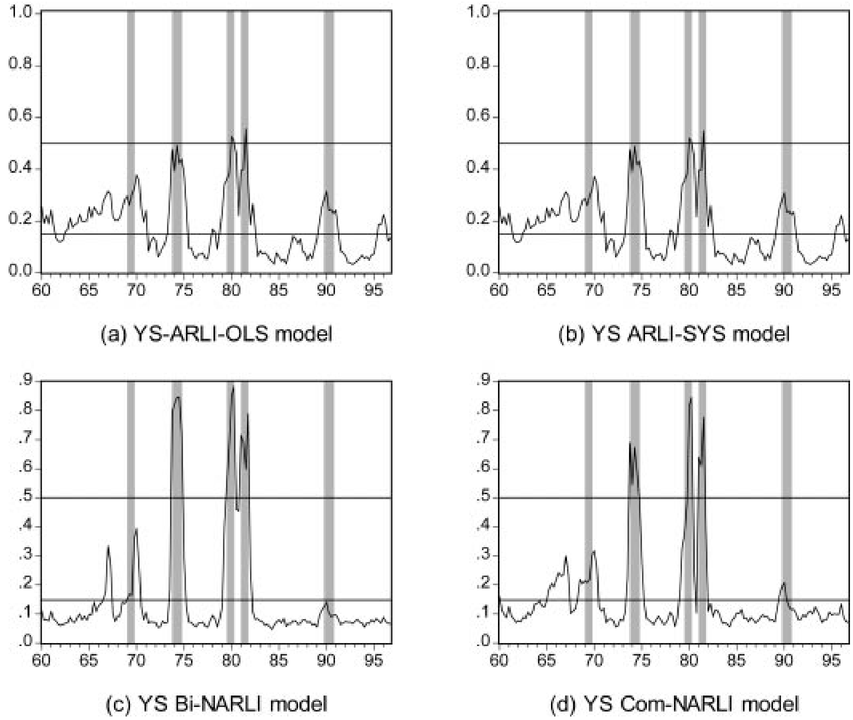

FIGURE 3A. Performance of output–spread models in predicting event A. The shaded areas indicate the times that the event occurs. The unconditional probability of event A is 0.149.

A comparison of the performance of the bivariate ARLI models with the univariate models shows that the addition of the spread to the output equation improves the accuracy measure over an AR(2) by more than 25% in all cases. It is informative to compare these improvements with those reported by Fair (1993), even though his sample period (1954:1–1990:1) is not exactly the same as ours (1960:1–1996:4) and his real GDP series is also different from ours (we use the new BEA real GDP series in 1992 chained dollars). Fair's model is a structural model with 30 stochastic equations, 98 identities, and 82 exogenous variables. When the exogenous variables are predicted by adding autoregressive specifications for them to the system, Fair's model predicts event A slightly worse and event B slightly better than his AR(2) model. When the exogenous variables are taken as known, his model can predict events A and B better than the AR(2) model, by 20% and 30%, respectively.

Having established that the use of the spread in a linear specification increases the ability to predict recessions, we next ask whether the data support the linear specification of the AR-LI models. We find no evidence of an omitted lagged CDR term in the AR-LI equation for output (the p-value for this is 0.22). However,

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TABLE 3. p-Values of STAR nonlinearity tests

Transition variable	Output equation	Spread equation
y_{t-1}	0.712	0.006
y_{t-2}	0.272	<0.001
y_{t-3}	0.528	0.030
s_{t-1}	0.004	0.001
s_{t-2}	0.002	0.045
s_{t-3}	0.065	<0.001

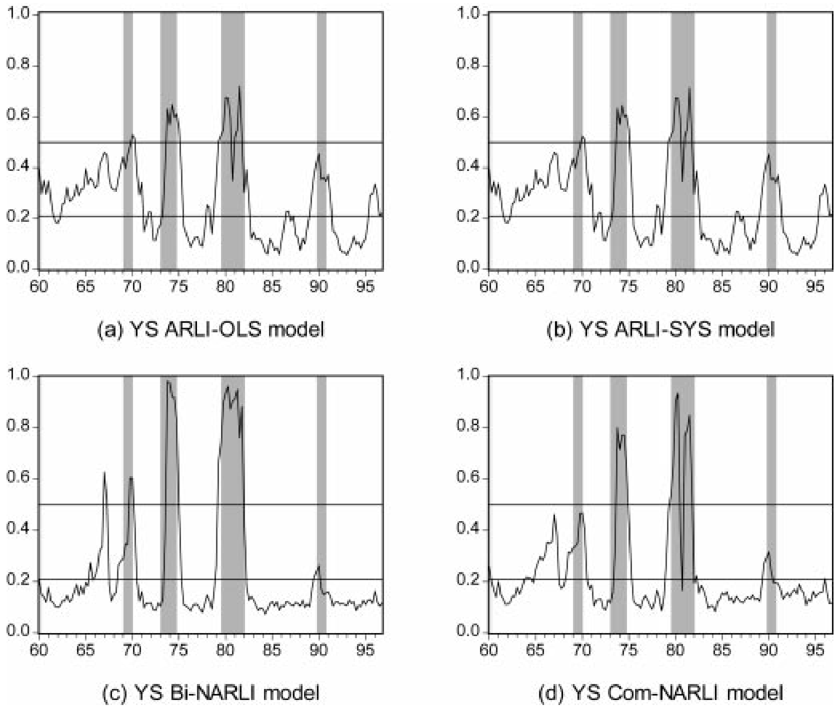

FIGURE 3B. Performance of output–spread models in predicting event B. The shaded areas indicate the times that the event occurs. The unconditional probability of event B is 0.209.

augmented first-order tests[1] for STAR nonlinearities [Luukkonen et al. (1988)] reject the null of linearity in each equation, as shown in Table 3.

Often, in practice, these linearity tests can reject the null for more than one hypothesized transition variable, especially when the transition variables are highly correlated. This happens here. In such cases, it is usual to choose the transition variable that minimizes the p-value of the linearity tests for each equation, but

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sometimes a common nonlinear factor model might account for the nonlinearity in both equations [Anderson and Vahid (1998)]. Here, a general test that allows for any of the three lags of the spread to be the transition variable rejects the hypothesis of common nonlinearity. The p-value for this test is 0.001.

Based on minimum p-values, the transition variables in the output and spread equations are the second and third lags of the spread. We estimate this bivariate nonlinear leading-indicator (YS Bi-NARLI) model equation-by-equation, and report the coefficients in Appendix B. The reported equation for output incorporates the restriction that all coefficients on lagged terms except the AR(1) term are zero[2] when $f_{yt}(s_{t-2}) = 1$. Thus, in normal times when s_{t-2} is positive, y_t is essentially an AR(1) process, but when the yield curve is steep and negatively sloped, then the dynamic behavior of y_t becomes more complicated.

The event-forecast performance of this nonlinear output–spread model is illustrated in Figures 3A(c) and 3B(c). Relative to the linear models, this model provides much stronger signals for the 1974, 1979, and 1981 recessions, but a weaker signal for the 1990 recession. One reason our model [and those of many others including Sims (1993) and Stock and Watson (1993)] produced a weak signal for the 1990 recession is that this recession was very short (and mild). Another reason that many models (including ours) had difficulty predicting this recession is that the 1990 recession was quite atypical. Unlike other recessions, it was not preceded by tight monetary policy, and although the spread fell prior to 1990, this fall was due to a fall in the long rate, rather than a rise in the short rate that normally accompanies tighter monetary policy. The linear models produce a stronger signal for the 1990 recession, but then the linear models also signal recessions on every occasion that the spread falls.[3] Thus, the linear models give misleading probability predictions of recession all through the 1960's and in 1995. The probability scores for our YS Bi-NARLI model are reported in the third row of Table 4. Unlike the univariate case, moving from a linear to a nonlinear specification, makes a noticeable difference in the accuracy of the predictions.

Although the general common nonlinearity test rejected the null hypothesis, we followed a referee's suggestion and conditioned on both equations having the same transition variable. Conditional on this assumption, common nonlinear factor tests [Anderson and Vahid (1998)] supported the null only when s_{t-2} was used as a common transition variable. The estimated common nonlinear factor

TABLE 4. Performance of output–spread models in predicting recessions

	Event A		Event B	
Model	QPS	LPS	QPS	LPS
YS ARLI-OLS	0.172	0.296	0.223	0.375
YS ARLI-SYS	0.172	0.295	0.223	0.374
YS Bi-NARLI	0.127	0.238	0.171	0.301
YS Com-NARLI	0.149	0.265	0.206	0.350

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(YS Com-NARLI) model is reported in Appendix B, its event predictions are shown in Figures 3A(d) and 3B(d), and its probability scores are in the last line of Table 4. This model does not perform as well as the YS Bi-NARLI model.

LM tests for additional nonlinearity [Eitrheim and Teräsvirta (1996)] find no evidence of further nonlinearity in the output equation, but they indicate unmodeled nonlinearity in the spread equation. The p-values for these tests are 0.88 and 0.001. A system test for additional nonlinearity in our YS Com-NARLI model has a p-value of 0.0001. We discuss this issue further in our conclusion.

5.2. Does M2 Help?

Economists have started to exclude M2 from the list of the leading indicators, believing that the relationship between money and output has changed in the past decade or so. Figure 1(d) indicates why this might be so. Clearly, the long-run relationship between money and output has changed. However, this does not necessarily imply that money does not Granger-cause output in the short run. The evidence that monetary growth Granger-causes output growth is still strong. Zellner and Min (1999) find that autoregressive models that include monetary growth as a leading indicator can predict turning points well. Also, the probit regressions reported by Estrella and Mishkin (1998, Appendix) show that M2 growth can predict recessions in addition to interest-rate spreads. This leads us to ask whether we can improve our bivariate models by adding M2.

The starting point for the three-variable model is a trivariate VAR(3). Our modeling methodology is the same as for the bivariate model. In Appendix C, we report the equation-by-equation OLS, and the system-estimated linear models. The evidence of nonlinearity in the output equation is weak (the minimum p-value for LSTAR tests is 0.05 with s_{t-2} as the transition variable, and the p-value for an omitted lagged CDR term is 0.89). However, there is strong evidence of nonlinearity in the money and spread equations. The final estimated trivariate nonlinear leading-indicator (Tri-NARLI) model is reported in Appendix C, and since the evidence of nonlinearity in the linear output equation is weak, we have also reported a model (Tri-NARLI0) that uses a linear specification for the output and nonlinear specifications for money and spread.

The probability forecasts obtained from the trivariate models are illustrated in Figures 4A and 4B. These models are able to predict the recessions of 1974, 1979, and 1982, but relative to the output–spread models they provide misleading indications of recession around 1966 and 1995. The probability scores of the trivariate models are provided in the first four rows of Table 5. Comparing Tables 4 and 5, we conclude that even though adding money improves the predictive ability of the linear ARLI model, it does not improve the performance of the nonlinear ARLI model. The same conclusion can be drawn from a comparison of the forecast MSE measures for 1997:1–1999:1, reported below each model in the appendices.

Table 5. Performance of models that include M2

	Event A		Event B	
Model	QPS	LPS	QPS	LPS
ARLI-OLS	0.169	0.284	0.231	0.377
ARLI-SYS	0.169	0.284	0.232	0.379
Tri-NARLI	0.160	0.264	0.202	0.328
Tri-NARLI0	0.165	0.277	0.224	0.367
YM ARLI-OLS	0.226	0.365	0.303	0.464
YM Bi-NARLI	0.227	0.368	0.305	0.466

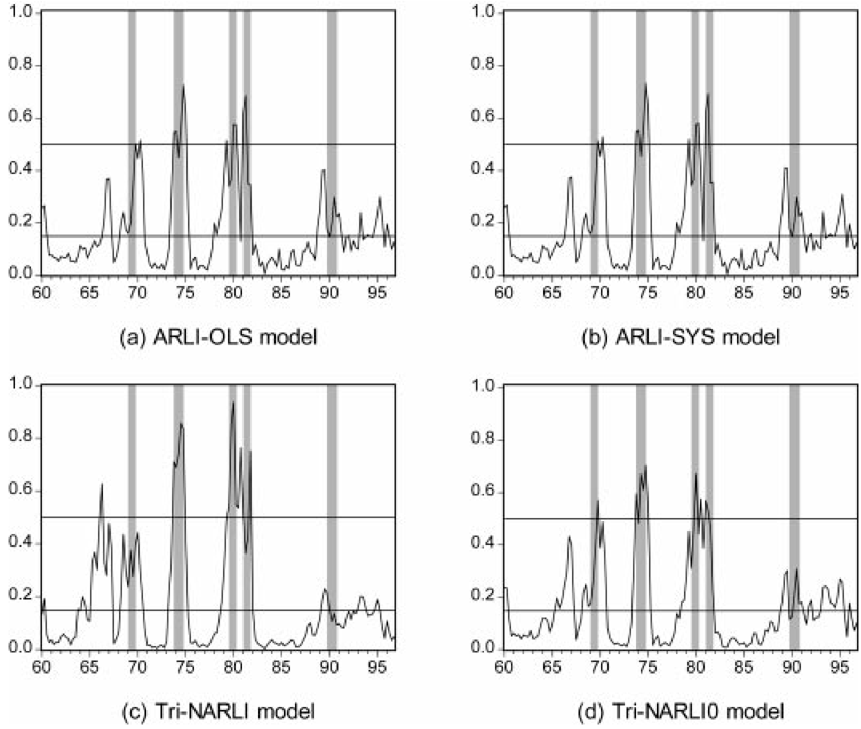

Figure 4A. Performance of trivariate models in predicting event A. The shaded areas indicate the times that the event occurs. The unconditional probability of event A is 0.149.

We conclude that the addition of M2 does not help in predicting recessions. To check that this conclusion is not influenced by the order in which we added the spread and M2, we also estimated bivariate linear and nonlinear output–money models. We do not report the estimated models, but their probability forecasts are in Figure 5, and their probability scores are in the last two rows of Table 5.

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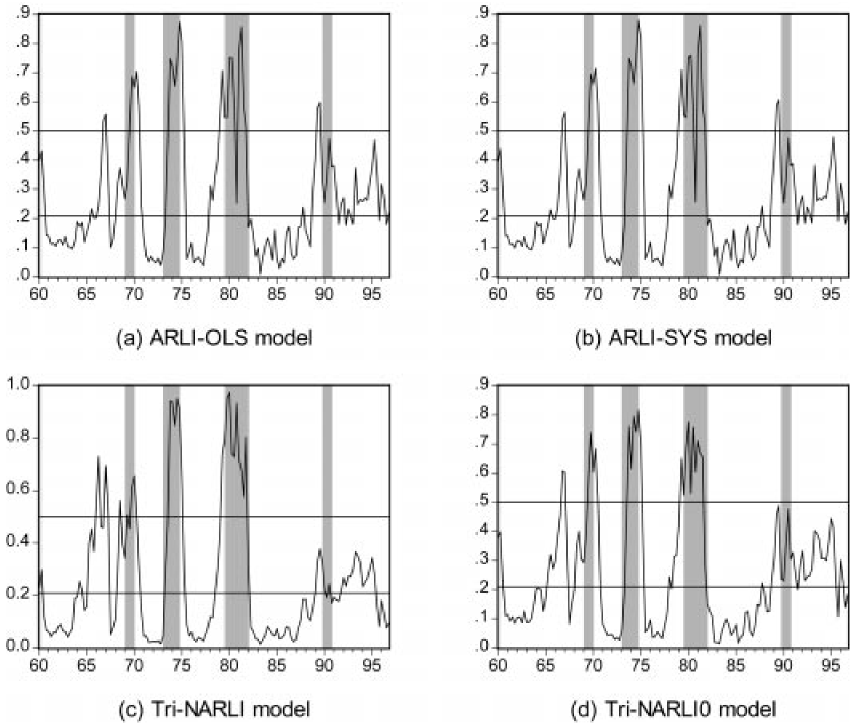

FIGURE 4B. Performance of trivariate models in predicting event B. The shaded areas indicate the times that the event occurs. The unconditional probability of event B is 0.209.

It is clear that the output–money models are inferior to output-spread models, and that our conclusion was not influenced by the order of our investigation. The output–money models are only slightly better than the univariate models, and their event-probability forecasts from the late eighties onward are erratic. This provides further evidence that the relationship between M2 and output has changed.

6. CONCLUSION AND DIRECTIONS FOR FURTHER RESEARCH

In this paper we develop a bivariate nonlinear model of output and the interest-rate spread, and compare its ability in predicting recessions with linear and nonlinear models of output. We corroborate the recent results in the literature that the spread is a better leading indicator for output growth than money. We also establish that, conditional on the spread, the marginal contribution of M2 growth in predicting recessions is negligible. We find that a nonlinear model of output and the spread seems to give fewer false warnings of recession than a linear model. This leads us to conclude that future research should concentrate on developing and comparing alternative bivariate nonlinear models of output and the spread.

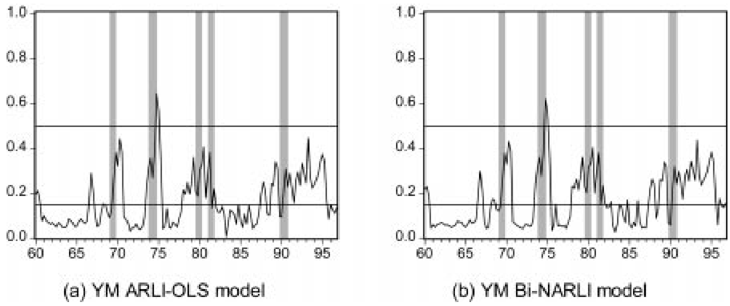

FIGURE 5A. Performance of output–money models in predicting event A. The shaded areas indicate the times that the event occurs. The unconditional probability of event A is 0.149.

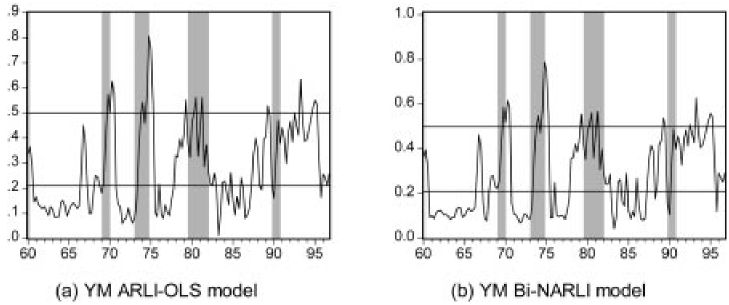

FIGURE 5B. Performance of output–money models in predicting event B. The shaded areas indicate the times that the event occurs. The unconditional probability of event B is 0.209.

Our method of evaluation uses the same data that are used to estimate the parameters, but a different loss function from the loss function that is minimized to estimate the parameters. Even though this is not a genuine out-of-sample evaluation, given the sparsity of events of particular interest such as recessions, we think that it is an informative exercise in evaluating the practical usefulness of nonlinear models. In practice, we did not observe that this evaluation criterion moved in any systematic way as the number of parameters increased. Nevertheless, more theoretical research should be done on characterizing the exact nature of the dependence between the event-probability scores and the sum of squared errors of a model.

In this paper we only consider logistic smooth-transition models for our multivariate nonlinear models. The strategy that we use for developing these models starts from a linear specification and augments the model in the direction that is most favored by the results of LSTAR nonlinearity tests. This method leads to models for output that show no further evidence of nonlinearity. However, there is still strong evidence of further nonlinearity in the residuals of the spread equation. One direction is to patch up the nonlinear model of spread with a second transition function. This strategy, however, would quickly exhaust the degrees of freedom in

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a multivariate model, and lead to a very complicated likelihood function with many flat regions. An alternative direction might be to consider the joint data generation process of short and long interest rates. Such an approach might incorporate nonlinearities in the cointegration between these two interest rates, as in Balke and Fomby (1997) and Anderson (1997).

NOTES

1. This test is based on the joint significance of a set of nonlinear test regressors in a regression of the residuals of the linear model on the explanatory variables in the linear model and these test regressors. The set of nonlinear regressors consists of the products of the hypothesized transition variable and the lagged explanatory variables, and the cubic power of the transition variable.

2. The first draft of this paper did not impose this restriction. Following a referee's suggestion, we tested this restriction and could not reject it. Both models lead to very similar predictions, and we report only the restricted version.

3. It is interesting to observe that the graphs of probabilities predicted by the linear models resemble the upside-down image of the graph of the spread.

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APPENDIX A: UNIVARIATE MODELS OF OUTPUT (1960:1–1996:4)

Note that MSE is the mean squared forecast error for 1997:1–1999:1.

AR(2) model of output:

$$\hat{y}_t = \underset{(0.11)}{0.49} + \underset{(0.08)}{0.25} y_{t-1} + \underset{(0.08)}{0.13} y_{t-2}$$

$$\hat{\sigma}_{\text{MLE}} = 0.89, \ \text{MSE} = 0.12$$

TAR model of output:

$$\hat{y}_t = \underset{(0.12)}{0.52} + \underset{(0.08)}{0.26} y_{t-1} + \underset{(0.09)}{0.20} y_{t-2} - \underset{(0.08)}{0.16} y_{t-5} + f_t \times (\underset{(0.20)}{0.43} y_{t-5})$$

$$f_t = (y_{t-2} < 0)$$

$$\hat{\sigma}_{\text{MLE}} = 0.87, \ \text{MSE} = 0.15$$

LSTAR model of output:

$$\hat{y}_t = \underset{(0.88)}{-1.51} - \underset{(0.64)}{1.40} y_{t-2} + f_t \times (\underset{(0.90)}{2.04} + \underset{(0.09)}{0.26} y_{t-1} + \underset{(0.64)}{1.50} y_{t-2})$$

$$f_t = [1 + \exp\{-11\,(y_{t-2} + 0.55)\}]^{-1}$$

$$\hat{\sigma}_{\text{MLE}} = 0.86, \ \text{MSE} = 0.12$$

CDR model of output:

$$\hat{y}_t = \underset{(0.13)}{0.35} + \underset{(0.08)}{0.24} y_{t-1} + \underset{(0.10)}{0.22} y_{t-2} + \underset{(0.13)}{0.20}\, \text{CDR}_{t-1}$$

$$\text{CDR}_t = \max\{\text{CDR}_{t-1}, y_t\} - y_t$$

$$\hat{\sigma}_{\text{MLE}} = 0.89, \ \text{MSE} = 0.15$$

Markov-switching model of output:

$$\hat{y}_t = \mu_{s_t} + \underset{(0.14)}{0.24}\left(y_{t-1} - \mu_{s_{t-1}}\right) + \underset{(0.11)}{0.11}\left(y_{t-2} - \mu_{s_{t-2}}\right)$$

$$\mu_{s_t=1} = \underset{(0.51)}{-0.80}, \qquad \mu_{s_t=0} = \underset{(0.12)}{0.95}, \qquad P = \begin{bmatrix} \underset{(0.28)}{0.52} & 0.05 \\ 0.48 & \underset{(0.03)}{0.95} \end{bmatrix}$$

$$\hat{\sigma}_{\text{MLE}} = 0.76, \qquad \text{MSE} = 0.12$$

APPENDIX B: BIVARIATE MODELS OF OUTPUT AND THE SPREAD (1960:1–1996:4)

Note that MSE is the mean squared forecast error for 1997:1–1999:1.

ARLI-OLS model of output and spread:

$$\hat{y}_t = \underset{(0.12)}{0.26} + \underset{(0.08)}{0.19}y_{t-1} + \underset{(0.08)}{0.12}y_{t-2} + \underset{(0.06)}{0.20}s_{t-2}$$

$$\hat{\sigma}_{\text{MLE}} = 0.86, \ \text{MSE} = 0.16$$

$$\hat{s}_t = \underset{(0.08)}{0.28} - \underset{(0.05)}{0.14}y_{t-2} + \underset{(0.08)}{1.05}s_{t-1} - \underset{(0.12)}{0.36}s_{t-2} + \underset{(0.08)}{0.20}s_{t-3}$$

$$\hat{\sigma}_{\text{MLE}} = 0.56$$

ARLI-SYS model of output and spread:

$$\hat{y}_t = \underset{(0.12)}{0.26} + \underset{(0.08)}{0.17}y_{t-1} + \underset{(0.08)}{0.13}y_{t-2} + \underset{(0.06)}{0.21}s_{t-2}$$

$$\hat{\sigma}_{\text{MLE}} = 0.86, \ \text{MSE} = 0.16$$

$$\hat{s}_t = \underset{(0.08)}{0.28} - \underset{(0.05)}{0.14}y_{t-2} + \underset{(0.08)}{1.04}s_{t-1} - \underset{(0.11)}{0.35}s_{t-2} + \underset{(0.08)}{0.18}s_{t-3}$$

$$\hat{\sigma}_{\text{MLE}} = 0.56$$

Bi-NARLI model of output and spread:

$$\hat{y}_t = \underset{(0.50)}{-0.45} - \underset{(0.24)}{0.55}y_{t-1} + \underset{(0.24)}{0.51}y_{t-2} + \underset{(0.29)}{0.55}y_{t-3} - \underset{(0.32)}{0.59}s_{t-1} + \underset{(0.56)}{1.04}s_{t-2}$$

$$+ f_{yt} \times (\underset{(0.52)}{1.22} + \underset{(0.26)}{0.77}y_{t-1} - \underset{(0.24)}{0.51}y_{t-2} - \underset{(0.29)}{0.55}y_{t-3} + \underset{(0.32)}{0.59}s_{t-1} - \underset{(0.56)}{1.04}s_{t-2})$$

$$f_{yt} = [1 + \exp\{-7.66\,(s_{t-2} - 0.009)\}]^{-1}$$

$$\hat{\sigma}_{\text{MLE}} = 0.75, \ \text{MSE} = 0.10$$

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$$\hat{s}_t = \underset{(0.69)}{1.15} + \underset{(0.20)}{0.66}y_{t-1} - \underset{(0.04)}{0.16}y_{t-2} + \underset{(0.24)}{1.54}s_{t-1} - \underset{(0.39)}{2.89}s_{t-2} + \underset{(0.66)}{2.16}s_{t-3}$$

$$+ f_{st} \times (\underset{(0.70)}{-0.88} - \underset{(0.20)}{0.73}y_{t-1} - \underset{(0.25)}{0.46}s_{t-1} + \underset{(0.40)}{2.67}s_{t-2} - \underset{(0.66)}{2.09}s_{t-3})$$

$$f_{st} = [1 + \exp\{-13.75\,(s_{t-3} + 0.555)\}]^{-1}$$

$$\hat{\sigma}_{\text{MLE}} = 0.43$$

Com-NARLI model of output and spread:

$$\hat{y}_t = \underset{(0.44)}{-1.48} - \underset{(0.25)}{0.40}y_{t-1} + \underset{(0.25)}{0.76}y_{t-2} + \underset{(0.18)}{0.30}s_{t-3} - \underset{(0.58)}{1.96}com_t,$$

$$\hat{\sigma}_{\text{MLE}} = 0.77, \ \text{MSE} = 0.11$$

$$\hat{s}_t = \underset{(0.30)}{1.20} + \underset{(0.12)}{0.24}y_{t-1} - \underset{(0.17)}{0.44}y_{t-2} + \underset{(0.08)}{1.03}s_{t-1} - \underset{(0.12)}{0.20}s_{t-2} + com_t,$$

$$\hat{\sigma}_{\text{MLE}} = 0.52$$

$$com_t = [1 + \exp\{-2.14\,(s_{t-2} + 0.55)\}]^{-1}$$

$$\times [\underset{(0.39)}{-1.19} - \underset{(0.13)}{0.33}y_{t-1} + \underset{(0.18)}{0.37}y_{t-2} + \underset{(0.08)}{0.18}s_{t-3}]$$

APPENDIX C: TRIVARIATE MODELS OF OUTPUT, SPREAD, AND MONEY (1960:1–1996:4)

Note that MSE is the mean squared forecast error for 1997:1–1999:1.

ARLI-OLS model of output, spread, and money:

$$\hat{y}_t = \underset{(0.11)}{0.26} + \underset{(0.06)}{0.17}s_{t-2} + \underset{(0.10)}{0.17}m_{t-1} + \underset{(0.10)}{0.27}m_{t-2}$$

$$\hat{\sigma}_{\text{MLE}} = 0.80, \ \text{MSE} = 0.10$$

$$\hat{s}_t = \underset{(0.08)}{0.28} - \underset{(0.05)}{0.14}y_{t-2} + \underset{(0.08)}{1.05}s_{t-1} - \underset{(0.12)}{0.36}s_{t-2} + \underset{(0.08)}{0.20}s_{t-3}$$

$$\hat{\sigma}_{\text{MLE}} = 0.56$$

$$\hat{m}_t = \underset{(0.09)}{0.14} + \underset{(0.07)}{0.15}s_{t-1} - \underset{(0.07)}{0.13}s_{t-3} + \underset{(0.07)}{0.63}m_{t-1} + \underset{(0.07)}{0.10}m_{t-3}$$

$$\hat{\sigma}_{\text{MLE}} = 0.67$$

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ARLI-SYS model of output, spread, and money:

$$\hat{y}_t = \underset{(0.10)}{0.26} + \underset{(0.06)}{0.17}s_{t-2} + \underset{(0.09)}{0.14}m_{t-1} + \underset{(0.09)}{0.31}m_{t-2}$$

$$\hat{\sigma}_{\text{MLE}} = 0.80, \ \text{MSE} = 0.10$$

$$\hat{s}_t = \underset{(0.08)}{0.28} - \underset{(0.06)}{0.13}y_{t-2} + \underset{(0.08)}{1.04}s_{t-1} - \underset{(0.11)}{0.34}s_{t-2} + \underset{(0.08)}{0.19}s_{t-3}$$

$$\hat{\sigma}_{\text{MLE}} = 0.56$$

$$\hat{m}_t = \underset{(0.09)}{0.14} + \underset{(0.07)}{0.16}s_{t-1} - \underset{(0.07)}{0.13}s_{t-3} + \underset{(0.07)}{0.64}m_{t-1} + \underset{(0.07)}{0.10}m_{t-3}$$

$$\hat{\sigma}_{\text{MLE}} = 0.67$$

Tri-NARLI model of output, spread, and money:

$$\hat{y}_t = \underset{(0.31)}{-1.13} + \underset{(0.26)}{0.88}y_{t-2} + \underset{(0.09)}{0.24}m_{t-2} - \underset{(0.49)}{1.57}m_{t-3}$$

$$+ f_{yt} \times (\underset{(0.33)}{1.83} - \underset{(0.27)}{0.89}y_{t-2} + \underset{(0.50)}{1.67}m_{t-3})$$

$$f_{yt} = [1 + \exp\{-9.94\,(s_{t-2} + 0.036)\}]^{-1}$$

$$\hat{\sigma}_{\text{MLE}} = 0.74, \text{MSE} = 0.09$$

$$\hat{s}_t = \underset{(0.08)}{0.29} - \underset{(0.05)}{0.09}y_{t-1} - \underset{(0.04)}{0.16}y_{t-2} + \underset{(0.07)}{1.06}s_{t-1} - \underset{(0.07)}{0.14}s_{t-2} + \underset{(0.04)}{0.06}m_{t-2}$$

$$+ f_{st} \times (\underset{(0.61)}{-2.80} + \underset{(0.60)}{2.40}y_{t-1} - \underset{(0.32)}{0.86}y_{t-3} + \underset{(0.40)}{1.12}s_{t-1} - \underset{(0.92)}{4.98}s_{t-2} - \underset{(0.81)}{2.49}m_{t-2})$$

$$f_{st} = [1 + \exp\{-9.23\,(s_{t-3} + 0.593)\}]^{-1}$$

$$\hat{\sigma}_{\text{MLE}} = 0.42$$

$$\hat{m}_t = \underset{(1.44)}{1.02} + \underset{(0.29)}{0.91}s_{t-1} - \underset{(0.39)}{0.61}s_{t-2} + \underset{(0.31)}{0.85}s_{t-3} + \underset{(0.11)}{0.79}m_{t-1} - \underset{(0.17)}{0.29}m_{t-3}$$

$$+ f_{mt} \times (\underset{(10.62)}{-15.18} + \underset{(0.24)}{0.33}y_{t-3} - \underset{(0.70)}{3.07}s_{t-1} + \underset{(1.72)}{7.78}s_{t-2} - \underset{(0.70)}{3.21}s_{t-3}$$

$$- \underset{(0.37)}{0.64}m_{t-1} + \underset{(0.51)}{1.38}m_{t-3})$$

$$f_{mt} = [1 + \exp\{-0.82\,(s_{t-2} - 3.26)\}]^{-1}$$

$$\hat{\sigma}_{\text{MLE}} = 0.52$$

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