From Lithium to Uranium: Elemental Tracers of Early Cosmic Evolution
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Improved nucleosynthetic yields

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Abstract. Theoretical estimates of nucleosynthesis yields are widely used to infer the meaning of abundance trends, and to predict behavior of abundances in the first stars. We show that the standard prescriptions are incomplete, and illustrate some simple improvements. We consider the effects of internal gravity waves (induced in radiative regions by convective zones) on the evolution of slowly rotating (nonrotating) stars. We demonstrate that such modifications to the standard evolutionary algorithms give better agreement with observation. They imply new phenomena: interations between carbon and oxygen burning shells, detached flames in convective layers, and intermittency, for example. We argue that such hydrodynamic behavior must be part of realistic stellar evolution. Some preliminary results for the Sun and Sirius are discussed.

Keywords. Nuclear reactions, nucleosynthesis, abundances, stars: evolution, gravitational waves, Sun: general

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1. Introduction

In order to provide plausible nucleosynthesis yields it is necessary to implement reasonably complete physics in stellar models. There are some well-known deficiencies: the lack of rotation, and the treatment of convection and mixing. We examine the effects of gravity waves (g-modes) on the evolution of stars, primarily through induced mixing. Although g-modes transfer angular momentum, we first consider the simpler case of no rotation. We use both theory and multi-dimensional numerical simulations (both 2D and 3D) to elucidate the coupling of g-modes in radiative regions to convective zones (which drive them).

2. Predictive Stellar Evolution

Parameterized stellar evolution will be inherently unreliable when extrapolated to new conditions (for example, the first stars).

We have developed two tools to make stellar theory more predictive. TYCHO is a 1D stellar evolution code with an adaptive (now 177 nuclei) nuclear reaction network, Raucher and Thieleman reaction rates, OPAL opacities and at lower temperatures, Alexander opacities, Thoul-Bahcall diffusion, Timmes-Arnett equation of state or OPAL equation of state, intermediate screening, and a graphics toolbox.

PROMPI is a 3D compressible hydrodynamics PPM code, with a 27 nuclei network, Timmes-Arnett equation of state, radiation diffusion, parallelized in MPI by domain decomposition. Models can be accurately mapped from TYCHO to PROMPI. The two codes use the same microphysics.

This allows us to test directly the 1D algorithms against 3D simulations. Progress with implementing wave physics has occured for wide eclipsing binaries (Young *et al.* (2005), Young *et al.* (2003), Young *et al.* (2001)), Sirius A and B (Liebert *et al.* (2005)), the Sun (Arnett, Meakin & Young (2005)), and precollapse models (Young *et al.* (2005).

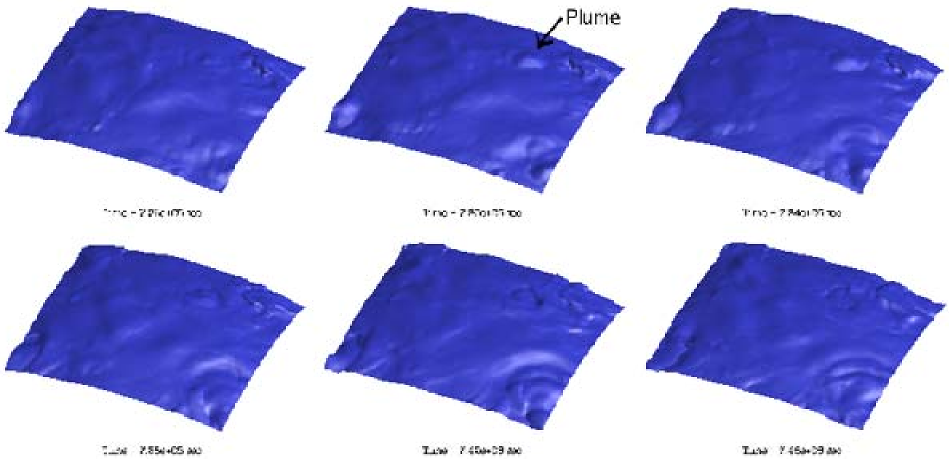

Figure 1. Dynamics of isocontours of composition at the boundary of a convective zone.

3. Dynamic versus Static Interfaces

Stellar evolution has be conceived as a succession of slowly changing static states. Historically convection has been fitted into this picture as a region of adiabatic structure, with no sources or sinks of entropy. Although convection is dynamic by definition, the notion was that the average behavior was slowly varying on evolutionary time scales. Cowling (1941) found that stellar models were subject to wave motion, and classified these in two families, p-modes (sound waves) and g-modes (gravity waves). For completeness, we note that there is a third type, f-modes, which correspond to surface waves at an interface. Helioseismology has demonstrated the reality of p-modes, and because g-modes are exponentially attenuated in a convective regions (evanescent), they are expected to be unobservable at the solar surface, and have not been detected unambiguously (Christensen-Dalsgaard (2002)).

Early numerical hydrodynamical simulations (2D) of oxygen burning in stars indicated a wealth of wave behavior (Arnett 91994), Bazan & Arnett (1998), Asida & Arnett (2000)). We have refined and extended this work to 3D, and confirm these results; this work will be published separately in more detail (Meakin & Arnett (2005)). While many of the examples below refer to massive stars (a 23 $M_\odot$ population I star like EM Car), we also consider intermediate and solar mass stars, and early burning stages as well as oxygen burning.

In Figure 1 is shown the lagrangian (co-moving) contour of the top edge of a convective zone, in six snapshots. Nominally this would be a sphere at which the Schwarzschild discriminant is zero; instead it is neither spherical nor static. This behavior is much clearer in simulations which describe multi-fluid components. A plume collides with the interface in panel 2 and then disappears by panel 6. The size of the region is much smaller than a pressure scale height, and the corresponding frequencies higher than predicted from mixing length turnover times (Goldreich & Keeley (1977), Goldreich & Kumar (1988), Spruit (1987)).

The motion of the convective-radiative interface induces p-modes and g-modes in the adjacent radiative region, which is no longer static. Figure 2 shows a convective zone overlaid by a radiative zone; the variable indicated is velocity. In the first panel, the radiative zone is motionless and only the convective zone shows velocity. The motion of the interface gradually fills the radiative region with g-modes. While lower in amplitude

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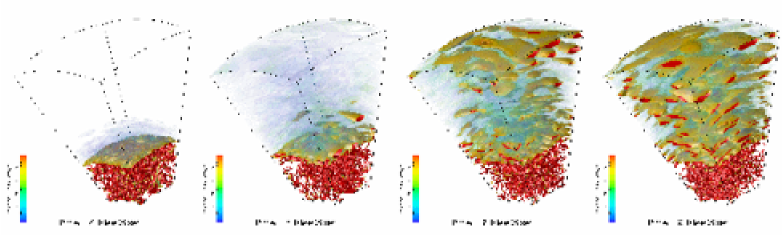

Figure 2. Generation of g-modes above a convective region.

than the convective velocities, the g-modes can cause mixing by nonlinear processes (wave breaking, wave-wave interaction, etc.), and in general can transport angular momentum.

While these examples involve a convective core and a radiative overlying layer, we have also examined the effects at the bottom of a convective layer. It is much the same story. The primary symmetry breaking seems to be due to the manner by which the entropy is changed (raised in a burning shell to cause rising plumes, lowered below the photosphere to give descending plumes).

Turbulent mixing is an unsolved problem (Dimotakis (2005)). The available resolution now and in the forseeable future will give numerical dissipation greater than the real dissipation in the stars we simulate. Despite this, simulations do give a self-consistent picture of the evolution of a complex nonlinear system. Laboratory systems which are analogous show intermittency, in which large scale modes dominate, break up, and reform. These large scale modes are captured by our simulations, and drive the g-modes. In this restricted sense we believe the simulations are appropriate to the stellar case.

4. Mixing in a stellar evolution code

Young & Arnett (2005), Young *et al.* (2003) have introduced descriptions of stellar mixing based upon earlier numerical hydrodynamic simulation (Bazan & Arnett (1998), Asida & Arnett (2000)). Mixing length theory ignores the momentum of moving plumes and has symmetric velocities in up and down flows. Simulations suggest that both assumptions are wrong. The kinetic energy flux is finite as the convective boundary is approached. We have used the convective speed from mixing length theory to make a crude estimate of the flux, and assume that it is converted into g-mode waves (we are studying this directly with 3D simulations and linear wave theory now, and hope to report an improved description soon).

Using this picture, Young & Arnett (2005) then used the analysis of Press (1981), Press & Rybicki (1981) to define an enhanced mass diffusion coefficient, and applied this to the evolution of well-observed wide eclipsing binaries. Both the radii and the apsidal motions predicted agreed much better with observations than did models calculated with the canonical assumptions. Figure 3 shows the improvement in the chi-squared values for luminosity and radius for fifteen systems, with masses ranging from solar up to $23M_\odot$. The largest source of error is now the abundance determinations in these stars. The cause of the improvement is an increase in the size of the stellar cores.

An interesting consistency test is indicated in Figure 4, which shows the size of convective oxygen burning shells in a $23M_\odot$ star (an older EM Car), with lines indicating the convective region in the original 1D calculation. This is a work in progress; we have already found that a better discriminant for the size of a convective region is the Richardson number, which takes account of the ability of convective shear to eat into the radiative region.

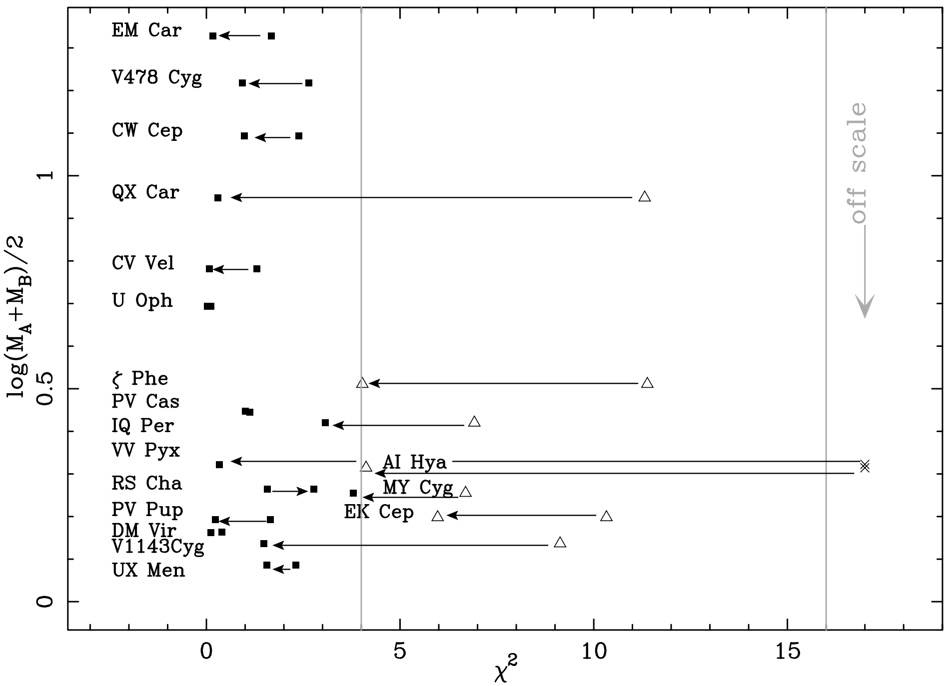

Figure 3. χ^2 for coeval predictions of luminosities and radii, for 15 selected wide eclipsing binaries. Wave mixing gives much better agreement with observations (indicated by arrows).

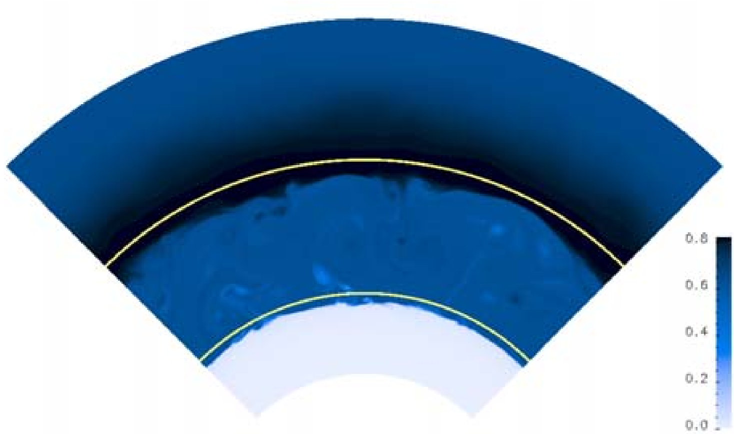

Figure 4. Hydrodynamic simulations can be used to test algorithmic mixing. The abundance of ^{16}O is shown.

We take the point of view that rotational mixing and g-mode mixing are both "hydrodynamics", and expect a variation in behavior with increasing stellar rotation, so that a theory should include both as aspects of a comprehensive picture.

5. Sirius

A necessary ingredient of any nucleosynthesis prediction is the ability to explain the initial-final mass relation for white dwarfs because most of the mass returned to the interstellar medium is processed in such stars. Liebert *et al.* (2005) have compared

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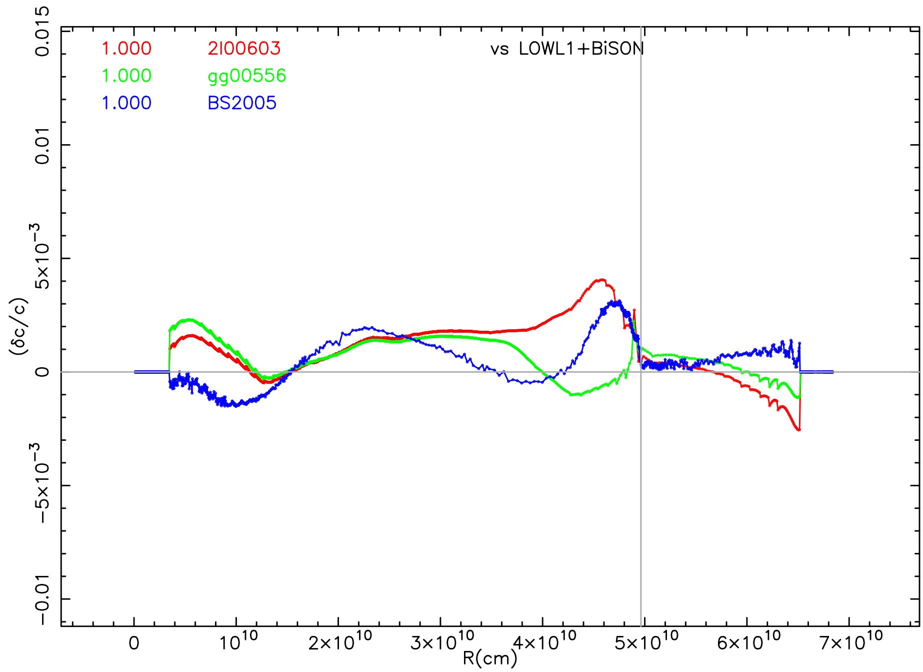

Figure 5. Bahcall-Serenelli solar model, our equivalent, and one with wave induced mass diffusion. Hydrallically induced opacity not yet included.

TYCHO predictions to the well observed pair Sirius A and B. Because Sirius B is a massive white dwarf, this test is particularly important for nucleosynthesis theory. A consistent solution was found; see Liebert *et al.* (2005) for detail.

6. Solar Models

The best observed star is the Sun, and a stellar nucleosynthesis code should be able to describe the Sun if it is to be plausible. There is a conflict between new solar abundance determinations (Asplund *et al.* (2000), Allende-Prieto *et al.* (2001)), helioseismology (Christensen-Dalsgaard (2001), Basu & Antia (1997)), and the standard solar models (Bahcall & Pinsonneault (2004)).

Bahcall *et al.* (2005) favor an ad hoc increase in the opacity by 11% just below the convection zone. Bahcall, Basu, & Serenelli (2005) have suggested an ad hoc increase in the neon abundance above the standard value (Asplund, Grevesse, & Sauval (2005)) by 0.4-0.6 dex.

In order to examine such subtle differences we must first demonstrate that we can reproduce the standard solar model with the same physics; this is shown in Figure 5, which gives sound speed discrepancies for three models, as a function of stellar radius. Note that the errors are all less than half a percent. All models used the standard (old) solar abundances (metallicity about 0.018). The top two models are Bahcall & Serenelli (2005) and our TYCHO model with almost identical physics. The third model, which does not have such a marked peak in errors below the convection zone (at radius $r \approx 5 \times 10^{10}$cm), includes the enchanced mass diffusion coefficient described above. While it seems to help the inconsistency, the effect is inadequate to remove it for metallicity of 0.014. However, Press (1981), Press & Rybicki (1981) describe another effect, "hydrallically enhanced opacity", which we have not included. This effect is small for massive stars,

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but may be significant in this case (it is proportional to $\nabla - \nabla_{ad}$, which increases as stellar mass decreases, on average).We are in the process of examining this effect.

7. Conclusions

Our "extra mixing", due to g-mode induced mass diffusion, improves stellar models, at least from 1 to 23$M_\odot$, and requires no recalibration over this range. Stars are dynamic systems, even in quasi-hydrostatic evolution. Wave processes induce mixing in radiative regions, even in slowly rotating stars. Combined numerical hydrodynamic simulations and analytic methods promise to improve our ability to predict stellar behavior (see also the contributions by Charbonnel, Maeder, Meynet, and Zahn, these proceedings). Our preliminary algorithms already indicate significant differences in prediction for presupernova structure (Young *et al.* (2005)). This, and the lack of an agreed-upon explosion mechanism for supernova (both Types I and II), indicate that stellar nucleosynthesis, instead of being a solved problem, is becoming a more interesting challenge than ever.

Acknowledgements

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