Journal of the International Neuropsychological Society (1997), **3**, 260–268.

The abstraction of numerical relations: A role for the right hemisphere in arithmetic?

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(RECEIVED October 16, 1995; REVISED July 1, 1996; ACCEPTED July 8, 1996)

Abstract

Arithmetical reasoning ability has been investigated in a group study of patients with unilateral cerebral lesions. Two series of 38 and 39 patients, who had suffered unilateral cerebral lesions of the right and left cerebral hemisphere, respectively, were investigated. They completed a neuropsychological battery that included a test of computation (Graded Difficulty Arithmetic, GDA; Jackson & Warrington, 1986), and a new test of numerical series completion (Arithmetical Reasoning Test, ART). Whereas the left-hemisphere lesion group were markedly more impaired on the GDA compared to both the right-hemisphere lesion group and a standardization sample, both lesion groups were equally severely impaired on the ART. It is suggested that the abstraction of numerical relations, which is essential to numerical series completion, relies on the integrity of the right hemisphere. A global model of arithmetic processing that incorporates these findings is proposed. (*JINS*, 1997, *3*, 260–268.)

Keywords: Dyscalculia, Right hemisphere, Arithmetic

INTRODUCTION

The structure and organization of the cognitive processes that subserve calculation are increasingly well understood; however, the cognitive systems required for arithmetic reasoning have yet to be studied intensively. Similarly, a consistent body of work now indicates that the left cerebral hemisphere takes a crucial role in the storage and application of number knowledge and rules. In contrast, despite a few reports scattered through the literature suggesting right cerebral hemisphere involvement in some arithmetic operations, the precise nature of its contribution remains unclear.

Dyscalculia is frequently observed in conjunction with language disorders. An early account by Henschen (1919) described patients who had difficulty in reading and writing numbers. However, a selective impairment of dyscalculia was also described, independent of any aphasia. Berger (1926) distinguished between patients in whom the impairment of calculation was secondary to other cognitive impairments and those patients with a primary impairment in calculation processes. He terms the latter disorder *anarithmetria*. Calculation has been demonstrated by subsequent group studies to be dependent on the integrity of the left hemisphere.

Hécaen et al. (1961) reported that patients with posterior lesions of the left hemisphere were impaired at calculation. Grafman et al. (1982) were able to show that patients with intact reading and writing of numbers could be poor calculators, and this was especially true of those who had suffered posterior damage to the left hemisphere. The same disadvantaged calculation of patients with left-hemisphere lesions was demonstrated by Jackson and Warrington (1986), using their graded difficulty test (Graded Difficulty Arithmetic, or GDA) of oral addition and subtraction. In the study, patients with left-hemisphere damage were significantly more impaired than patients with right-hemisphere lesions, all of whom scored within the range of controls. In a large retrospective study (Warrington et al., 1986), patients with left-hemisphere lesions were again significantly more impaired than those with right-hemisphere lesions on the WAIS Arithmetic subtest. Interestingly, within the left-hemisphere lesion group, those patients with left-parietal involvement were more impaired than those patients with intact parietal lobes, which supports observations made by Henschen (1919).

Although the preeminent role of the left hemisphere in the utilization of arithmetic rules and number knowledge is now well established, intriguingly some studies have impli-

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cated the right hemisphere and spatial skills in certain aspects of arithmetical problem-solving. Henschen (1926) is credited with one of the first discussions on this topic, in which he suggested that the right hemisphere could act as a substitute for damaged left-hemisphere processes in simple arithmetic. His analysis of cases led him to conclude that a "visual factor" that related to spatial processing at some level was important.

Cohn (1961), in a qualitative account of 40 patients presenting with disturbances in calculation ability, described how his subjects performed written multiplication problems and drew simple geometric figures. He noted that whenever arithmetical functions were markedly disturbed, geometric ability was also profoundly affected. The Grafman et al. (1982) study of written addition, subtraction, multiplication, and division problems found that patients with both right- and left-hemisphere lesions were impaired when compared to control subjects, although the left posterior lesion group were especially weak.

In an early attempt to identify the processes that might underlie arithmetical functions, Hécaen (1962) tested large groups of unilateral lesion patients for three types of impairment. He demonstrated that the reading or writing of digits, simple calculation, and visuospatial processing of arithmetic tasks could each be disrupted by unilateral lesions to either the left or right hemisphere, with varying frequency. Hécaen concluded that a plurality of mechanisms underlie calculation tasks.

The different cognitive processes subserving arithmetic became more distinct with the advent of information-processing models. Based on the selective pattern of impairments observed in patients who had suffered cerebral lesions, a modular cognitive architecture was proposed. For example, Warrington (1982) described a patient with a specific impairment of the retrieval of arithmetic facts, who nevertheless enjoyed preserved arithmetic processes during comprehension and comparative judgments. Other workers (e.g., McCloskey et al., 1985) have supported Warrington's conclusions of two functionally independent neural mechanisms: one for number facts, and the other for number procedures.

Evidence for a third type of numerical ability has been described by Dehaene (1992). He suggested that comparison and approximation do not depend on linguistic competence, but rather constitute an analogical representation of numerical quantities. The approximation skills are close to the numerical competence demonstrated in preverbal children and animals, leading to the conclusion that they may function as a separate preverbal system of arithmetic processes (Gallistel & Gelman, 1992). Dehaene and Cohen (1995) have developed the idea of magnitude representation to encompass both the direct comparisons of numerical quantities and the relative positions of numbers on a number line.

Apart from the specific high-level arithmetical operations that are required to solve arithmetic problems, it seems likely that there would also be some general intelligence or reasoning contribution to the problem solution. As early as 1923, Spearman had noted that tests of arithmetic separate on *g* depending on whether they require the solving of a problem or merely the application of arithmetic rules. In problem arithmetic, where the arithmetic operations are not explicit and the subject must analyze the information given and decide which rules to apply, the *g* loading is very high ($r = .7–.8$). However, in the mechanical arithmetic items, the operations required are explicitly stated and the *g* loading is moderate ($r = .4–.6$).

Each of the validated tests of arithmetic referred to so far examine a specific aspect of arithmetic operations, but none specifically target arithmetical reasoning. For example, the WAIS Arithmetic subtest items (Wechsler, 1955) consist of one or several sentences that are presented aurally to the subject (on a maximum of two occasions). It clearly relies on the subject being able to exercise good attentional skills and a competent syntactic analysis. The WAIS Arithmetic subtest thus tests many skills in addition to arithmetical reasoning. The GDA targets addition and subtraction operations by presenting specific sums aurally to the subject. No decision is required by the subject regarding which arithmetic rules to apply, or to which numerals. The arithmetical reasoning component is deliberately minimal, to produce a more precise measure of the computation skills involved in addition and subtraction.

The WAIS Arithmetic and GDA are both well-validated measures of aspects of calculation competence, but a more focused measure of arithmetical reasoning, less confounded by language processing demands, would enable a more precise exploration of arithmetic reasoning to be undertaken. In particular, its relative resistance to the effects of mild aphasic deficits might afford the opportunity to disentangle the usually interactive roles of language, calculation and reasoning processes in a group of patients who had suffered unilateral cerebral lesions.

A test of arithmetical reasoning that minimizes attentional and language skills has recently been developed (Langdon & Warrington, 1995). The arithmetical reasoning test is one of six separately validated sections designed to test specific types of reasoning, presented in matched sets of verbal and spatial format stimuli (together called the Verbal and Spatial Reasoning Test or VESPAR).

The aim of this study was to investigate the performance of patients with unilateral cerebral lesions on a numerical series completion task, the ART, and to compare arithmetical reasoning abilities with arithmetical computation skills. In particular, the contributions of the right and left cerebral hemispheres to these arithmetical procedures would be explored.

METHOD

Experimental Groups

Two consecutive series of 38 and 39 patients with unilateral left and right cerebral lesions were tested. The mean ages for the left and right hemisphere lesion groups were 43.8 years ($SD = 16.8$) and 48.6 years ($SD = 14.3$) respectively. There were 28 males in the left-hemisphere group and 24 in the right-hemisphere group. All patients physically fit enough to be tested in the Psychology Department and able to cooperate with the task demands were included (severe dysphasia was not a basis for exclusion). All patients who had not been educated in the normal English school system (at present a minimum of 11 years) were excluded, as were patients with a previous medical history that might compromise the cerebrum. The distribution of social economic status (SES; Office of National Statistics, 1990) was also similar in both lesion groups. The percentage for the standardization sample and the left and right cerebral lesion patients were, respectively; 5, 8, and 14 for SES 1; 39, 23, and 24 for SES 2; 44, 46, and 27 for SES 3; 3, 18, and 22 for SES 4; 8, 3, and 3 for SES 5; and 0, 3, and 0 for SES 6. This suggested that there were no systematic differences in premorbid status levels between the two lesion groups, and furthermore, the two experimental groups' premorbid status levels were similar to the current status levels of the standardization sample.

In all cases a CT scan became available, and the right- and left-lesion hemisphere groups were allocated to an anterior or posterior group (3 patients with extensive lesions could not appropriately be designated anterior or posterior and were excluded from the precise localization analysis). Patients classified as anterior had frontal, frontoparietal, or frontotemporal lesions (13 had left-hemisphere lesions and 11 right-hemisphere lesions). Those classified as posterior had parietal, temporal, temporoparietal, occipitoparietal, or occipitotemporal lesions (23 had left-hemisphere lesions and 25 had right-hemisphere lesions). In order to investigate the possibility of a systematic age bias in lesion location, Pearson product–moment correlation coefficients were calculated, and two-tailed probabilities derived. Neither the laterality ($r = .1546$, $p = .179$) nor the anterior–posterior position of lesions ($r = .1754$, $p = .138$) were significantly related to age.

Standardization Sample

A representative cross section of the population was assessed contemporaneously with the experimental subjects. This sample consisted of 155 normal controls ages 18 to 70 years. The controls had a mean age of 45.1 years ($SD = 14.9$), and 59 were male. A measure of intelligence in this sample was derived from the National Adult Reading Test (NART; Nelson, 1982). Performance on the NART for each of three age bands (<40 years, 40–55 years, 55–70 years) was normally distributed about mean IQ equivalents of 105.3 ($SD = 12.3$), 104.8 ($SD = 12.1$), and 105.3 ($SD = 12.4$), respectively (for further details see Langdon and Warrington, 1995).

Procedures

Baseline tests

The following four subtests of the WAIS were administered:

1. Similarities: to provide a measure of verbal reasoning.
2. Digit Span: to provide a measure of short-term auditory verbal memory.
3. Block Design: to provide a measure of spatial intelligence.
4. Picture Completion: to provide a measure of "perceptual" intelligence.

Arithmetic tests

1. Graded Difficulty Arithmetic (GDA): The GDA consists of 12 additions and subtractions, each graded in difficulty from pairs of single digits to two- and three-digit numbers. Each test is preceded by two practice items, and there is a 10-s time limit for each item of the GDA.
2. Arithmetical Reasoning Test (ART): The ART consists of 25 graded-difficulty reasoning items. Each item consists of three numbers from a progressive series. They range in difficulty from a simple arithmetic series (i.e., 1, 2, 3) to a complex geometric series (2, 4, 8). The subject is required to select a fourth number to complete the series from a multiple choice of four numbers (i.e., 2, 4, 6, ?—10, *8*, 6, 12). Our aim was to have a sufficient spread of difficulty in order to obtain a normal distribution of scores and to avoid floor and ceiling effects. In addition there are three practice items and two "screen" items. Any subject failing both screen items was excluded from the study. The test items together with the verbatim instructions are given in Appendix I.

RESULTS

Baseline Tests

The mean and standard deviation of the raw scores of each of the baseline tests for each of the hemisphere lesion and location subgroups are given in Table 1. Age effects were assessed by combining the baseline test raw scores of the two lesion series and then calculating Pearson product–moment correlation coefficients with age and deriving two-tailed probabilities. Although only Block Design reached significance ($r = -.4157$, $p < .001$), the other three baseline tests were close to significance (Similarities $r = -.2134$, $p = .062$; Digit Span $r = -.2211$, $p = .055$; Picture Completion $r = -.2178$, $p = .060$). Age was therefore included in laterality by location analyses of covariance (ANCOVAs), which were computed (SPSS, 1992) for each of the four baseline tests (see Table 2). There was a main effect of laterality on three of these tests: Digit Span, Similarities and Picture Completion. There was not a main effect of laterality on the Block Design Test. There were significant main effects of location on Similarities, Digit Span, and Block Design. There were no significant Laterality $\times$ Location interactions.

Performance on these four measures in the two experimental groups was compared by reference to the performance of the standardization sample. The cumulative

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Table 1. Raw scores of the baseline tests for each of the hemisphere lesion and location subgroups

	Similarities	Digit Span	Block Design	Picture Completion
Group or Subgroup	*M* (*SD*)	*M* (*SD*)	*M* (*SD*)	*M* (*SD*)
Left-hemisphere group ($N = 39$)	9.80 (6.24)	8.79 (3.57)	28.00 (11.21)	12.41 (4.51)
Right-hemisphere group ($N = 38$)	14.47 (4.24)	11.05 (2.14)	29.13 (10.39)	14.03 (3.69)
Left anterior ($N = 13$)	12.69 (2.93)	10.23 (2.28)	33.58 (9.04)	13.58 (3.61)
Right anterior ($N = 11$)	15.82 (2.48)	12.09 (2.39)	33.55 (9.18)	15.55 (2.54)
Left posterior ($N = 23$)	8.33 (7.00)	8.13 (4.07)	26.13 (11.18)	12.09 (4.91)
Right posterior ($N = 25$)	13.80 (4.87)	10.68 (1.99)	27.04 (10.65)	13.48 (4.03)
Standardization sample ($N = 155$)	15.5 (4.1)	12.2 (2.6)	36.2 (8.8)	15.3 (3.0)

Means and standard deviations of the standardization sample are given for comparison.

percentile frequency of the scores of the normative sample were used to derive 5th, 25th and 50th percentile cutoff scores. Each experimental subject's score was thus allocated to 1 of 4 grades of competence.

In order to compare the experimental data to the control sample, expected frequencies were calculated with reference to the 5th, 25th and 50th percentile cutoff scores of the control sample. These expected frequencies were compared to the observed frequencies using the chi square for linear trend. This statistic was appropriate for this analysis for two reasons. First, it allows the comparison of groups of unequal size. Second, it is influenced by the trend of the data, which results in special weight being given to a systematic skew toward poorer performance in the experimental groups, rather than a simple difference between cells.

Chi squares for linear trend were performed (Epi-stat, 1984) to compare the two lesion series, first with the control group and second with each other (see Table 3). These findings largely corroborate the results of the ANCOVAs. In comparison with the normal sample, the left-hemisphere lesion group is impaired on all four tests, whereas the right hemisphere lesion group is only impaired on Block Design. In the comparison of the right- and left-hemisphere lesion groups, a significant difference has been found on the two verbal tests, Similarities and Digit Span.

Arithmetic Tests

Graded Difficulty Arithmetic (GDA)

The means and standard deviations of the raw scores for each of the lesion hemisphere groups and the location subgroups on the GDA are given in Table 4. As a reference point, the 50th percentile score of the control group is also given. The age effects on the GDA were assessed by combining the GDA raw scores of the two lesion series and then calculating a Pearson product–moment correlation coefficient with age, and deriving a two-tailed probability ($r = -.2392$, $p = .037$). Age was therefore included in a Laterality × Location ANCOVA of the raw GDA scores (see Table 5). There was a main effect of laterality but not for location, and the Laterality × Location interaction was not significant.

Performance on the GDA in the two experimental groups was compared by reference to the performance of the standardization sample, using the same method as described above for the baseline tests. The observed and expected frequencies of each grade on the GDA for each lesion group are given in Appendix II.

Chi squares for linear trend were performed (Epi-stat, 1984) to compare the two hemisphere lesion groups, first

Table 2. Laterality × Location ANCOVA

		Similarities	Digit Span	Block Design	Picture Completion
Factor	*df*	*F*	*F*	*F*	*F*
Laterality	3	15.34**	12.56**	0.73	4.23*
Location	3	4.29*	4.04*	5.08*	1.78
Interaction	7	0.13	0.004	0.45	0.64

*$p < .05$.
**$p < .001$.

Table 3. Comparison of experimental groups: Values of chi squares for linear trend

	Similarities	Digit Span	Block Design	Picture Completion
Groups	χ^2	χ^2	χ^2	χ^2
L *vs.* S	20.73**	15.42**	4.93*	5.61*
R *vs.* S	0.01	0.78	4.42*	0.55
L *vs.* R	18.03**	11.80**	0.98	3.18

*$p < .05$, **$p < .001$. L is left-hemisphere group, R is right-hemisphere group, S is standardization sample.

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Table 4. Raw scores of the two arithmetic tests for each of the hemisphere lesion and location subgroups

		GDA	ART
Group or subgroup	*N*	*M* (*SD*)	*M* (*SD*)
Left-hemisphere group	38	6.9 (6.1)	13.6 (4.6)
Right-hemisphere group	39	11.2 (5.5)	14.2 (4.1)
Left anterior	13	8.4 (5.9)	14.3 (5.1)
Right anterior	11	12.6 (6.8)	15.2 (3.6)
Left posterior	23	6.4 (6.4)	13.3 (4.6)
Right posterior	25	10.5 (5.0)	13.5 (4.3)
Standardization sample	155	14.4 (5.6)	17.4 (3.8)

Means and standard deviations of the standardization sample are given for comparison.

Table 6. Comparison of experimental groups: Chi squares for linear trend

	GDA	ART
Comparison	χ^2	χ^2
L *vs.* S	17.03**	13.79**
R *vs.* S	4.84*	11.96**
L *vs.* R	5.96*	0.24

*$p < .05$, **$p < .001$, L is left-hemisphere group, R is right-hemisphere group, S is standardization sample.

with the control group and secondly with one another. The left-hemisphere lesion group was demonstrated to be more severely impaired, both when compared with the control group and with the right-hemisphere lesion group (see Table 6).

Arithmetical Reasoning Test

The means and standard deviations for the hemisphere lesion groups and the location subgroups are given in Table 4. As a reference point, the 50th percentile score of the control group is also given. The age effects on the ART were assessed by combining the ART raw scores of the two lesion series and then calculating a Pearson product–moment correlation coefficient with age and deriving a two-tailed probability ($r = -.2673$, $p = .020$). Age was therefore included in a Laterality × Location ANCOVA of the raw ART scores (see Table 5). There was no main effect of either laterality, or location, and the Laterality × Location interaction was not significant.

Performance on the ART in the hemisphere lesion groups was compared by reference to the performance of the standardization sample, using the same method as described above for the baseline tests. The observed and expected frequencies of each grade in the ART for each lesion group are given in Appendix II. Chi squares for linear trend were performed (Epi-stat, 1984) to compare the two lesion series, first with the control group and secondly with one another (see Table 6). However, there was not a significant difference between the right- and the left-hemisphere lesion groups, although both these lesion groups were significantly impaired in comparison with the standardization sample.

The relationship between GDA and ART scores within the two lesion series was initially assessed by calculating two partial correlation coefficients, controlling for age, and deriving a two-tailed probability. The scores of the left-hemisphere lesion group on the GDA and ART were highly correlated ($r = .7505$, $p < .001$). In contrast, the correlation of the scores of the right hemisphere lesion group on this was not significant ($r = .3063$, $p = .065$).

The format of the ART is lines of numbers. The question arises as to whether any spatial-skills deficits or neglect syndromes might have systematically biased the responses of the right-hemisphere lesion group. To investigate this, the number of responses selected from each of the four serial positions was calculated for the two lesion groups. Because correct answers might have a special salience, correct and incorrect responses were analyzed separately. Chi squares for linear trend, which would be especially sensitive to a systematic lateral position bias, revealed no significant difference in the position of chosen responses that were correct [$\chi^2() = 0.13$, $p = .716$] or incorrect ($\chi^2() = 0.08$, $p = .784$], when the positions of answers chosen by the left- and right-hemisphere lesion groups were compared.

Table 5. Laterality × Location ANCOVA of the GDA and ART

		GDA	ART
Factor	*df*	*F*	*F*
Main effects			
Laterality	3	12.1**	0.9
Location	3	0.9	0.7
Interaction of Laterality × Location	7	0.5	0.8

*$p < .05$.
**$p < .001$.

DISCUSSION

A new test of arithmetical reasoning (ART), which requires the completion of a numerical series, has been devised and administered to patients who have suffered unilateral cerebral lesions. Their performance has been compared with a standardization sample. Patient groups who had suffered both left- or right-hemisphere lesions were significantly impaired when compared to the standardization sample. In addition, the two unilateral lesion groups did not differ significantly from each other when their scores on the numerical series completion task were directly compared. There was no effect of lesion location when patients with anterior lesions were compared with patients with posterior lesions in either the right- or the left-hemisphere groups.

The lack of a statistically significant difference between the right- and left-hemisphere lesion group on the ART was a rather unexpected result. The question arises as to whether this could be accounted for by an unrepresentative patient sample.

Performance on established tests of cognitive function suggested that the two unilateral lesion groups were representative. Predictably, the left-hemisphere lesion group was significantly impaired on the WAIS Similarities and Digit Span subtest. In comparison and equally predictably, the right-hemisphere lesion group was not impaired on the two verbal subtests of the WAIS. Both right- and left-hemisphere lesion groups were impaired on the Block Design subtest, another finding that is in line with previous reports (Warrington et al., 1986). The only possible anomaly was the finding that the left-hemisphere lesion group was impaired on the Picture Completion subtest, whereas the right-hemisphere lesion group was not. However, a similar finding has been reported in a large retrospective analysis of patients with cerebral lesions (Warrington et al., 1986). In that study, the left-hemisphere lesion group's weakness on the Picture Completion subtest was attributed to a subclinical visual agnosia causing difficulties for some patients with left-hemisphere lesions. Overall, we would argue that there was nothing remarkable about the cognitive profile of the two experimental lesion groups.

Before turning to a detailed discussion of the pattern of arithmetic test results, we will consider possible confounding factors for the right-hemisphere lesion group's impairment on the ART. First, there is the question of task difficulty. Might the equal impairment of the left- and right-hemisphere lesion groups on the ART simply reflect a nonspecific brain damage effect? Such an explanation would not account for the right-hemisphere lesion group performing better than the left-hemisphere lesion group on Digit Span, Similarities and, more importantly, on the GDA, all demanding graded-difficulty tests on which normal subjects do not perform at ceiling. Thus, their susceptibility to global task difficulty cannot explain similar performance of the right- and left-hemisphere lesion groups on the ART.

A second possible explanation for the right-hemisphere lesion group's poor performance on the ART would be a global deficit in series completion. This is an unlikely possibility in view of the right-hemisphere lesion group's normal performance on a verbal series completion task (Langdon, 1993).

Third, there is the question of a deficit in spatial processing and neglect syndromes having contributed to the poor performance of the right-hemisphere lesion group on the ART. An analysis of the chosen responses by each lesion group, in terms of the serial position of their selections, was performed. No significant differences were demonstrated between the right- and left-hemisphere lesion groups for either correct or incorrect responses. Because the spatial processing of the left-hemisphere lesion group are unlikely to have been significantly compromised, the similar performance of the right-hemisphere lesion group indicates that no systematic bias relating to response position occurred.

Turning now to a more detailed discussion of the two arithmetic tests: we note that the pattern of performance of the two lesion groups differed between the GDA and the ART. The left-hemisphere lesion group was significantly more impaired on the GDA than was the right-hemisphere lesion group. In contrast, there was no significant difference between the two lesion groups' performance on the ART, both groups having a significant impairment when compared with the standardization sample. The left-hemisphere lesion group was significantly disadvantaged on the relatively pure computations of the GDA compared to the right-hemisphere lesion group. This poses the problem of what additional cognitive processes the ART may require that are vulnerable to right-hemisphere damage.

Any arithmetic computation must at some point require the application of arithmetic facts and procedures for its solution, however sophisticated the reasoning processes that support their selection and operation. It could be argued that a primary computation deficit underlay the left-hemisphere lesion group's poor performance on the ART. In contrast, the right-hemisphere lesion group were more competent on the more straightforward computational task (GDA) than the left-hemisphere lesion group. Other aspects of the ART must have compromised the performance of the right-hemisphere lesion group.

The ART numerical series completion task requires not only computational ability but also the solution of an arithmetic reasoning problem. The subject must determine how the three numbers given in the series relate to each other, abstract a relationship to calculate the missing fourth text item, and then verify their own calculated solution against the four offered solution alternatives. If no match is found, the whole process must be repeated, perhaps informed by the discrepancy between the magnitude of the incorrect, calculated solution and the magnitude of each of the four possible answers. Insofar as the right-hemisphere lesion group was very impaired on the ART, but only mildly impaired on the GDA, could it be that the more the abstraction of relationships is required by a numerical task, the more vulnerable it might be to right-hemisphere damage?

Although it is obvious how right-hemisphere dysfunction might compromise reasoning problems presented in a spatial format, it is less clear why right-hemisphere dysfunction should impair the abstraction of relations in the context of numerals. There is evidence from both developmental studies (Hermelin & O'Connor, 1986) and normal adults (Benbow et al., 1983; Casey et al., 1990) that mathematical talent is linked to spatial skills (and hence, in all probability, to right-hemisphere function). The role of the right hemisphere in spatial processing is established in the lesion literature (e.g., Benton et al., 1978; Warrington & James, 1988). The studies of Hécaen (1962) and Cohn (1961), which were discussed in the introduction, emphasized the importance of spatial skills in the solution of some arithmetic problems. It seems plausible that the abstraction of numerical relations should be at least as vulnerable to spatial skills as written calculation problems. Therefore it may be that the mechanism of the right-hemisphere lesion group's impairment on number series completion was allied to compromised spatial skills.

We wish to propose a tentative speculation to account for our finding of a significant deficit in the performance of the right-hemisphere lesion group on the ART. Dehaene (1992) highlighted the role of comparison and approximation, which suggests that some representations of numerical quantities may not be primarily linguistic. Such comparisons may rely on a general appreciation of numeral magnitude. Hence the solution of a numerical series may rely on a general appreciation of the magnitude of the intervals involved as part of the abstraction of the relation. These "sizing" operations might be linked to spatial analysis, which could require the integrity of the right hemisphere.

Dehaene and Cohen (1995) have suggested that both cerebral hemispheres possess magnitude representation that is at least sufficient for each to determine which of two quantities are larger or smaller. They suggest that the right hemisphere may be superior to the left in the processing of quantities. They have suggested that magnitude comparison may rely on a representation in the form of a number line. A number line could be used to approximate a numerical series, by first locating the given numbers and then assessing and comparing the magnitude of the steps between them. However, a two-dimensional array is a more efficient way of representing a series, which can then be described by a single line. Perhaps the representation of two-dimensional number space places greater demands on spatial processing.

Taking account of the role of spatial skills in the abstraction of numerical relations allows a plausible account of why both right- and left-hemisphere lesion groups were equally impaired on the numerical series completion task. An essential contribution is required from each hemisphere for an efficient solution. Their individual contributions might be as follows: the spatial processing of the right hemisphere contributes to an initial rough appreciation of the series progression (the slope of the function or, more basically, the magnitude of the intervals). The verbal computation processes of the left hemisphere generate and test likely formulae. The most efficient mode of operating would be for the approximation, formula generation, computational rules, and calculation processes to operate interactively. The generating and testing procedures would probably be performed iteratively, but not to the exclusion of the approximation processes. This speculation is described in Figure 1.

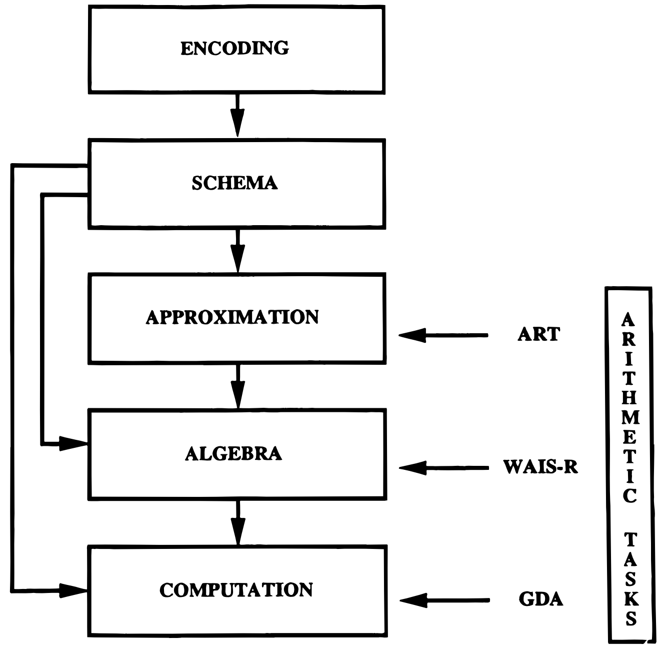

Fig. 1. Suggested model of cognitive processes involved in arithmetic tasks.

If the involvement of the two cerebral hemispheres in arithmetic series completion is as suggested above, then an account of our findings would be as follows. Patients with right-hemisphere lesions would not be able to approximate a "ball-park" function for the left-hemisphere formulas and computation processes to derive and verify. Possibly the left-hemisphere formula-generation processes, deprived of a reliable framework to guide their formulas, would be reduced to generating possible solutions in a more random way, and thus be less likely to hit on the correct formula and subsequent solution. Thus the relatively spared computation processes of the right-hemisphere lesion group would confer no advantage on ART performance. In contrast, those patients with left-hemisphere lesions, despite having good approximation skills, would be compromised on the ART by their inefficient computation processes, essential for the derivation and verification of the correct solution.

At this point it is of interest to consider the work of Fassotti et al. (1992) in relation to the model. Fassotti and colleagues demonstrated the augmentation of encoding and schema formation for verbal arithmetic problems to the specific advantage of patients who had suffered left-frontal and bilateral-frontal cerebral lesions (there was no advantage to patients with right-frontal or left-posterior lesions, nor to control subjects). This work suggests that a set of encoding and schema processes are involved in the solution of some arithmetic problems, which apparently require the integrity of the left frontal lobe. The encoding and schema processes are therefore added to the model, which is described in Figure 1.

https://doi.org/10.1017/S1355617797002609 Published online by Cambridge University Press

Other researchers have devised more fine-grained models of specific processes involved in arithmetic. The more general processing structure that we propose does not conflict with these detailed schemes. Gallistel and Gelman (1992) proposed that preverbal arithmetic skills are based on magnitude appreciation. They maintained that these processes remain active alongside adult numerical arithmetic, providing approximate verification of the answers produced by verbal computations. Our model extends the role of the magnitude-judgement processes to the approximation of mathematical functions. This could occur by means of appreciating the magnitude of intervals between pairs of consecutive series items.

For example, McCloskey's (1992) model of calculation describes the relationship between transcoding processes and computation processes, but does not discuss the operation of the computational module. The elaboration of McCloskey's model put forward by McNeil and Warrington (1994) suggests possible dissociations within the transcoding and computation stages, which again can be accommodated within this more general account. Nor is there any discrepancy between the main stages that we propose and Dehaene's (1992) triple-code model of number processing. Dehaene's representation models fit comfortably within the processing stages we define. The schema formation stage has not previously been made explicit in this context, but we would support its inclusion as a major part of the global processing structure for arithmetic.

In conclusion, our findings suggest that the integrity of both the right and left hemispheres is necessary for the solution of arithmetical reasoning. The performance of the right-hemisphere lesion group on this test was unexpected, and clearly in future studies one would predict and test for a high correlation with spatial-processing tasks. Our notion of magnitude comparison could be put to a more direct empirical test. Future work could also address the question of how different types of arithmetic reasoning, such as arithmetic and geometric progressions, relate to our proposed model.

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APPENDIX I

Test Instructions

Here are the numbers followed by a blank. On each of these practice items, your job is to look at the first three numbers, on the left, and then choose a number, from the four on the right, to fit on the end of the first three. For the first one, 7, 8, 9.

The instructions are repeated as necessary for each of the five practice items. Help and prompts are given according to the specified administration details.

Now you have finished the practice items, here are the 25 main items. As you have practised, you must choose one of the four numbers on the right, to complete the series given on the left. If you are not sure, just have a guess.

Practice Items

a.	7	8	9	—	6	**10**	12	23
b.	2	4	6	—	**8**	5	7	0
c.	5	4	3	—	1	**2**	10	0

Screen items

d.	1	2	3	—	7	**4**	9	8
e.	5	10	15	—	1	25	**20**	18

Test items

1.	3	6	9	—	10	**12**	15	7
2.	97	98	99	—	**100**	101	110	109
3.	3	5	7	—	14	10	8	**9**
4.	20	16	12	—	**8**	4	2	1
5.	28	21	14	—	0	**7**	10	8
6.	8	11	14	—	35	22	**17**	16
7.	9	7	5	—	2	1	**3**	4
8.	7	5	3	—	14	6	**1**	2
9.	1	5	9	—	**13**	14	10	15
10.	1	4	7	—	**10**	9	11	12
11.	20	17	14	—	7	10	**11**	12
12.	10	7	5	—	3	**4**	1	0
13.	2	4	8	—	**16**	10	12	20
14.	1	3	7	—	7	16	10	**15**
15.	15	7	3	—	2	**1**	0	3
16.	1	9	13	—	16	20	**15**	18
17.	2	5	11	—	15	**23**	22	16
18.	1	3	9	—	18	15	**27**	10
19.	3	5	8	—	10	11	**12**	9
20.	5	6	4	—	3	2	8	**7**
21.	1	1	2	—	**3**	4	1	5
22.	1	3	6	—	9	**10**	12	8
23.	5	6	9	—	16	15	**18**	10
24	1	2	6	—	8	10	14	**22**
25.	1	2	5	—	9	11	15	**26**

APPENDIX II

Number of patients scoring within each percentile band

Group and percentile band	GDA *n*	ART *n*	Expected *n*
Left-hemisphere group			
At or below 5th %ile	18	9	2
6th–25th %ile	9	13	8
26th–50th %ile	3	12	10
51st–100th %ile	8	4	18
Right-hemisphere group			
At or below 5th %ile	5	7	2
6th–25th %ile	15	15	8
26th–50th %ile	7	12	10
51st–100th %ile	12	5	19

https://doi.org/10.1017/S1355617797002609 Published online by Cambridge University Press